Integration of rain gauge measurement errors with the overall rainfall uncertainty estimation using kriging methods

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Research Problem

- Often very poor rainfall information is used operationally, and uncertainty is neglected.
- Rain gauge uncertainty can increase due to poor network operation and data management.
- For urban applications, the spatial variability of rainfall needs to be captured at fine scale.
- In order to reach the rain gauge density necessary for urban studies, different networks, even with poorer data quality, need to be used.

Can we better estimate rainfall considering rain gauge uncertainty?

Proposed solution

- Use of kriging interpolation methods for uncertainty estimation.
- Different uncertainty for different rain gauges can be included as different nugget effects in the covariance function.
- The use of radar rainfall estimates, merged with all the available rain gauge information weighted on their accuracy, is used to achieve the best rainfall estimation.

Method

Variogram

A variogram was calculated from the data for each of the 5 considered rainfall events, at hourly and daily accumulation. Each variogram has been fitted with the following exponential model:

$$\gamma(d) = c\left(1 - \exp\left(-\frac{3d}{r}\right)\right)$$

Where d is the distance, c is the sill, and r is the range.

Table 1: sill and range calculated obtained fitting the exponential model to the data

	Event 1		Event 2		Event 3		Event 4		Event 5	
	Sill	Range	Sill	Range	Sill	Range	Sill	Range	Sill	Range
Hourly	1.050	39.205	0.577	100.277	0.624	22.184	0.670	151.408	0.145	68.928
Daily	0.064	51.310	0.014	37.500	0.037	23.528	0.018	105.785	0.013	272.497

Covariance function

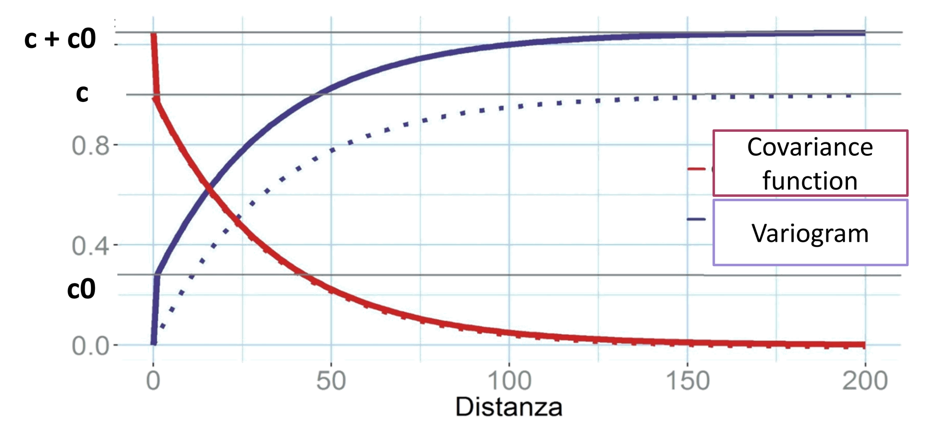

Figure 2: Variogram and covariance function compared, with and without nugget

The effects of measurement errors results in a nugget effect in the variogram. Working with a covariance function $C(d)$ rather than a variogram $\gamma(d)$ allows the nugget effect to appear only for distance zero:

$$C(d) = c - \gamma(d)$$

$$C(d) = \begin{cases} c + c_0 & for\ d = 0 \\ c - c\left(1 - \exp\left(-\frac{3d}{r}\right)\right) & for\ d > 0 \end{cases}$$

Where c_0 is the nugget.

Kriging application

Ordinary Kriging (OK):

$$\boldsymbol{C} = \begin{bmatrix} C(d_{11}) & \cdots & C(d_{1N}) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ C(d_{N1}) & \cdots & C(d_{NN}) & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix} = covariance\ of\ the\ distances\ between\ all\ observation\ points$$

$$\boldsymbol{D} = \begin{bmatrix} C(d_{10}) \\ C(d_{20}) \\ \vdots \\ C(d_{N0}) \\ 1 \end{bmatrix} = covariance\ of\ the\ distances\ between\ observation\ points\ and\ prediction\ point$$

Universal Kriging with radar as covariate (UK):

$$\boldsymbol{C} = \begin{bmatrix} C(d_{11}) & \cdots & C(d_{1N}) & 1 & Rad_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ C(d_{N1}) & \cdots & C(d_{NN}) & 1 & Rad_N \\ 1 & \cdots & 1 & 0 & 0 \\ Rad_1 & \cdots & Rad_N & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} C(d_{10}) \\ C(d_{20}) \\ \vdots \\ C(d_{N0}) \\ 1 \\ Rad_0 \end{bmatrix}$$

The elements on the diagonal are the only ones at distance 0, therefore the only ones where the nugget effect appears.

