Sampling design optimisation for rainfall prediction using a non-stationary geostatistical model

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Introduction

Conventional geostatistical models assume that the property being monitored is the realisation of a second-order stationary random process

$$Z(s) = \mu + \varepsilon(s)$$

$\mu = constant$

$Cov(\varepsilon(s), \varepsilon(s+h)) = C(h)$

if $h = 0 => Cov(\varepsilon(s), \varepsilon(s)) = Var(\varepsilon(s)) = C(0)$

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But this is often an invalid assumption
$=>$ can be checked with exploratory analysis of the observed data

Expectation

Objectives...

1. Account for non-stationarity in the mean and variance of rainfall
2. Optimize the sampling locations of rain gauges for mapping rainfall over time

Introduction

Simple solutions exist for non-stationarity

In the mean

$$Z(s) = m(s) + \varepsilon(s)$$

and in the variance

$$Z(s) = m(s) + \sigma(s) \cdot \varepsilon(s)$$

- Mean rainfall at location s

$$Z(s) = \sum_{k=0}^{K} \beta_k f_k(s) + \sum_{l=0}^{L} \kappa_l g_l(s) \cdot \varepsilon(s)$$

Nonstationary variance model

- Mean rainfall at location s

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- Multiplier for error at location s

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- Multiplier for error at location s
- Standardized random error

In matrix notation

$$\mathbf{Z} = \mathbf{F}\beta + \underbrace{\mathbf{G}\boldsymbol{\kappa} \cdot \varepsilon}$$

$\mathbf{C} = diag\{\mathbf{G}\kappa\} \cdot \mathbf{R} \cdot diag\{\mathbf{G}\kappa\}^T$ is the variance-covariance matrix

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Predictions at new location

$$\hat{\mathbf{z}}(\boldsymbol{s}_0) = \mathbf{f}(\boldsymbol{s}_0)^T\hat{\beta} + \mathbf{g}(\boldsymbol{s}_0)^T\hat{\boldsymbol{\kappa}} \cdot \hat{\varepsilon}(\boldsymbol{s}_0)$$

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Prediction error variance at new location

$$\sigma^2(\boldsymbol{s}_0) = \underbrace{\boldsymbol{c}(0) - \boldsymbol{c}_0^T \mathbf{C}^{-1} \boldsymbol{c}_0}_{\text{prediction error variance of the residuals}}$$

$$+ \underbrace{(\boldsymbol{f}_0 - \mathbf{F}^T \mathbf{C}^{-1} \boldsymbol{c}_0)^T (\mathbf{F}^T \mathbf{C}^{-1} \mathbf{F})^{-1} \boldsymbol{f}_0 - \mathbf{F}^T \mathbf{C}^{-1} \boldsymbol{c}_0)}_{\text{error variance of the trend}}$$

Parameter estimation

With exponential correlogram,

$$r(h) = c_0 + (1 - c_0)\{exp(\frac{-3h}{a})\}$$

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Independant of β_i, Restricted loglikelihood:

$$\ell(\Phi|\mathbf{z}) = Constant - \frac{1}{2}ln|\mathbf{C}| - \frac{1}{2}ln|\mathbf{X}^T\mathbf{C}^{-1}\mathbf{X}| - \frac{1}{2}\mathbf{y}^T\mathbf{C}^{-1}(\mathbf{I} - \mathbf{Q})\mathbf{z}$$

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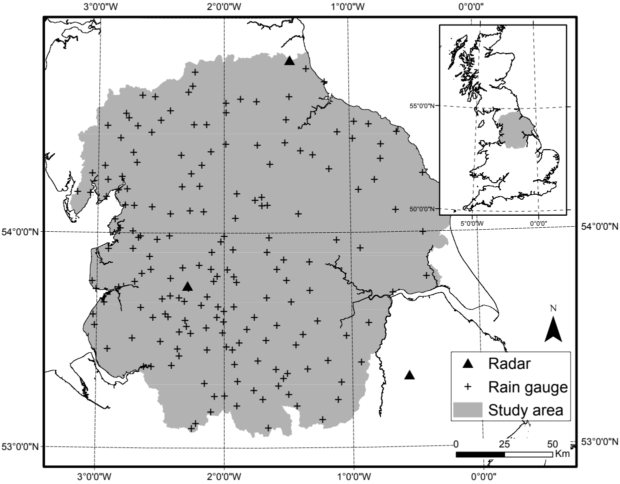
β_i are estimated with GLS using REML estimates of kappa, c_0 and a.

