

# Analysis of Structural Uncertainty in the Analytical Solution of ADE in River Impacted By CSOs

## Goal

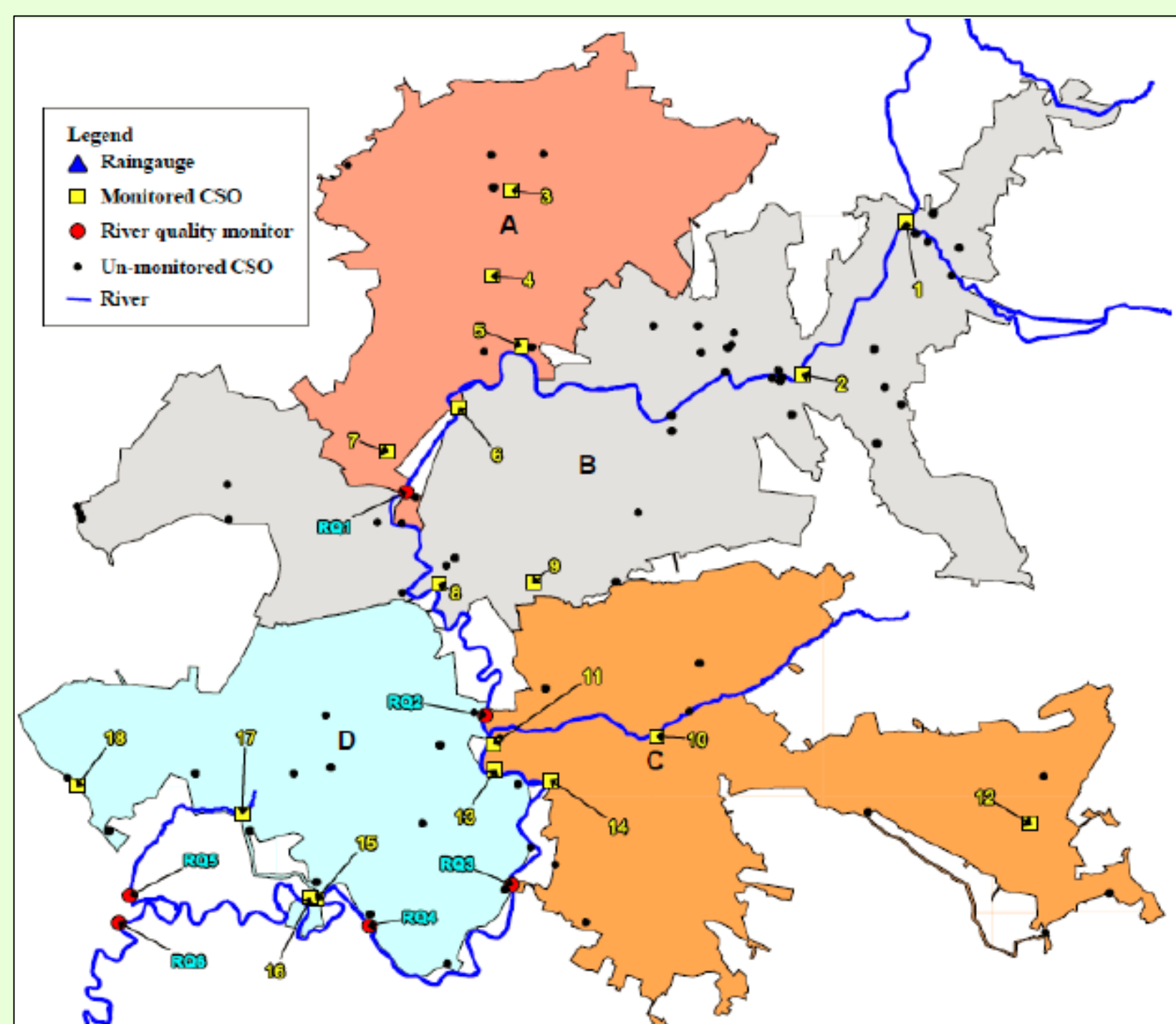
To develop a framework of various pollutant transport models to analyse and quantify the structural and parametric uncertainty of the pollutant transport models

## Objectives

- To evaluate the pollutant transport models individually
- To evaluate model uncertainty in relation to parametric uncertainty
- To determine the temporal and spatial scales where structural and parameter uncertainty are significant

