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Kostas Mouratidis, Georgios Papapanagiotou, and Christoph Thoenissen

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Kostas Mouratidis
University of Sheffield

Georgios Papapanagiotou
University of Macedonia

Christoph Thoenissen
University of Sheffield

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Abstract

The level of productivity is an important macroeconomic indicator that informs monetary policy decisions. Since the 2008 financial crisis, UK productivity has plunged and remained below its pre-crisis level for more than a decade. Evidence that the European debt crisis was generated by a fall of productivity and subsequent current account deficit of South euro area countries raises challenging questions about the impact of productivity on the UK's business cycle. We measure this impact by accounting for both permanent and transitory shocks to total factor productivity (TFP) and investment-specific technology (IST). Our results suggest that permanent positive TFP and IST shocks worsen the trade balance by increasing domestic absorption and appreciating the real exchange rate. However, the real exchange rate itself has no effect on net trade. In contrast, cyclical productivity shocks do not impact trade or the real exchange rate. Finally, our findings show that the financial crisis had a long-run negative impact on both output and trade.

Keywords: Productivity, Trade Balance, International Business Cycle, Cointegration and GVAR

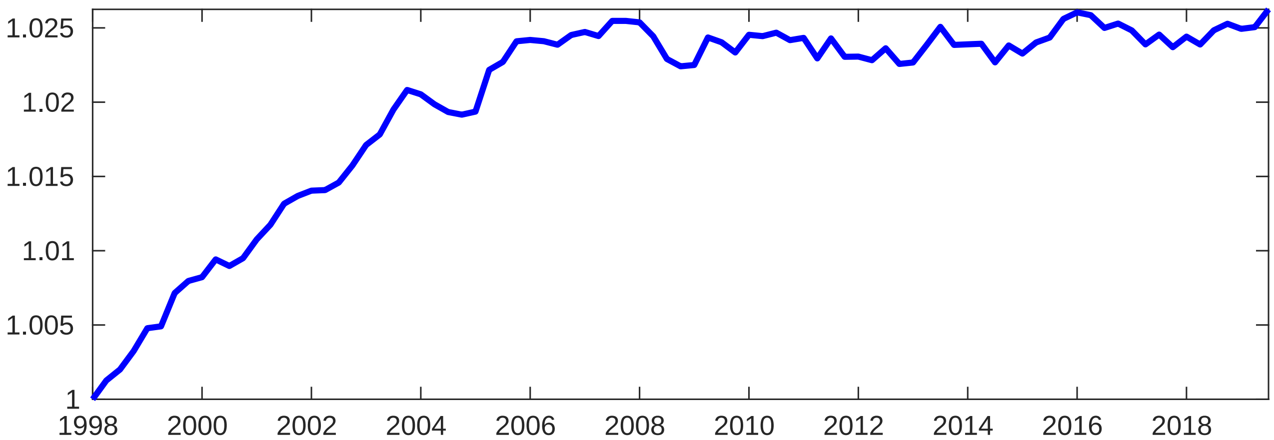
JEL Classification: C11, C23, C32, F14, F21, F32 and F44

*Mouratidis: Department of Economics, University of Sheffield, Email: k.mouratidis@sheffield.ac.uk; Papapanagiotou: Department of Economics, University of Macedonia, Greece, Email: papapgeorge@uom.edu.gr; Thoenissen: Department of Economics, University of Sheffield, Email: c.thoenissen@sheffield.ac.uk. This work is part of a project that received funding from Leverhulme Trust: RF-2021-429. This work is part of a project that received funding from Levehulme Trust: RF-2021-429. We thank seminar participants at Sheffield and the AMEF 2024 in Thessaloniki for useful comments. All remaining errors are our own.

1 Introduction

The level of productivity is an important macroeconomic indicator, since it matters both for welfare and for economic policy. Productivity is, for example, a key determinant of how much demand can grow without generating inflation.[1] Since the 2008 financial crisis, the growth rate of productivity in the United Kingdom, measured as the ratio of gross value added (GVA) to hours worked, has fallen sharply and has remained significantly below its pre-crisis trend for more than ten years.[2] This seemingly permanent decline in growth, often referred to as the "Productivity Puzzle" (PP), has stimulated a range of research.[3] Evidence from the European debt crisis—a period marked by a productivity slowdown and subsequent trade deficits in Southern euro area countries prior to the crisis—raises a key question: how will the UK's "productivity puzzle" impact economic variables like consumption, investment, and the trade balance?[4]

Figure 1: Gross value added per hour of work for the UK.



1.1 Contribution

Our contribution to the understanding of how permanent and transitory productivity shocks impact the business cycle is twofold. First, we propose an empirical framework that has never been applied before to identify the permanent component of these shocks. More formally, in the first stage of our econometric framework, we employ cointegration analysis to identify permanent shocks as those affecting strongly exogenous variables.[5] For example, if the TFP and IST variables, both domestic and

[1]Welfare gains of productivity are not only financial. For example, Tenreyro (2018) shows that countries with 20 percent higher productivity enjoy higher life expectancy and lower child mortality.

[2]In particular, labour productivity, a decade before the financial crisis of 2008, grew on average by 2.3 percent per year.

[3]Barnett et al. (2014) and Weale (2014) show that productivity downturn might be driven by cyclical factors related to demand conditions and more persistent factors related to factors affecting persistently the capacity of the economy to supply goods and services: capital deepening and efficient allocation of resources.

[4]Giavazzi and Spaventa (2011) show that the European countries subject to debt crisis suffer a productivity loss prior to crisis. McAdam et al. (2022) also stress the role of competitiveness as a mechanism to ameliorate trade imbalance between North and South euro area countries.

[5]Note that the strongly exogenous variables do not receive any feedback from any shock in the system. For more detailed analysis see Juselius (2006).

foreign, are found to be strongly exogenous then we can estimate the impact of permanent productivity shocks on the rest of the variables included in our model. Although cointegration analysis has been used widely to estimate the long-run properties of macro and financial variables, to the best of our knowledge this is the first paper that identifies, using strong exogeneity test, permanent productivity shocks.[6] Identification of strongly exogenous variables allows us to simulate the impact of permanent shocks using impulse response function (IRF) analysis. We can then estimate the long-run effects of TFP and IST shocks on the trade balance and GDP using their coefficients on the cointegrating vectors. These vectors are uniquely identified and normalized with respect to net trade and real GDP per capita, reflecting the restrictions implied by a standard international business cycle model.

Our second contribution is related to the second stage of our econometric framework where we focus not only on the cyclical effects of productivity shocks but also on the spillover effects from the euro-area to the UK. We do so by employing a Bayesian structural global VAR (GVAR). The GVAR developed by Pesaran et al. (2004) considers the multi-country dimension of the global economy and allows us to capture spillover effects that might exist across countries. Unlike bilateral (two-country) models used in the literature to capture international spillover effects, the GVAR model accounts for higher-order spillover effects. For example, while the two-country model used by Görtz et al. (2022) captures potential productivity spillover effects from the US to the UK, it misses indirect spillover effects from the EU. Georgiadis (2017) demonstrates that spillover estimates from a bilateral model are inconsistent, prone to greater bias and mean square error (MSE) than estimates generated by a multilateral model.[7] A further advantage of Bayesian GVAR is that it accounts both for parameter and model uncertainty using the Stochastic Search Variable Selection (SSVS) approach introduced by George et al. (2008). We also account for shock uncertainty by allowing the errors of a country-specific VAR to be time-varying: the estimated errors follow a stochastic volatility (SV). Notably, Tenreyro (2018) identifies uncertainty as the principal factor explaining the low level of business investment and productivity during the post-crisis and post-Brexit periods. In this paper, we also contribute to the theoretical literature by developing a theoretical two-country international business-cycle model akin to that in Görtz et al. (2022), which is used to disentangle structural shocks by imposing sign restrictions on the impulse response function generated by the GVAR model. It is rather striking that the IRFs from the calibrated theoretical model are very close to the IRFs generated by the structural GVAR model. This implies that our model is cable to recover the underlying true data generated process (DGP).

The role of productivity on UK economic activity has gained prominence in the post-Brexit era. A fall in capital deepening is one of the key reasons for the UK's productivity downturn, whereas faster capital accumulation explains the recovery of the euro area productivity performance.[8] Given the productivity gains of capital deepening, why did business investment in the UK remain subdued

[6]For example, Benati (2014) test for cointegration between IST and TFP, but the aim of his study was not to identify permanent shocks. Alternatively, Ireland (2013) investigates differences between the euro area and the US by calibrating a two-country stochastic growth model in which both IST and TFP shocks were non-stationary but cointegrated across countries. Görtz et al. (2022) used a novel state-space model to identify the permanent and transitory component of TFP and IST shocks but identification was based on the assumption that TFP and IST shocks were not cointegrated.

[7]Chudik and Pesaran (2016) also consider the estimation of VAR models where the number of economies and the sample size converge to infinite. Their contention is that a multilateral model framework is the most suitable for conducting spillover analysis when the set of neighbouring economies comprises of more than one economy.

[8]Tenreyro (2018) demonstrates that one third of productivity downturn was due to the fall of capital deepening: the ratio of capital services to hours of worked.

in the aftermath of 2008 financial crisis? Tenreyro (2018) argues that one of the reasons why the UK might be lagging behind is the added uncertainty over the future trading relationship with the European Union. In the short term, this is likely to continue to weigh on business investment and capital accumulation.

Unlike the existing literature, the aim of this paper is not to explain the UK's productivity puzzle, but to investigate the impact of the UK's productivity slowdown on the UK business cycle.[9] To this end, we pose the following research question: How relevant are domestic and international productivity shocks in driving the domestic business cycle?

A major factor in the slow recovery of UK productivity following the financial crisis was the fall in business investment (see Tenreyro, 2018). Greenwood et al. (1997) and Fisher (2006) show that a positive trend of investment is driven by continuous improvements in investment-specific technology (IST).[10] The relative importance of TFP and IST shocks has been the subject of extensive research which, however, mainly focuses on the US.[11] The literature concerning the impact of productivity on the UK's business cycle is far less voluminous. We aim to fill this gap by using a two-stage econometric framework which allows us to account both for the nature of productivity shocks -permanent vs transitory- and for international spillover effects.[12]

The empirical analysis is conducted using quarterly data from 1998Q1 to 2019Q4. We classify our key findings concerning the impact of productivity on the variables included in our model into three groups: i) the steady-state effects of productivity on trade and output growth – the estimated coefficients of productivity on the cointegration space; ii) the impact of permanent productivity shocks on the variables included in our analysis; and iii) the impact of cyclical productivity shocks on the UK economy. In what follows, and unless stated otherwise, the term domestic refers to the United Kingdom.

In steady-state (cointegration space) the estimated coefficients are consistent with our priors. More formally, domestic TFP has positive effects both on output and net trade while consumption affects the latter negatively. We also observe that the real exchange rate, defined such that an increase denotes a depreciation, has a positive impact on trade. Our results imply that, in the long-run, TFP can lead to an improvement of the trade balance via a depreciation of real exchange rate. There is also evidence that the financial crisis had a strong negative long-run effect on both output and trade. This is consistent with the UK's PP literature, which shows that productivity fell sharply following the financial crisis and has since then remained below its pre-crisis level.

When we asses how productivity trend shocks influence the domestic business cycle, we draw three key conclusions. First, a positive domestic permanent TFP shock increases output, consumption and investment and causes net trade to deteriorate while the behaviour of real exchange rate depends on the specification of foreign variables. When the weighted average of the US and EU variables are

[9]The key findings concerning the research on the UK productivity puzzle is that while there was labour misallocation during the credit boom but not after the financial crisis. For example, Tenreyro (2018) and Goodridge et al. (2018) show that although labour reallocation contributes to the productivity slowdown prior to financial crisis the reverse was true for the post-crisis period. Furthermore, Broadbent (2013) and Barnett et al. (2014) show that capital investment did not flow to sectors with higher returns to capital.

[10]IST is proxied by the inverse of relative price of investment, which is measured as the ratio of gross fixed capital deflator to consumption deflator.

[11]the exception to this is Görtz et al. (2022) who mainly focus on the UK.

[12]International shocks are identified as shocks originating from the euro area and the US, which together account for more than 80 percent of the UK's trade.

used as foreign variables then there is a mild appreciation of real exchange rate. Alternatively, when EU variables are used as foreign variables then there is a persistent depreciation of the real exchange rate. While the deterioration of trade balance, in the former case, is driven by an increase of domestic absorption in the latter case uncertainty about the future trade relationship between the UK and the EU might have enhanced the negative effects of an increase of domestic demand on trade. This is so because the potential positive effects of a depreciation of the real exchange rate on net trade have been outweighed by the negative effects generated by an increase of domestic absorption.

Second, a domestic IST shock increases both consumption and investment while the response of the real exchange rate is either weak or an appreciation in the model where only the EU variables were used as proxy of foreign variables. Therefore, the negative response of net trade reflects both the increase of domestic demand following the domestic IST shock and the appreciation of real exchange rate. Output measured as the sum of consumption and investment increases but not enough to counterbalance the negative effect of aggregate demand and appreciation of real exchange rate on net trade.

Third, when we focus on the international spillover effects, we observe that a US TFP shock has a positive effect on domestic output and absorption - consumption and investment - which in turn affect net trade negatively. The negative response of net trade to a US TPF shock is due to an increase of domestic demand which is not offset by an unresponsive real exchange rate.

The variance decomposition reveals that the permanent (trend) domestic TFP and IST shocks explain 27 percent and 19 percent respectively of the UK GDP business cycle fluctuation while the US TFP shock explain another 22 percent. The remaining 32 percent is explained by foreign IST and TFP shocks that have long-run impact on domestic output, consumption and investment.

Finally, when we turn to cyclical productivity shocks, our empirical results are consistent with those of a two-country international real business cycle model. More formally, the impulse response functions of the theoretical model and those of the GVAR model are qualitatively consistent: they provide the same policy implications. The main take away of this exercises is that although both domestic and foreign productivity - TFP and IST - shocks affect most of the domestic variables, net trade is unresponsive. There is evidence that a positive domestic TFP shock leads to an increase of domestic output, consumption and investment. The relative price of investment increases due to higher demand for investment. Although, there is a depreciation of the real exchange rate, net exports remain unresponsive. When we look at the responses following a domestic IST shock, we observe, as implied by the underlying model, a positive response of real GDP, consumption, and investment while real exchange rate falls (appreciates) and once again net trade is muted. The responses to a foreign TFP shock are symmetric to those of a domestic TFP shock. We observe an appreciation of real exchange rate, which due to positive wealth effects leads to an increase in consumption. Investment also increases due to an appreciation of the real exchange rate, output increases but net trade is neutral.[13] Finally, a foreign IST shock does not have much of an impact on domestic variables. An increase of foreign (EU) IST leads (as expected) to a depreciation of UK's real exchange rate, which in turn will generate negative wealth effects and fall of domestic consumption albeit only on impact. Zero is out of the credible set. The rest of the variables remain insignificant.

[13] Corsetti and Müller (2006) show that in small open economies an appreciation of the real exchange rate can increase the rate of return of investment.

2 Data

We use quarterly data from 1998Q1 to 2019Q3. For output (y_t), consumption (c_t) and investment (i_t) we consider GDP, private final consumption expenditure and the gross fixed capital formation, respectively. The variables are in logarithmic form and in real per capita units. For the investment specific technology variable (ist_t), we consider the consumption deflator over the gross fixed capital deflator. Net trade (ntr_i) is defined as the difference between real exports and imports as percentage of the GDP. All data regarding the aforementioned variables are obtained from the OECD Quarterly National Accounts. The real effective exchange rate data is obtained from the IMF International Finance Statistics.

Our measure of total factor productivity (tfp_t), defined as a utilisation-adjusted Solow residual is defined as follows:

$$dy_t = dtfp_t + \alpha(du_t + dk_t) + (1-\alpha)(de_t + dh_t) \tag{1}$$

where d denotes the difference operator $(1-L)$. Data for the capital utilisation rate (u_t) and the total hours worked per emploee (h_t) are obtained from the OECD Main Economic Indicators. Following Kamber et al. (2017), we set the change in effort (e_t) equal to change in hours worked: $de_t = dh_t$.[14] Since data on capital stock (k_t) are not available on a quarterly basis, we use annual capital data from FRED and interpolate it using the change in quarterly investment (i_t). Finally, we set the capital share $\alpha = 0.3$, as in Kamber et al. (2017). For the US, we use the utilisation adjusted TFP series constructed by Fernald (2014).

In line with our objectives, we use data for the UK, US and European countries. The European countries are treated as one entity (denoted EU) in the model and includes data from the so-called 'North euro area' (Austria, Belgium, Finland, Germany and the Netherlands); and the so-called 'South euro area' (Greece, Italy, Spain and Portugal); as well as France, Ireland and Norway.[15] We construct a VARX* model that includes seven endogenous variables: real per capita output (y_{it}), real per capita consumption (c_{it}), real per capita gross capital formation (gcf_{it}), the real exchange rate (rer_{it}), net trade (ntr_{it}), domestic total factor productivity (tfp_{it}) and the relative price of investment goods (ist_{it}). We also consider two foreign variables related to our main objective: foreign TFP (tfp^*_{it}) and foreign IST (ist^*_{it}). Therefore, the vectors of domestic and foreign variables are given by:

$$\begin{aligned}\boldsymbol{x}_{it} &= (y_{it}, c_{it}, gcf_{it}, rer_{it}, ntr_{it}, tfp_{it}, ist_{it})' \text{ and}\\ \boldsymbol{x}^*_{it} &= (tfp^*_{it}, ist^*_{it})'\end{aligned}$$

We construct the foreign variables using fixed weights based on the trade shares of foreign countries in total trade - exports and imports - over the period. For example, the foreign TFP is:

$$tfp^*_{it} = \sum_{j=1}^{11} w_{ij} tfp_{jt}$$

where the trade weight $w_{ij} \geq 0$ represent the trade share of country j to the total trade share of

[14] Kamber et al. (2017) actually consider $de_t = \zeta dh_t$, experiment with different values of ζ between 0.1 and 10 and conclude that the choice of ζ does not affect the results (given that it is different from 0).

