Parameter estimation of urban drainage models using binary observations from low-cost sensors

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Abstract
To better calibrate urban drainage models, it would be good to use not only dedicated flowmeters, but also cheap and robust sensors, such as binary level or overflow detectors. In this study, we suggest a formal likelihood function to efficiently extract the information content of these binary signals. We apply this methodology on data from a small urban drainage system, where we calibrate a hydrodynamic rainfall-runoff model on both continuous observations and the corresponding binary data in a Bayesian framework. For our case study, we find that the inference leads to a comparable model performance for both types of data - with Nash-Sutcliffe efficiencies of 0.80 using continuous and 0.78 using binary observations. As expected, model predictions based on binary data are much more informative than predictions with prior parameters, i.e. an uncalibrated model. However, in our case, the binary information describes the duration of an exceedance of a threshold and thus cannot capture peak flows very well. Therefore, an optimal experimental design will probably not rely exclusively on cheap and robust sensors, but use many of them together with a few accurate flow meters. The results could possibly be improved by extending the likelihood function with a reliable description of monitoring errors of binary sensors.

Keywords
Low-cost sensors, binary observations, urban drainage model, parameter estimation, Bayesian inference, likelihood function

INTRODUCTION

Traditionally, computer models of urban drainage systems are calibrated on continuous observations of physical variables, such as discharge or water levels. Unfortunately, the installation and management of monitoring devices in an urban drainage network is laborious and costly, so that typically only a few locations are equipped with such sensors (Siemers et al., 2011). However, often other sensors are available, which simply detect the occurrence of overflow events (Rasmussen et al., 2008) or exceedance of critical water levels, and thus provide only binary information. Recently, it has even been suggested to specifically develop such binary monitoring devices based on robust and low-cost sensors such as temperature probes (Hofer et al., 2014; Montserrat et al., 2013), motion detectors (Siemers et al., 2011) and electrical switches (Rasmussen et al., 2008) (Figure. 1). In the future, such sensors should help to better calibrate urban drainage models (Rasmussen et al., 2008; Siemers et al., 2011). The idea that many inaccurate sensors provide more information about a complex (urban drainage) system than few very accurate ones is very compelling. However, so far only ad-hoc approaches to model calibration and parameter estimation have been suggested and it is currently not clear what the information content of such binary observations is and how we can use them most efficiently to learn about model parameters.

In this paper, we therefore suggest a novel approach to use binary observations for the calibration of urban drainage models based on rigorous statistical principles. The main innovation is a sound likelihood function which allows for parameter inference using binary signals. Coupled with an error model, it also makes possible the explicit accounting of uncertainties in input variables and uncertainties arising from model structure deficits.

Figure 1: Different types of low-cost sensors for sewer system monitoring. From top left clockwise: i) Electrical switch at overflow crest (Rasmussen et al., 2008), ii) Motion detector at leaping weir (Siemers et al., 2011), iii) Motion detector iv) Flood float (www.123mc.com)

METHODS AND MATERIAL

A likelihood function for binary observations

To efficiently calibrate an urban drainage model, we must first construct a likelihood function. In our case, this function describes the likelihood, given a parameter set for our drainage model and the error model, of observing a set of binary signals. To be able to consider model structure deficits and input errors, we follow the suggestions of Dietzel and Reichert (2012) and describe the true system response Y_t at time t with a deterministic model M, and stochastic process B, the so-called "bias" term:

$$Y_t = M(\theta, X_t) + B(\theta, X_t) \tag{1}$$

Here θ is the parameter vector and $\mathrm{X_t}$ is the input variable vector. The bias term B captures the mismatch between system response and model predictions due to the uncertainty in input variables, deficiency in the structure of model equations. Then, a likelihood function $p_Y(\mathbf{Y} \mid \theta)$for the continuous system response $\mathbf{Y} = \{Y_{t_1}, \dots, Y_{t_n}\}$, such as water level in a combined sewer overflow tank or discharge, can be formulated. However when only a binary signal is observed Y_t can be written as Z_t such that:

$$Z_t = \begin{cases} 1 & \mathrm{Y_t} > y_{threshold} \\ 0 & Y_t \leq y_{threshold} \end{cases} \tag{2}$$

The likelihood function for $\mathbf{Z} = \{Z_{t_1}, \dots, Z_{t_n}\}$ becomes

$$p_{\mathbf{Z}}(\mathbf{Z} \mid \theta) = \int_{l_1}^{u_1} \dots \int_{l_n}^{u_n} p_Y\left(Y_{t1}, \dots, Y_{t_n} \mid \theta\right) dY_{t1} \dots dY_{t_n} \quad (3)$$

where u and l are the upper and lower limits of **Y** respectively.

