Accounting for correlation in uncertainty propagation, a copula approach.

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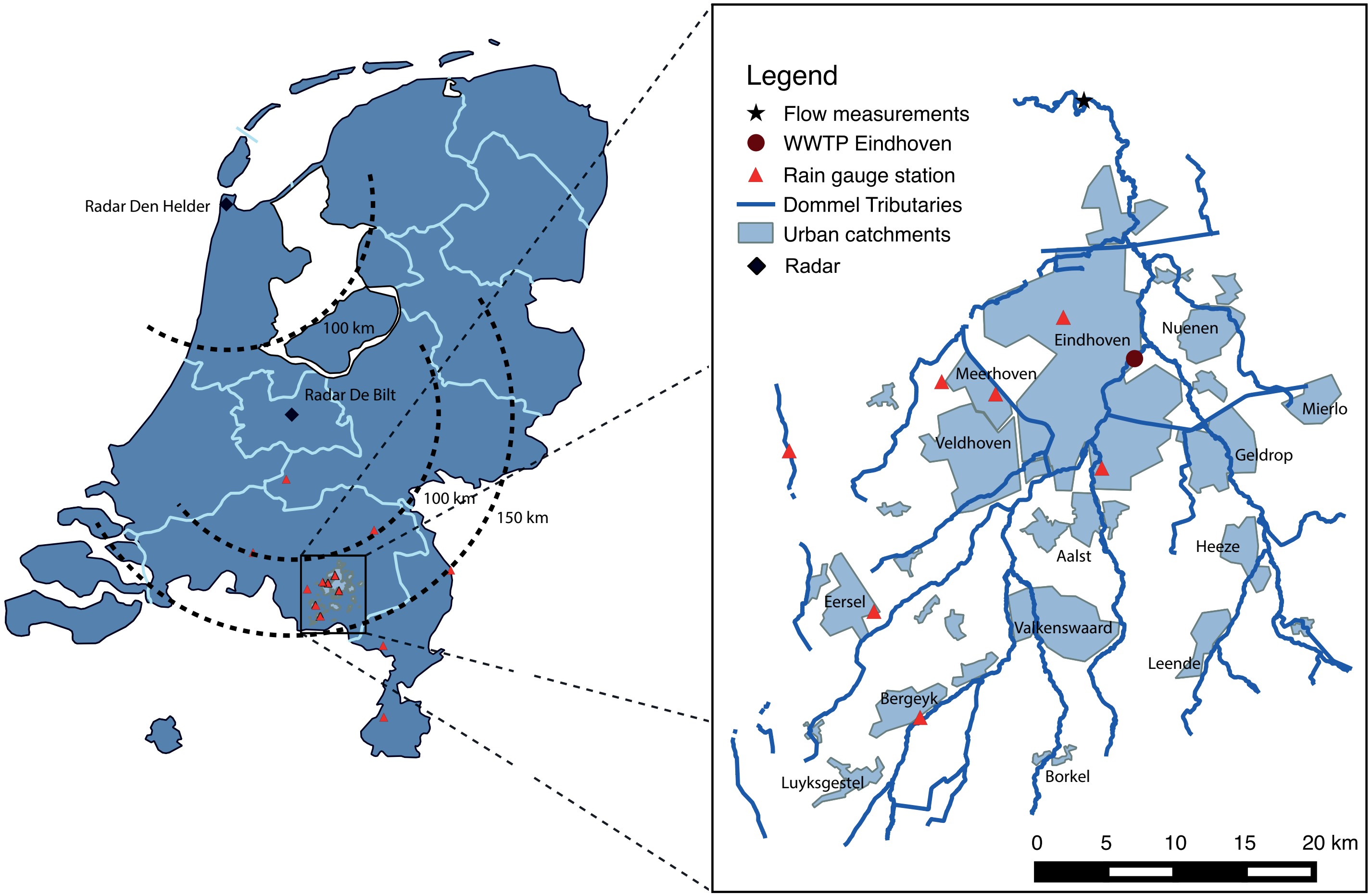
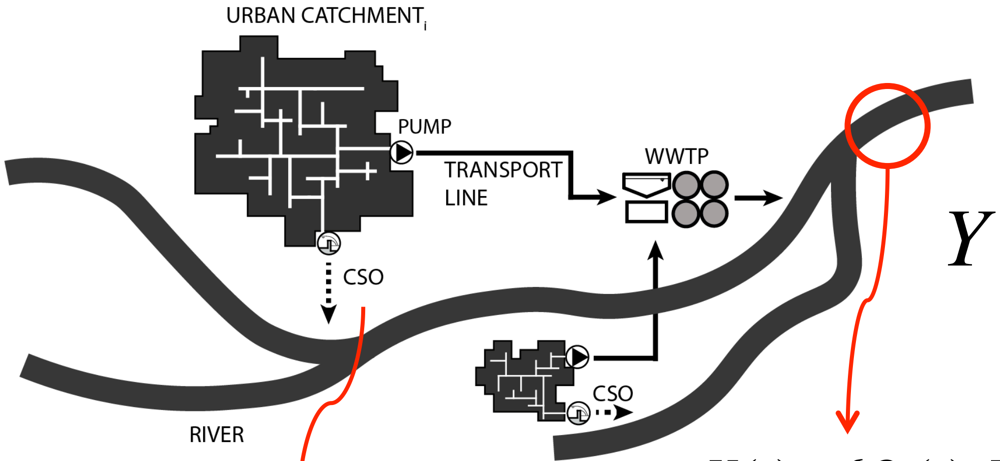
TU Delft QUICS ICUD 2017