[15] The countries included in our GVAR model account for 80 percent of the UK's trade.

country i such that $\sum_{j=1}^{N} w_{ij} = 1$ and $w_{ii} = 0$. We average the data from the euro area countries and Norway based on a weighted scheme of each individual country. Table 1 reports the trade weights used for the construction of foreign variables.

An issue that can arise in empirical studies is the presence of structural breaks. We employed a sequence of tests to determine the stability of the country specific models.[16] In doing so, we proxy structural breaks that are common across countries or occur in periods that are close to each other. Therefore, we create a shift dummy ($dshift$) to account for the impact of financial crisis on labour productivity; a permanent dummy ($d16Q1$) to proxy the impact of Brexit on trade and investment and; a permanent dummy to consider the impact of a structural break identified by a series of test at the second quarter of 2015: $d_i = (dshift, d16Q1, d15Q2)'$. It is worth noting that although the Brexit dummy was designed to capture the impact referendum on domestic variables the period included in our sample after the Brexit referendum is rather short. Therefore, due to sample availability the effects of Brexit might have been underestimated.

Table 1: Trade flows among the countries considered in the analysis

	BEL	FIN	FRA	DEU	GRC	IRL	ITA	NLD	NOR	PRT	ESP	EU	GBR	USA
BEL	0.00	0.01	0.19	0.23	0.00	0.04	0.06	0.22	0.01	0.01	0.03	0.00	0.09	0.09
FIN	0.07	0.00	0.07	0.31	0.01	0.01	0.06	0.17	0.06	0.01	0.04	0.00	0.09	0.10
FRA	0.14	0.01	0.00	0.28	0.01	0.01	0.12	0.10	0.01	0.02	0.11	0.00	0.09	0.09
DEU	0.09	0.02	0.15	0.00	0.01	0.02	0.10	0.19	0.02	0.01	0.06	0.00	0.11	0.13
GRC	0.06	0.01	0.10	0.23	0.00	0.02	0.23	0.10	0.01	0.01	0.08	0.00	0.07	0.06
IRL	0.11	0.00	0.09	0.11	0.00	0.00	0.03	0.06	0.01	0.00	0.03	0.00	0.26	0.29
ITA	0.07	0.01	0.19	0.28	0.02	0.01	0.00	0.08	0.01	0.01	0.10	0.00	0.08	0.11
NDL	0.17	0.01	0.10	0.33	0.01	0.02	0.05	0.00	0.03	0.01	0.04	0.00	0.12	0.09
NOR	0.05	0.03	0.09	0.22	0.00	0.02	0.04	0.14	0.00	0.01	0.03	0.00	0.27	0.09
PRT	0.04	0.01	0.13	0.17	0.00	0.01	0.06	0.06	0.00	0.00	0.40	0.00	0.06	0.04
ESP	0.05	0.01	0.23	0.21	0.01	0.01	0.12	0.07	0.01	0.10	0.00	0.00	0.09	0.07
EU	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50
UK	0.08	0.01	0.11	0.21	0.00	0.07	0.06	0.13	0.05	0.01	0.05	0.81	0.00	0.19
US	0.08	0.01	0.12	0.26	0.00	0.08	0.10	0.10	0.02	0.01	0.04	0.17	0.83	0.00

Notes: i) The element (i, j) of the Table denotes the total trade share of country j to country i, over the period 2010-2019, as a percentage of the total trade of country i with the countries presented in the table. ii) We use the ISO-3 code to denote the European countries. iii) Trade flows for euro area and Norway (EU) are constructed as the sum of trade share for the European countries reported in this Table. For the sake of brevity, Austria is not included in the Table.

3 Empirical methodology

We address our objective –identification of the influence of productivity shocks on the UK's business cycle– by employing an econometric methodology that includes two stages. The first stage focuses on long-run relationships and consists of two steps. In the first step, using cointegration analysis, we identify the long-run relationships. We also identify strongly exogenous variables used as proxies of

[16] Following Ploberger and Krämer (1992), our set of structural stability tests is based on the cumulative sums of the OLS residual tests denoted by PKsup and PKmsq. Furthermore, we also implemented the Nyblom (1989) test for time-varying parameters and sequential Wald tests such as QLR, MW and APW. QLR refers to the likelihood ratio test suggested by Quandt (1960) while MW is a Wald statistic based on Hansen (2002) and Andrews and Ploberger (1994), APW is also based on the work of Andrews and Ploberger (1994).

permanent shocks. In the second step, we simulate the impact of permanent shocks, by implementing an IRF analysis, on the variables included in our model.

In the second stage, we gauge the influence of cyclical productivity shocks by estimating a structural Bayesian Global VAR (GVAR) model. In doing so, we also account for the spillover effects of cyclical shocks across countries. Identification of cyclical shocks is achieved using sign restrictions implied by the Dynamic Stochastic General Equilibrium (DSGE) model presented in section 5.

It is worth clarifying that while we follow a frequentist approach to implement cointegration analysis- identification of unique cointegrating vectors and strongly exogenous variables- we adopted a Bayesian approach to model and estimate the impact of permanent shocks on the rest of the variables included in our model.[17] We do so because to the best of our knowledge identification of strongly exogenous-trend variables- in Bayesian framework has not been developed. We also use a Bayesian framework for the BGVAR analysis: estimation and identification of transitory (cyclical) productivity shocks.[18]

4 The Long-run Analysis of Productivity Shocks

This section consists of two parts. The first part describes the econometric framework used to analyse long-run relationships among the variables included in our model and identify strongly exogenous variables as proxies of permanent shocks. The second part presents and discusses empirical results concerning both steady-state relationships and the impact of permanent productivity shocks (domestic and foreign) on domestic variables.

4.1 Cointegration analysis

We consider a model with N countries, indexed $i = 0, 1, \ldots, N-1$, where $i = 0$ stands for the reference country. For each country, we denote $\mathbf{x}_{it}$ the $k_i \times 1$ vector of domestic variables, $\mathbf{x}^*_{it}$ the $k^*_i \times 1$ vector of country-specific foreign variables and $\mathbf{d}_{it}$ the $k_{d_i} \times 1$ vector of global variables. The VARX*(p_i,q_i) model for country i is:

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \sum_{l=1}^{p_i} \mathbf{\Phi}_{il}\mathbf{x}_{i,t-l} + \sum_{l=0}^{q_i} \mathbf{\Lambda}_{il}\mathbf{x}^*_{i,t-l} + \sum_{l=0}^{q_i} \delta_{il}\mathbf{d}_{i,t-l} + \mathbf{u}_{it} \tag{2}$$

where $\mathbf{a}_{i0}$ and $\mathbf{a}_{i1}$ are $k_i \times 1$ vectors of intercept terms and trend coefficients, $\mathbf{\Lambda}_{i0}$ and δ_{i0} are the $k_i \times k^*_i$ and $k_i \times k_{d_i}$ matrices of the contemporaneous effect that foreign and global variables have on the domestic variables and $\mathbf{\Phi}_{il}$, $\mathbf{\Lambda}_{il}$ and δ_{il} are the matrices of the lagged coefficients of domestic, foreign and global variables (of dimensions $k_i \times k_i$, $k_i \times k^*_i$ and $k_i \times k_{d_i}$, respectively). For the vector of country-specific shocks $\mathbf{u}_{it}$, we assume that $E(\mathbf{u}_{it}\mathbf{u}_{jt}) = \sigma_{ij}$ and $E(\mathbf{u}_{it}\mathbf{u}_{js}) = \mathbf{0}$ for $t \neq s$.[19] One of the key assumption underlying the country-specific VARX* is that the foreign variables are weakly exogenous. Dees et al. (2009) show that country-specific foreign variables can be treated as weakly exogenous

[17] To be more concrete, once strongly exogenous variables were identified, we model their marginal distribution, which are then used for an impulse response function analysis.

[18] McAdam et al. (2022) estimate cointegrated VECM model with stochastic volatility in a Bayesian framework but they only focus on identification of cointegrated space and not on strongly exogenous variables.

[19] Equation (2) shows that foreign variable affects domestic economies through two separate but interconnected channels: i) contemporaneous dependence between $\mathbf{x}_{it}$ and $\mathbf{x}^*_{it}$ and ii) contemporaneous dependence of shocks in country i with shocks in country j: σ_{ij}.

even though they are endogenous to the system as a whole. Here, we treat both domestic and foreign variables as endogenous. Therefore, we create a new vector of endogenous variable $\boldsymbol{z}_{it} = (\boldsymbol{x}'_{it}, \boldsymbol{x}^{*\prime}_{it})'$, which for the sake of clarity replaces $\boldsymbol{x}_{it}$. Then the vector error correction model (VECM) can be written as:

$$\boldsymbol{\Delta z}_{it} = \boldsymbol{c}_{i0} + \boldsymbol{\alpha}_i \boldsymbol{\beta}'_i [\boldsymbol{z}_{it-1} - \boldsymbol{d}_{t-1} - \boldsymbol{\lambda}_i (t-1)] + \sum_{j=1}^{p-1} \boldsymbol{\Gamma_{il} \Delta z}_{i,t-j} + \boldsymbol{\delta}^*_{i1} \Delta \boldsymbol{d}_{t-1} + \boldsymbol{u}_{it} \tag{3}$$

where $p = \max(p_i, q_i)$, $\boldsymbol{\alpha}_i$ is a $(\kappa_i + \kappa^*_i \times r_i)$ matrix of speed of adjustment coefficients and $\boldsymbol{\beta}_i = (\boldsymbol{\beta}'_{ix}, \boldsymbol{\beta}'_{ix^*}, \boldsymbol{\beta}'_{id})'$ is a $(\kappa_i + \kappa^*_i + m_d) \times r_i$ matrix of cointegrating vectors of rank r_i.[20]

We test for weak exogeneity along the lines of Johansen (1992) and Harbo et al. (1998). This involves a joint significance test of the estimated speed of adjustment coefficients of the error correction terms. In particular, for each l^{th} element of $\boldsymbol{z}_{it}$ the following auxiliary regression is carried out

$$\boldsymbol{\Delta z}_{it,l} = \boldsymbol{c}_{i0,l} + \sum_{j=1}^{r_i} \alpha_{ij,l} ECM^j_{i,t-1} + \sum_{j=1}^{p_i-1} \Gamma_{il} \boldsymbol{\Delta z}_{i,t-j} + \boldsymbol{\delta}^*_{i1} \Delta \boldsymbol{d}_{t-1} + \boldsymbol{u}_{it} \tag{4}$$

where $ECM^j_{i,t-1}$ for $j = 1, 2, \ldots, r_i$ are the estimated error correction terms associated with the cointegrating relationships. A weak exogeneity test for a variable $z_{it,l}$ is given by the hypothesis that $\alpha_{ij,l} = 0$ for $j = 1, 2, \ldots r_i$. While weak exogeneity of a variable indicates that its accumulated residual can be considered as potential trend, the series itself is not necessarily the trend. For this to hold a further condition must be satisfied: *strong exogeneity*. We can test for *strong exogeneity* by writing (3) in moving average representation (MA):

$$\boldsymbol{z}_{it} = \boldsymbol{C}_i \sum_{t=1}^{t} \boldsymbol{u}_{it} + \boldsymbol{\tau}_1 t + \boldsymbol{\tau}_0 + \boldsymbol{X}_t$$

where $\boldsymbol{C}_i = \boldsymbol{\beta}_{i\perp} (\boldsymbol{\alpha}'_{i\perp} \boldsymbol{\Gamma} \boldsymbol{\beta}_{i\perp})^{-1} \boldsymbol{\alpha}'_{i\perp}$, $\boldsymbol{\Gamma} = -(\boldsymbol{I} - \sum_{l=1}^{p-1} \boldsymbol{\Gamma}_{il})$, $\boldsymbol{\tau}_0 = \boldsymbol{C z}_{i0}$, $\boldsymbol{\tau_1} = \boldsymbol{C c}_{i0}$ and $\boldsymbol{X}_t$ is the stationary-cyclical-component. Strong exogeneity of a weakly exogenous variables say $\boldsymbol{z}_{it,l}$ requires that the corresponding rows of $\boldsymbol{\Gamma}_i$ matrices are not statistically different from zero. This implies that the relevant variable is affected only by its own shocks. Given that $\boldsymbol{z}_{it,l} \sim I(1)$, the equation of a strongly exogenous variable becomes $\Delta z_{it,l} = u_{it,l}$ which implies that $z_{it,l} = \sum_{t=1}^{t} u_{it,l}$.

Here, it is worth noting that a variable found to be strongly exogenous in a country-specific VARX* model is not necessarily strongly exogenous in the global model. Therefore, we treat strongly exogenous variables as trend variables only for the country-specific model. In doing so, we gauge the impact of strongly exogenous variables based on the country-specific model that is found to be strongly exogenous. More formally, if there are s strongly exogenous variables - $\boldsymbol{w}_{it}$ - then we can write (3) in levels conditional on strongly exogenous- variables:

$$\boldsymbol{z}^*_{it} = \boldsymbol{c}_{i0} + \boldsymbol{c}_{i1} t + \sum_{l=1}^{p_i} \boldsymbol{\Phi}_{yi,l} \boldsymbol{z}^*_{t-l} + \boldsymbol{\Theta}_{i0} \boldsymbol{w}_{it} + \sum_{l=1}^{q_i} \boldsymbol{\Theta}_{il} \boldsymbol{w}_{t-l} + \boldsymbol{\delta}_{i0} \boldsymbol{d}_t + \boldsymbol{u}_{it} \tag{5}$$

[20]Note that (3) allows for cointegration among the domestic foreign and global variables.

$$\boldsymbol{w}_{it} = \boldsymbol{\mu}_w + \sum_{l=1}^{k_i} \boldsymbol{\Phi}_{i,wl} \boldsymbol{w}_{it-l} + \boldsymbol{\omega}_{it},, \ \boldsymbol{\omega}_{it} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_w) \tag{6}$$

where $\boldsymbol{z}_{it}^*$ is a $(k_i + k_i^* - s) \times 1$ vector of the endogenous variables: it is the same as $\boldsymbol{z}_{it}$ excluding the strongly exogenous variables: $\boldsymbol{w}_{it}$. The $\boldsymbol{\mu}_w$ is an $s \times 1$ vector of constant coefficients; $\boldsymbol{\Phi}_{yi,l}$ is $(k_i + k_i^* - s) \times (k_i + k_i^* - s)$ matrix of the conditional model autoregressive coefficients; $\boldsymbol{\Theta}_{il}$ is the $(k_i + k_i^* - s) \times s$ matrix of coefficients reflecting the impact of strongly exogenous variable on the endogenous variables; and $\boldsymbol{\Phi}_{i,wl}$ is an $s \times s$ matrix of autoregressive coefficients. We use (5) and (6) to implement an impulse response function analysis suggested by Pesaran et al. (2004) for the global - strongly exogenous - variables $\boldsymbol{w}_{it}$:

$$GI(h) = \frac{1}{\sigma_{w,ii}} \boldsymbol{\Theta}_{i0} \boldsymbol{\Phi}_{i,w}^h \boldsymbol{\Sigma}_w e_j \tag{7}$$

Economic theory is typically more informative about the long-run relationships than it is on the short-run period.[21] Identification of long-run relationships boils down to estimation of unique cointegration vectors. Note that if there are r_i cointegrating vectors then we can choose a non-singular matrix $\boldsymbol{Q}$ such as $\boldsymbol{\alpha}_i\boldsymbol{\beta}_i' = \boldsymbol{\alpha}_i\boldsymbol{Q}^{-1}\boldsymbol{Q}\boldsymbol{\beta}_i' = \boldsymbol{\alpha}_i^*\boldsymbol{\beta}_i^{*\prime}$ are observationally equivalent. In order to identify the long-run structure, we need to impose restrictions on each cointegrating relationship. If there are r_i cointegrating relationships then for exact identification of country-specific model i we need to impose $r_i - 1$ restrictions on each cointegrating vector. For the set of restrictions to be valid, we apply the Johansen (1995) rank conditions, which test for the restricted cointegrating vector to be linearly independent.[22] If some of the estimated coefficients, in the cointegration space, are not significant, we impose further exclusion restrictions: empirical identification. In doing so, we also test for over-identification. If the estimated coefficients of the restricted - over-identified - model is consistent with the two-country international business cycle model developed in section 5 then the estimated cointegration space is considered to be identified in terms of economic identification.

4.2 Empirical results from cointegration analysis: the steady-state effects of productivity

Here, in line with our empirical strategy we identify the long-run relationships among the variables included in our model. Prior to identification, we implement a specification test and test for the number of cointegrating relationships. After determining the number of cointegrating vectors, we facilitate identification by implementing both exclusion (from the cointegration space) restriction tests and tests for weak exogeneity.[23] We then write the model in a moving average form and test for strong exogeneity. If more than one variable is found to be strongly exogenous then we perform a

[21] Garratt et al. (2012) argues that this is because economic theory is frequently silent concerning the sequence of economic decisions, the structure of information sets across agents and the nature of rigidities that arise from transaction costs.

