Some relations between standard distributions

Notation from distributions handout. Some relations are recorded there too.

Relations to the normal

1. If Z is $\mathrm{N}(0,1)$, then Z^2 is χ_1^2.
2. If $Z \sim \mathrm{N}(0,1)$ is independent of $W \sim \chi_\nu^2$, then

$$T = \frac{Z}{\sqrt{W/\nu}} \sim \mathrm{t}_\nu$$

   i.e. has a t -distribution with ν degrees of freedom.
3. If $Z_1, Z_2, \ldots, Z_n$ are independent $\mathrm{N}(0,1)$ r.v's, then

$$Z_1^2 + Z_2^2 + \ldots + Z_n^2 \sim \chi_n^2$$

4. If $X_1, X_2, \ldots, X_n$ are independent $\mathrm{N}\left(\mu, \sigma^2\right)$ and $\bar{X} = \frac{1}{n}\sum_1^n X_i$ then

$$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

5. If $X_1, X_2, \ldots, X_n$ are independent $\mathrm{N}\left(\mu, \sigma^2\right)$ then

$$\frac{\left(X_1 - \bar{X}\right)^2 + \left(X_2 - \bar{X}\right)^2 + \ldots + \left(X_n - \bar{X}\right)^2}{\sigma^2} \sim \chi_{n-1}^2$$

   and it is independent of $\bar{X}$.
6. If $X_1, X_2, \ldots, X_n$ are independent $\mathrm{N}\left(\mu, \sigma^2\right)$,

$$\bar{X} = \frac{1}{n}\sum_1^n X_i \text{ and } S^2 = \frac{1}{n-1}\sum_1^n \left(X_i - \bar{X}\right)^2$$

   then, combining 2, 4 and 5,

$$T = \frac{\bar{X} - \mu}{\sqrt{S^2/n}} \sim \mathrm{t}_{n-1}$$

7. If $W_1 \sim \chi_{\nu_1}^2$ and $W_2 \sim \chi_{\nu_2}^2$ with W_1, W_2 independent, then

$$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim \mathrm{F}_{\nu_1,\nu_2}$$

   i.e. has F -distribution with ν_1, ν_2 degrees of freedom.
8. From 1, 2 and 7 above, if $T \sim \mathrm{t}_\nu$, then $T^2 \sim \mathrm{F}_{1,\nu}$.

Other relations

1. If $X \sim \mathrm{Ga}(a,b)$ then $\lambda X \sim \mathrm{Ga}(a, b/\lambda)$. In particular (with $a = 1$) if $X \sim \mathrm{Ex}(b)$ then $\lambda X \sim \mathrm{Ex}(b/\lambda)$.
2. If $X_i \sim \mathrm{Ga}(a_i, b)$ and are independent, then

$$\sum X_i \sim \mathrm{Ga}\left(\sum a_i, b\right).$$

In particular $\left(a_i = \nu(i)/2, b = 1/2\right)$, $X_1 \sim \chi^2_{\nu(1)}$, if $X_2 \sim \chi^2_{\nu(2)}, \ldots, X_n \sim \chi^2_{\nu(n)}$ and are independent, then

$$X_1 + X_2 + \ldots + X_n \sim \chi^2_{\nu(1)+\nu(2)+\ldots+\nu(n)}.$$

3. If $Y_i \sim \mathrm{Po}(\mu_i)$, independent then

$$\sum Y_i \sim \mathrm{Po}\left(\sum \mu_i\right)$$

4. If $Y \sim \mathrm{Po}(\mu)$ and μ is large than $Y \sim \mathrm{N}(\mu, \mu)$ approximately.