Multivariate autoregressive modelling and conditional simulation of precipitation time series for urban water models

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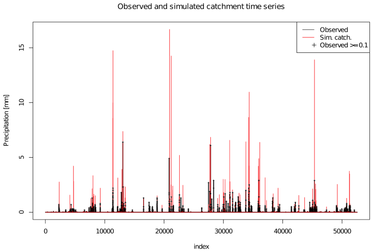

EWRA, Athens, Greece, 6th July 2017

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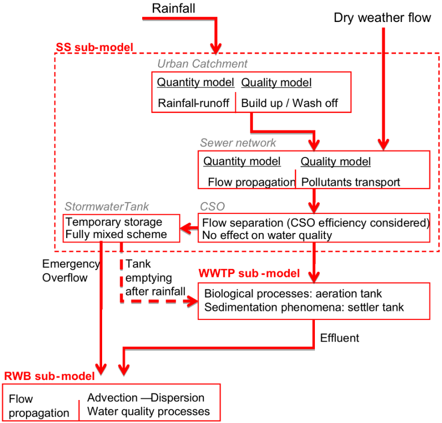
Challenges in Urban Drainage Modelling, UDM

- Sub-models at different spatial and temporal scales
- Requires up- and down-scaling to connect sub-models correctly
- Uncertainties associated to model inputs, model parameters, and model structure
- Uncertainties propagate across scales to effect the final model outputs

QUICS linkage on integrated UDM

Different sub-models, processes and interconnections

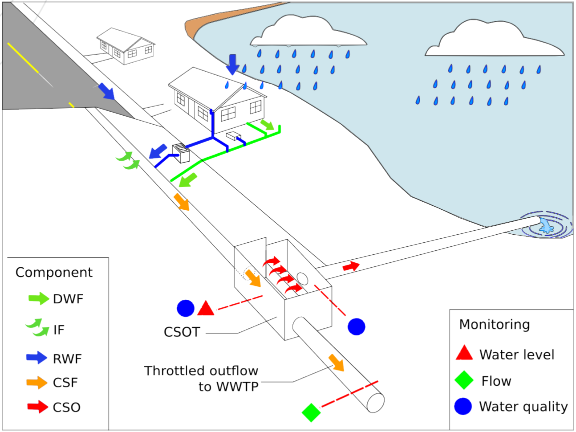


[Freni and Mannina, 2010].

Current research

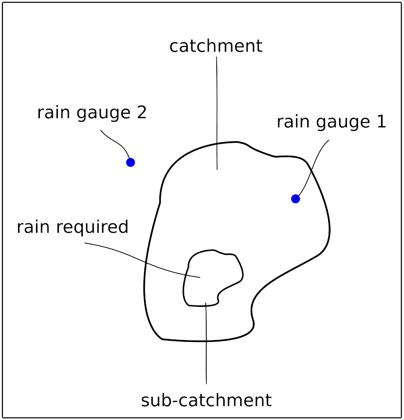
- Specific research questions:
  - How can we identify and characterise the main sources of uncertainty within an Urban Drainage Model (EmiStatR)?
  - How do we propagate input uncertainties through water quality Urban Drainage Models?
  - Catchment average precipitation is a major driving force and key component in UDM. How can we deal with the average catchment precipitation which is not always accurately known when measured at rain gauge?
  - How to translate uncertainty analysis to environmental quality assessment and decision making?

The EmiStatR model: fast and parallelised computing



1) Dry Weather Flow (DWF) including Infiltration Flow (IF); 2) Pollution of DWF; 3) Rain Weather Flow (RWF); 4) Pollution of RWF; 5) Combined Sewer Flow (CSF) and pollution; and 6) Combined Sewer Overflow (CSO) and pollution.

