Analysis of sub-kilometre variability of rainfall in the context of urban runoff modelling

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INTRODUCTION

It is an uncommon practise to measure rainfall on a sub-kilometre scale considering the cost and effort required to collect such data. Nevertheless such small scale rainfall variability proves to be important when it comes to runoff prediction, especially for urban catchments where the proportion of impervious area is high *(Gires et al., 2012).* In this study, such variability is studied in detail using rainfall measurements collected from 8 measurement stations covering an area of 400m*200m in Bradford. The objective of this study is to describe the correlation between rainfall collected from these stations as a function of their separation distance and then to analyse the effect of this sub kilometre spatial variability of rainfall on urban runoff. Inter-gauge correlation is described using Pearson's product moment for different averaging interval. Further, the behaviour of these correlation functions are analysed with relation to rainfall intensity. A similar study was previously carried out by *Ciach & Krajewski, 2006* using rainfall measurements over an area of 3km*3km in central Oklahoma. But in our study we focused on rainfall variability on a sub-kilometre scale. In addition, it is planned to study the effect of small scale rainfall variability on urban runoff and the results will be presented in the final paper.

DATA COLLECTION

Rainfall data was collected from 8 pairs of rain gauges installed on selected roofs of Bradford University buildings (Ref Figure 1) covering an area of 400m*200m from April, 2012 to August 2013. All rain gauges are ARG100 tipping bucket type with a resolution of 0.2mm. Measurements were recorded at every minute and frequent checks were carried out every 4-5 weeks to make sure that the instruments were free of dirt and debris. Paired gauges were used to ensure the reliability of the measurement. Measurement from one gauge is checked against its paired gauge and the mean value of these paired gauges is used for further analysis. Any pairs with absolute difference percentage exceeding 4% were removed. Data from April, 2012-August, 2012 with the total accumulated rainfall of around 600mm is selected for the final analysis.

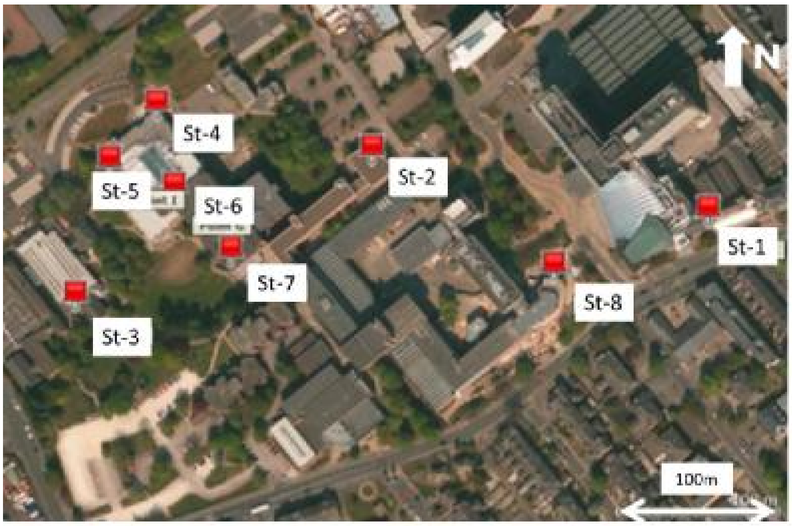

Figure 1. Measurement locations (stations 1 to 8) at Bradford University, UK covering an area with a high percentage of impermeable (>70%) surface

ANALYSIS AND MODELLING METHODS

To develop the relationship between inter-gauge correlation and separate distance, firstly an inter-gauge correlation co-efficient matrix is generated using Pearson's product moment, as given in equation 1.

$$P_{ij} = \frac{\overline{R_i . R_j} - \overline{R_i} . \overline{R_j}}{\sqrt{\left(\overline{R_i^{\,2}} - \bar{R}_i^{\,2}\right)\left(\overline{R_j^{\,2}} - \bar{R}_j^{\,2}\right)}} \qquad Equation\ (1)$$

where i, j are rain gauge indices, R is rainfall intensity for a given time scale and the bar indicates the mean value.

A sample matrix of correlation co-efficient for 15 min time scale is presented in Table 1.

Table 1. Symmetric matrix of inter-gauge correlation coefficient for a time scale of 15 min

	[,1]	**[,2]**	**[,3]**	**[,4]**	**[,5]**	**[,6]**	**[,7]**	**[,8]**
[1,]	1.000	0.982	0.967	0.971	0.977	0.964	0.968	0.958
[2,]	0.982	1.000	0.979	0.984	0.990	0.975	0.978	0.962
[3,]	0.967	0.979	1.000	0.977	0.981	0.965	0.968	0.948
[4,]	0.971	0.984	0.977	1.000	0.985	0.981	0.984	0.972
[5,]	0.977	0.990	0.981	0.985	1.000	0.972	0.977	0.954
[6,]	0.964	0.975	0.965	0.981	0.972	1.000	0.989	0.972
[7,]	0.968	0.978	0.968	0.984	0.977	0.989	1.000	0.971
[8,]	0.958	0.962	0.948	0.972	0.954	0.972	0.971	1.000

Having generated the co-efficient matrix for different averaging intervals ranging from 2 min to 3 hours, linear models are applied to represent relationship between separated distances and the correlation co-efficient. Linear models are chosen considering the number of sample points and simplicity in the representation. But one disadvantage of using linear model is its inability to represent the correlogram in the region of 0- 50m. But given that there are no data points in that region except at zero separation where correlation coefficient is 1, and the region of interest is mainly >50 m, the linear model is still fit for purpose.