To have a convenient mathematical formulation, we describe B with a Gaussian process, which makes $p_Y(\mathbf{Y} \mid \theta)$ a multivariate normal distribution. In addition, a Gaussian process is a suitable model for the involved errors, because it can capture the autocorrelated differences between the simulation results and observations which are usually found in hydrological applications.

Parameter inference, implementation and performance assessment

The parameters of the likelihood function in Eq. 3 are difficult to estimate due to the problem of identifiability between *M* and *B*. Inference is possible if we include our prior knowledge on probable parameter values in the analysis. And incorporation of prior knowledge is generally done in a Bayesian framework.

Although the integrals in the likelihood function (Eq.3) are known to be analytically intractable for normal distributions, a very efficient numerical solution has been suggested (Genz, 1992). We implemented the inference in the programming language R (CRAN, 2015) and used the *pmvnorm* function (Genz et al., 2015) to compute these integrals. The mode of the posterior can be estimated with an effort comparable to the use of an informal objective function. Markov Chain Monte Carlo (MCMC) methods are used for sampling from the posterior. Samples converge to the distribution and allow for the computation of the moments and other statistics of the distribution. The best parameter estimate is made by getting the parameter vector corresponding to the maximum posterior distribution. Here, we assess the predictive performance of the model based on the Nash-Sutcliffe efficiency (NSE) (Nash and Sutcliffe, 1970).

Catchment and data

We demonstrate the potential of the suggested technique by calibrating a rainfall-runoff model for the small urban drainage system of Adliswil near Zurich. Adliswil is a small city that lies on the western side of Lake Zurich. The catchment area has size of about 1km × 3 km. In a dedicated monitoring campaign over 1.5 years we collected detailed rainfall information with a dense network of 6 weighting rain gauges. Also, level and flow measurements were done in several manholes. As our corresponding monitoring campaign is still ongoing, we transform the continuous readings into binary observations by “detecting” an exceedance as all values above a threshold of 100 l/s (Figure 2, dotted horizontal line). Here, we only focus on the observations at the outlet of the catchment.

Model and inferred parameters

We model the urban drainage system of Adliswil with the semi distributed model - EPA SWMM, version 5.1 (EPA, 2015). For our study case, SWMM employs a conceptual hydrologic model for the runoff estimation in the small subcatchments and routes the discharge in the pipes using the dynamic wave equation. The model has 101 sub catchments, 458 junction nodes, and 461 conduit links. We chose ten SWMM parameters for estimation based on sensitivity analysis, which include i) the subcatchment imperviousness, ii) subcatchment width, iii) subcatchment slope, iv) manning roughness of the subcatchment and v) manning roughness of the conduit. The other parameters are related to the storages and weir heights.

Defining prior distributions

The prior distributions were defined based on the physical constraints of the calibration parameters. Normal truncated distribution was used as priors and the mean of these distributions were chosen based on expert opinion. This eliminates unrealistic parameter estimates and allows for the incorporation of accumulated knowledge about a system like a drainage model. As it is challenging to formulate the priors of the bias parameters, we have good experiences with defining them based on preliminary analysis of residuals of past observation periods.

Numerical experiments and performance assessment

We perform two different numerical experiments. First, we compared the NSE of the model that has been calibrated on the binary dataset to that which has been calibrated on the continuous data. Second, we assessed the information content of different thresholds.

The second experiment is interesting, although in most real applications, binary sensors would detect overflow events at a certain weir with given height, or a predetermined critical levels which for example would lead to local flooding. In our case study, however, there is no such one critical exceedance threshold and we could test how far the information content in the binary data depends on the threshold of our binary sensor. In a real sewer, this would correspond to installing an electrical switch, or motion sensor, at different heights in the cross-section. In the extreme cases, binary sensors would never (or always) “detect” events by choosing an unrealistically high or low installation height.