We can modify the diagonal to add a different nugget for each observation point, according to the error of each rain gauge:

$$c_{0_i} = err_i^2$$

$$C(d_{ii}) = c + c_{0_i}$$

$W = C^{-1} \cdot D =$ Kriging weights

$R(x_0) = W^T \cdot R(x_\alpha) = prediction\ at\ x_0,\ given\ \alpha = 1,2,\ldots,N$

$\sigma^2(x_0) = c - W^T \cdot D = variance\ at\ x_0$

Study area

Eindhoven catchment, river Dommel

Available data:

- **7 KNMI rain gauges:**
  - *High quality*
  - *Automatic, floating device*
- **6 Dommel Water Board and Eindhoven Municipality rain gauges:**
  - *Lower quality*
  - *Tipping bucket*
- **35 Amateur rain gauges:**
  - *Daily*
  - *Tipping bucket*
  - *Used for validation*
- **KNMI Radar composites:**
  - *2 single-pol radars 70 and 170 km away*
  - *spatial resolution: 1x1 km*
  - *temporal resolution: 5 min*

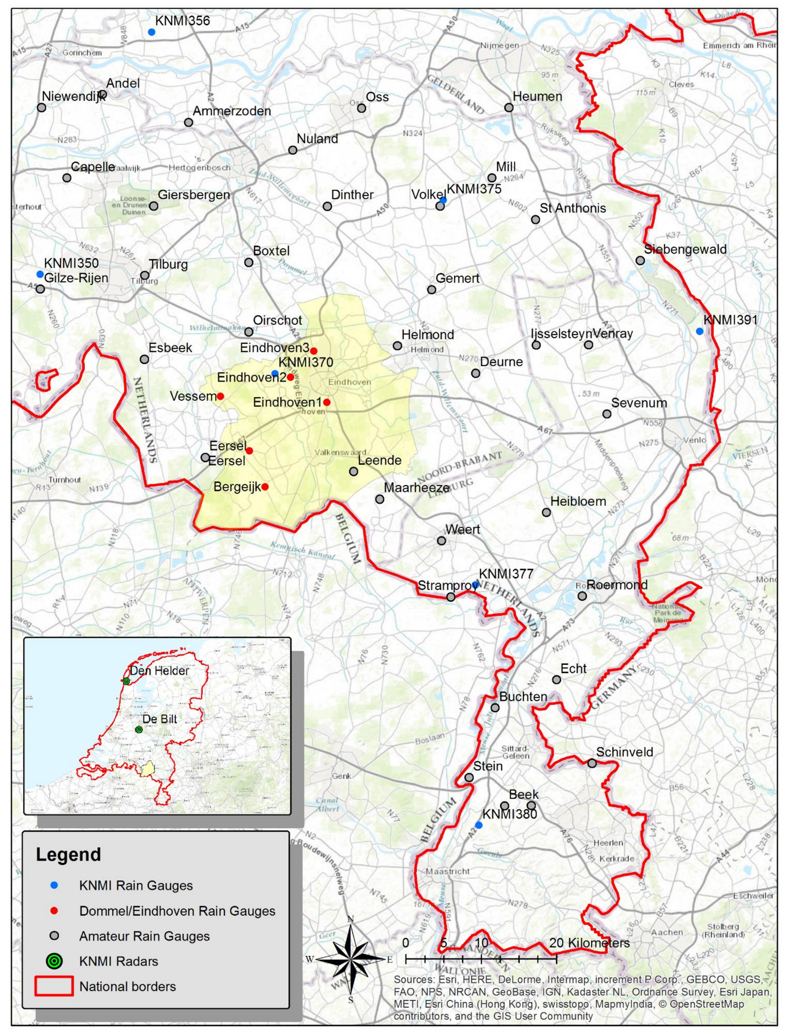

Figure 1: study area. The image reports the smaller area of interest, around the Eindhoven municipality, sharing the same urban drainage system and the broader area where the used rain gauges are located

Rain gauge errors

The observed accuracy of KNMI gauges is less than 3% (Wauben, 2006), independently on the rainfall rate. Considering operational use, we round it to 5%.

Using the formulation in Ciach, 2003 ($\epsilon_{rel} = e_0 + \frac{R_0}{R}$) fitted on our data, the tipping bucket error is estimated as a function of the rain rate, using the KNMI gauges as a reference.