The following sections describe the characteristic of the models by taking into consideration the effect of averaging interval and rainfall intensity separately. But regardless of time scale and intensity, declination of correlation with increasing distance is a common characteristic and it can be seen in both Figure. 2 and Figure. 3.

Dependency on averaging interval
As expected, Figure 2 shows that correlation gets better with increasing time scales and after a certain time scale the improvement is hardly visible. This behaviour is anticipated considering how small the measurement area is. It is also interesting to note the significant improvement in correlation from 2 min to 5 min time scales. The big drop of correlation in 2 min time scale is a good indicator of the importance of small scale spatial variability in urban drainage modelling as the time step of such models can be as small as 2 min.

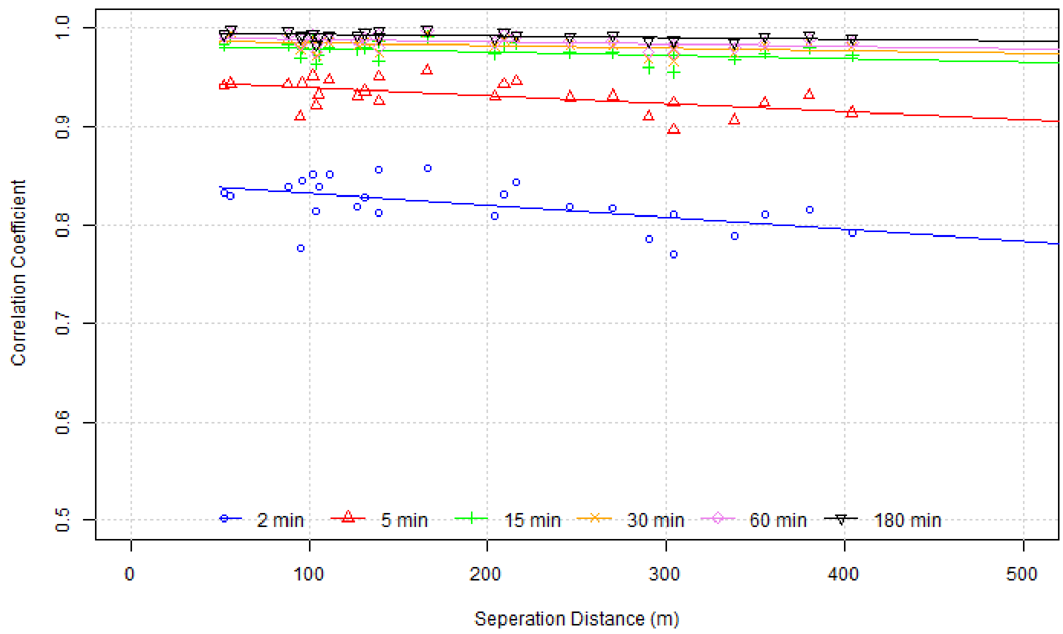

Figure 2. Spatial correlograms for different time scales ranging from 1 min to 3 hours

Dependency on rainfall intensity

In this section, the behaviour of correlation is studied against different intensity thresholds. Figure 3 shows the different behaviour of correlation of stronger (>5mm/hr) and weaker (<5mm/hr) rainfall events for time scales of 5min and 15min. It can be seen that the correlation is high for heavy rainfall events compare to mild rainfall events/drizzle. In a previous similar study done by *(Ciach & Krajewski, 2006)*, it was concluded that this behaviour is threshold dependent. But in our studies, it is observed that this behaviour is consistent regardless of intensity threshold values. This is mainly due to the difference in the size of the area covered in both studies. In our study, the maximum inter-gauge separated distance is around 0.4km whereas *Ciach & Krajewski, (2006*) covered a maximum distance of around 4km in their studies.