First, we generated sixteen different binary data sets from the continuous observations by increasing the threshold values from 0 to 300 l/s in increments of 20 l/s. Second, we estimated the best model parameters from each of these data sets. Initially, to save computing time, we only evaluated the prediction performance of the model, in terms of NSE, at the maximum of the posterior distribution. To find the maximum, we used a general simulated annealing algorithm (Gubian, 2015). At this point, we were only interested in the NSE and have not yet been assessing the coverage and sharpness of the predictions achieved with the different binary datasets.

RESULTS AND DISCUSSION

In general, we find that the data from the binary sensors are informative. Thus, it is possible to learn about model parameters (Figure 2. left) and the resulting posterior distributions (grey) are narrower than the corresponding priors (dotted lines). The spread of posterior distribution captures the parametric uncertainty. Numerically, we approximated the joint posterior distribution of the model parameters and error parameters by drawing 5000 sampling from the posterior with an MCMC algorithm (Chivers, C., 2012; Scheidegger, A., 2012; Vihola, M., 2012).

Interestingly, in our case study, the model can be calibrated almost equally well on the binary data (NSE= 0.78) when compared to continuous data (NSE= 0.80) (Figure 2 right). This is the parameter set which corresponds to maximum posterior probability density value from the sampled set (Figure 2 left); whereas the NSE from the optimization of posterior distribution is obtained using general simulated annealing (Figure 3), which manages to only delivers local optima.

The performance of model calibrated on binary data based on Nash efficiency seems decent, however looking at the discrepancies between model results and continuous observations, it can be seen that the binary data lose a lot of information regarding observed peak flows.

Apart from capturing the system behaviour we are also able to capture the parametric uncertainty using the likelihood description (Figure 2. left). If the inferred parameters of the drainage model and the error model are used for future predictions, the uncertainty arsing due to model structure deficits and input will also be covered.