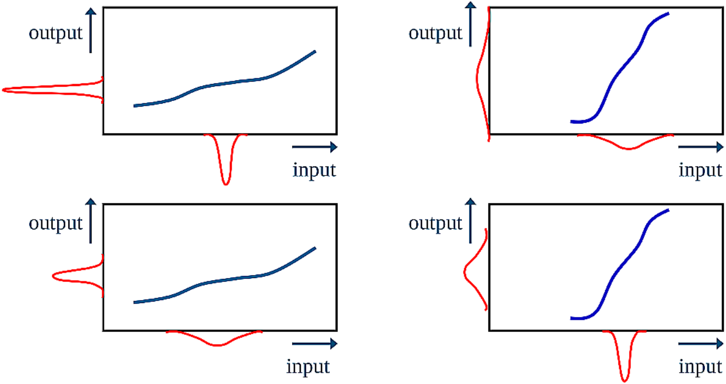
Case study set-up



Schematic definition of the case study set-up. Rain gauge 1 = $RM(t)$; rain gauge 2 = $RM2(t)$; rain required = $R(t)$

Identification of sensitivity WQ model inputs

Model sensitivity and magnitude of input uncertainty



(With kind permission of Gerard Heuvelink)

Screening of sensitive model inputs

Change of model output to model input, variability in $\pm 10\%$

- Quantity: $VTank$, V_{Ov}, Q_{Ov}
  - $Impervious\ area$
  - $Precipitation$
  - $Pass\ forward\ flow$
  - $Volume\ CSO\ tank$

- Quality: $Load_{COD,Ov}$, $C_{COD,Ov}$
  - $Impervious\ area$
  - $Precipitation$
  - COD_{runoff}
  - $Pass\ forward\ flow$
  - $Volume\ CSO\ tank$
  - COD_{DWF}

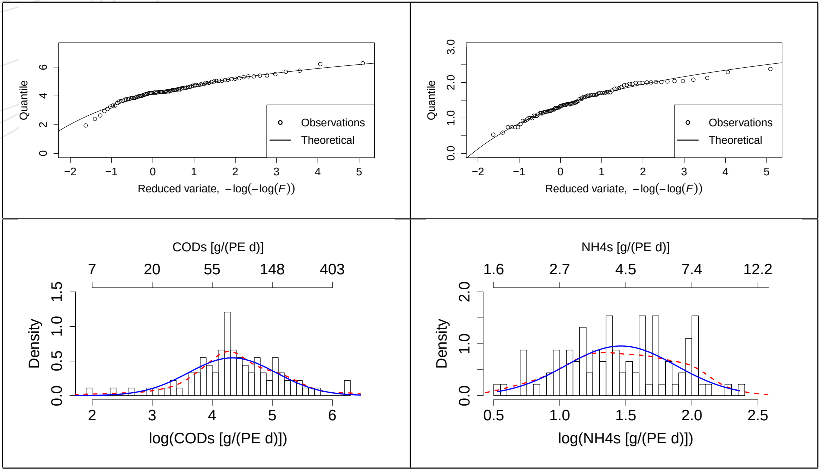
- Quality: $Load_{NH4,Ov}$, $C_{NH4,Ov}$
  - $Impervious\ area$
  - $Precipitation$
  - $Pass\ forward\ flow$
  - $Volume\ CSO\ tank$
  - $NH4_{runoff}$
  - $NH4_{DWF}$

Selection of model inputs for U. propagation

Input variable	Input uncertainty	Model sensitivity	Uncertainty analysis
Wastewater			
1. *water consumption*	+	++	no
2. COD_{DWF}	++	++	**yes**
3. $NH4_{DWF}$	++	++	**yes**
Infiltration water			
4. qf	++	+	no
5. $CODf$	−−	−−	no
6. $NH4f$	−−	−−	no
Rainwater			
7. *Precipitation*	++	++	**yes**
8. COD_{runoff}	++	++	**yes**
9. $NH4_{runoff}$	+	++	no
Storm water runoff			
10. tf	−−	−−	no
Sub-catchment			
11. *Land use*	+	−−	no
12. *Total Area*	+	−−	no
13. *Impervious Area*	+	++	no
14. *Population equivalents*	+	++	no
15. tc	−	−−	no
CSO structure			
16. *Pass forward flow*	−	++	no
17. *Volume CSO Tank*	−	++	no