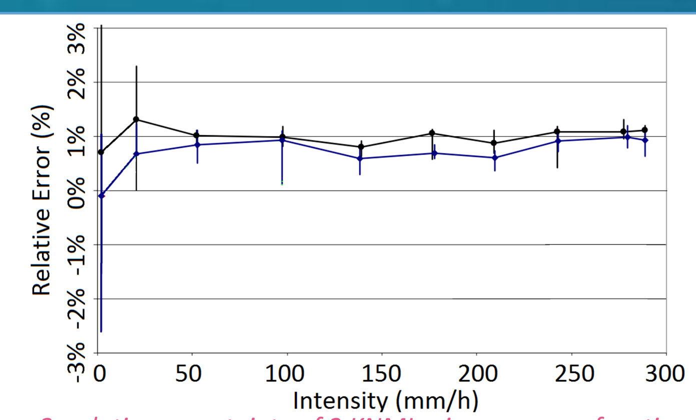

Figure 3: relative uncertainty of 2 KNMI rain gauges as function of rainfall rate (KNMI technical report TR-287, Wauber, 2006)

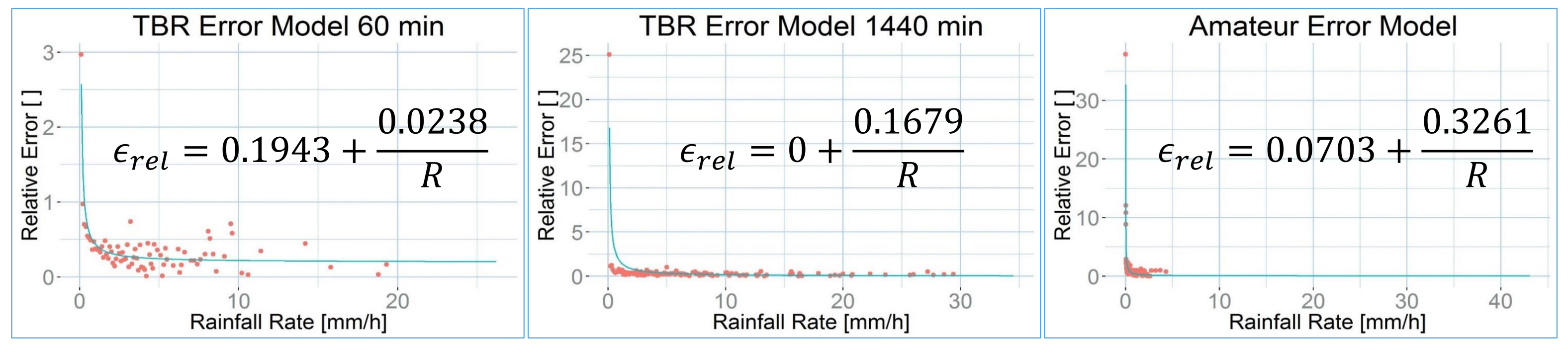

Figure 4: Error models for the tipping bucket rain gauges in the case study, derived fitting the Ciach 2003 model on the observations. The observation is obtained comparing the rain gauge "Eindhoven 2" and the reference "KNMI 370" and for the amateur network comparing the rain gauge "Volkel" with the "KNMI 375".

Conclusions

- As expected, universal kriging performs better than ordinary kriging, which performs better than the use of a single rain gauge;
- The consideration of rain gauge uncertainty further improves the universal kriging results;
- Surprisingly, the consideration of rain gauge uncertainty worsen the ordinary kriging results: this is due to the fact that, in an already data-scarce situation, we disregard part of the information because of its quality;
- Universal kriging is able to capture the spatial distribution of rainfall, the shape of storms and their precise location and intensity;
- The universal kriging results are not only better prediction, but are also more accurate, having a lower kriging variance;
- It is highly advisable to use universal kriging products with rain gauge uncertainty consideration, for modelling, even in small-scale urban applications.