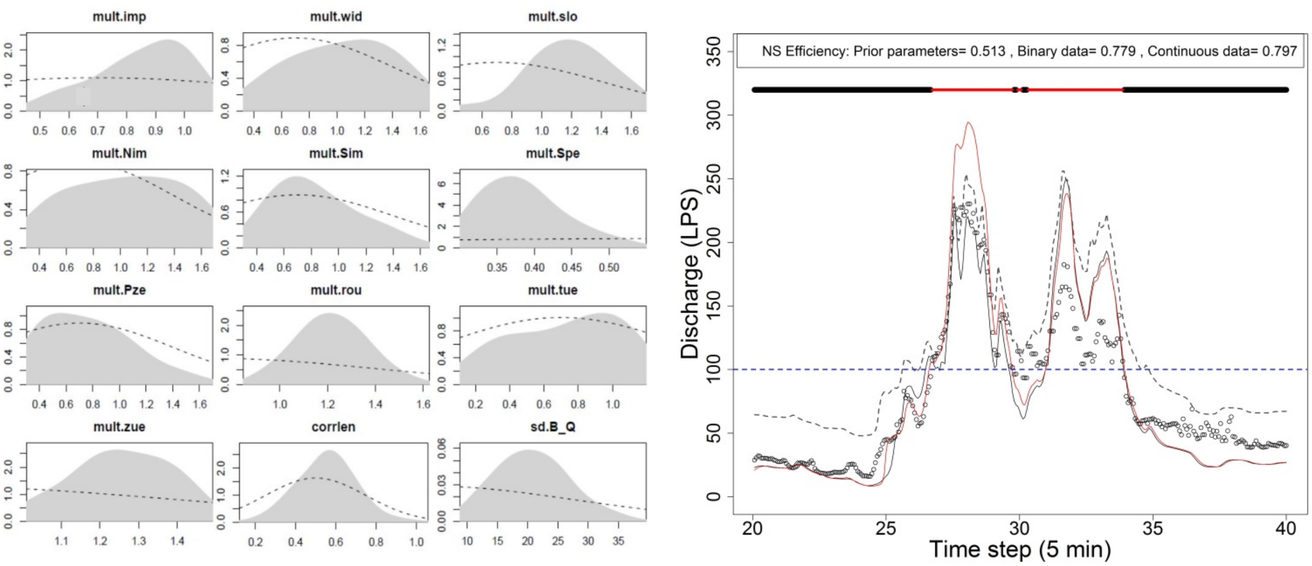

Figure 2 (Left): Prior (dashed lines) distributions of urban drainage model parameters and error model parameters (last two graphs). Posteriors (light grey) estimated from binary data. The y axis is the probability density and the x axis shows the value of multiplicative parameters.
(Right) Model predictions for the calibration phase based on continuous (thin black line), binary observations (thin red line) and prior parameter values (dashed grey line). The continuous data are plotted as black circles and the binary observations as a red horizontal and black line at the top of figure. The dashed blue line depicts the threshold of the sensor, 100 l/s.

It is observed that the information content in a binary data time series first increases with increasing threshold and then decreases. This is an expected result as less or no variability is captured by the binary sensor towards low and high values of threshold. For a centrally located threshold, the binary signals can calibrate a model pretty well (Figure 3.).

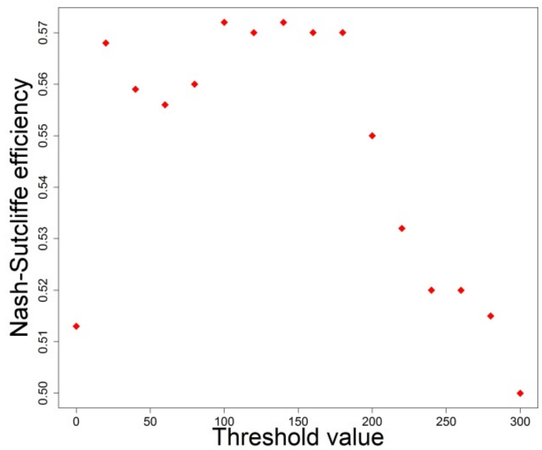

Figure 3. Information content, expressed as Nash-Sutcliffe efficiency, of observations from binary sensors with different detection thresholds.

It is assumed that the binary signals $\mathbf{Z}$ are measured without error. Nevertheless, by distinguishing between the observed binary signals $\mathbf{Z_o}$ and the "true" signal $\mathbf{Z}$, an observation model can be introduced $p_{\mathbf{Z_o}}(\mathbf{Z_o} \mid \mathbf{Z})$ which accounts for the uncertainty in binary observations. The likelihood function requires marginalization over $\mathbf{Z}$ (Eq. 4)

$$p_\theta(\theta \mid \mathbf{Z_o}) \propto \sum_{z_{t1}=0}^{1} \dots \sum_{z_{tn}=0}^{1} p_{\mathbf{Z_o}}(\mathbf{Z_o} \mid \mathbf{Z}) \cdot p_{\mathbf{Z}}(\mathbf{Z} \mid \theta) \qquad (4)$$

resulting in summing 2^n terms. Even for moderate n this is likely to be not feasible. Alternatively, samples from $p_\theta(\theta, \mathbf{Z} \mid \mathbf{Z_o}) \propto p_{\mathbf{Z_o}}(\mathbf{Z_o} \mid \mathbf{Z}) \cdot p_{\mathbf{Z}}(\mathbf{Z} \mid \theta)$ could be generated. The marginalization is then achieved trivially by ignoring the $\mathbf{Z}$ dimensions of the samples. However, this can be computationally still challenging. Thus, the incorporation of observational uncertainty in binary sensors has been left out for future research. Our analysis still helps in making a preliminary value judgement on binary data in the context of calibration.

Apart from the potential for a reasonable parameter estimation, the use of binary likelihood function allows for the quantification of uncertainty arising from the unknown parameters, input errors and model structure deficits. Thus apart from the model predictions, we also get an indication of the reliability of these predictions. We have not included the uncertainty estimates here as the emphasis is on parameter estimation, but previous research shows (Del Giudice, 2013) that an autocorrelated error process makes it possible to cover data well and produce reliable (albeit largely varying) predictions using similar bias description

We repeated the same analysis as done above for level data only (instead of discharge) and it does not produce as good a calibration as the data from a flowmeter. Nevertheless, it gives the expected trend, where the model calibrated on continuous data is better than that calibrated on binary data, which in turn is better that the model with prior estimates of the parameters. Theoretically, at least, this relative performance of calibration should not depend on the system response variable and binary data should provide information usable in parameter inference.