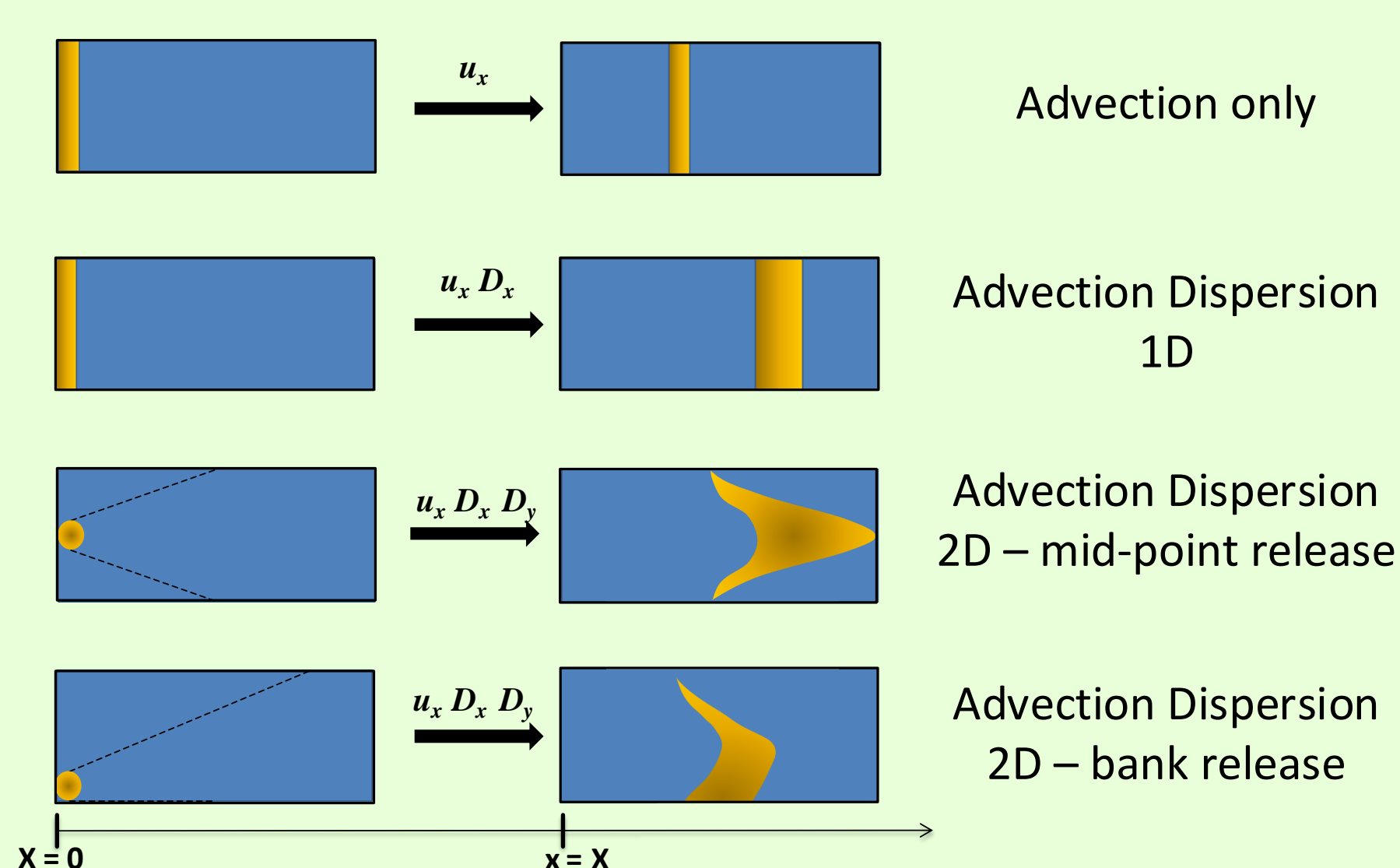
## Site Description

- Urban catchment
- CSO discharge during wet weather conditions
- Flow and quality data describing the CSO spill has been collected as part of a wider integrated model verification study (Norris et al., 2014).
- The receiving water is modelled using the DUFLOW package
- Boundary conditions for study were obtained from DUFLOW



Parameter	Value
River velocity $V_x$ ( $\text{ms}^{-1}$ )	1.0
Pollutant mass $M$ (kg)	1.0
River average depth $d$ (m)	2.5
River cross section area $A$ ( $\text{m}^2$ )	50
Longitudinal dispersion $D_x$ ( $\text{m}^2\text{s}^{-1}$ )	0.2
Transverse dispersion $D_y$ ( $\text{m}^2\text{s}^{-1}$ )	0.002