Results

1	OK with 1 RG and no RG error	2	OK with 1 RG and with RG error
3	OK with all RG and no RG error	4	OK with all RG and with RG error
5	UK with all RG + radar and no RG error	6	UK with all RG + radar and with RG error

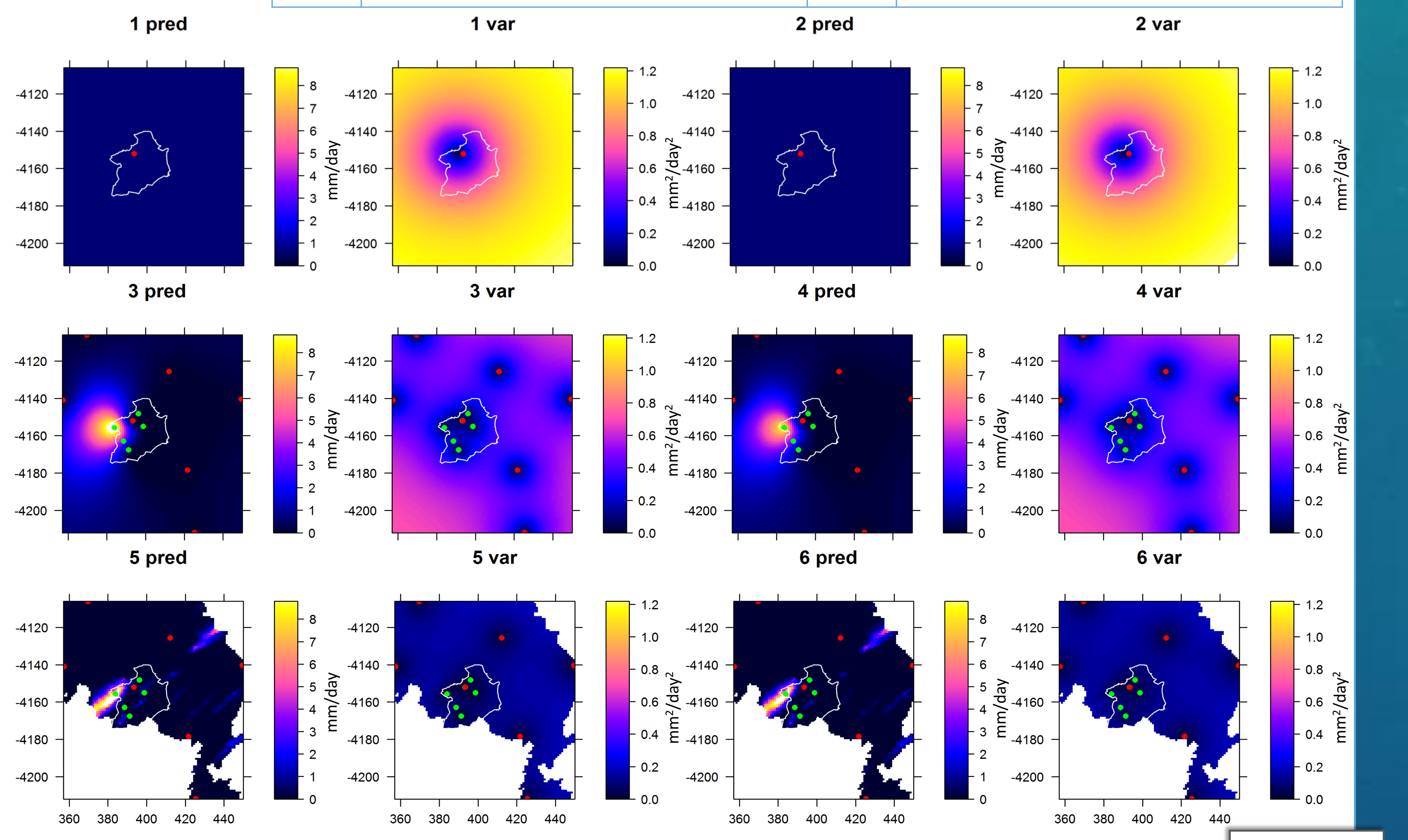

Figure 5: Hourly rainfall predictions and variance with and without rain gauge measurement errors at 14:00 on 13th July 2012