[22] In particular, Johansen (1995) shows that a set of restrictions are formally identified if for each restricted cointegrating vector j and for $g = r - 1$ holds

$$rank(\boldsymbol{R}_j'\boldsymbol{H}_{j_1}, \ldots, \boldsymbol{R}_j'\boldsymbol{H}_{j_g}) \geq g$$

where $\boldsymbol{R}_j$ is $(k_i + k_i^* + k_d) \times m_i$ restriction matrix and $\boldsymbol{H}_j = \boldsymbol{R}_j\perp$.

[23] Exclusion restriction tests are applied to non-significant estimated coefficients of the unrestricted model

join exogeneity test for all variables that are found to be strongly exogenous. We identify long-run relationships (cointegrating vectors) conditional on both exclusion restrictions and on the strongly exogenous variables.

Strong exogeneity tests based on $\boldsymbol{C}_i$, the estimated long-run impact matrix, are shown in Table 2, which includes three panels of results.[24] The upper panel of Table 2 presents the long-run impact matrix of the model where the foreign variables- foreign TFP and IST- were constructed using the weighted average of the relevant EU and the US variables. The long-run impact matrices of the models, where the foreign variables are proxied by the EU and the US variables, are shown in the second and final panels of Table 2, respectively. The column of the $\boldsymbol{C}_i$ matrix indicates how the accumulated residual from each VAR equation loads into the other equations. Alternatively, the rows of the $\boldsymbol{C}_i$ matrix gives the weights with which each variable has been affected by the accumulated residual of the other variables included in our model. For example, while the first column of Table 2 shows the impact that an output shock has on the rest of the variables, the first row of Table 2 shows the impact that accumulated shocks on the rest of the variables have on output. Therefore row-wise, if the only shock that is significant is the shock of the relevant equation then the variable is considered to be strongly exogenous. The rows of the upper panel of Table 2 shows that domestic TFP and IST are strongly exogenous in the sense that they have been influenced only by their own accumulated shocks. Panel B of Table 2 shows that only domestic TFP and IST shocks are strongly exogenous. The final panel C indicates that beside the domestic TFP and IST shock the foreign TFP variable is also strongly exogenous. Conditional on the strongly exogenous variables, we impose exclusion restrictions to identify the two cointegrating vectors.[25]

[24]Prior to the strong exogeneity test, a joint test for weak exogeneity accept the null that domestic TFP, IST and foreign TFP are weakly exogenous at 10 percent significant level.

[25]Note that in all three cases Johansen cointegration test (the Lmax and Ltrace statistic) for the number of cointegrating vectors accepted the null of two cointevecting vectors

Table 2: The long run impact matrix, the case of UK.

	y	c	gcf	tfp	ist	ntr	rer	tfp^*	ist^*
Panel A									
y	0.027	0.148	0.732***	20.523***	-0.222	-0.198**	0.360	-1.675	2.909**
c	-0.328	0.796***	0.442***	16.079***	-0.334	-0.180**	0.249*	0.042	1.895*
gcf	0.078	-0.043	0.904***	3.401	0.114	-0.281***	0.227	-2.731*	2.967**
tfp	-0.001	0.007	0.004	1.119***	-0.019	0.000	0.009	0.015	0.042
ist	-0.040	0.009	0.019	-1.109	0.931***	-0.021	-0.015	-0.025	0.403
ntr	0.616***	-0.393***	-0.228*	-15.599***	0.476	0.393***	0.846***	-0.074	-0.213
rer	0.289*	0.244**	0.122	-8.697**	0.283	0.203***	0.883***	-0.126	1.031
tfp^*	0.008	0.008	-0.010	0.501	-0.063**	0.007	0.004	1.029***	-0.151
ist^*	-0.046*	0.007	0.018	1.109**	0.035	-0.008	0.064**	0.182	1.045***
Panel B									
y	0.237*	0.042	0.739***	16.660***	-1.061***	-0.127*	0.371*	0.845	2.612***
c	-0.400**	0.707***	0.460***	15.383***	-0.672**	-0.163**	0.226	1.817	1.602**
gcf	0.216	-0.011	0.899***	1.751	-0.401	-0.190***	0.133	3.500	1.495*
tfp	0.000	0.007*	0.004	1.120***	-0.012	-0.000	0.008	-0.178	0.043
ist	-0.036	0.003	0.015	-1.288	0.887***	-0.017	-0.026	0.718	0.195
ntr	0.832***	-0.391***	-0.314**	-15.702***	0.421	0.335***	0.810***	-1.983	-1.455
rer	0.369**	0.249**	0.088	-9.586***	0.500*	0.155**	1.009***	2.580	0.854
tfp^*	-0.007*	-0.003	-0.000	0.286***	-0.025	-0.002	0.005	0.942***	-0.006
ist^*	-0.026	0.003	0.008	0.835*	0.025	-0.012	0.064***	0.844	0.866***
Panel C									
y	0.035	0.163	0.734***	20.075***	-0.257	-0.199***	0.373*	-0.339	2.906**
c	-0.342	0.793***	0.443***	15.934***	-0.313	-0.190**	0.261	-0.021	1.954*
gcf	0.089	-0.029	0.899***	3.181	0.053	-0.279***	0.230	-0.597*	2.971**
tfp	-0.001	0.006	0.004	1.115***	-0.019	0.000	0.010	0.006	0.040
ist	-0.041	0.009	0.020	-1.105	0.933***	-0.022	-0.016	-0.014	0.412
ntr	0.620**	-0.386***	-0.232*	-15.749***	0.467	0.394***	0.836***	-0.005	-0.244
rer	0.290*	0.235**	0.120	-8.553**	0.288	0.200***	0.891***	-0.059	1.077
tfp^*	0.041	0.028	-0.054	1.542	-0.185	0.036	0.016	1.002***	-0.740*
ist^*	-0.048*	0.006	0.018	1.130**	0.041	-0.009	0.064**	0.026	1.061***

Notes: i) In Panel A, UK foreign variables are the weighted average of their respective EU and US domestic variables, in Panel B, UK foreign variables are the respective EU domestic variables and in Panel C, UK foreign TFP is the US domestic TFP while UK foreign IST is the same as in Panel A. ii) *, ** and *** denote statistical significance at the 10, 5 and 1 % significance level.

Table 3 presents the identified long-run relationships of the three models corresponding to the three panels of results presented in Table 2. Panel A of Table 3 shows that there are two cointegrating vectors normalised with respect to trade and with respect to real GDP per capita. In the first cointegration vector normalised with respect to trade, we observe that while consumption has a negative effect on trade, domestic TFP has a positive impact. An increase in domestic TFP will increase the supply of domestic output, as this is observed in the second cointegration vector, which in turn will lead to lower domestic prices and a depreciation of real exchange rate. Corsetti et al. (2008) show that if the elasticity of substitution between home and foreign produced goods is greater than one then a depreciation of the real exchange rate will lead to an increase of global demand for the former goods. Therefore, a depreciation of the real exchange rate will improve the domestic trade balance.[26] There is also strong evidence that the financial crisis of 2008 - the shift dummy - has a strong negative effect

[26] Corsetti et al. (2008) show that a depreciation of real exchange rate will have a positive substitution effects on both domestic and foreign demand for domestic good while it will have negative income effects only on domestic demand. If the elasticity of substitution between domestic and foreign tradables is higher than one then the substitution effects will out weight the income effects. Therefore a depreciation of real exchange rate will lead to higher export and lower imports improving the net trade.

both on trade and on GDP growth. The financial crisis' harmful impact on trade and the real GDP per capita are congruent with the well-researched literature on the UK productivity puzzle. Tenreyro (2018) among others, argues that the inefficient allocation of resources prior to the crisis in conjunction with a decline of TFP and capital deepening, after the crisis, has undermined the recovery of aggregate productivity in the UK.[27] Tenreyro (2018) also claims that the weak growth of business investment is due to the uncertainty concerning the future relationship with the European partners. In line with the argument of weak investment, we do observe in the second cointegration vector that investment has positive and significant effect on real GDP per capita. The positive effects of both investment and TFP on the steady state values of real GDP and on trade underlying the importance of productive investment on the UK trade balance.

Table 3: Testing for exogeneity in long-run relations for the UK.

	y	c	gcf	tfp	ist	ntr	rer	tfp^*	ist^*	$dsift$	$trend$
Panel A											
Cointegrating vectors											
β_1'	0.000	0.905***	-0.004	-7.830***	0.000	1.000	-1.129***	0.000	0.000	0.530***	-0.013***
β_2'	1.000	-0.081*	-0.698***	-15.389***	0.000	0.000	-0.042	0.000	0.000	0.423***	-0.017***
Test for exogeneity of tfp, ist and tfp^*: $\mathcal{X}^2(6) = 11.183^*$											
Test of restricted model: $\mathcal{X}^2(12) = 19.197^*$											
Panel B											
Cointegrating vectors											
β_1'	0.000	0.985***	-0.039	-8.203***	0.680	1.000	-1.012***	0.000	1.173*	0.510***	-0.014***
β_2'	1.000	0.096	-0.801***	-13.758***	0.797**	0.000	-0.076	0.000	-1.239**	0.469***	-0.019***
Test for exogeneity of tfp and ist: $\mathcal{X}^2(4) = 8.625^*$											
Test of restricted model: $\mathcal{X}^2(6) = 9.241$											
Panel C											
Cointegrating vectors											
β_1'	0.000	0.876***	0.005	-7.010***	0.000	1.000	-1.120***	0.000	0.000	0.519***	-0.013***
β_2'	1.000	-0.094*	-0.699***	-15.023***	0.000	0.000	-0.047	0.000	0.000	0.417***	-0.017***
Test for exogeneity of tfp, ist and tfp^*: $\mathcal{X}^2(6) = 8.088^*$											
Test of restricted model: $\mathcal{X}^2(12) = 16.947$											

Notes: i) In Panel A, UK foreign variables are the weighted average of their respective EU and US domestic variables, in Panel B, UK foreign variables are the respective EU domestic variables and in Panel C, UK foreign are the respective US domestic variables. ii) *, ** and *** denote statistical significance at the 10, 5 and 1 % significance level. iii) In all three cases, β_1 and β_2 are normalised with respect to y and ntr, respectively.

The second panel of Table 3 focuses on the steady state-cointegration-relationships between UK and EU variables: EU IST and EU TFP. Results are similar to those presented in the upper panel of Table 3: Consumption has negative impact on net trade; TFP has positive effects both on net trade and real GDP per capita; investment, once again, has a strong positive impact on real GDP. The shift dummy shows that the financial crisis had a strong negative effect on both net trade and real GDP. It is remarkable that the positive effects of the EU IST outweighed the negative effects of domestic IST. This might reflect the plunge of domestic investments following the financial crisis while the EU business investment rebound quickly leading to an noticeable increase of EU productivity. A rise in IST stimulates EU investment and leads to an appreciation of the domestic real exchange rate,

[27] Unlike in the euro area where an increase of capital deepening explains a significant part of the stronger post-crisis European productivity performance, in the UK a slowdown of capital investment drag on aggregate productivity.

enhancing relative demand and, subsequently, the supply of home-produced goods. The demand for domestic goods increases due to foreign demand for home-produced investment goods. The final panel of Table 3 shows results from the steady state relationship between UK and US variables. Results here reflect those from the the two previous panels.

We provide further validation of the identified cointegrating vectors by presenting in Figure A1 (Appendix A) the persistence profiles (PPs) of a system-wide shock corresponding to the long-run restrictions displayed in Table 3. The PPs examine the impact that a system-wide shock has on the the dynamics of the cointegrating relations in the VARX*. If the imposed cointegrating vector is valid, the value of these PPs is one on impact and it should converge to zero as the forecast horizon increases. In our case, the PPs of system-wide shocks reflects shocks to strongly exogenous variables. The PPs show the speed at which the equilibrium errors return to zero after a shock. All PPs are well behaved confirming that the identified long-run relationships are valid. For example, the first column of Figure A1 shows the PPs corresponding to the long-run relationships presented in the upper panel of Table 3. We observed that all variables return to their long-run equilibrium within four to five years.

4.2.1 The impact of permanent shocks: an impulse response function analysis

The UK productivity puzzle has emphasised the importance of permanent shocks as a primary drivers of the business cycle. We simulate permanent shocks as shocks to strongly exogenous variables. A strongly exogenous variable is affected only by its own shocks. Given that the variables included in our model are I(1), a strongly exogenous variable is equal to the accumulated shocks, i.e., the series itself is the trend. Table 2 shows that whenever EU variables were used to construct foreign variables - Panels A and B - strong exogeneity tests show that only domestic TFP and IST were found to be strongly exogenous. Alternatively, when the US variables are treated as foreign, tests for strong exogeneity show that not only domestic TFP and IST found to be strongly exogenous but also foreign (US) TFP was found strongly exogenous. Therefore, we simulate the time profile of domestic TFP, domestic IST and of the US TFP shock.

In this section we investigate the empirical relevance and transmission of domestic and foreign productivity shocks. Table 4 provides a summary of the median forecast error variance decomposition allowing for a formal assessment of a shock's contribution to UK business cycle fluctuations. Although we present the forecast error-variance decomposition of all three models considered in Table 2, there is little variation across models. We observe that 66 percent of the UK's GDP fluctuation is explained by domestic and foreign (permanent) productivity shocks: the role of domestic TFP is 27 percent; the contribution of domestic IST is 18 percent and; the foreign (US) TFP adds another 21 percent. The remaining 34 percent of GDP's fluctuation is explained by a linear combination of shocks of the rest of the endogenous variables included in our model. For example, foreign IST and TFP may influence domestic business cycle by having long-run impact on domestic consumption and investment, which consistently enter the cointecration space. Note that the long-run impact matrices presented in Table 2 show that foreign IST and TFP affect domestic output consumption and investment. An important role is also played by the shift dummy, which in line with the literature on the productivity puzzle had a long-run negative impact on productivity, which in turn affects GDP negatively. When we compare panels B and C of Table 4, we observe that the influence of productivity shocks on the UK business cycle fluctuations is stronger in the model where foreign variables are proxied by the EU IST and EU

TFP (Panel B) rather than the corresponding US variables (Panel C). Table 1 shows that this might be a reflection of the EU's greater trade weight (81 percent) on the UK's net trade compared to the trade weight of US (19 percent).

Table 4: Forecast error variance decomposition

	Panel A		Panel B		Panel C		
	tfp	ist	tfp	ist	tfp	ist	tfp^*
y	0.272	0.186	0.274	0.186	0.216	0.143	0.218
c	0.270	0.193	0.274	0.185	0.215	0.142	0.218
gcf	0.270	0.193	0.277	0.185	0.219	0.142	0.214
ntr	0.271	0.192	0.275	0.186	0.217	0.144	0.217
rer	0.261	0.193	0.266	0.179	0.210	0.139	0.210

Notes: i) In Panel A, UK foreign variables are the weighted average of their respective EU and US domestic variables, in Panel B, UK foreign variables are the respective EU domestic variables and in Panel C, UK foreign are the respective US domestic variables. ii) The values denote the median forecast error variance decomposition over a horizon of 20 quarters.