## Conceptual diagram

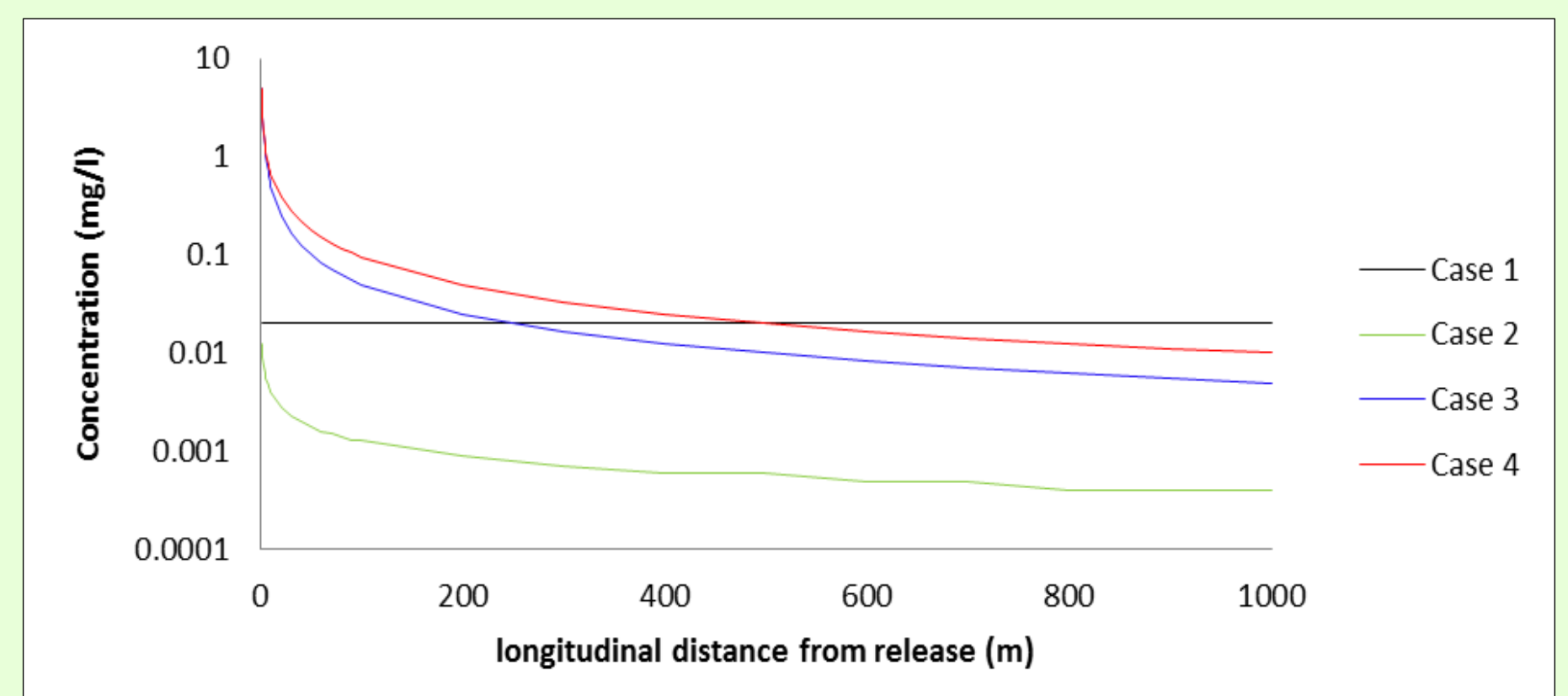


## Background

- Analytical and numerical models can be used to represent the advection-dispersion processes governing the transport of pollutants in rivers (Fan et al., 2015; Van Genuchten et al., 2013).
- Simplifications and assumptions in these models result in various uncertainties while estimating pollutant concentrations.
- One common simplification is the assumption that when a pollutant is released into a river location (for example from a CSO discharge), the pollutant is instantaneously fully mixed over the river cross section (Kannel et al., 2011; Sharma et al., 2013).
- The scale and significance of these uncertainties has not previously been examined.

## Results – peak concentrations

Estimated peak concentrations vs. distance. It is observed that by considering only advection processes, the initial concentrations are underestimated while at longer distances, concentrations are overestimated.