1- Dissolved oxygen predictions in integrated urban water systems



1- Dissolved oxygen predictions in integrated urban water systems

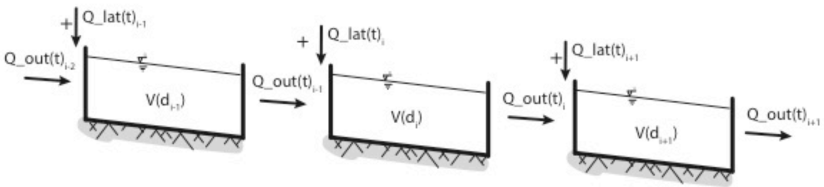


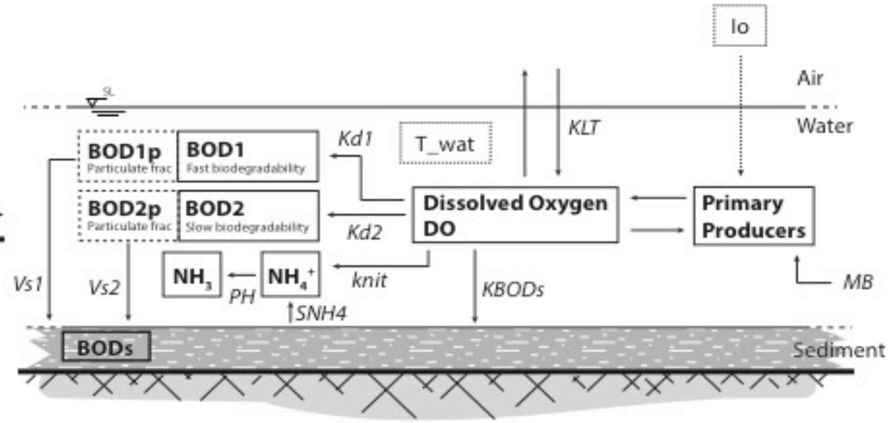
$$\tilde{Y}(t) = M(x_0, I(t), \theta)$$

$$Y(t) = M(x_0, I(t), \theta) + \varepsilon$$

$$Y(t) = \{Q_1(t), DO_1(t)\}$$

$$\bar{\theta} \rightarrow \{k_{BOD}, k_{COD}, k_{NH4} ....\}$$

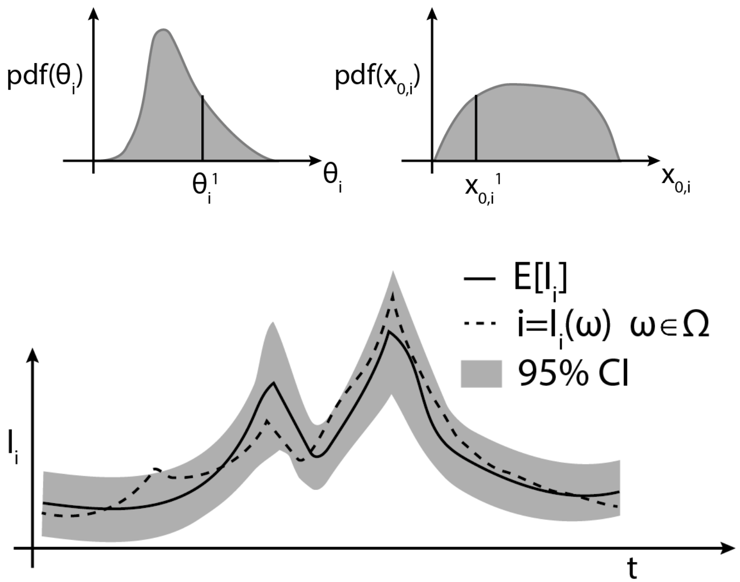




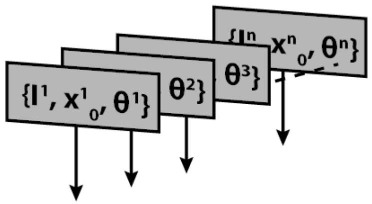
2- Uncertainty propagation

Monte Carlo Forward (pseudo-random Sampling)