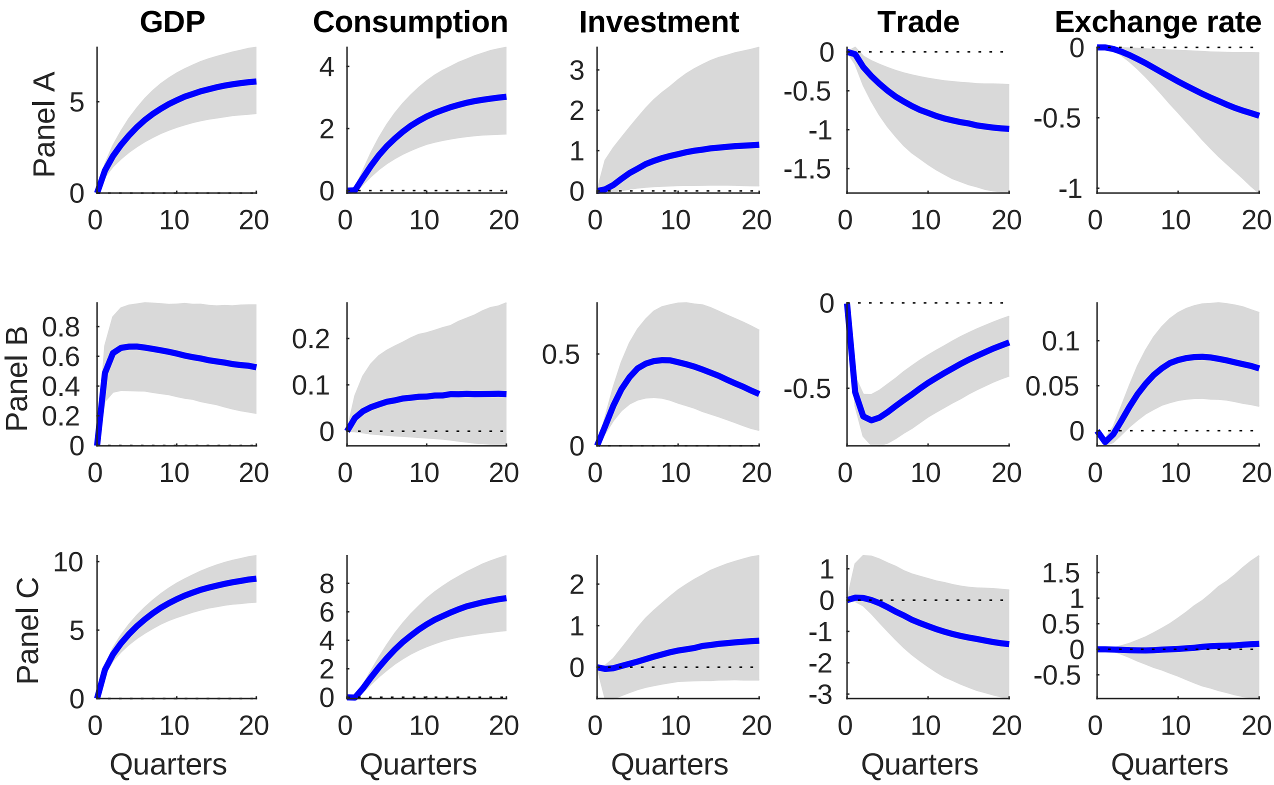
Figure 2 displays the impulse responses to domestic TFP shocks corresponding to the three models presented in panel A, B and C of Tables 2. The first row of Figure 2 shows that a domestic TFP shock has positive effects on real output, consumption and investment. This is consistent with evidence from the steady state analysis where positive productivity shocks increase output, which lead agents to increase consumption and investment. Following the domestic TFP shock, we also observe a modest real exchange rate appreciation, which might reflect a low trade elasticity.[28] An increase in domestic absorption and an appreciation of the real exchange rate seems to have a negative impact on trade balance. Similar responses to domestic TFP shock were observed in the second and third row of Figure 2 where we used as foreign variables the corresponding EU and US TFP and IST variables. For example, in the second row Figure 2 there is evidence of a strong positive response of domestic demand variables (investment and consumption) and of GDP. However, unlike what we observed in the first row of Figure 2, real exchange rate depreciates. Although the real exchange rate depreciates for more than two years following the shock there is evidence of persistent fall of trade balance. The negative relationship between real exchange rate and trade is driven by the strong positive responses of consumption and investment, which outweighed the positive effects of real exchange rate depreciation on net trade. Furthermore, the negative response of trade to a positive TFP shock might also reflect the productivity slowdown observed in industries with strong productivity performance prior to financial crisis: manufacturing, information technology and communication (ICT) as well as financial and insurance services.[29] In the same vein the third row of Figure 2 shows that when we treat US TFP and IST as foreign variables there is a strong positive response of output and consumption while trade deteriorates following a domestic (UK) TFP shock. Unlike what we observed in the first

[28] Corsetti and Müller (2006) argue that if the income effects from a low trade elasticity are strong enough, then an increase of domestic supply should be followed by an appreciation in the terms of trade, which in turn will have positive wealth effects to boost domestic demand and clear domestic good markets.

[29] Goodridge et al. (2018) and Tenreyro (2018) among others show that two thirds of the UK's productivity gap is explained by a plunge of productivity in manufacturing, information technology and communication (ICT) as well as financial and insurance services. They also show that two-third of market's sector productivity downturn is due to a decline of TFP while the other one third is due to a fall capital deepening after the financial crisis.

and second row of Figure 2 investment and real exchange rate remained muted following a domestic TFP shock. The weak response of investment and trade is consistent with the literature of the UK productivity puzzle: a permanent drop of the UK productivity level compare to its pre-crisis level. Weak productivity and flexible real wages led to substitution of capital from labour, discouraging of capital investment.

Figure 2: Impulse responses to a shock in domestic TFP.



Notes: i) Domestic TFP is considered as exogenous variable in all models. ii) In panel A, foreign variables –TFP and IST– are a weighted average of the respective EU and US variables. In panels B and C the foreign variables are the respective EU and US variables, respectively. iii) In each panel, the blue line denotes the median response and the grey area the 68% credible set.

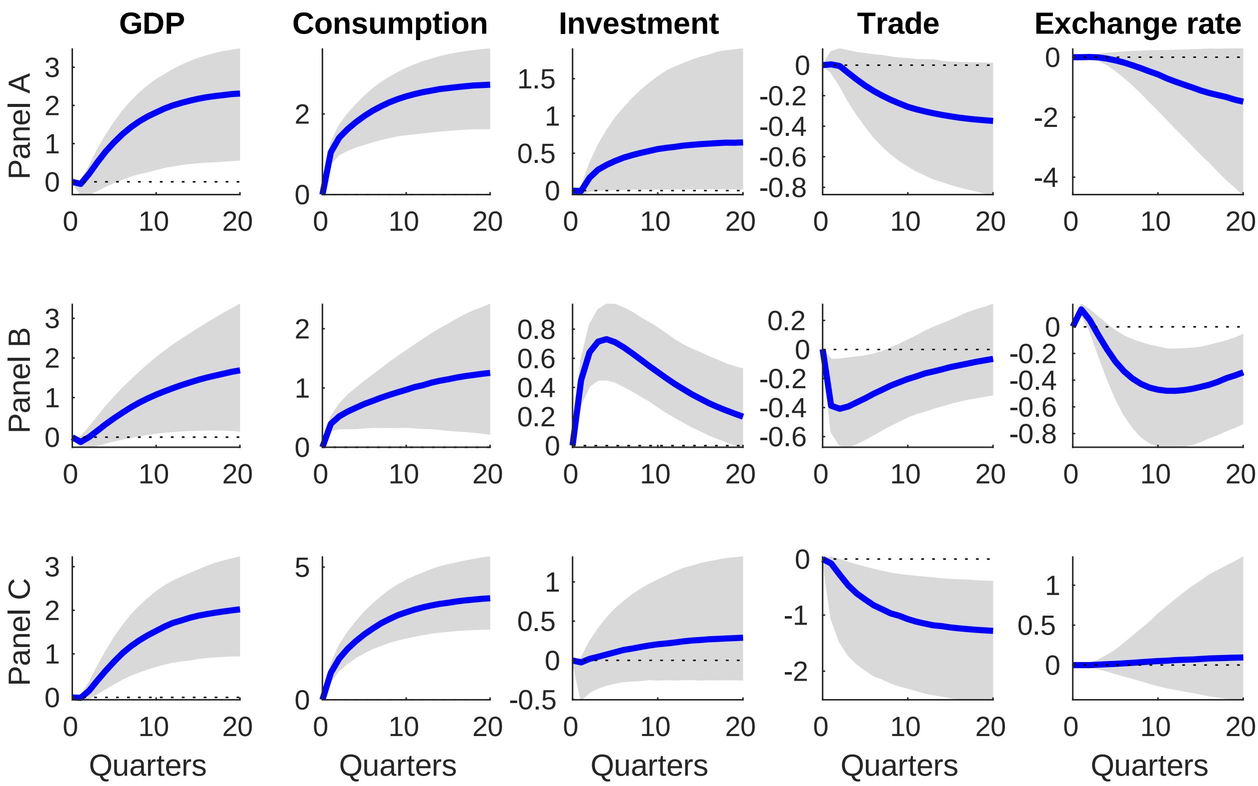
Figure 3 shows the responses to domestic IST shocks. The first row of Figure 3 shows the impulse responses of UK variables to a domestic IST shock. The responses are qualitatively similar to the responses due to domestic TFP shock presented in the first row of Figure 2.[30] We observe that there is positive response of output, consumption and investment, albeit the response of investment is marginally outside the credible set.[31] The weak response of investment to a positive IST shock may be due to the uncertainty generated by the Brexit referendum. More formally, Tenreyro (2018) argues that the higher level of uncertainty, post-crisis, led firms to opt for labour over capital rather than investing on capital: fall of capital deepening. Recent data point towards evidence of a recovery for EU and US business investment but not for the UK. Tenreyro (2018) claim that the reason why the UK investment lagging behind is the uncertainty over the trading relationship with the EU. Alternatively,

[30] A key difference is that responses are driven by a fall of relative price of investment.

[31] The zero line is lightly below the credible set. The credible set specifies the region where the 95 percent of the probability of the posterior distribution concentrate.

when foreign variables -foreign TFP and IST- are proxied by the corresponding EU variables, the second row of Figure 3 shows that unlike the first row of Figure 3 the responses of investment and real exchange rate to domestic IST shock are significant and persistent. We do observe a persistent appreciation of real exchange rate, which in turn has positive effect on the return and the level of domestic investments.[32] The strong positive responses of domestic investment and consumption outweighed the increase of output and lead to a deterioration of net trade. When we use the US TFP and IST as foreign variables, the last row of Figure 3 shows that the responses to domestic IST are qualitatively similar to those observed in the first row of Figure 3: output, consumption and investment increase while net trade deteriorate. However, unlike in the first row of Figure 3 the response of real exchange rate to a domestic IST shock is muted.

Figure 3: Impulse responses to a shock in domestic IST.



Notes: i) Domestic IST is considered as exogenous variable in all models. ii) In panel A, foreign variables –TFP and IST– are a weighted average of the respective EU and US variables. In panels B and C the foreign variables are the respective EU and US variables, respectively. iii) In each panel, the blue line denotes the median response and the grey area the 68% credible set.

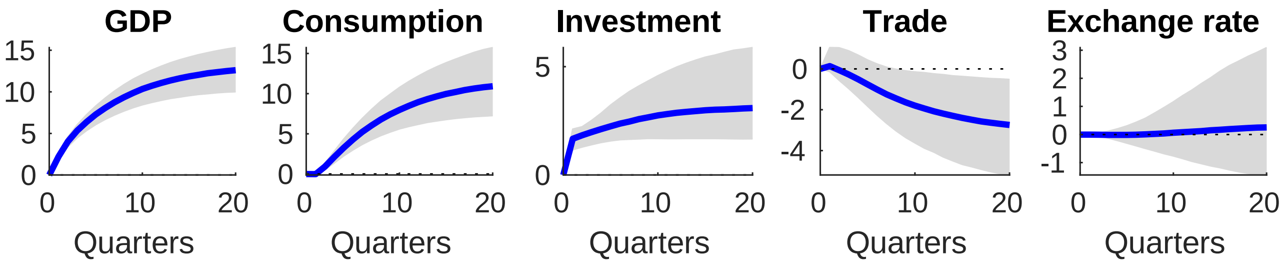
Finally, we also simulate the time-profile of foreign (US) TFP shock.[33] Figure 4 shows that a positive foreign –US– TFP shock has strong positive effects on real output, consumption and investment while trade becomes negative after two years following the shock. The positive response of domestic output, consumption and investment is consistent with the fact that the exogenous part of technology used in both countries share a common stochastic trend. Deterioration of the trade balance is due to

[32] Corsetti and Müller (2006) show that an appreciation of real exchange rate increases the capital return of domestic investment. This is so because investment include an exchange rate component: an appreciation reduce the production cost of home investment goods.

[33] Note that strong exogeneity test fort the US TFP shock accept the null of strong exogeneity.

the positive response of domestic demand while real exchange rate remains muted.

Figure 4: Impulse responses to a shock in foreign TFP.



Notes: *Notes:* i) Foreign TFP is considered as exogenous variable in the model. ii) Foreign variables –TFP and IST– the respective US variables, respectively. iii) The blue line denotes the median response and the grey area the 68% credible set.

5 Short-Run Analysis

This section estimates the impact of cyclical domestic and foreign productivity shocks. We account for international spillover effects using a structural BGVAR. Structural productivity shocks are identified based on sign restrictions extracted from a two-country international real business cycle model. This section comprises four subsections. The first subsection describes the structural model while the subsequent section calibrates the model and provides the sign restrictions imposed on BGVAR. The third subsection explains the econometric methodology -structural BGVAR- and the final part present and discuss empirical results.

5.1 An international business cycle model

This subsection presents the structural model which is used to obtain the sign restrictions imposed on the econometric estimation of cyclical domestic and foreign productivity shocks to total factor productivity, TFP, and investment-specific technology,IST, shocks. The model follows the basic structure of Gortz et al. (2023). More specifically, we generate sign restrictions from a two-sector, two-country flexible price, dynamic stochastic general equilibrium model. Each economy consists of a representative household consuming a final consumption good, made up of home and foreign-produced consumption goods, and supplying labour. Households in both countries are able to smooth consumption across time by trading in one-period non-state contingent bonds. In each economy, there are two production sectors producing intermediate consumption goods and investment goods, respectively. Firms in both of these sectors produce output using labour and sector-specific capital. There are two types of shocks hitting the economy: shocks to total factor productivity affecting both sectors at the same time (TFP) and productivity shocks specific to the investment goods-producing sector (IST).

5.1.1 Households

The representative household in the domestic economy derives utility from the consumption of final goods, C_t, and dis-utility from supplying labour, L_t according to preferences described by the expected utility function. Our specification for preferences follows Jaimovich and Rebelo (2009) which nests GHH and KPR type preferences and is consistent with balanced growth.

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_t - X_t \frac{\phi_0}{1+\phi} L_t^{1+\phi} \right)^{1-\sigma} \tag{8}$$

$$X_t = C_t^{\varsigma} X_{t-1}^{1-\varsigma} \tag{9}$$

X_t is a preference shift factor such that preferences are equivalent to GHH preferences when $\varsigma = 0$ and KPR type preferences when $\varsigma = 1$. The precise specification of preferences matters for the dynamics of consumption following an IST shock. β denotes the discount factor applicable to households. The parameter $\phi > 0$ is the inverse of the Frisch elasticity, respectively. $\phi_0 > 0$ determines the dis-utility of labour. Households face a cost of shifting their labour effort between sectors:

$$L_t = \left(\chi_c^{-\frac{1}{\xi}} L_{c,t}^{\frac{1+\xi}{\xi}} + \chi_i^{-\frac{1}{\xi}} L_{i,t}^{\frac{1+\xi}{\xi}} \right)^{\frac{\xi}{1+\xi}} \tag{10}$$

where $\chi_c = L_c/L$ and $\chi_i = L_i/L$ are the steady state shares of sectoral labour supplies and ξ is the elasticity of sectoral labour supply.

Households maximise their expected utility given by equation (8) subject to the labour adjustment cost function (10) and the following flow budget constraint

$$C_t + \frac{P_{c,t}^*}{P_{c,t}} B_t = (1 + r_{t-1}) \frac{P_{c,t}^*}{P_{c,t}} B_{t-1} + W_{c,t} L_{c,t} + W_{i,t} L_{i,t} + \pi_{c,t} + \pi_{i,t} \tag{11}$$

$P_{c,t}^*$ denotes the price of the foreign final consumption good and $P_{c,t}$ the domestic consumption goods price index. $w_{j,t} L_{j,t}$ denotes the representative household's wage income from working in sector $j = c, i$ and $\pi_{j,t}$ is the dividend income received due to ownership of firms. Households are able to smooth consumption risk by holding non-state contingent bonds, B_t, denominated in terms of the foreign-produced consumption good, that pay a quarterly yield of r_t.[34]

5.1.2 Final consumption goods

Final goods, used for consumption, C_t, are produced by combining home and foreign-produced intermediate goods according to a constant elasticity of substitution (CES) technology

$$C_t = \left[\eta_c^{\frac{1}{\theta}} \left(C_{H,t} \right)^{\frac{\theta-1}{\theta}} + (1 - \eta_c)^{\frac{1}{\theta}} \left(C_{F,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{12}$$

with the corresponding price index:

$$(P_{c,t})^{1-\theta} = \eta_c \left(P_{c,t}^H \right)^{1-\theta} + (1 - \eta_c) \left(P_{c,t}^F \right)^{1-\theta} \tag{13}$$

[34] As in Schmitt-Grohe and Uribe (2003) we allow the interest rate on domestically held foreign bonds to differ from the rate applicable to foreign agents by a small debt-elastic premium. Specifically, $(1+r_t) = (1+r_t^*)e^{-\phi_b b_t}$, the premium decreases with the net foreign asset position of the home country. This small bond holding cost eliminates the unit root in bond holdings and closes the model.