## ADE Analytical Solutions

Point injection at  $x = 0, t = 0$

$$C(v_x, x, t) = \begin{cases} \frac{M}{V} & \text{where } v_x = \frac{x}{t} \\ 0 & \text{where } v_x \neq \frac{x}{t} \end{cases}$$

Advection Dispersion 1D (Fischer 1973)

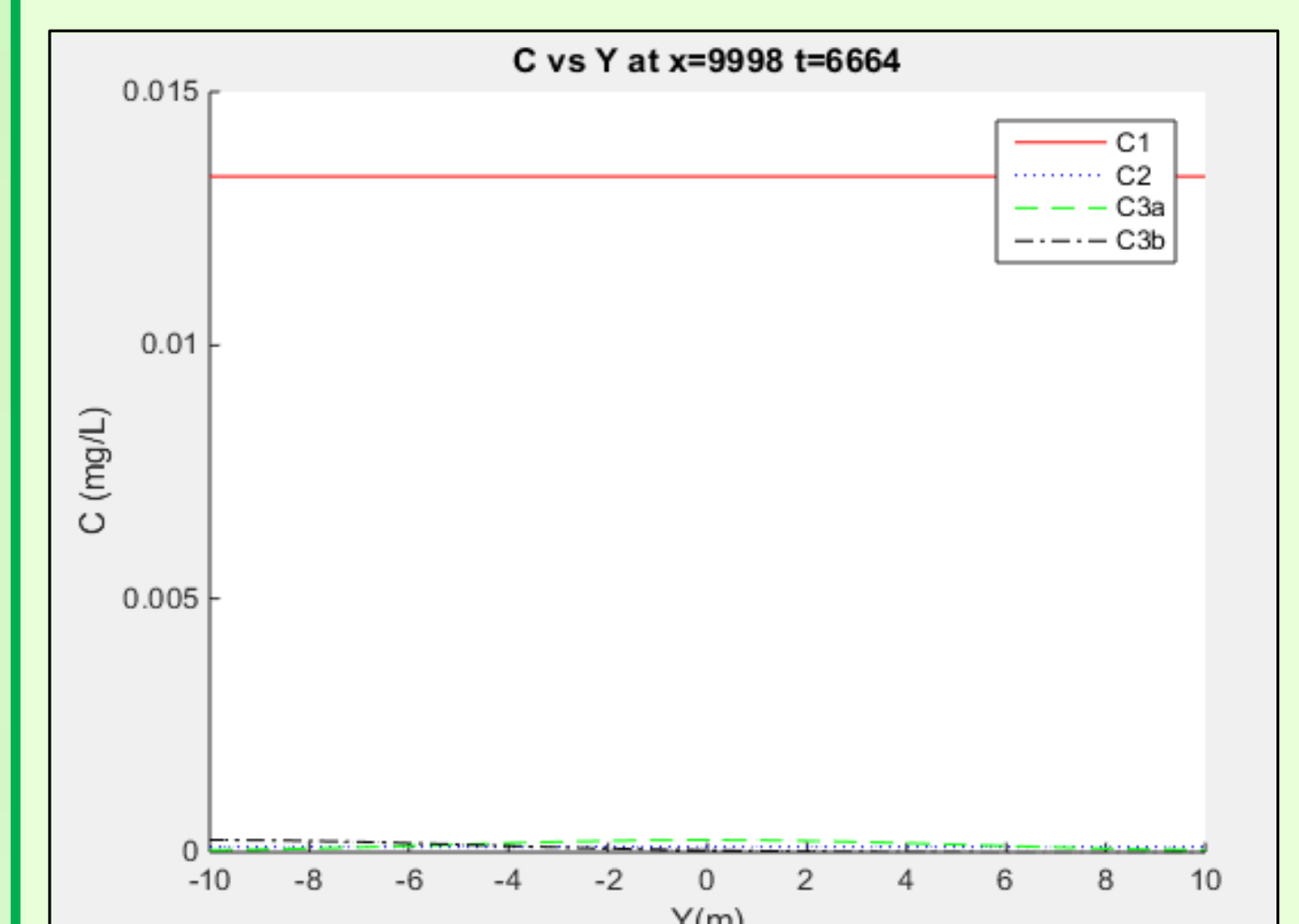
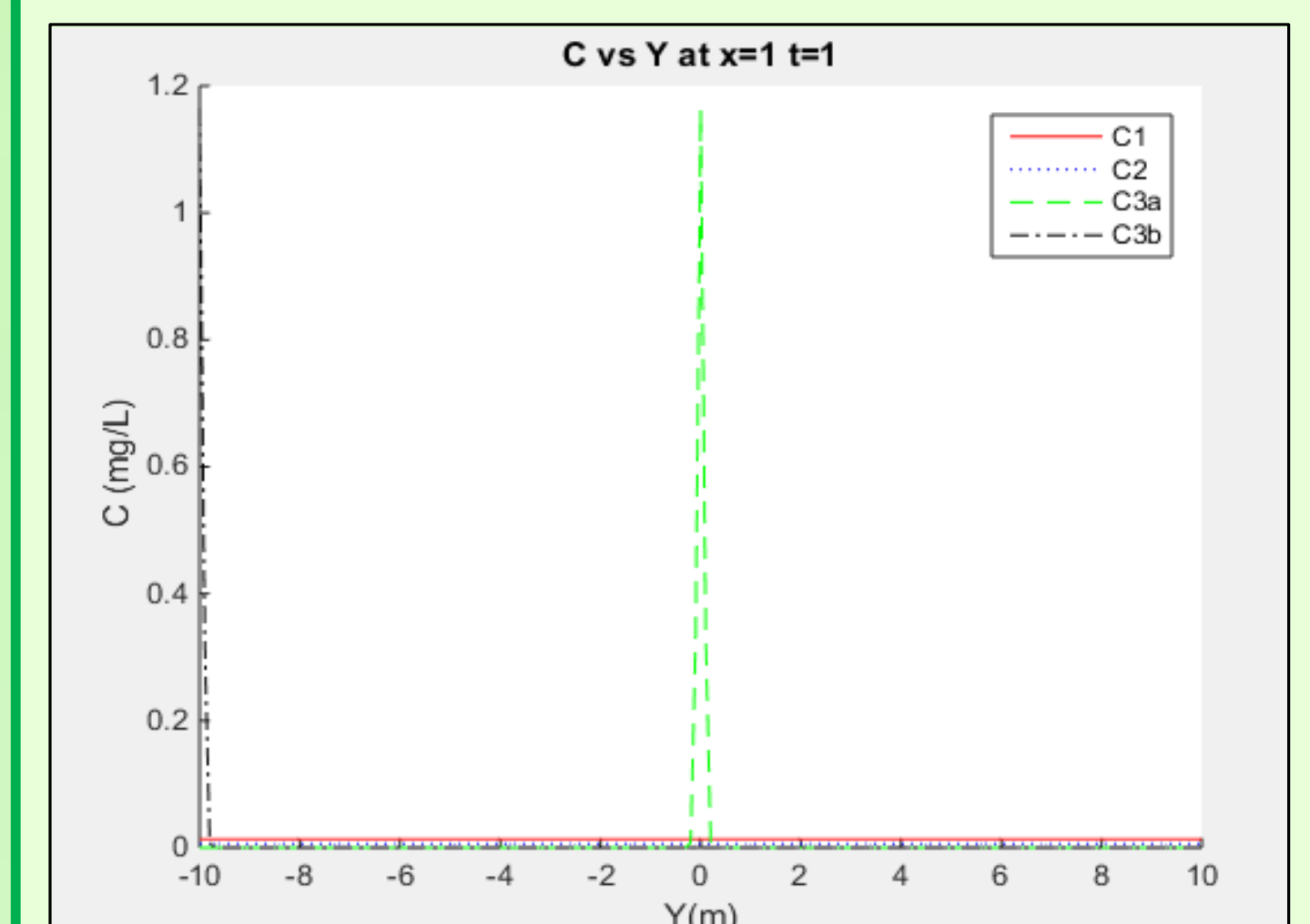
$$C(x, t) = \frac{M}{A\sqrt{4\pi D_x t}} \exp\left(-\frac{(x - v_x t)^2}{4D_x t}\right)$$