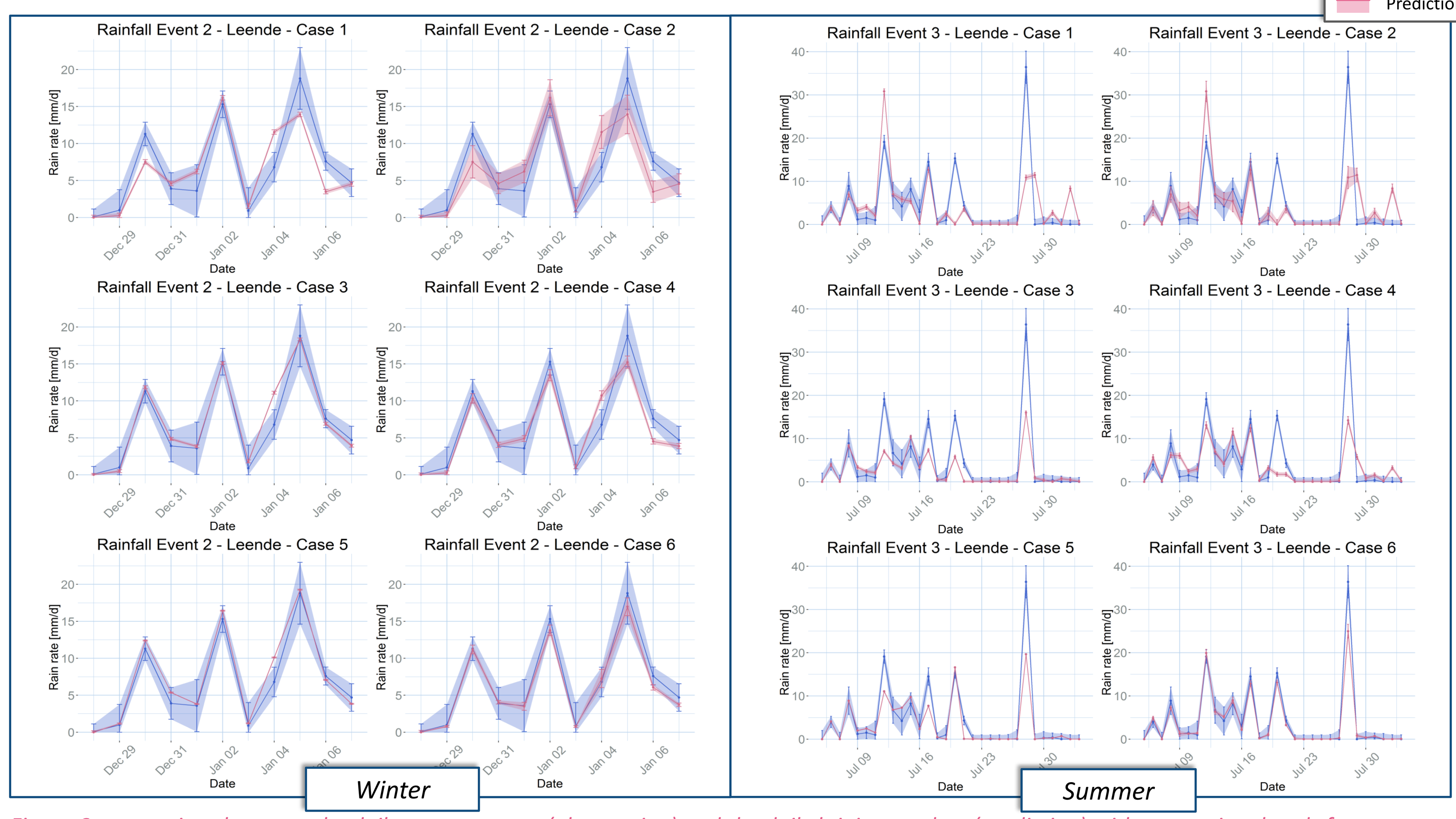

Figure 6: comparison between the daily measurement (observation) and the daily kriging product (prediction) with uncertainty bands for a winter and a summer event, for the amateur rain gauge named "Leende", the only one available in the smaller area of interest.

Table 2: average NSE coefficient and percentage of time the prediction with uncertainty band covers the observation with uncertainty band for daily predictions

NSE coefficient

		1	2	3	4	5	6
Event 1	Summer	-0.196	-0.196	0.700	0.506	0.818	0.861
Event 2	Winter	0.698	0.698	0.833	0.791	0.897	0.917
Event 3	Summer	-0.035	-0.035	0.625	0.633	0.829	0.903
Event 4	Summer	-0.173	-0.173	0.742	0.102	0.777	0.797
Event 5	Winter	0.905	0.905	0.914	0.920	0.934	0.959

Prediction coverage

		1	2	3	4	5	6
Event 1	Summer	0.668	0.710	0.745	0.721	0.767	0.848
Event 2	Winter	0.766	0.870	0.839	0.813	0.834	0.943
Event 3	Summer	0.676	0.737	0.774	0.764	0.822	0.890
Event 4	Summer	0.750	0.801	0.835	0.807	0.865	0.905
Event 5	Winter	0.831	0.914	0.831	0.851	0.857	0.914

This presentation participates in OSPP

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