5.1.3 Final Investment Goods Producers

We assume that domestically-produced investment goods are only used by home firms.

$$I_{c,t} + I_{i,t} = Y_{i,t} \tag{14}$$

5.1.4 Intermediate Consumption Goods Producing Firms

Domestic intermediate goods-producing firms produce country-specific output goods, $Y_{c,t}$, that are used in the production of final consumption goods. These firms maximise cash-flow

$$\pi_{c,t} = \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} - W_{c,t} L_{c,t} - \frac{P_{i,t}}{P_{c,t}} I_{c,t}, \tag{15}$$

where $\frac{P^H_{c,t}}{P_{c,t}}$ denotes the price of the home-produced intermediate consumption good relative to the domestic final good, and $\frac{P_{i,t}}{P_{c,t}}$ denotes the price of investment goods relative to consumption goods; subject to the representative firm's production function and the capital accumulation constraint. The firm produces output goods according to a standard Cobb-Douglas production function

$$Y_{c,t} = \left(u_{c,t} K_{c,t-1}\right)^{\alpha} \left(Z_t L_{c,t}\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{16}$$

where $K_{c,t-1}$ denotes physical capital services used in the production of consumption goods, $u_{c,t}$ is the rate at which physical capital is utilised, $L_{c,t}$ is hours worked and Z_t is a stationary productivity process that affects TFP in both, the consumption and the investment goods-sector. The parameter α determines the share of capital in production. The capital accumulation constraint is given by

$$K_{c,t} = (1 - \delta_{c,t}) K_{c,t-1} + \left(1 - \frac{\psi}{2}\left(\frac{I_{c,t}}{I_{c,t-1}} - 1\right)^2\right) I_{c,t}. \tag{17}$$

The function δ_{c_t} captures time-varying costs associated with the usage of capital,

$$\delta_{c,t} = \bar{\delta} + \nu_0 \left(u^{\nu}_{c,t} - 1\right) \tag{18}$$

5.1.5 Intermediate Investment Goods Producing Firms

Domestic intermediate goods-producing firms produce country-specific investment goods, $Y_{i,t}$, that are used in the production of final investment goods. These firms maximise cash-flow

$$\pi_{i,t} = \frac{P_{i,t}}{P_{c,t}} Y_{i,t} - W_{i,t} L_{i,t} - \frac{P_{i,t}}{P_{c,t}} I_{i,t}, \tag{19}$$

subject to the representative firm's production function and the capital accumulation constraint. The firm produces output goods according to a standard Cobb-Douglas production function

$$Y_{i,t} = V_t \left(u_{i,t} K_{i,t-1}\right)^{\alpha} \left(Z_t L_{i,t}\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{20}$$

where $K_{i,t-1}$ denotes physical capital services, $u_{i,t}$ is the rate at which physical capital is utilised, $L_{i,t}$ is hours worked, Z_t is a stationary productivity process affecting both production sectors and V_t is a stationary productivity process specific to the investment goods-producing sector. The parameter α determines the share of capital in production. The capital accumulation constraint is given by

$$K_{i,t} = (1-\delta_{i,t})K_{i,t-1} + \left(1 - \frac{\psi}{2}\left(\frac{I_{i,t}}{I_{i,t-1}} - 1\right)^2\right) I_{i,t}. \tag{21}$$

Analogous to the consumption goods producing sector, the depreciation rate of capital is an increasing function of the utilisation rate of the sector specific capital stock.

$$\delta_{i,t} = \bar{\delta} + \nu_0 \left(u_{i,t}^{\nu} - 1\right) \tag{22}$$

5.1.6 The Foreign Economy and Market Clearing

The foreign economy is symmetric to the home economy outlined above. It differs only by country size. We assume that the home country is of relative size N such that the foreign country is of size $1-N$. The market clearing conditions for home and foreign-produced consumption goods are given by:

$$NY_{c,t} = NC_{H,t} + (1-N)C^*_{H,t}, \qquad (1-N)Y^*_{c,t} = NC_{F,t} + (1-N)C^*_{F,t}. \tag{23}$$

The market clearing conditions for home and foreign-produced investment goods are given by:

$$Y_{i,t} = I_{c,t} + I_{i,t} \qquad Y^*_{i,t} = I^*_{c,t} + I^*_{i,t}. \tag{24}$$

The labour market, in both economies, clears when:

$$L_t = \left(\chi_c^{-\frac{1}{\xi}} L_{c,t}^{\frac{1+\xi}{\xi}} + \chi_i^{-\frac{1}{\xi}} L_{i,t}^{\frac{1+\xi}{\xi}}\right)^{\frac{\xi}{1+\xi}}. \tag{25}$$

Real gross domestic product is defined as:

$$GDP_t = \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} + \frac{P_{i,t}}{P_{c,t}} Y_{i,t}. \tag{26}$$

Finally, the current account equation is derived by combining the households' budget constraints and the firms' cash flow functions along the the investment sector market clearing condition:

$$C_t + \frac{P^*_{c,t}}{P_{c,t}} B_t = (1+r_{t-1}) \frac{P^*_{c,t}}{P_{c,t}} B_{t-1} + \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} \tag{27}$$

5.2 Calibration

Table 5 summarises the calibration for the DSGE model's deep parameters. The top half of Table 5 lists parameter values that are easy to to determine from the data or for which there is a consensus in the literature. These include the discount rate, the relative size of the home economy, our measure of trade openness, the share of capital in output and the depreciation rate. As is standard in the literature, the bond holding cost, ϕ_b, is assumed to be small, its only role is to rule out a unit root in bond holdings. For the purposes of the calibration, we treat the UK as the home country. The home country is small in the sense that the shocks originating in the UK have no effect on the dynamics of the foreign economy. The bottom half of the table lists a set of model parameters for which we want to explore a range of potential values. For each parameter, we determine a mean as well as an upper and a lower bound. We then take 50,000 random parameter draws and solve the model for each of these draws. The impulse responses in Figures 5 to 8 show the median as well as the 5 - 95% of the distribution of IRFs.[35]

Table 5: DSGE model parameters

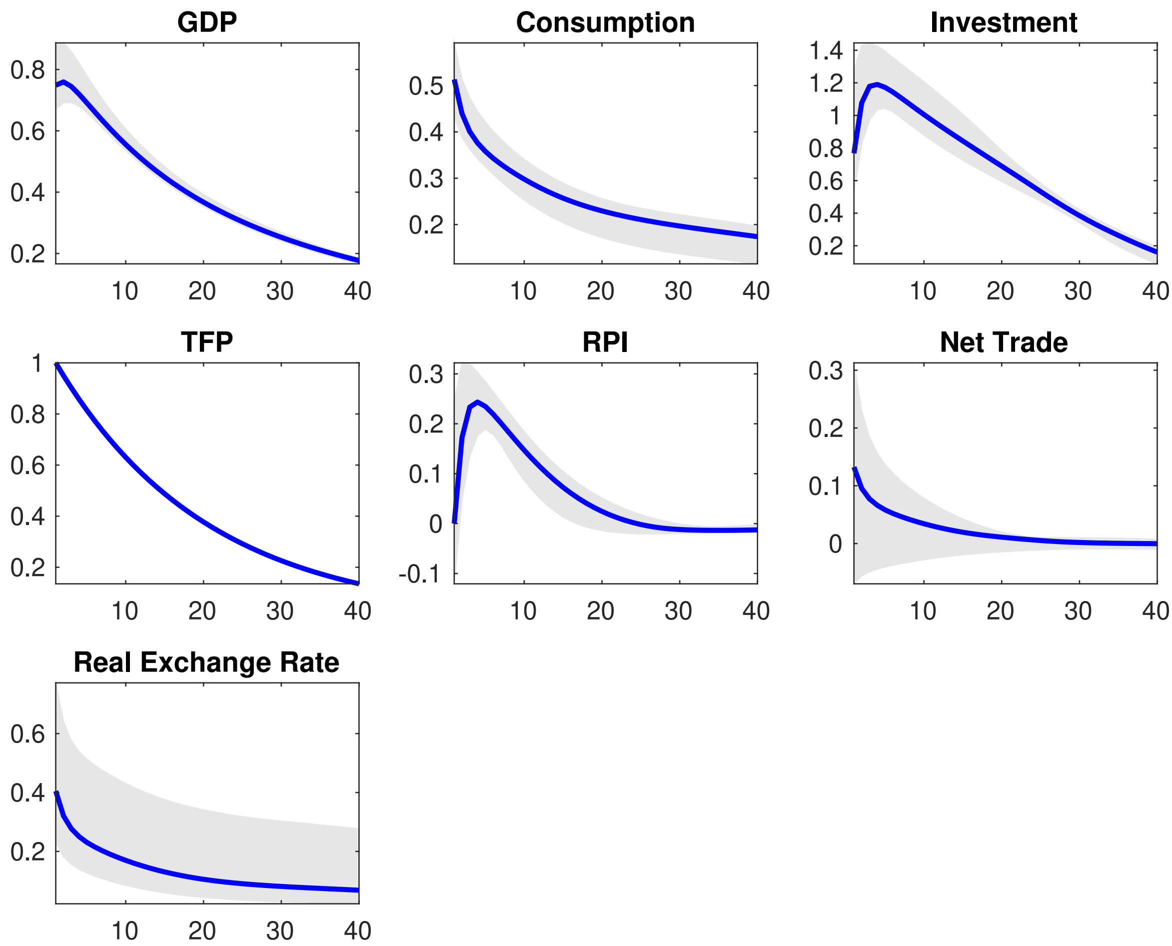
Parameter	Description	Value (mean)	(lower, upper)/Source
β	Discount rate	0.995	real interest rate of 2%
N	Relative size of the home economy	0	UK small open economy
γ	Trade openness	0.24	Import-to-GDP ratio for UK
α	Share of capital in output	0.3	
δ	Depreciation rate	0.02	standard value
ϕ_b	Bond holding cost	0.001	standard value (Kamber et al. (2017))
ϕ_0,ϕ_0^*	Utility function parameter	to yield L=1/3	standard value
σ	Relative risk aversion	1	standard value
ψ	Home investment adjustment cost (c,i)	1	(0,10)
ψ^*	Foreign investment adjustment cost (c,i)	1	(0,10)
ν_c	Home capital utilisation parameter (c,i)	10	(1.01, 20)
ν_c^*	Foreign capital utilisation parameter (c)	1.01	(1.01, 20)
ϕ	Home inverse of Frisch elasticity	2.5	(0.25, 5)
ϕ^*	Foreign inverse of Frisch elasticity	2.5	(0.25, 5)
θ	Trade elasticity	1	(0.6,6)
ξ	Home sectoral labour supply elasticity	50	(0.01,100)
ξ^*	Foreign sectoral labour supply elasticity	50	(0.01,100)
ς,ς^*	Parameter in utility function	0.5	(0.01, 0.99)

Figure 5 shows the responses of the model variables to a positive shock domestic TFP shock: a shock to Z_t, that directly affects output in both sectors. The solid blue lines are the 50th percentile impulse responses (IRFs) of the model. The grey shaded area represent the range between the 5th and the 95th percentile of the distribution of impulse responses.

A temporary shock to Z_t raises GDP, ie it increases output of both production sectors. Consumption as well as economy-wide investment rises. The relative price of investment goods, P_i/P_c, rises, given the increase in demand for investment goods. The real exchange rate depreciates (increases) following a domestic TFP shock. These results are robust throughout the parameter range analysed. The model is somewhat more ambiguous about the response of net trade. The 5th to 95th percentile range allows for both a deterioration as well as an improvement to net trade following a positive TFP shock.

[35] Figure A1 in the Appendix show the distribution of parameter draws

Figure 5: Responses of the DSGE model to a domestic TFP shock.

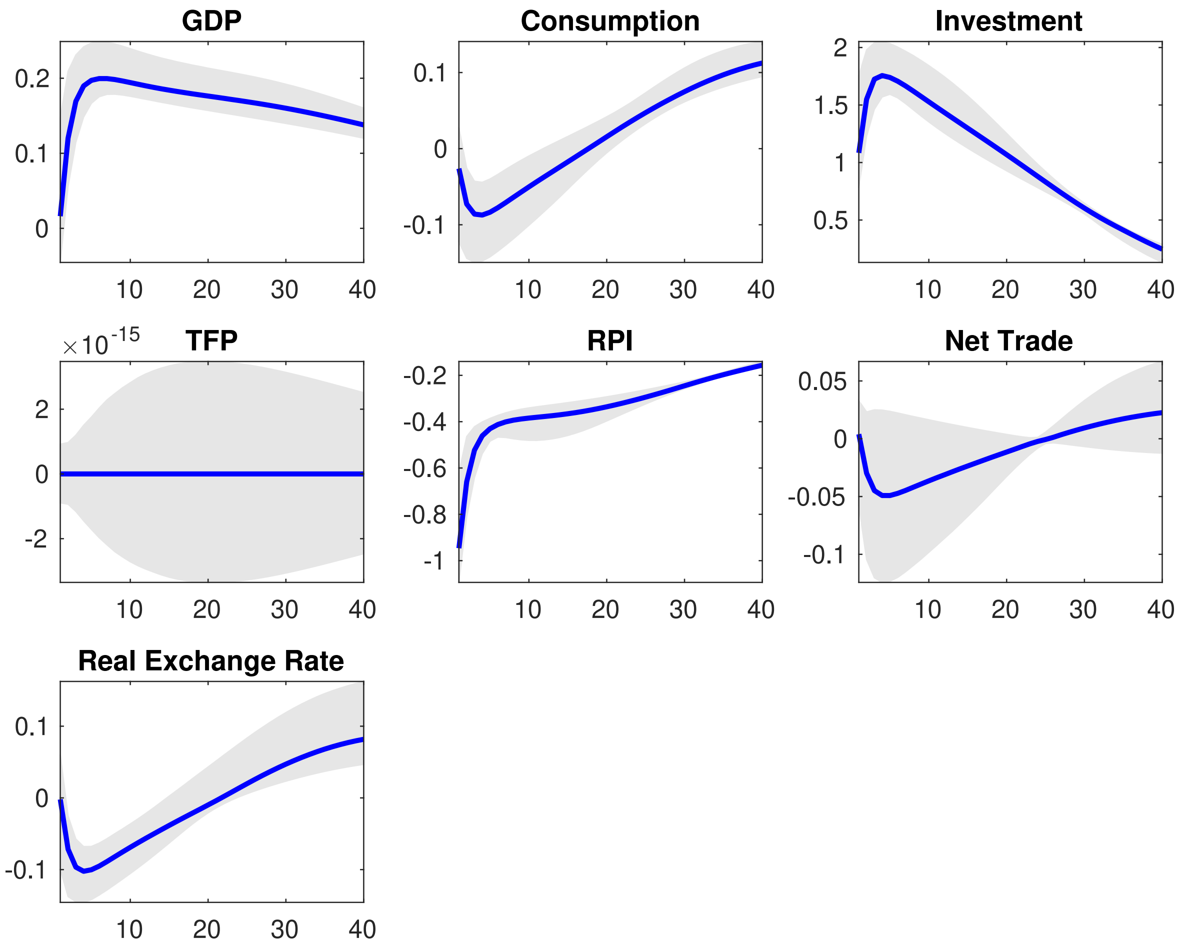


Notes: i) The solid black lines are the median impulse responses of the DSGE model to a temporary shock to domestic economy-wide TFP. ii) The shade areas show the 5 to 95% distribution of parameter draws.

Figure 6 shows the impulse responses to an unexpected and temporary increase in investment-sector specific technology, V_t. This raises output of the investment goods producing sector and reduces the relative price of investment goods. This decline in the relative price stimulates investment in both the consumption and the investment goods producing sectors. Real GDP, measured as the sum of consumption and investment goods output (where real GDP is deflated by the domestic consumption deflator) rises in response to a positive productivity shock specific to the investment goods sector. Consumption declines on impact, but rises after about 20 quarters and does so for virtually all parameter combinations. The consumption-based real exchange rate appreciates. The response of the real exchange rate following temporary domestic supply shocks, be it shocks to the consumption or the investment goods producing sectors, is in line with consumption risk-sharing. Given the small open economy assumption that domestic shocks do not affect the foreign economy, the real exchange rate and domestic consumption co-move. The response of net trade is somewhat more ambiguous. The parameter space under consideration allows for both an initial improvement as well as an initial deterioration of net trade. A rise in V_t shifts labour inputs from the consumption into the investment goods sector, causing output of the consumption goods sector to decline. Depending on the size of this decline in output, relative to the decline in consumption, the net trade can move in either direction.

The real appreciation will add to the tendency of net trade to deteriorate by shifting demand away from home-produced consumption goods to foreign-produced goods.

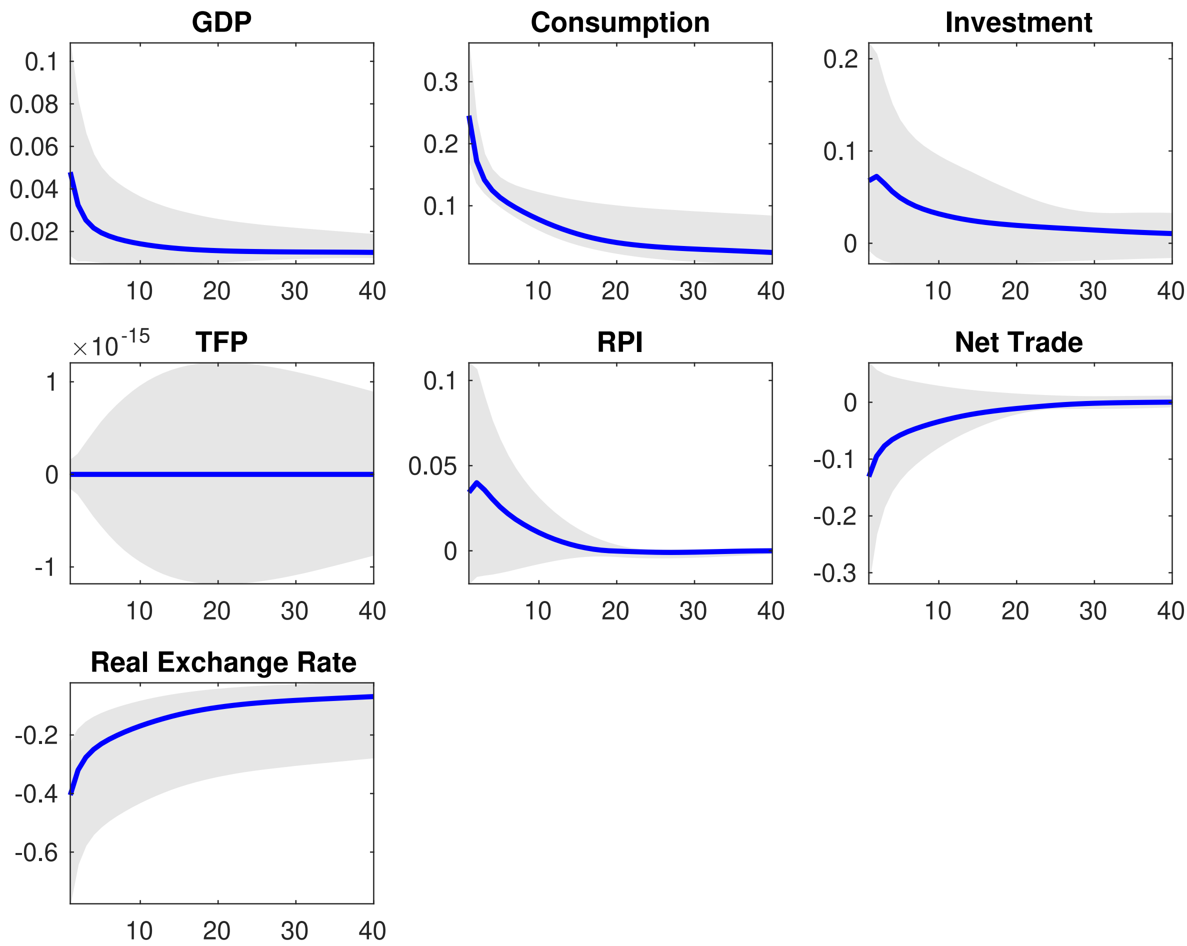
Figure 6: Responses of the DSGE model to a domestic IST shock.



