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Christoph Gortz, Konstantinos Theodoridis, and Christoph Thoenissen

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Christoph Görtz
University of Birmingham

Konstantinos Theodoridis
European Stability Mechanism
Cardiff University

Christoph Thoenissen
University of Sheffield

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Abstract

We estimate a novel empirical (state-space) model to study the effects of international and domestic technology trend shocks on the UK economy. We jointly identify anticipated and unanticipated domestic and international technological innovations arising from changes in total factor productivity (TFP) and investment specific technology (IST). The long-run restrictions used to jointly identify the structural trends in the data are informed by a standard two-country structural model. Our results point to large and persistent swings in productivity. International non-stationary TFP and IST shocks explain about 30% and 24% of the variance of UK GDP, respectively. UK-specific TFP and IST shocks are somewhat less important, but still a relevant factor. Notably, it is the anticipated components of these international and domestic productivity shocks, rather than their unanticipated counterparts, which account for the bulk of the volatility in the data. We dissect the historical role of different shocks as drivers of UK labor productivity growth. We find that a decline in the contribution of international IST shocks, combined with weak domestic TFP growth, can explain the widely documented slowdown in UK labor productivity after the financial crisis. A standard two-country model implies widely-used restrictions on the relative price of investment which we find to be inconsistent with our empirical evidence that relies on a minimum of structure. We show that a two-sector version of this model with adjustment cost in investment and costly sectoral labor reallocation can capture the empirical dynamics.

Keywords: International Transmission of Productivity Shocks, Total Factor Productivity, Investment Specific Technology, Small Open Economy Dynamics, News Shocks, State Space Model.

JEL Classification: E32, E3, F41, F44.

*Görtz: University of Birmingham, Department of Economics, Email: c.g.gortz@bham.ac.uk. Theodoridis: Cardiff University, Business School, Email: theodoridisk1@cardiff.ac.uk. Thoenissen: University of Sheffield, Department of Economics, Email: c.thoenissen@sheffield.ac.uk. The views expressed in this paper solely represent those of the authors and should not be interpreted as the views of the European Stability Mechanism. We thank Christopher Gunn, Christoph Himmels, Konstantinos Mouratidis and Robert Kirkby as well as conference participants of the 2023 Computing in Economics and Finance Conference and seminar participants at the University of Sheffield, the Reserve Bank of New Zealand, Victoria University of Wellington and University of Canterbury for comments. All remaining errors are our own.

1 Introduction

How important are international productivity trend shocks for small open economies? How relevant are total factor productivity (TFP) and investment specific technology (IST) as international and country-specific sources for productivity advancements? Do these innovations in TFP and IST typically come as a surprise or are they anticipated in advance? We address these questions by estimating a novel state-space model for the United Kingdom economy. This framework allows us to uncover the dynamic effects and the empirical relevance of domestic and international productivity shocks. One of its strengths is that it can jointly identify anticipated and unanticipated IST and TFP types of international and domestic productivity shocks. This allows for new insights into the spillover effects of international productivity shocks and the relevance of domestic technological advances as drivers of aggregate fluctuations in small open economies.

We focus on the UK economy and international technological advances are identified as shocks originating in the United States. International non-stationary technology shocks explain the majority of fluctuations of UK GDP, while their domestic counterparts and other cyclical shocks also play a non-negligible role. Our results suggest that it is important to jointly account for the role of IST and TFP as both types of technological advances are relevant drivers of UK macroeconomic aggregates. Notably, this holds in particular for their anticipated components. We find the relevance of their unanticipated counterparts is extremely limited, in contrast to the prominent role they often play in structural frameworks. Our rich empirical setup points to a drop-off in international IST, combined with weak domestic TFP growth, as forces behind the widely documented slowdown in UK labor productivity since the 2007 financial crisis.

This paper sheds light onto the role of international and domestic IST and TFP shocks for UK aggregate fluctuations due to a novel empirical setup. The strength of our empirical methodology is that it allows for the joint identification of various types of non-stationary technological advances: anticipated and unanticipated international and domestic TFP and IST shocks. In particular, we consider a linear state-space Gaussian model estimated using Bayesian techniques. Prior information about the model parameters is employed during the estimation via Minnesota-type prior distributions, while the posterior distribution is approximated using Gibbs sampling and simulation smoothing techniques. The international and domestic non-stationary (TFP and IST) productivity shocks are identified during the estimation process by restricting them to be the only exogenous forces that can have a permanent effect on the economy. At the same time, small open economy restrictions are imposed to ensure that domestic shocks do not affect the international block of the model. The cyclical shocks, which are not explicitly identified, can also explain variations in macroeconomic variables. To be precise, the model includes four unobserved variables — US TFP growth, US IST growth, UK TFP growth and UK IST growth — that display a stochastic trend. These four I(1) trends are the long-term driving forces of the observables, while the remaining shocks can account for cyclical variation in the observables.