1- Describe probability distribution of Inputs/ parameters

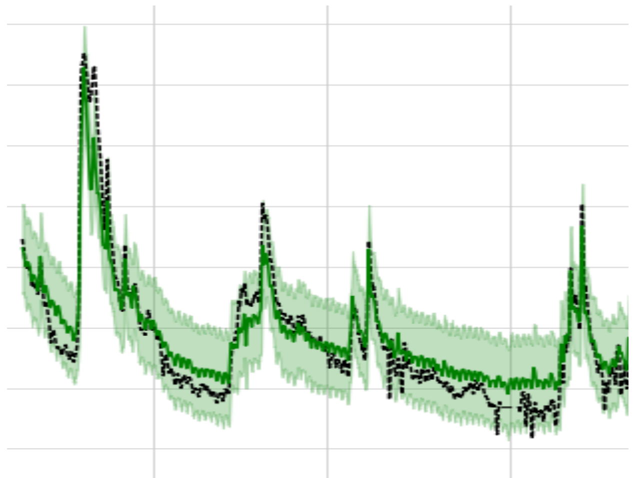


2- Draw samples from uncertain variables and evaluate the model



$$M(x_0, I(t), \theta)$$

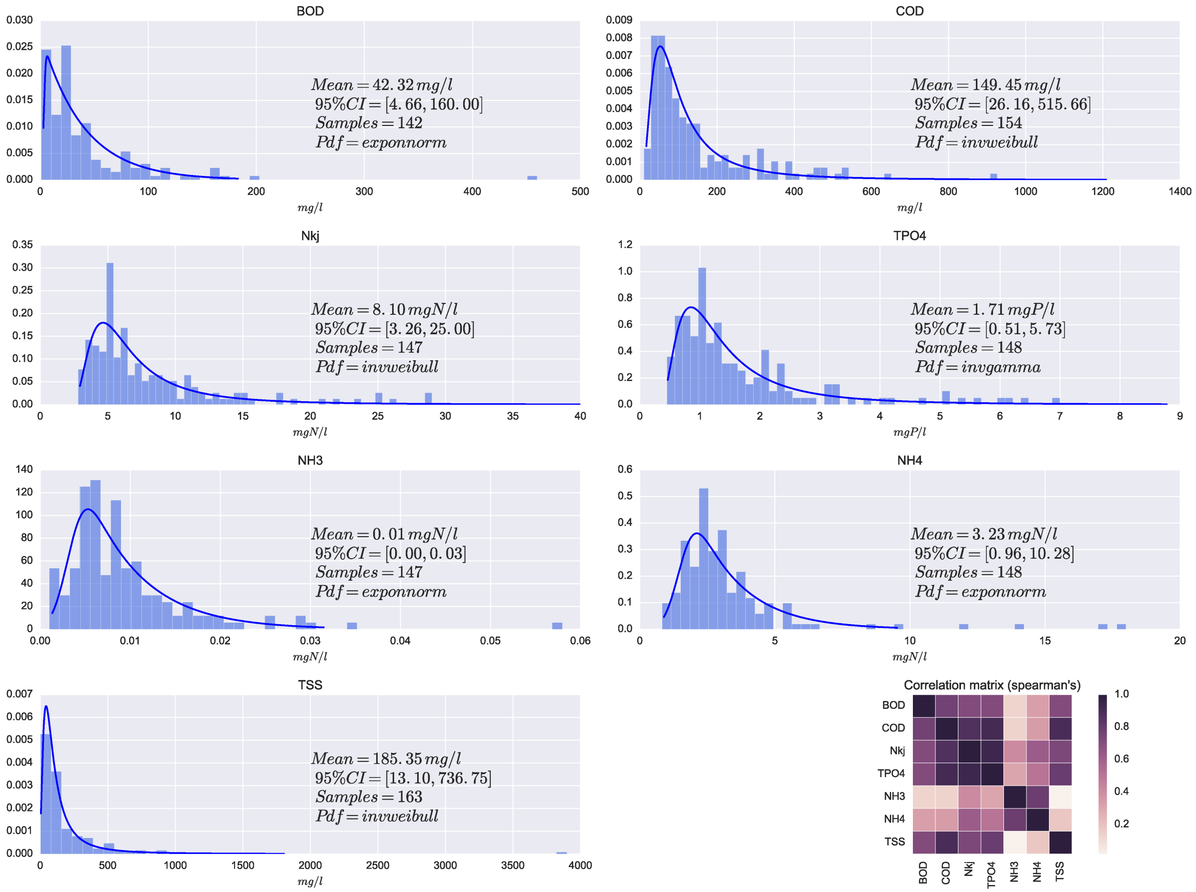
3- Analysis of outputs



2- Uncertainty propagation

Monte Carlo Forward (pseudo-random Sampling)

Step 1- Describe probability distribution pollutant mean concentrations



- Pollutant natural variability
- Non-Gaussian marginals
- Highly correlated variables

4- Step 2 Sampling from the joint parameter distribution

Create samples from a set of variables which have *arbitrarily distributed marginals* and present a certain structural *correlation*.

Building a Copula distribution

Marginal Distribution Information

Uniform joint distribution with correlation information

$$X = [X_1, X_2, \ldots, X_n]^T$$

$$F_i(X_i) = P[X_i \leq x]$$

$$(U_1, U_2, \ldots, U_n) = (F_1(X_1), F_2(X_2), \ldots, F_n(X_n))$$

$$C : [0,1]^n = P[U_i \leq u]$$

4- Step 2 Sampling from the joint parameter distribution

Create samples from a set of variables which have *arbitrarily distributed marginals* and present a certain structural *correlation*.