Notes: i) The solid black lines are the median impulse responses of the DSGE model to a temporary shock to TFP in the domestic investment goods producing sector (an IST shock). ii) The shaded areas show the 5 to 95% distribution of parameter draws.

Figures 7 and 8 show the responses of domestic variables to foreign economy-wide and investment-sector specific productivity shocks. There are two ways in which foreign shocks affect domestic economy: directly via an increase in demand for home-produced goods and indirectly via the real exchange rate. Since a domestic shock to economy-wide TFP causes the real exchange rate to depreciate, a shock originating in the foreign economy, Figure 7, will cause the real exchange rate to appreciate. An appreciation of the real exchange rate acts like a positive wealth shock, increasing demand for both home and foreign-produced goods. An appreciation will also increase domestic output and demand for investment goods - for a good proportion of the parameter space. The ambiguity in the response of investment is reflected in the response of the relative price of investment goods. Both an increase as well as a decrease in the relative price are possible in the model. The response of net trade is ambiguous, depending to the precise combination of model parameters.

Figure 7: Responses of the DSGE model to a foreign TFP shock.

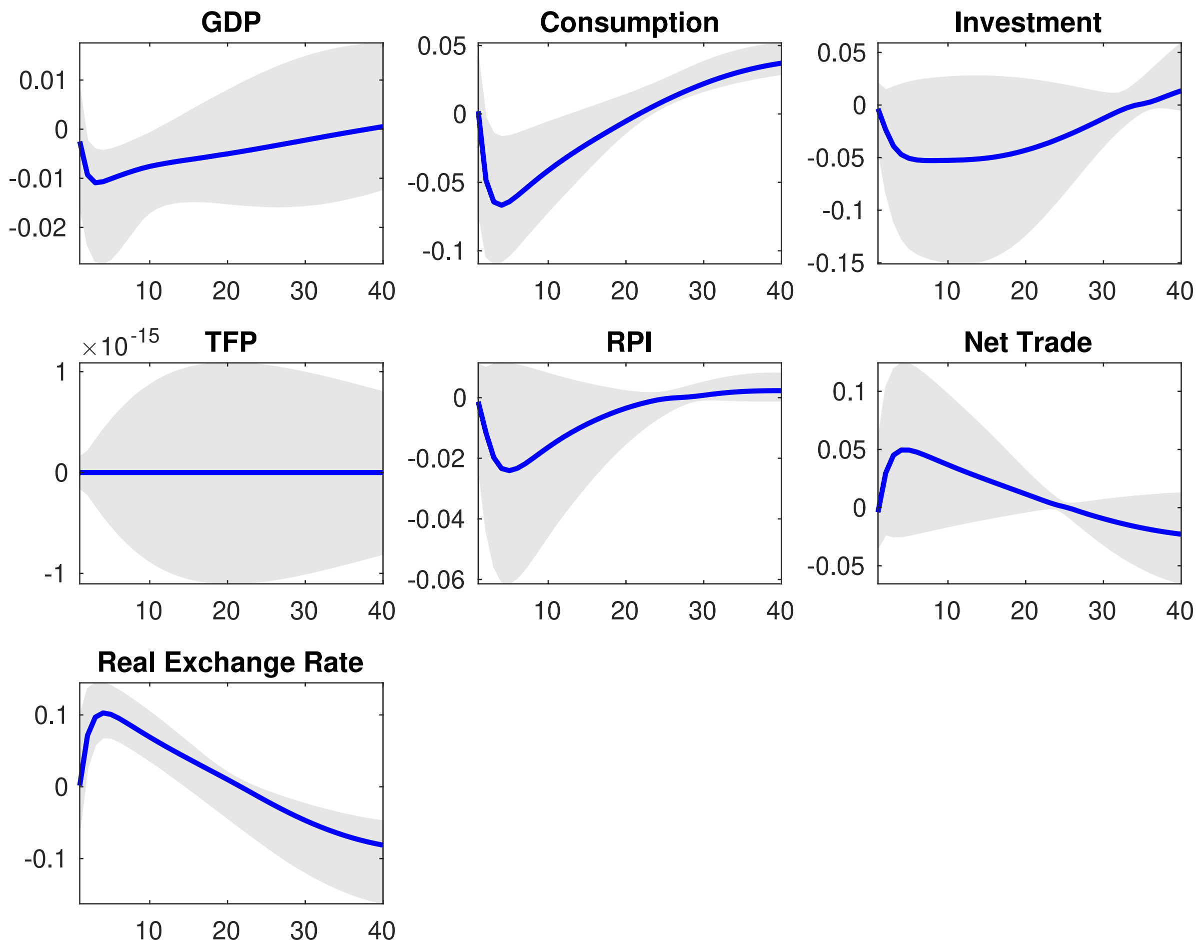


Notes: i) The solid black lines are the median impulse responses of the DSGE model to a temporary shock to foreign economy-wide TFP. ii) The shaded areas show the 5 to 95% distribution of parameter draws.

A rise in foreign investment-sector specific technology, as shown in Figure 8 is associated with a depreciation of the real exchange rate. The depreciation acts like a negative wealth shock, reducing consumption, investment as well as GDP. The response of investment, GDP, the relative price of investment goods and net trade are ambiguous. There are calibrations consistent with both increase as well as declines in these variables.

Having analysed the structural model's responses to home and foreign TFP and IST shocks, for a large parameter space, we can use these these impulse responses to derive a set of sign restrictions for our BGVAR.

Figure 8: Responses of the DSGE model to a foreign IST shock.



Notes: i) The solid black lines are the median impulse responses of the DSGE model to a temporary shock to TFP in the foreign investment goods producing sector (an IST shock). ii) The shade areas show the 5 to 95% distribution of parameter draws.

5.3 Identification of the short-run cyclical shock: GVAR model

This section use a Bayesian GVAR model to identify and simulated the impact of transitory domestic and foreign TFP and IST shock. The GVAR developed by Pesaran et al. (2004) and extended by Dees et al. (2007) considers the multi country dimension of the global economy and the estimation of spillover effects that might exist across countries. A further advantage of Bayesian GVAR is that it accounts both for parameter and model uncertainty using the Stochastic Search Variable Selection (SSVS) approach introduced by George et al. (2008). We also consider shock uncertainty by allowing the variance covariance matrix of area-specific errors to follow a stochastic volatility (SV) process. Detailed description of the GVAR model and identification of domestic and foreign productivity shocks are presented in Appendix B.

Alternative models that could be used to account for the multi-country dimension of the problems are FAVAR, panel VAR (PVAR) and large Bayesian VAR (BVAR) models. However, while PVARs become operational through parameter shrinkage (when dynamic interdependence is present), FAVAR models condense information of a large number of variables into a small number of unobserved factors.[36] In contrast, the GVAR gets around the problem of dimensionality by decomposing large-dimensioned VARS into smaller conditional models connected by cross-sectional averages. Therefore, rather than restricting the dynamics of distinct national sub-models, the GVAR imposes an intuitive structure on cross-sectional interlinkages (Pesaran, 2015).

5.4 The impact of cyclical productivity shocks: a GVAR impulse responses

While there is strong evidence that productivity trend shocks have important level or long-run effects on domestic variables; stationary (cyclical) productivity shocks can be an important source of business cycle fluctuations. Based on GVAR estimates, we gauge the influence of domestic productivity-TFP and IST-shocks. We also measure the impact of international productivity shocks originating from EU countries. We identify both domestic and international shock using sign restrictions. Table 6 presents the sign restrictions implied by our structural model. We use these sign restrictions to identify the impulse responses of short-run shocks generated by the estimation of the GVAR model.

Table 6: Identification of shocks through sign restrictions

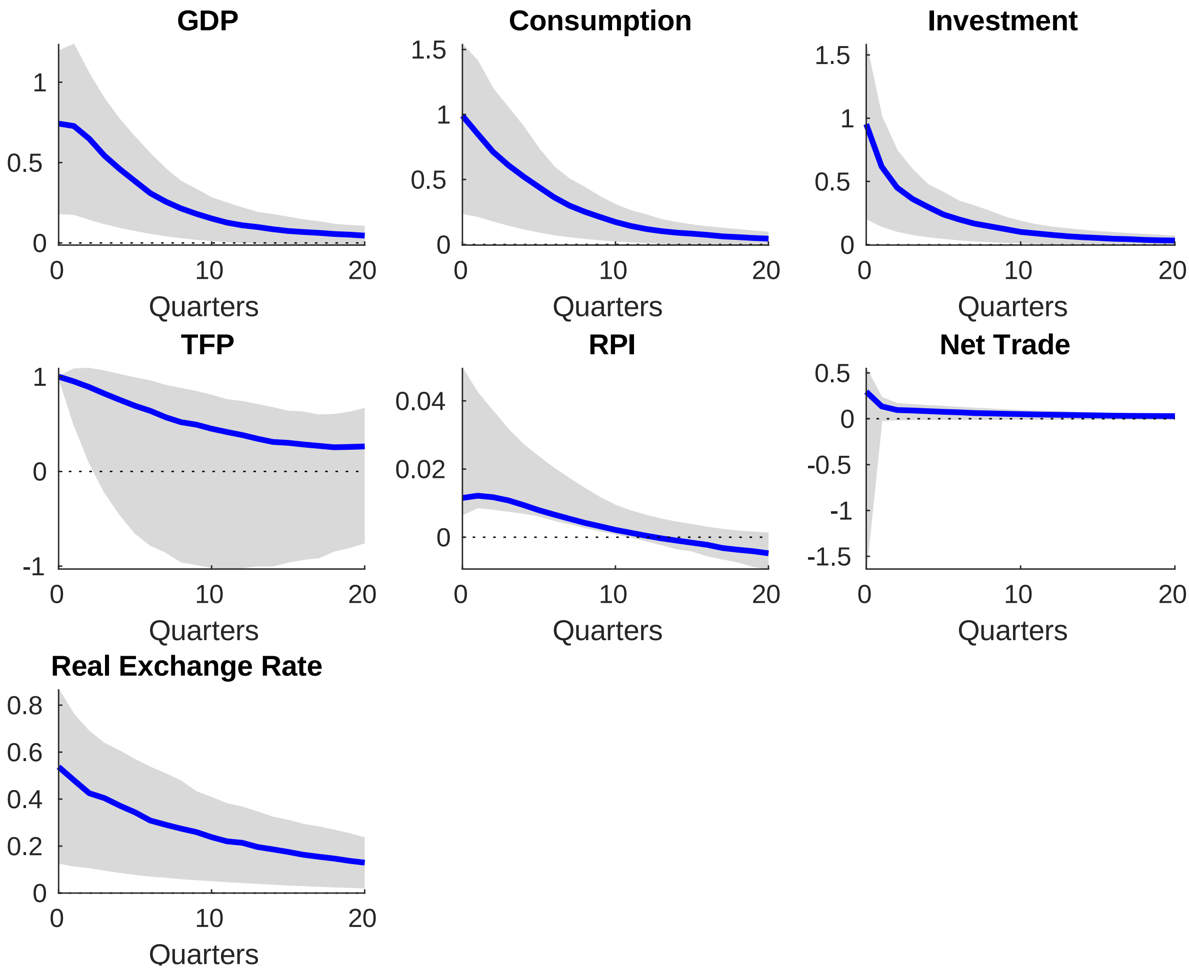
Scenario/Variable	GDP	Consumption	Investment	TFP	RPI	Exchange rate
Positive shock to TFP						
Response to home variables	+	+	+	+	+	+
Positive shock to IST						
Response to home variables	+	-	+	0	-	-
Positive shock to TFP*						
Response to home variables	+	+	+	0	+	-
Positive shock to IST*						
Response to home variables	0	-	0	0	0	+

Notes: i) + indicates a variable rises, - indicates a variable falls and 0 indicates that no restriction is imposed. ii) A fall in the real exchange rate indicates a real appreciation and a rise a real depreciation. iii) Home variables are the UK domestic variables and foreign variables are the EU and US domestic variables. iv) In all cases, the responses of net trade are left unrestricted.

Figure 9 presents the impulse responses to a domestic (UK) TFP shock. Results are qualitatively similar to the responses generated by the calibration of domestic TFP shocks presented in Figure 5. There is a strong and persistent positive response of output, which due to wealth effects leads to an increase consumption for more than two years following the shock. Investment increases due to higher returns on capital following a positive TFP shocks. The RPI increases because of an increased demand for investment. The real exchange depreciates and consumption rises. This co-movement suggests a degree of consumption risk-sharing. Although there is a depreciation of real exchange rate, net trade remains is not very responsive to the real exchange rate, probably reflecting the offsetting effects of an increase of aggregate demand.

[36] Furthermore, Pesaran (2015) contends that when the cross-sectional dimension is big, FAVAR cannot be computed consistently.

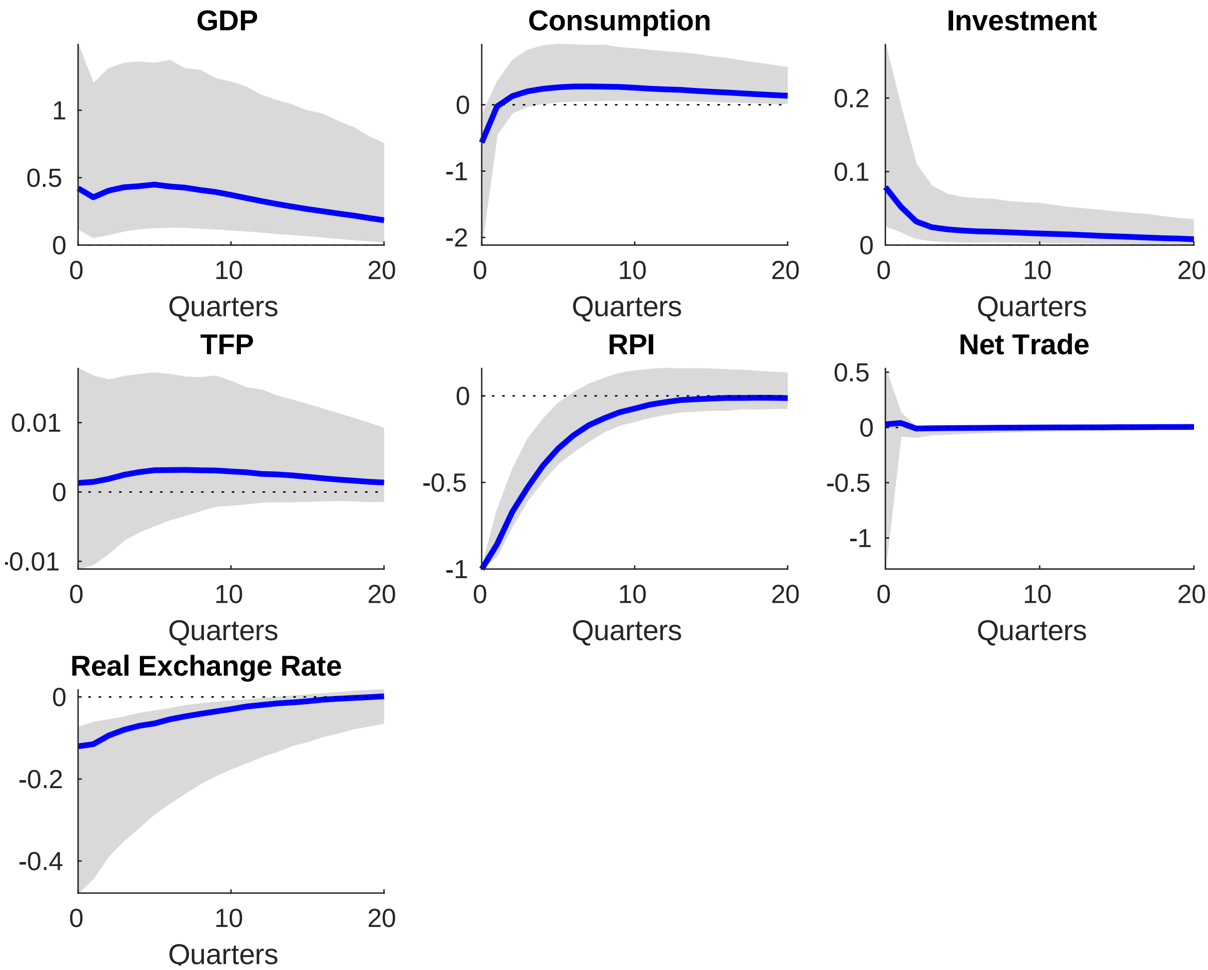
Figure 9: Responses of UK variables to a shock in the domestic TFP.



Notes: i) The blue lines denotes the median response and the grey area the 68% credible set.