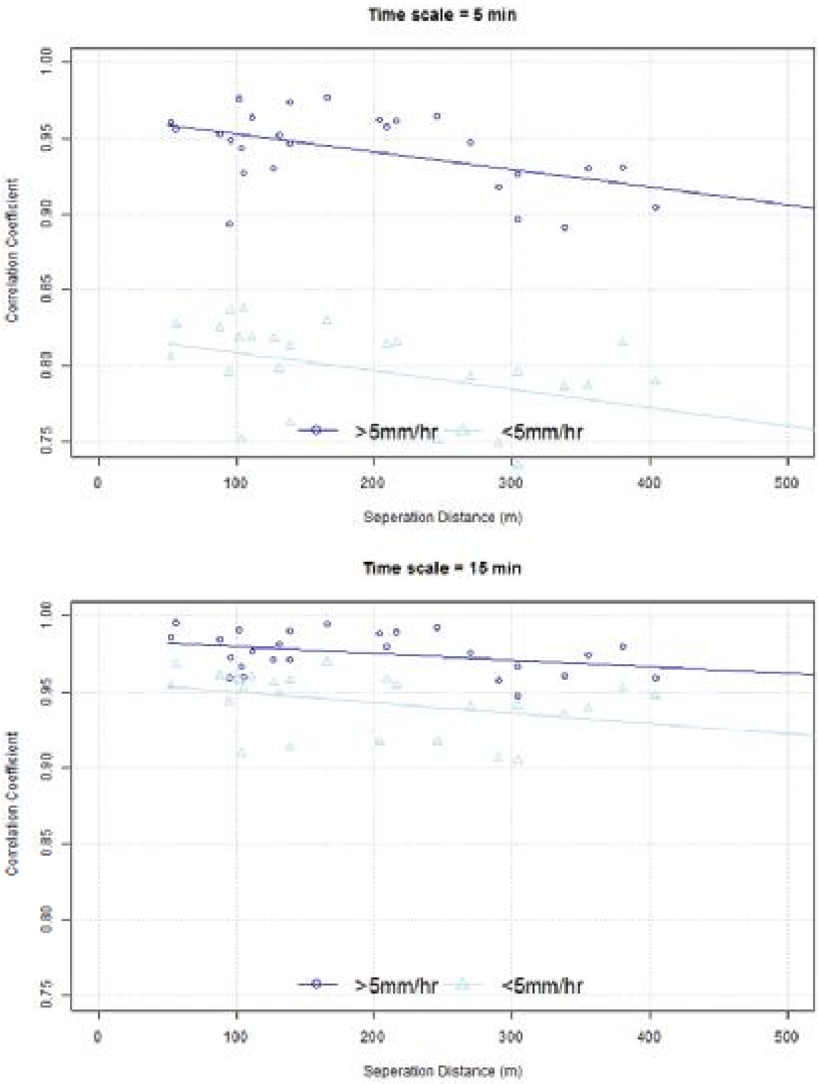

Figure 3. Spatial correlograms of weaker (<5mm/hr) and stronger (>5mm/hr) rainfall for time scales of 5 min and 15 min

EFFECT ON URBAN RUNOFF

As mentioned before, the small scale spatial variably of rainfall can be crucial in urban drainage modelling, especially runoff predictions in small urban catchments. *Gires et al., (2012)* found up to 20% uncertainty in peak flows due to unmeasured small scale variability over a 900 ha urban catchment in East London, UK. Similar attempt is made in this study using different rainfall inputs (averaging interval-15 min) generated using Thiessen polygons by taking i (i – 1, 2…, 8) number of rain gauges at a time. All possible realization of rainfall field is then fed in to a rainfall runoff model to produce catchment runoffs for a fictitious impermeable basin of size 200m*400m. InfoWorks CS is used for rainfall runoff modelling. Results show significant variation in peak runoff when using only one rain gauge in comparison to using all 8 rain gauges. The variation in peak runoff corresponds to one of the analysed rainfall event is shown below

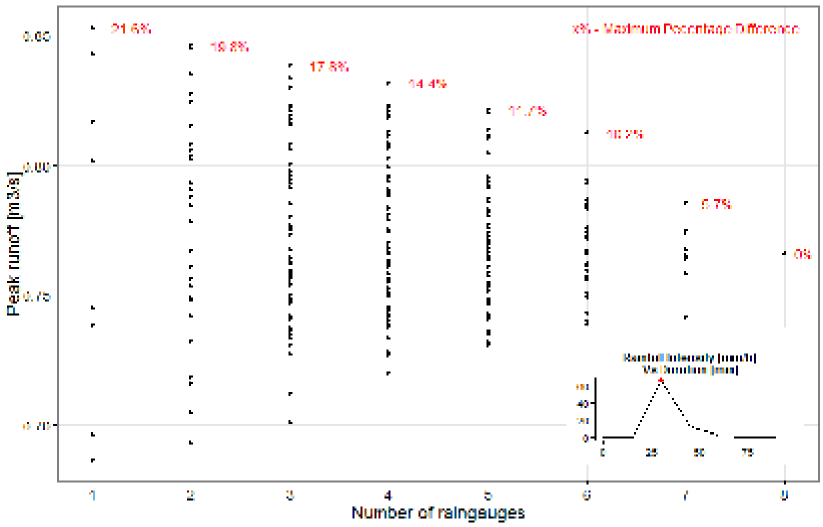

Figure 4. Peak runoffs derived from different rainfall inputs for a single event. Rainfall inputs are generated using Thiessen weights by taking i (i – 1, 2…, 8) number of rain gauges at a time. Corresponding weighted rainfall derived using all 8 rain gauges is given in the subplot.

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