Building a Copula distribution

Gaussian Copula

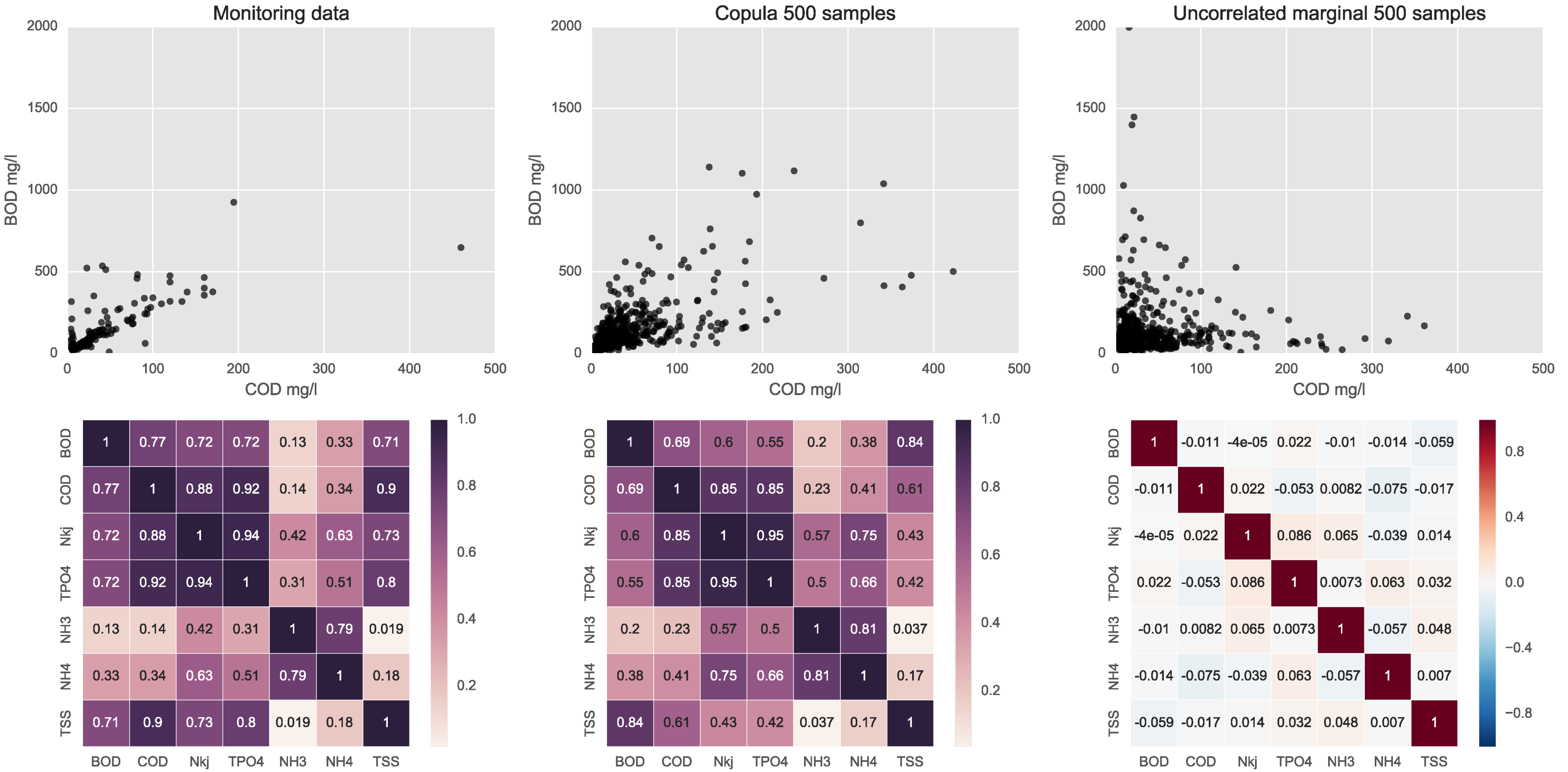