Figure 10 shows impulse responses of the UK variables to a domestic IST shock. A positive shock to IST implies a decline in the relative price of investment goods, which causes investments to increase in both sectors: consumption and investment good producing sectors. Real GDP increases reflecting the positive response of consumption and investment, real GDP consisting mostly of consumption and investment. Consumption falls on impact (albeit the credible set includes zero) but becomes positive and persistent four quarters following the RPI shock. The real exchange rate appreciates on impact and takes around 10 quarters to return to its steady state value. The positive correlation between real exchange rate and consumption is again consistent with consumption risk-sharing. The response of net trade is muted, which consistent with the calibrated responses presented in Figure 6. While the increase of output is higher than the aggregate response of consumption and investment, the appreciation of the real exchange rate will deteriorate net trade by switching demand from domestically-produced consumption goods to foreign-produced goods. Therefore, countervailing responses of supply and demand variables used in our model lead to a muted response of net trade. Finally, in line with the model's predictions, domestic TFP remains unaffected by a domestic RPI shock.

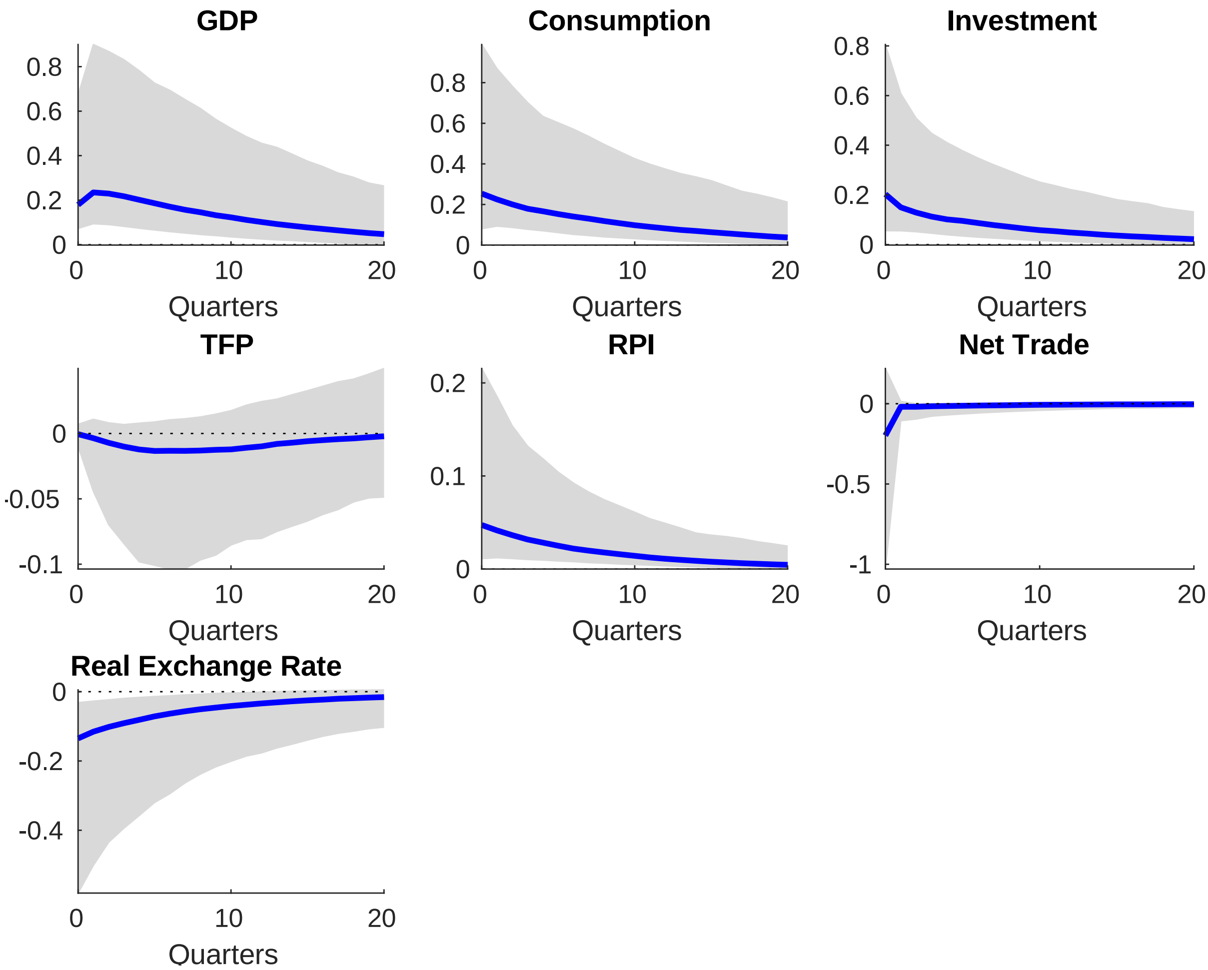
Figure 10: Responses of UK variables to a shock in the domestic IST.



Notes: i) The blue lines denotes the median response and the grey area the 68% credible set.

Figure 11 show the impulses responses following a foreign (EU) TFP shock. Consistent with our structural model, a TFP shock originating to a foreign country will lead to a domestic real appreciation. There is also a strong positive response of consumption probably driven by the positive wealth effects of real exchange rate appreciation. We also observe a positive response of investment consistent with the argument of Corsetti and Müller (2006) that an appreciation of real exchange will will increase the return and therefore the demand for domestic investment. GDP increases following a EU TFP shock reflecting the positive response of consumption and investment. Domestic TFP and net trade as predicted by the model remain unresponsive to foreign TFP shock.

Figure 11: Responses of UK variables to a shock in the EU TFP.

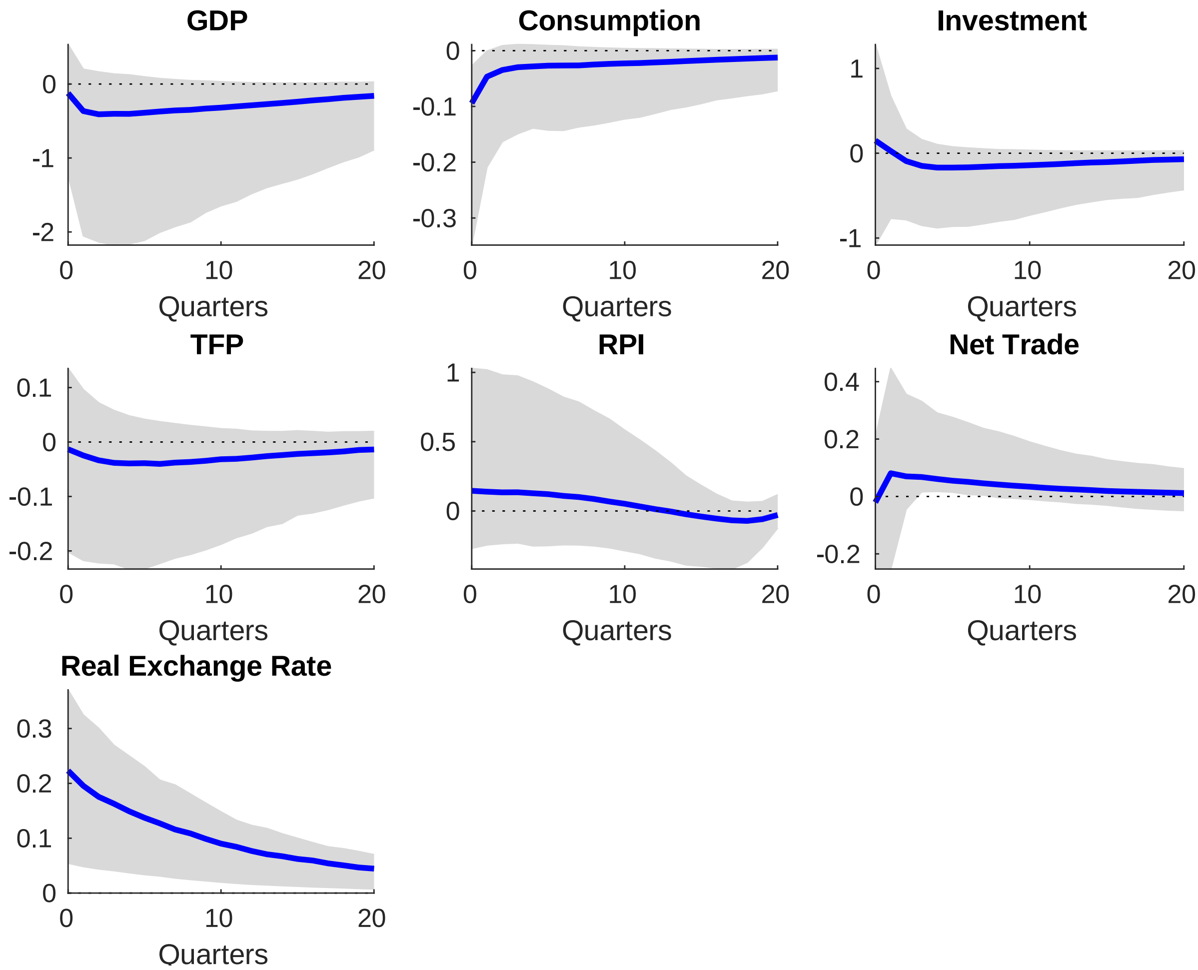


Notes: i) The blue lines denotes the median response and the grey area the 68% credible set.

Figure 12 shows the response of domestic (UK) variables to a foreign (EU) IST shock. A rise in foreign RPI is associated with depreciation of domestic real exchange rate, which is consistent with the observed positive response of the UK real exchange rate as shown in Figure 12. A real exchange rate depreciation has a negative wealth effects and can have negative effects on consumption and investment.[37] Although following a foreign IST shock consumption, as shown in Figure 12, falls, it is not significant in the sense that the credible set includes zero. The rest of the domestic variables- real GDP, net trade,domestic TFP and RPI- as predicted by our structural model remain unaffected by a foreign IST shock.

[37]The later are driven by the negative effect that a depreciation of real exchange rate has on demand for investment.

Figure 12: Responses of UK variables to a shock in the EU IST.



Notes: i) The blue lines denotes the median response and the grey area the 68% credible set.

6 Conclusions

The level of productivity is a key input into the policy-making decision process: it shows how much demand can grow without lifting inflation. Evidence also that the main factor that underlined the European debt crisis was the plunge of productivity in South euro area countries raise major concerns about the impact that the UK's post-crisis productivity downturn on the UK's business cycle. The role of productivity becomes predominant in the post-Brexit era. This so because while stronger capital accumulation explains the recovery of productivity in the euro area, low capital investment was one of the main cause of the UK's productivity puzzle which in turn undermined domestic absorption -consumption, investment- and net trade.

In this paper we have analysed the impacts of productivity shocks (TFP and IST) on the business cycle dynamics of a small open economy, using the UK as an example. We do so by accounting for the role of domestic and foreign TFP and IST shocks as a source of the UK's business cycle fluctuations. We use an econometric framework that distinguishes between trend and cyclical shocks. In particular, we use a two-step approach to account both for the long-run and short-run effects of productivity shocks. In the first step, we use a cointegration analysis where trend shocks are proxied by shocks to strongly exogenous variables. To the best of our knowledge, this is the first paper where permanent shocks are identified based on strong exogeneity tests. Most of the paper in the literature either calibrate permanent productivity shocks or identify them based on untested assumption of cointegration between the TFP and IST variable.[38] In the second step, we use a Bayesian structural GVAR to simulate stationary productivity shocks identified based on theory-consistent sign restrictions. In both steps, we account for spillover effects by the construction of foreign variables. When we focus on the impact of cyclical shocks, the GVAR model provides a multi-country framework that accounts for second-order spillover effects.[39] We classified our results into three groups: i) the steady-state impact of productivity; the impact of productivity trend shocks; and the effects of cyclical productivity shocks.

In steady-state, there is evidence that domestic TFP and the real exchange rate have strong positive effects on trade. Our results implies that positive TFP shock increases output and leads to a depreciation of real exchange and improvement of trade balance. We also observed that an increase of TFP will lift domestic consumption, which, however, affects trade negatively.[40] It is also conspicuous the negative effects of financial crisis both on output and trade. This is might be driven by the plunge of labour productivity in the post-crisis period.

When we focus on the impact of trend shocks there are three main observations. First, there is evidence that a positive domestic TFP shock leads to an increase in output, consumption and investment and trade balance deteriorate. The real exchange rate appreciates when foreign variables constructed as the weighted average of the corresponding EU and US variables and depreciates when the EU variables used as proxies of foreign variables. Deterioration of the trade balance is driven by an increase in domestic absorption. Second, in a model where only EU factors were employed as proxies for foreign variables, a domestic IST shock raises both consumption and investment but the real exchange rate response is either weak or appreciates. Therefore, the observed negative response of net trade is due to both the upward lift of domestic demand following the domestic IST shock and the appreciation of real exchange rate. Third, spillover effects from the US TFP shock leads to higher domestic output and domestic demand -consumption and investment- but trade weaken reflecting the positive response of domestic absorption and the muted response of real exchange rate following a TFP trend shock in the US.

The important role of permanent productivity shocks on the UK's business cycle fluctuation is justified by a variance decomposition which shows that both domestic and foreign TFP and IST shocks account for 68 percent of the UK's GDP variability. The remaining 32 percent is probably explained by foreign IST and TFP shocks, which have long-run impact on domestic consumption and investment.

When we turn to the impact of cyclical productivity shocks to the UK economy, we mainly observe that while productivity shocks -domestic and foreign- affect most the domestic variables the net trade remains unresponsive. Therefore, unlike permanent productivity shocks, transitory shock do not affect

[38] Benati (2014) test for cointegration between TFP and IST in bivariate framework. Ireland (2013) calibrate a two-country stochastic growth model where both TFP and IST were non-stationary and cointegrated.

[39] Although bi-country models used in the literature to simulate international spillover effects captures direct spillover effects they overlooked indirect effects that might exist.

[40] IST shocks, both domestic and foreign, do not seems to have a long-run impact on domestic variables. Exception to this is the model where the foreign variables are proxied by the euro area and Norway variables. Then there is evidence that domestic IST shock has negative impact both on trade and real GDP per capita.

the UK's trade balance. However, transitory productivity shock remain a source of domestic business cycle fluctuation by affecting domestic demand and supply.

In summary, our findings demonstrate that permanent productivity shocks lead the real exchange rate to appreciate and domestic absorption to rise, both of which have a negative impact on the trade balance. Alternatively, transitory shocks affect only domestic variables -output, consumption and investment - without having any impact on trade. Our results also suggest that uncertainty generated mainly by the financial crisis and potentially by the Brexit referendum might have hindered investment and undermined productivity. Therefore, there are two policy implications drawn from our analysis. First, it is of paramount importance for policy makers to identify the nature of productivity shocks: permanent vs transitory. Second, policymakers should ensure financial stability and minimise any factors that could raise uncertainty about the economic outlook.

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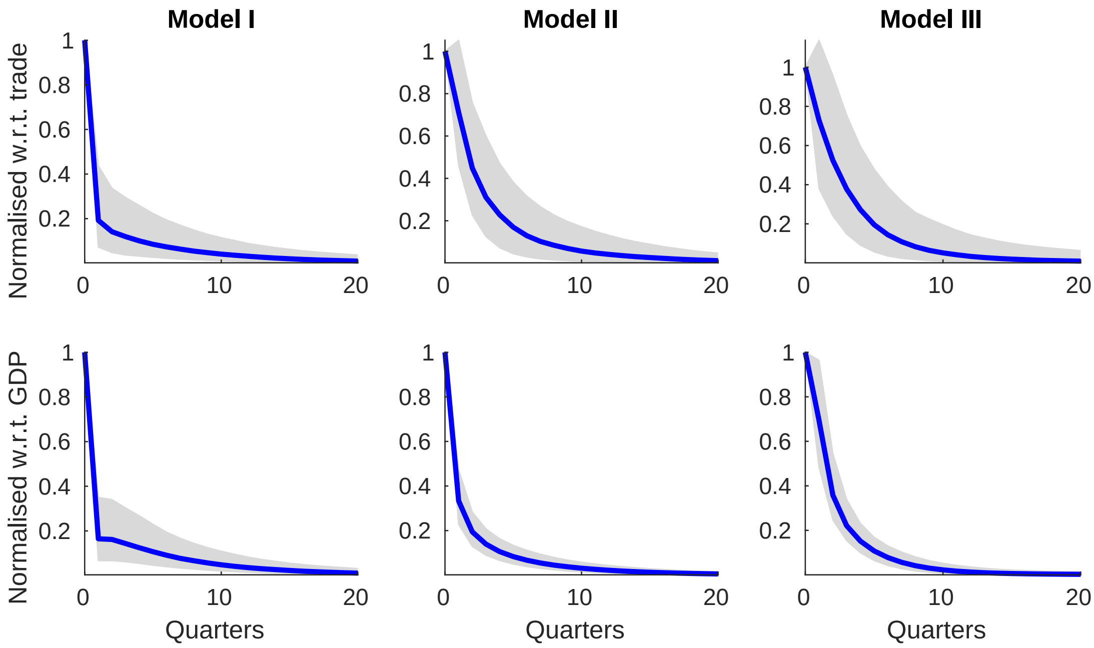
A Additional Tables and Figures

Figure A1: Persistence profiles of system wide shocks for the identified cointegrating vectors.



Notes: In model I the UK foreign variables are a weighted average their respective EU and US domestic variables. In models II and III the UK foreign variables are the domestic EU and US variables, respectively. W.r.t stands for *with respect to*.