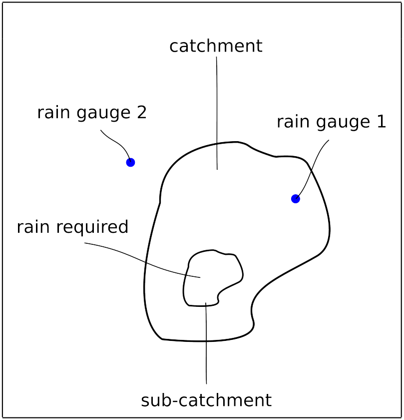
COD_{DWF} and $NH4_{DWF}$

Observed data [g/(PE*d)]



Quantile plot (top). Histogram, empirical density and theoretical normal density (bottom).

Case study set-up



Schematic definition of the case study set-up. Rain gauge 1 = $RM(t)$; rain gauge 2 = $RM2(t)$; rain required = $R(t)$

Precipitation time series as a multiplicative factor

$$R(t) = RM(t) \cdot \delta(t) \quad (1)$$

- $R(t)$, $RM(t)$ and $\delta(t)$ are log-normally distributed stochastic processes

$$\log[R(t)] = \log[RM(t)] + \log[\delta(t)] \quad (2)$$

equivalent to:

$$LR(t) = LRM(t) + L\delta(t) \quad (3)$$

Multivariate autogressive time series

Given Equation 3, it is only needed to model $LRM(t)$ and $L\delta(t)$ as [Luetkepohl, 2005]:

$$\begin{bmatrix} LRM(t+1) \\ L\delta(t+1) \end{bmatrix} = \begin{bmatrix} \mu_R \\ \mu_\delta \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \left(\begin{bmatrix} LRM(t) \\ L\delta(t) \end{bmatrix} - \begin{bmatrix} \mu_R \\ \mu_\delta \end{bmatrix} \right)$$

$$+ \begin{bmatrix} \varepsilon_R(t+1) \\ \varepsilon_\delta(t+1) \end{bmatrix} \quad (4)$$

Conditional time series simulation

We define:

$$X_1(t) = LRM(t) - \mu_R; \quad \varepsilon_1(t) = \varepsilon_R(t) \tag{5}$$

$$X_2(t) = L\delta(t) - \mu_\delta; \quad \varepsilon_2(t) = \varepsilon_\delta(t) \tag{6}$$

and therefore we have:

$$\begin{bmatrix} X_1(t+1) \\ X_2(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(t+1) \\ \varepsilon_2(t+1) \end{bmatrix} \tag{7}$$

Conditional time series simulation (II)

$$Y = \begin{bmatrix} X_1(t) \\ X_1(t+1) \\ X_2(t) \\ ----- \\ X_2(t+1) \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad (8)$$

with

$$Y_1 = \begin{bmatrix} X_1(t) \\ X_1(t+1) \\ X_2(t) \end{bmatrix}; \quad \text{and} \quad Y_2 = \left[X_2(t+1)\right] \quad (9)$$

Conditional time series simulation (III)