$$X = [X_1, X_2, \ldots, X_n]^T$$

$$F_i(X_i) = P[X_i \leq x]$$

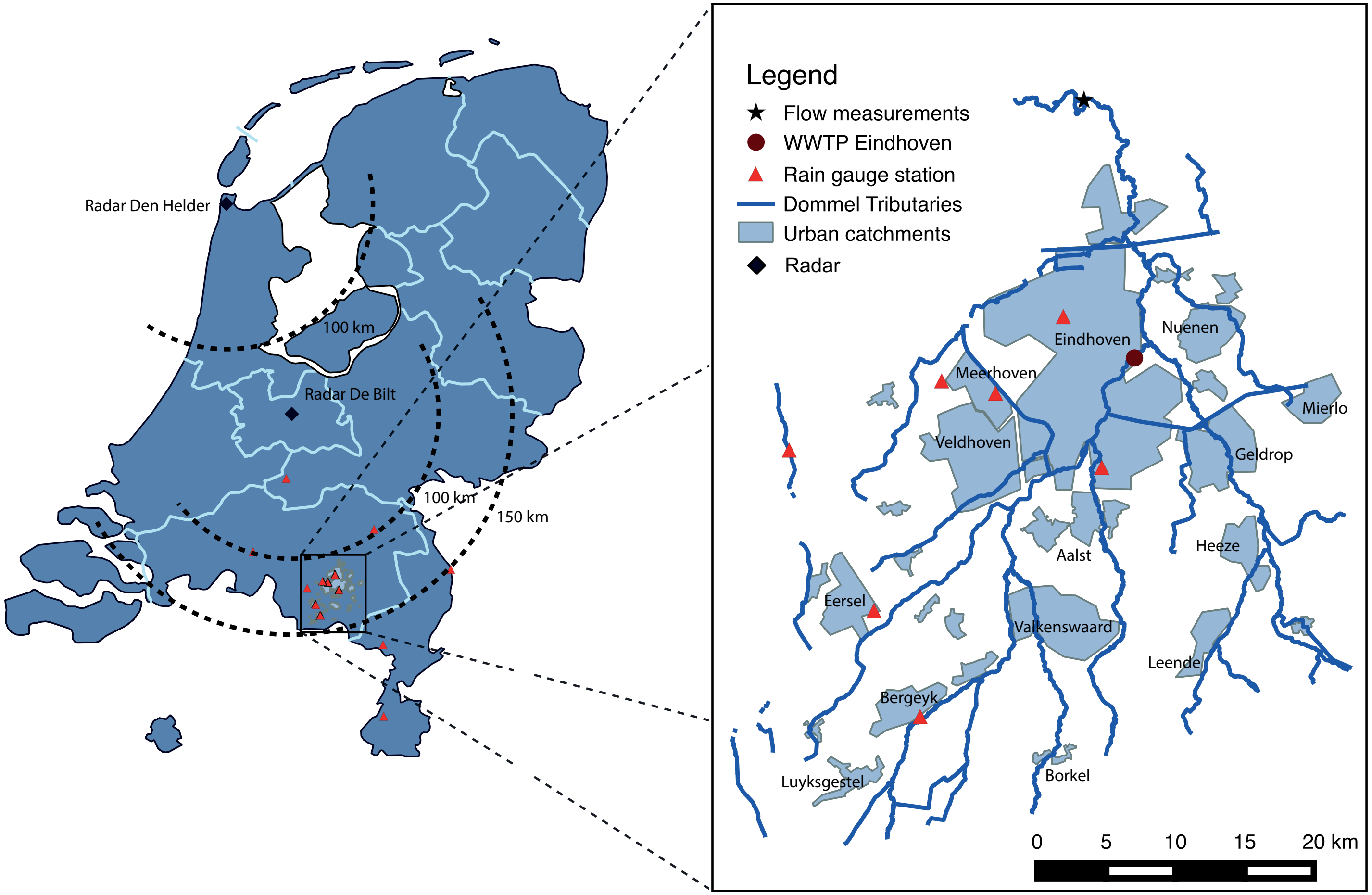
Pseudo-code:

1- Describe marginal CDF	F_i
2- Describe Rank Correlation	R
3- Perform Cholesky Decomp	$R=AA^T$
4- Generate n samples (Std Gaussian)	$Z \sim N_d(0,I)$
5- Correlate them	$X= ZA$
6- Generate Copula samples	$U=(\phi(X_i))$
7- Reshape to Marginals	$Y=(F_i^{-1}(U_i))$

4- Step 2 Sampling from the joint parameter distribution

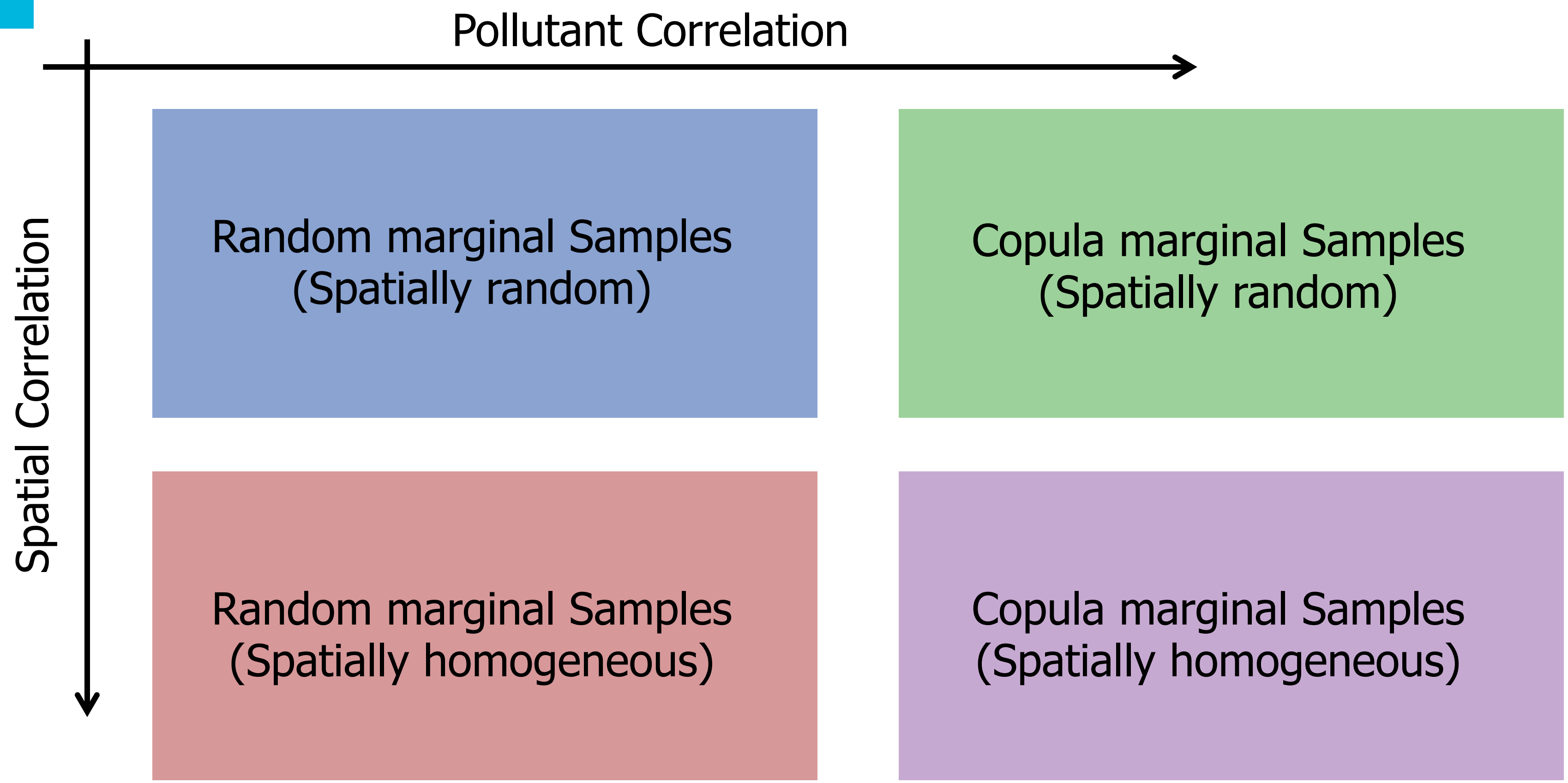


5- Step 3 Propagate samples: Does correlation matter?



5- Step 3 Propagate samples: Does correlation matter?

- Four stochastic pollutant concentration models:



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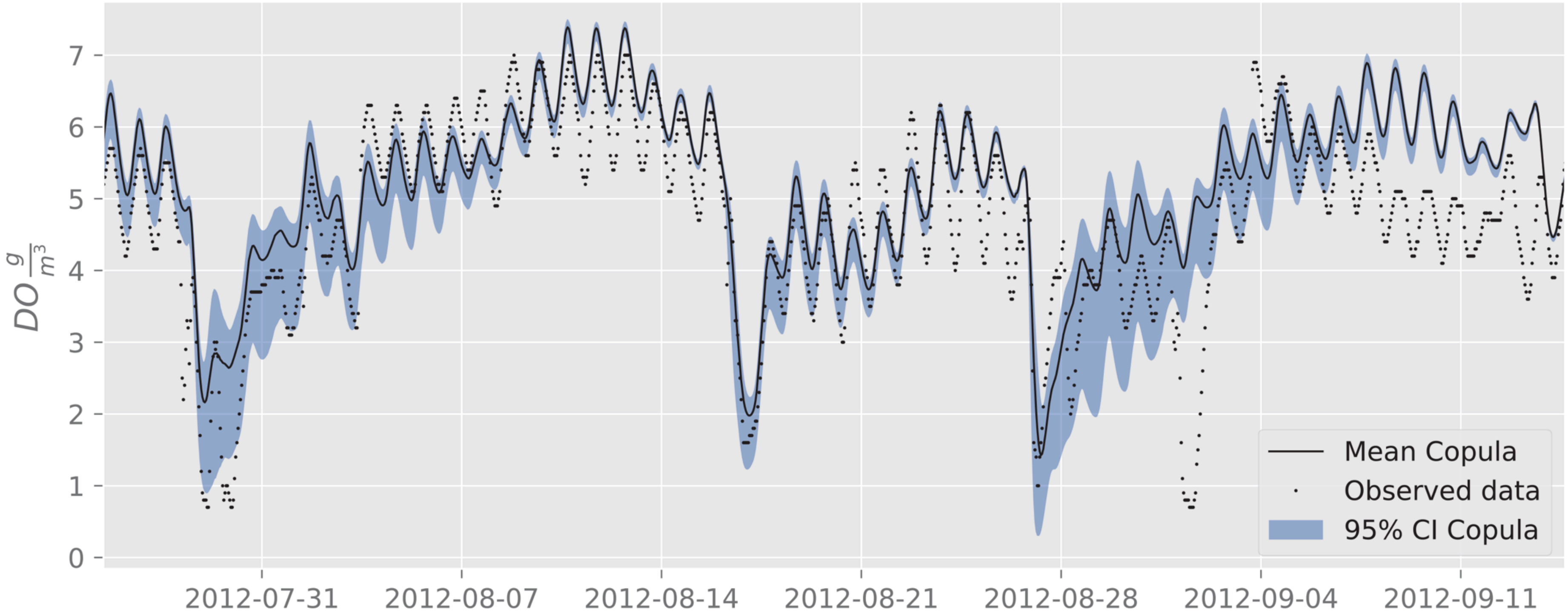
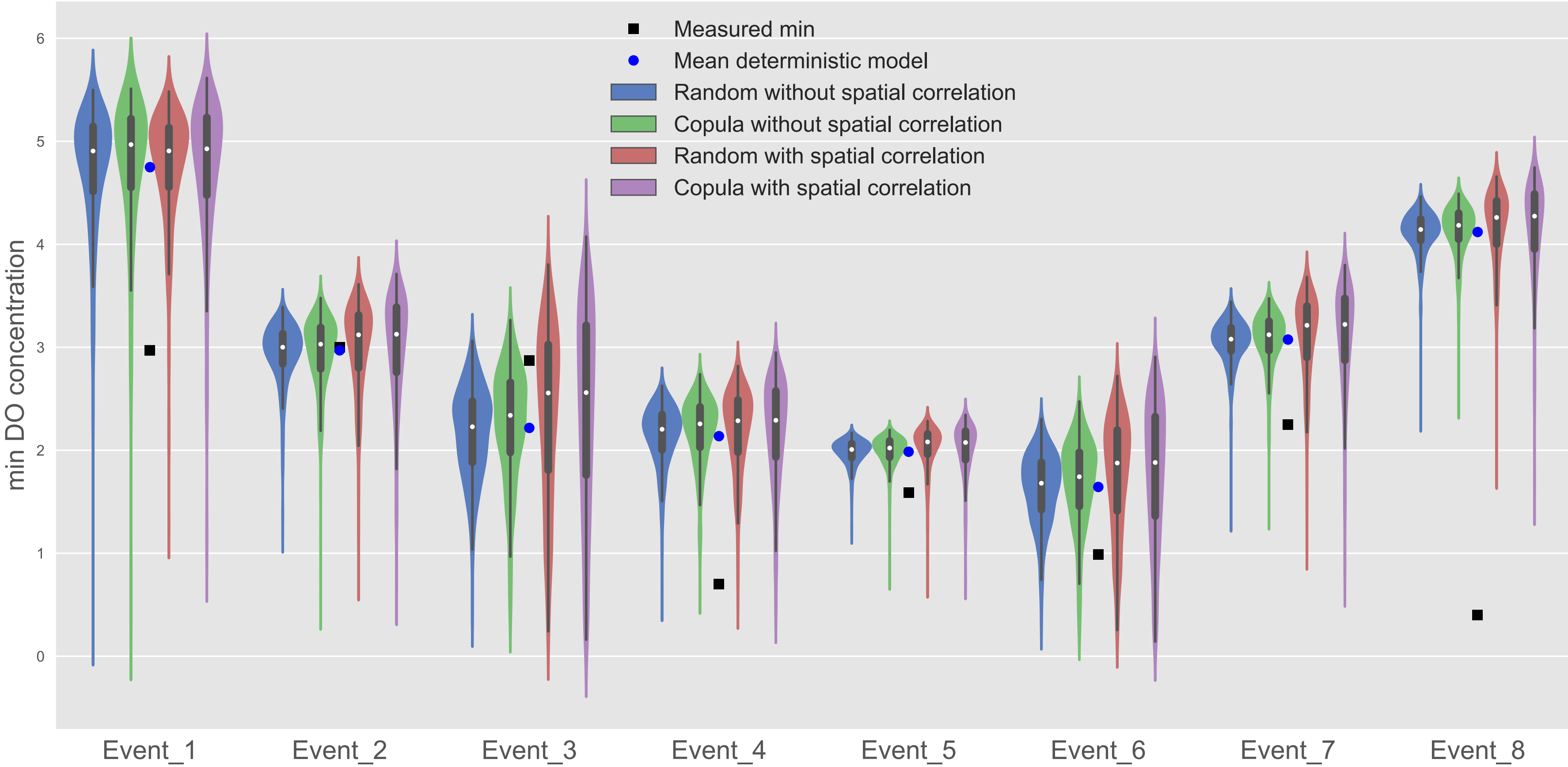


4- Example: Does correlation matter?



5- Summary

- Accounting for correlation at pollutant mean concentration vectors has an effect of the parametric uncertainty of DO dynamics.

- Copula distributions can easily be implemented in sampling schemes for non-Gaussian correlated multivariate spaces

Thanks for your attention

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