B Appendix: The GVAR model: Estimation and Identification

B.1 GVAR model

The GVAR introduced by Pesaran et al. (2004) and extended by Dees et al. (2007) consists of two steps. The first step accounts for cross-section heterogeneity by solving country- specific VARX* models while the second step stacked country-specific model VARX* model into a global model, which is used to simulate the dynamic propagation of shocks emanated by any country in our sample.

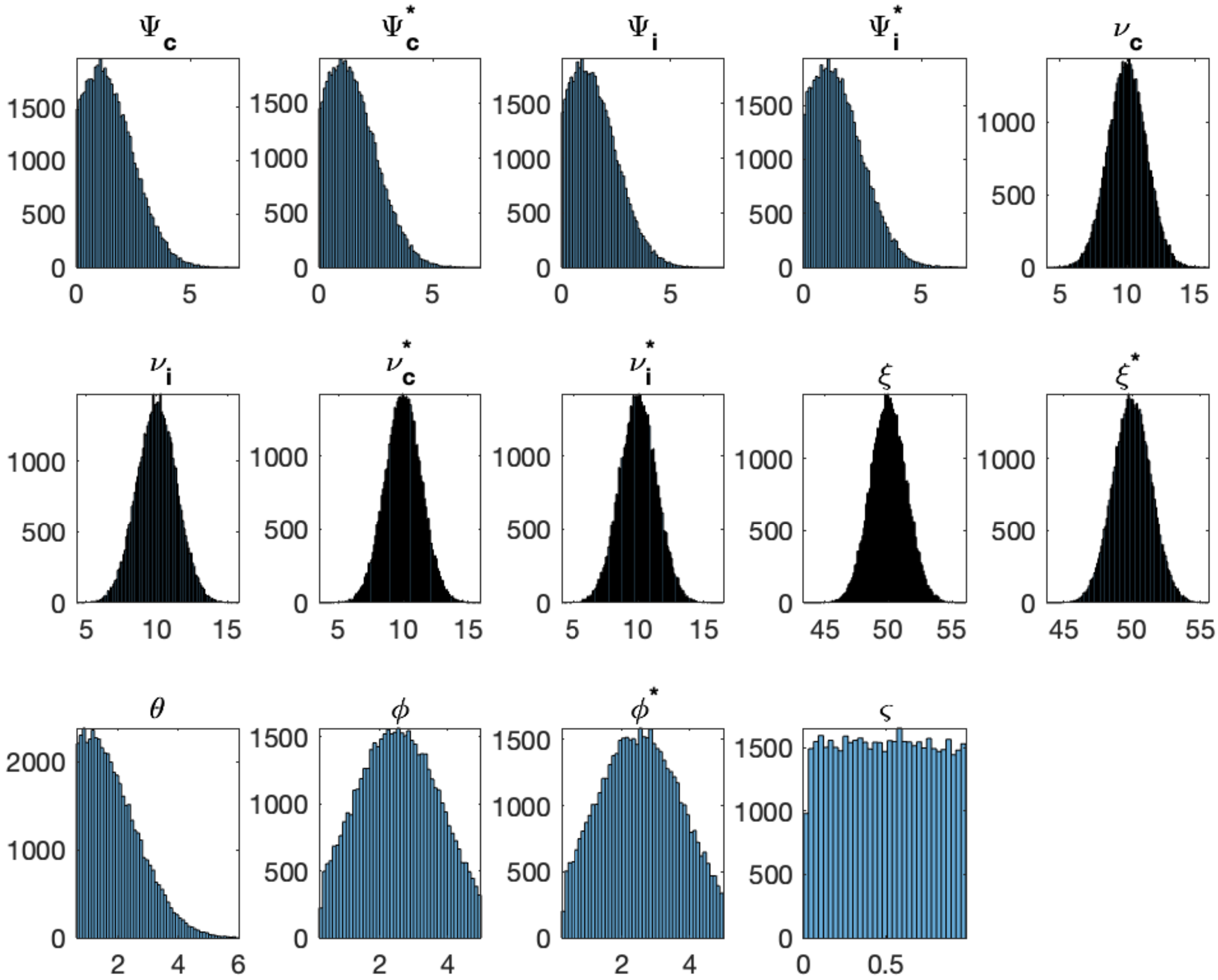
We consider in 2 that $p = q = 1$ and we solve the country-specific VARX*(1,1) models by writing (2) as:

$$\boldsymbol{A}_{i0}\boldsymbol{z}_{it} = \boldsymbol{a}_{i0} + \boldsymbol{a}_{i1}t + \boldsymbol{A}_{i1}\boldsymbol{z}_{i,t-l} + \boldsymbol{\delta}_i\boldsymbol{d}_t + \boldsymbol{u}_{it} \tag{28}$$

where $\boldsymbol{z}_{i,t} = (\boldsymbol{x}'_{i,t}\ , \boldsymbol{x}^{*\prime}_{i,t})'$ is a $k_i + k_i^*$ dimensional vector, $\boldsymbol{A}_{i0} = (\boldsymbol{I}_{ki}, -\boldsymbol{\Lambda}_{i0})$, $\boldsymbol{A}_{i1} = (\boldsymbol{\Phi}_{i1}, \boldsymbol{\Lambda}_{i1})$. Note that by using the $(k_i + k_i^*) \times k$ link matrix $\boldsymbol{W}_i = [\boldsymbol{E}'_i, \widetilde{\boldsymbol{W}}_i]$, where $\boldsymbol{E}_i$ and $\widetilde{\boldsymbol{W}}_i$ are $k \times k_i$ and $k \times k_i^*$ dimensional selection matrices respectively, we can write $\boldsymbol{z}_{it}$ in terms of a k-dimensional global vector $\boldsymbol{x}_t = (\boldsymbol{x}'_{0t}, \boldsymbol{x}'_{1t}, \ldots, \boldsymbol{x}'_{Nt})'$ where $k = \sum_{i=0}^{N} k_i$ as:

$$\boldsymbol{z}_{it} = \boldsymbol{W}_i\boldsymbol{x}_t : i = 0, \ldots, N \tag{29}$$

Figure A2: Distribution of parameter draws.



This allows us to write (28) as

$$\boldsymbol{A}_{i0}\boldsymbol{W}_i\boldsymbol{x}_t = \boldsymbol{a}_{i0} + \boldsymbol{a}_{i1}t + \boldsymbol{A}_{i1}\boldsymbol{W}_i\boldsymbol{x}_{t-1} + \boldsymbol{\delta}_i\boldsymbol{d}_t + \boldsymbol{u}_{it} \tag{30}$$

Stacking each country-specific model in (30) leads to:

$$\boldsymbol{G}_0\boldsymbol{x}_t = \boldsymbol{a}_0 + \boldsymbol{a}_1 t + \boldsymbol{G}_1\boldsymbol{x}_{t-1} + \boldsymbol{\Upsilon}\boldsymbol{D}_t + \boldsymbol{u}_t \tag{31}$$

where $\boldsymbol{a}_0 = (\boldsymbol{a}_{00}', \boldsymbol{a}_{10}', \ldots, \boldsymbol{a}_{N0}')'$, $\boldsymbol{a}_1 = (\boldsymbol{a}_{01}', \boldsymbol{a}_{11}', \ldots, \boldsymbol{a}_{N1}')'$, $\boldsymbol{u}_t = (\boldsymbol{u}_{0t}', \boldsymbol{u}_{1t}', \ldots, \boldsymbol{u}_{Nt}')'$, $\boldsymbol{G}_h = [(\boldsymbol{A}_{0h}\boldsymbol{W}_0)'$, $(\boldsymbol{A}_{1h}\boldsymbol{W}_i)', \ldots, (\boldsymbol{A}_{Nh}\boldsymbol{W}_N)']$, and $\boldsymbol{\Upsilon} = [\boldsymbol{\delta}_0', \boldsymbol{\delta}_1', \ldots, \boldsymbol{\delta}_N']$ for $h = 0, 1$. If $\boldsymbol{G}_0$ is invertible, then we pre-multiply (31) by $\boldsymbol{G}_0^{-1}$ to obtain the GVAR model

$$\boldsymbol{x}_t = \boldsymbol{F}_1\boldsymbol{x}_{t-1} + \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \boldsymbol{\Psi}\boldsymbol{D}_t + \boldsymbol{\epsilon}_t \tag{32}$$

where $\boldsymbol{F}_1 = \boldsymbol{G}_0^{-1}\boldsymbol{G}_1$, $\boldsymbol{b}_j = \boldsymbol{G}_0^{-1}\boldsymbol{a}_{0j}$ for $j = 0, 1$, $\boldsymbol{\Psi} = \boldsymbol{G}^{-1}\boldsymbol{\Upsilon}$ and $\boldsymbol{\epsilon}_t = \boldsymbol{G}_0^{-1}\boldsymbol{u}_t$. The GVAR model (32)

Table A1: Posterior inclusion probabilities and posterior distribution of the coefficients in the investment equation of the UK model.

	PIP	Q16	Q50	Q84
y_{t-1}	0.628	0.244	0.281	0.319
c_{t-1}	0.540	-0.004	0.004	0.017
gcf_{t-1}	0.910	0.328	0.404	0.470
tfp_{t-1}	0.542	-1.913	-0.149	0.075
ist_{t-1}	0.300	-0.050	-0.010	0.020
ntr_{t-1}	0.637	-0.046	-0.005	0.001
rer_{t-1}	0.528	-0.022	-0.004	0.003
$tfp*_t$	0.349	-3.880	-0.758	0.129
$ist*_t$	0.515	-1.235	-0.838	-0.122
$tfp*_{t-1}$	0.411	-4.019	-0.665	0.079
$ist*_{t-1}$	0.282	-0.158	-0.036	0.051
ecm_{1t}	0.335	-0.004	-0.000	0.003
ecm_{2t}	0.557	-0.512	-0.440	-0.368
$d16Q1$	0.609	-0.058	-0.001	0.000

Notes: i) All results refer to coefficients of the investment equation in the UK country model. ii) The first column presents the posterior inclusion probabilities. iii) Columns 2-4 present the 16%, 50% and 86% quantiles of the posterior distribution.

is solved recursively and used for the impulse response function analysis.

B.2 Model uncertainty and time-variation

Most GVAR literature assumes a fixed covariance matrix of $\boldsymbol{u}_{it}$. However, uncertainty after the financial crisis and the Brexit referendum raises questions about the assumption of a constant volatility.[41] We estimate a GVAR where we assume the variance covariance of $\boldsymbol{u}_{it}$ is time-varying. Based on Huber (2016), we consider a stochastic volatility model:

$$\begin{aligned}
\Sigma_{it} &= \exp(h_{it})\Sigma_i \\
h_{it} &= c_i + \rho_i(h_{it} - c_i) + e_{it}u_{it}, \qquad \text{with} \qquad e_{it} \sim N(0,1),
\end{aligned}$$

where h_{it} denotes the country-specific volatility, c_i is the is the unconditional variance, ρ_i is an autoregressive parameter and e_{it} is the variance of the log-volatility.

We also take into account the uncertainty about the variables included into country-specific model by using Stochastic Search Variable Selection (SSVS) approach introduced by George et al. (2008). In doing so, we estimate a Bayesian GVAR as introduced by Cuaresma et al. (2016) and Dovern et al. (2016). For convenience of prior implementation, we work with the stack $K_i = 2k_i + k_i^2 + 2(k_i \times k_i^*)$-dimensional vector of coefficients for country i: $\Xi_i = (\boldsymbol{a}_{i0}^{'}, \boldsymbol{a}_{i1}^{'}, vec(\boldsymbol{\Phi}_{i1})^{'}, vec(\boldsymbol{\Lambda}_{i0})^{'}, vec(\boldsymbol{\Lambda}_{i1})^{'})$. The

[41] Furthermore, Sims and Zha (2006) show that overlooking changing volatilities will lead to a model with time-varying parameters.

SSVS imposes a mixture of Normal on each coefficient j of Ξ_i :

$$\Xi_{ij}|\gamma_{ij} \sim (1-\gamma_{ij})N(0,\tau_{0,ij}) + \gamma_{ij}N(0,\tau_{1,ij}),$$

where γ_{ij} is a binary random variable equal to 1 if Ξ_{ij} is included in the country model and zero otherwise. The prior variances, $\tau_{0,ij}$ and $\tau_{1,ij}$, are chosen such as $\tau_{0,ij} < \tau_{1,ij}$.[42]

B.3 Identification of short-run shocks

Identification of structural shock v_{it} is related to the reduced form error in (2) through the transformation $v_{it} = B_{i0}u_{it}$:

$$\begin{aligned} \boldsymbol{B}_{i0}\boldsymbol{x}_{it} &= \boldsymbol{a}_{i0} + \boldsymbol{B}_{i0}\boldsymbol{a}_{i1}t + \boldsymbol{\Phi}_{i1}\boldsymbol{x}_{t-1} + \boldsymbol{B}_{i0}\boldsymbol{\Lambda}_{i0}\boldsymbol{x}^*_{it} + \boldsymbol{B}_{i0}\boldsymbol{\Lambda}_{i1}\boldsymbol{x}^*_{i,t-1} + \boldsymbol{B}_{i0}\boldsymbol{\delta}_{i0}\boldsymbol{d}_t + \boldsymbol{B}_{i0}\boldsymbol{u}_{it} \\ \boldsymbol{B}_{i0}\boldsymbol{x}_{it} &= \widetilde{\boldsymbol{a}}_{i0} + \widetilde{\boldsymbol{a}}_{i1}t + \widetilde{\boldsymbol{\Phi}}_{i1}\boldsymbol{x}_{t-1} + \widetilde{\boldsymbol{\Lambda}}_{i0}\boldsymbol{x}^*_{it} + \widetilde{\boldsymbol{\Lambda}}_{i1}\boldsymbol{x}^*_{i,t-1} + \widetilde{\boldsymbol{\delta}}_{i0}\boldsymbol{d}_t + \boldsymbol{v}_{it} \end{aligned} \tag{33}$$

where $\boldsymbol{v}_{it} \sim N(0, \boldsymbol{I}_{k_i})$. Identification of $\boldsymbol{v}_{it}$ boils down to identifying $\boldsymbol{B}_{i0}$. We set $\boldsymbol{B}_{i0}^{-1} = \boldsymbol{P}_i\boldsymbol{Q}_i$ where $\boldsymbol{P}_i$ is the Cholesky factor of $E(u_{it}u'_{it}) = \boldsymbol{\Sigma}_{u_{it}} = \boldsymbol{P}_i\boldsymbol{P}'_i$ and $\boldsymbol{Q}_i$ is a $k_i \times k_i$ orthogonal matrix, which is chosen such as $\boldsymbol{\Sigma}_{u_{it}} = \boldsymbol{P}_i^{-1}\boldsymbol{Q}_i\boldsymbol{Q}'_i\boldsymbol{P}_i^{'-1}$ satisfying theory-consistent sign restrictions.[43]

For the numeraire country region $i = 0$, we proceed to construct a $k \times k$ matrix $\boldsymbol{B}$, where the first rows and columns corresponds to $\boldsymbol{B}_{00}$: $\boldsymbol{B} = \text{diag}(\boldsymbol{B}_{00}, \boldsymbol{I}_{k1}, \ldots, \boldsymbol{I}_{k_n})$. The corresponding structural representation of GVAR is given by:

$$\begin{aligned} \boldsymbol{B}\boldsymbol{G}_0\boldsymbol{x}_t &= \boldsymbol{B}\boldsymbol{G}_1\boldsymbol{x}_{t-1} + \boldsymbol{B}\boldsymbol{\phi}_t + \sum_{l=0}^{1}\boldsymbol{B}\boldsymbol{\Delta}_1\boldsymbol{D}_{t-l} + \boldsymbol{B}\boldsymbol{u}_t \\ \widetilde{\boldsymbol{G}}_0\boldsymbol{x}_t &= \boldsymbol{F}\boldsymbol{x}_{t-1} + \widetilde{\boldsymbol{\phi}}_t + \sum_{l=0}^{1}\widetilde{\Delta}_1\boldsymbol{D}_{t-l} + \boldsymbol{v}_t \end{aligned} \tag{34}$$

where $\widetilde{\boldsymbol{G}}_0$, $\boldsymbol{F}$, $\widetilde{\boldsymbol{\Delta}}$ are coefficient matrices of structural coefficient and $\boldsymbol{v}_t$ is a $k \times 1$ vector of structural shocks with variance covariance matrix $\boldsymbol{\Sigma}_v = \text{diag}(\boldsymbol{I}_{k0}, \boldsymbol{\Sigma}_{u1}, \ldots, \boldsymbol{\Sigma}_{un})$. The impulse response function (IRF) of structural shocks $\boldsymbol{v}_t$, at horizon h, will be given by $\boldsymbol{\Psi}(h) = \boldsymbol{\Theta}(h)\boldsymbol{G}_0^{-1}\boldsymbol{P}$ where $\boldsymbol{\Theta}(h)$ can be estimated by the moving average representation of (32)

[42] The choice of small value for $\tau_{0,ij}$ implies that the variable j is constrained to be excluded from the country-specific model i. Alternatively, the choice of large value for $\tau_{1,ij}$ indicates relatively non-informative prior for Ξ_{ij}.

[43] To obtain a candidate rotation matrix we draw Q_i based on the algorithm of Rubio-Ramirez et al. (2010). For each posterior draw, we search for rotation matrix that satisfy the set of sign restrictions by considering 5000 tries. Note that for each shock we obtain, on average, 3000 rotation matrices.