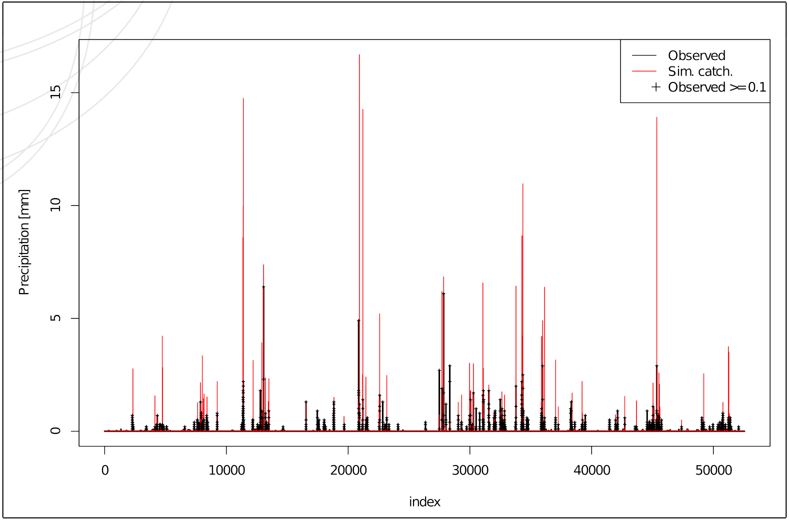
Y is a multivariate normal with mean vector μ and variance-covariance matrix $\sum$ [Box et al., 2008]:

$$Y = \begin{bmatrix} Y_1 \\ \\ Y_2 \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_1 \\ \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right) \tag{10}$$

So we can simulate from $Y_2 = X_2(t+1)$ by sampling from this conditional normal distribution:

$$\{Y_2|Y_1 = a\} \sim N \left(\mu_2 + \Sigma_{21} \cdot \Sigma_{11}^{-1} \cdot (a - \mu_1), \qquad \Sigma_{22} - \Sigma_{21} \cdot \Sigma_{11}^{-1} \cdot \Sigma_{12} \right) \tag{11}$$

Observations and simulation at Goesdorf



Input uncertainty contributions over time

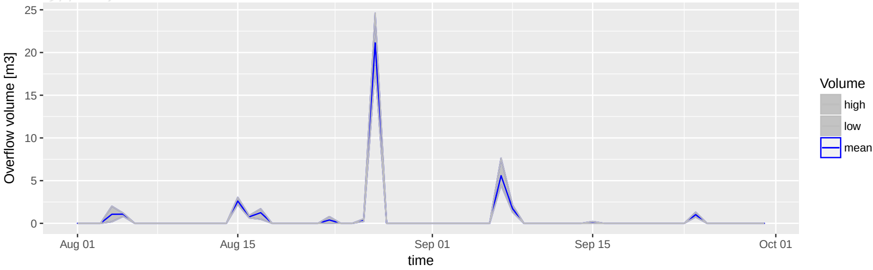


Figure : Temporal uncertainty of input variables to volume of combined sewer overflow (CSO)

Input uncertainty contributions over time

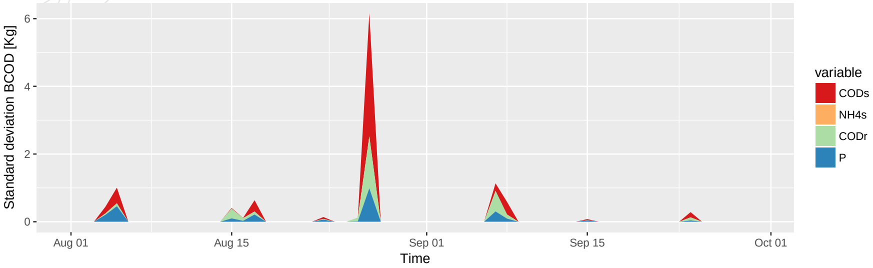


Figure : Temporal contributions of input variables to load of overflow COD in terms of standard deviation

Conclusions

- Catchment average precipitation is a major driving force and key component in UDM. However, average catchment precipitation is not always accurately known when measured at rain gauge.

- We developed a method to estimate the precipitation in a catchment given a known precipitation time series in a location outside of the catchment, while quantifying the uncertainty associated with the estimation.

Conclusions (II)

- A first-order multivariate autoregressive model for conditional simulation of input precipitation based on a multiplicative error model was proposed.
- The approach helps practitioners to better account for uncertainties for:
  - Design and dimensioning of Urban Drainage Systems
  - Pollution control of receiving water bodies

Thank you!

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