Also, we are currently analysing real binary observations of level data from a dedicated monitoring campaign that was conducted on a catchment in Lucerne, and in near future this analysis will be repeated for binary observation collected from there.

CONCLUSIONS

In sewers, usually many more sensors than traditional flow and water level measurement devices are available. Unfortunately, they often produce only binary observations, such as overflow detectors and it has not been known how to efficiently use such measurements in parameter estimation. In this paper, we present a statistically sound likelihood function which, for the first time, makes it possible to efficiently extract the information content from the binary data in a probabilistic framework. Our results, using Bayesian inference on a didactical example demonstrate that, although binary observations are inferior to continuous measurements, they contain substantial information to calibrate urban drainage models. As our models are always prone to deficiencies, we suggest to use error models, such as autocorrelated stochastic processes, which make it possible to capture errors in the model structure and input data.

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REFERENCES

Chivers, C., 2012. General Markov Chain Monte Carlo for Bayesian Inference using adaptive Metropolis-Hastings sampling. https://cran.r-project.org/web/packages/MHadaptive/index.html

CRAN, 2015. https://www.r-project.org/

Del Giudice, D., Honti, M., Scheidegger, A., Albert, C., Reichert, P., Rieckermann, J., 2013. Improving uncertainty estimation in urban hydrological modeling by statistically describing bias. Hydrology and Earth System Sciences 17(10): 4209-4225.

Dietzel, A., Reichert, P., 2012. Calibration of computationally demanding and structurally uncertain models with an application to a lake water quality model. Environmental Modelling & Software, 38: 129-46.

Environmental Protection Agency, United States, 2015. http://www.epa.gov/athens/wwqtsc/html/swmm.html

Genz, A., 1992. Numerical Computation of Multivariate Normal Probabilities. Journal of Computational and Graphical Statistics, Vol. 1, Iss. 2

Genz, A., Bretz, F., Miwa, T., Mi, X., Leisch, F., Scheipl, F., Bornkamp, B., Maechler, M., Hothorn, T., 2015. Multivariate Normal and t Distributions. https://cran.r-project.org/web/packages/mvtnorm/index.html

Gubian, S., Xiang, Y., Suomela, B., Hoeng, J., 2015. Functions for Generalized Simulated Annealing. https://cran.r-project.org/web/packages/GenSA/GenSA.pdf

Hofer, T., Gruber, G., Gamerith, V., Montserrat, A., Corominas, L., Muschalla, D., 2014. Using Temperature Sensors to Detect Occurrence and Duration of Combined Sewer Overflows. Presented at the 13th International Conference on Urban Drainage, Sarawak, Malaysia.

Montserrat, A., Gutierrez, O., Poch, M., Corominas, L., 2013. Field validation of a new low-cost method for determining occurrence and duration of combined sewer overflows. Sci. Total Environ. 463-464, 904–912. doi:10.1016/j.scitotenv.2013.06.010

Nash, J., Sutcliffe, J.,1970: River flow forecasting through conceptual models part I : A discussion of principles, J. Hydrol., 10, 282–290, doi:10.1016/0022-1694(70)90255-6.

Rasmussen, M.R., Thorndahl, S., Schaarup-Jensen, K., 2008. A low cost calibration method for urban drainage models. Presented at the 11th International Conference on Urban Drainage, Edinburgh, Scotland, UK.

Scheidegger, A., 2012. Implementation of a generic adaptive Monte Carlo Markov Chain sampler. https://cran.r-project.org/web/packages/adaptMCMC/index.html

Siemers, L., Dodd, J., Day, D., Kerr, D., LaGorga, J., Romano, P., 2011. Low Cost Overflow Monitoring Techniques and Hydraulic Modeling of A Complex Sewer Network. Proc. Water Environ. Fed. 2011, 571–583.

Vihola, M. 2012: Robust adaptive Metropolis algorithm with coerced acceptance rate, Stat. Comput., 22, 997–1008,.doi:10.2175/193864711802837363