Advection Dispersion 2D (Fischer 1973)

$$C(x, y, t) = \frac{M}{4\pi dt\sqrt{D_x D_y}} \exp\left(-\frac{(x - v_x t)^2}{4D_x t} - \frac{y^2}{4D_y t}\right)$$

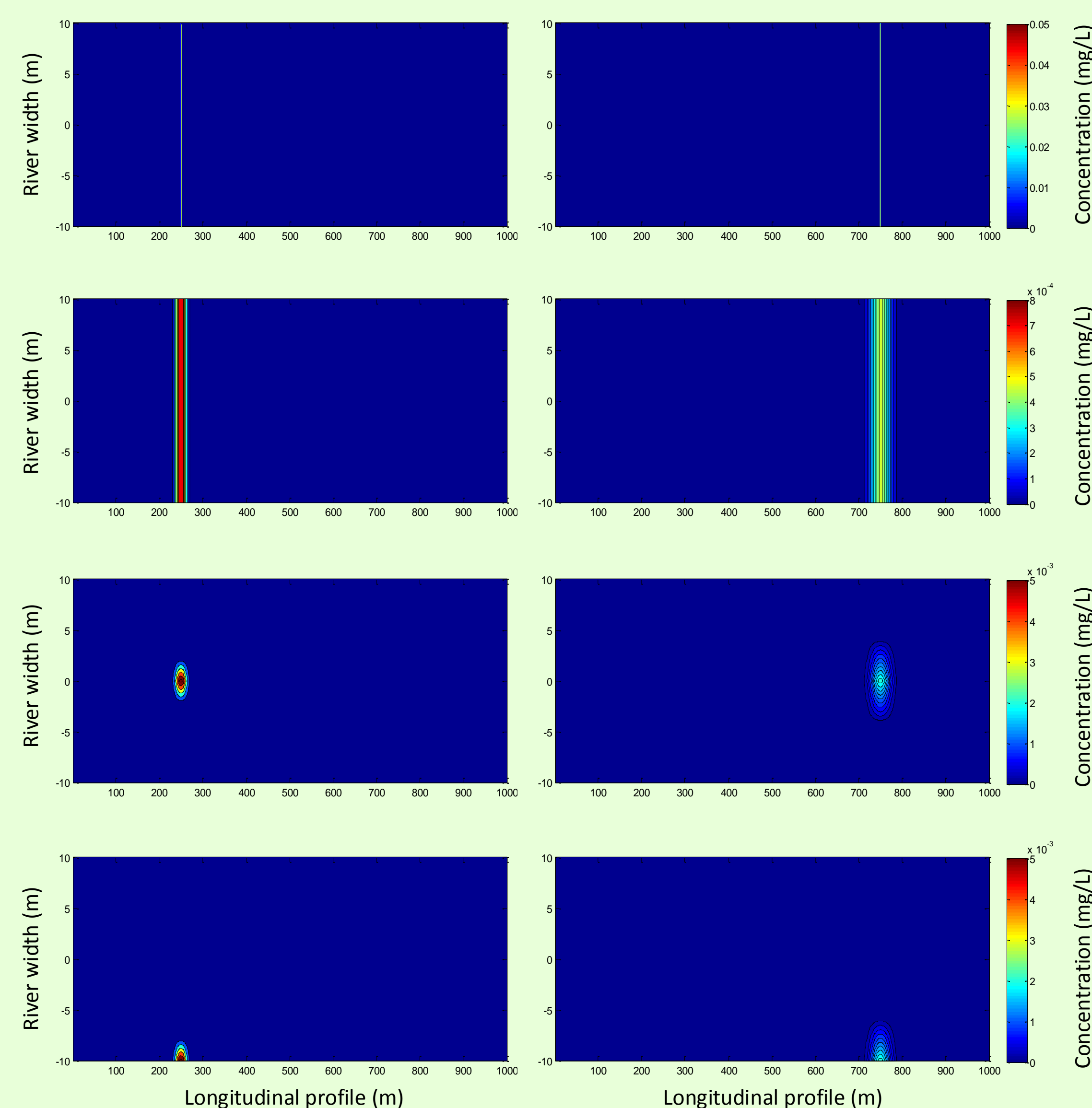
## Results - hydraulic conditions

With different river hydraulics, estimated pollutant concentrations for the different cases still show large differences depending on the discretization of time and space.



## Results – model differences

Modelled river BOD concentration profile in (mg/L) after CSO discharge after 250 and 750 seconds after release. Velocity = 1 m/s. When comparing cases 3a and 3b to case 1 and case 2 (one dimensional cases), a large difference in concentrations is observed reaching several orders in magnitude. As the pollutant travels in the longitudinal direction, the pollutant mixes completely along the cross section, and the difference between the predictions reduces.



## Conclusion

Initially, the pollutant is treated as an instantaneous release with a constant velocity. Future work will treat the pollutant discharge as a time series discharge, and compare the ADE analytical solutions to numerical solutions and other pollutant transport models such as the aggregated Dead Zone Model by Wallis et al (1989b). The work presented in this abstract will be extended to include a decay coefficient for BOD concentrations, a pollutant input discharge as a time series, a varying river velocity due to wet weather conditions, and a comparison with a commercial model and river quality verification data.

## Acknowledgements

The authors would like to thank United Utilities for sharing the data used for this study. This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement no 607000.