Illustration with a simple case, daily rainfall mapping with radar and rain-gauge



Covariates

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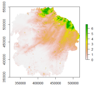
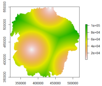
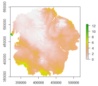


Rainfall from radar



Distance from radar



Previous predicted rainfall

f_k

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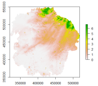

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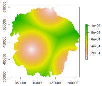

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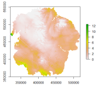

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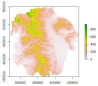
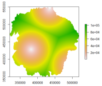
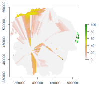


Elevation



Distance from radar



beam blockage

g_l ·

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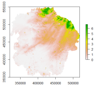

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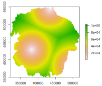

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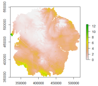

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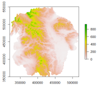

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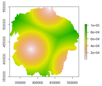

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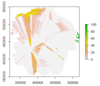

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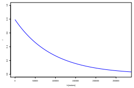
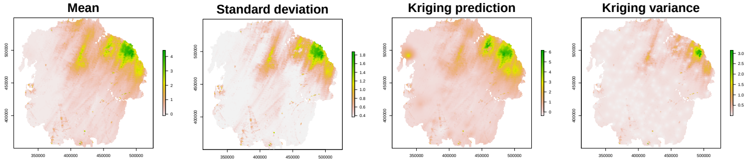


Correlogram

Model calibration

Example, February 11th, 2010...

Parameter	Estimated value	Associated to
c_0	0.0001278	*nugget*
a_1	8914	*range [meters]*
β_1	-0.02205	*intercept*
β_2	-0.1141	*radar image*
β_3	1.967e-05	*distance from radar*radar image*
β_4	0.1771	*previous estimated rainfall map*
κ_1	0.3699	*intercept*
κ_2	4.555e-11	*elevation*radar image*
κ_3	6.445e-06	*distance from radar*radar image*
κ_4	1.35e-10	*beam blockage*radar image*



Minimizing the variance criterion by a random search called Spatial Simulated Annealing (SSA)

$$Criterion = \frac{1}{T}\frac{1}{|A|}\int_{t=0}^{T}\int_{s\in A} Var(Z(s,t) - \hat{Z}(s,t))dsdt \tag{1}$$

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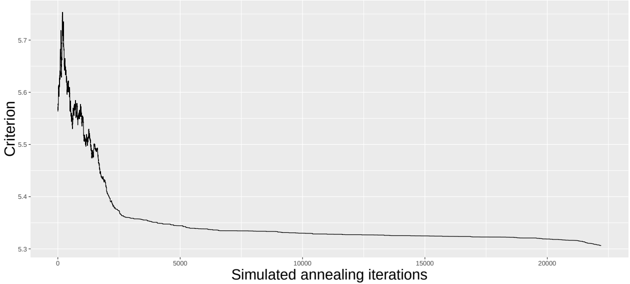
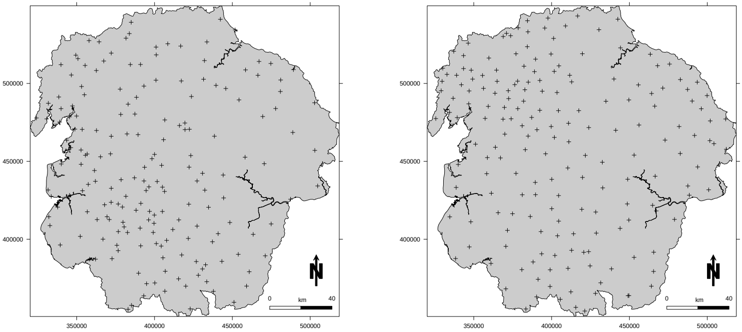


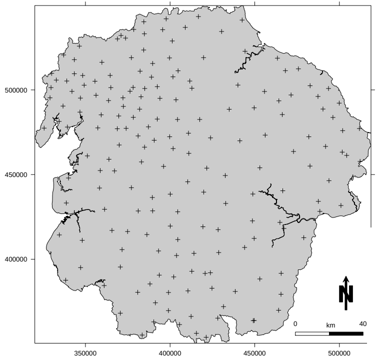
Sampling design optimisation

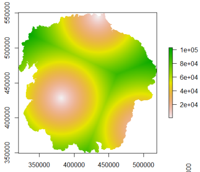


Initial

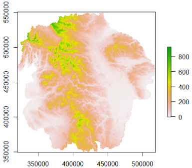
Optimized

Sampling design optimisation





Distance from radar



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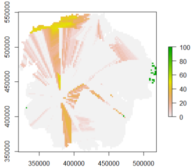

beam blockage

Final remarks

Decrease of the rainfall prediction error variance is obtained by the optimized rain-gauge network

1. It pays off to place rain-gauges at locations where the radar imagery is inaccurate
2. Uniform distribution of rain-gauge over the study area is also important

Interesting for:

- Fast radar-gauge merging accounting for the radar uncertainty (and soon the rain-gauge uncertainty too)
- The optimisation method could be applied to specific targets (flood forecasting)

Thank you for your attention

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