Benati (2014) finds evidence for the notion that US IST and TFP are not co-integrated and are best thought of as independent processes. Our econometric setup is informed by this evidence in that we jointly identify IST and TFP shocks, but do not impose any co-integrating relationships between these two technology shocks. Our decomposition between trend and cyclical components in the vector of observables is agnostic as it is informed by economic theory. In particular, it is informed by the relationships between non-stationary and cyclical components implied by standard two-country dynamic stochastic general equilibrium models, as in Backus et al. (1994) and Heathcote and Perri (2002).

We estimate the state-space-model for a 1971Q1-2018Q2 sample on US and UK data. The anticipated and unanticipated types of the international non-stationary TFP and IST shocks compete with the non-stationary anticipated and unanticipated domestic TFP and IST shocks, as well as cyclical shocks, in explaining variations in macroeconomic aggregates. A variance decomposition reveals that the international TFP and IST shocks explain about 30% and 24%, respectively of fluctuations in UK GDP at business cycle frequencies. The domestic TFP and IST shocks are somewhat less important as they are responsible for about 10% and 15% of the fluctuations in UK GDP. The remaining 21% are explained by the cyclical shocks. Importantly, for the domestic and international TFP and IST shocks, it is the anticipated rather than the unanticipated type that explain the vast majority of the fluctuations in UK GDP. International news (surprise) TFP and IST shocks explain 26.8% (3.5%) and 23.7% (0.5%), domestic news (surprise) TFP and IST shocks explain 9% (1%) and 13.3% (1.3%) of fluctuations in GDP. The dominance of the anticipated disturbances over the unanticipated technology shocks in the variance decomposition is not confined to UK GDP but extends to all other observables. These results stress the importance of distinguishing between international and domestic investment specific and total factor productivity for understanding the anatomy of UK economic growth. They particularly highlight also that it is paramount to allow for anticipated technology shocks — in addition to the often considered unanticipated innovations — when studying small open economies.

Our analysis reveals that UK GDP was driven until the 1990s to a large extent by domestic productivity advances, and particularly those of the TFP type. International IST shocks took center stage after the recession in the early 1990s and prevented a severe stagnation in UK GDP growth. The onset of the financial crisis marks a weakening in the effect of long-run IST on UK GDP growth and this change, together with the ongoing limited contribution of TFP, explains the UK's slow recovery after the financial crisis.

Similarly, domestic TFP shocks have also been an important driver for UK labor productivity in the first third of our sample, while they couldn't maintain these positive dynamics in the remaining two thirds. In contrast, international technology shocks dampened UK labor productivity in the 1970s and early 1980s, but continue to provide a substantial boost since the late 1980s into the early 2000s. We find that particularly the domestic TFP and international IST shocks are behind the slowdown in labor productivity after the 2007 financial crisis, also coined the productivity puzzle, while international TFP and domestic IST shocks supported labor productivity growth during this episode.

Consistent with the empirical importance of anticipated technology shocks, we document that positive international TFP and IST four quarter ahead news shocks generate an expansion in UK and US output, investment and labor productivity. The IST shock further leads to a persistent decline in the relative price of investment goods in both economies. The shock has a larger and more immediate effect on the RPI in the United States than in the United Kingdom. In response to positive news about future TFP, the UK and US relative prices of investment rise. These last two findings are at odds with the dynamics in standard two-country one-sector dynamic stochastic general equilibrium models, where the relative price of investment is simply the inverse of investment specific technology. Our empirical results suggest this may be a too restrictive constraint that will affect the relevance of the IST for explaining aggregate fluctuations when such models are taken to the data. This motivates the development of a two-country two-sector model with distinct sectors for the production of consumption and investment goods. In this model the RPI is a function of the inverse of investment specific technology, the sectoral capital services-to-labor shares and sectoral real wages. Hence, not only IST but in principle all shocks can explain variations of the RPI.

We employ a prior predictive analysis to investigate whether the two-sector model can resemble the empirical impulse response functions in response to anticipated IST and TFP shocks for realistic parameter ranges. Sampling the structural parameters from their uninformative prior distribution, we search for the set of values (i.e. structural models) that deliver predictions consistent with the stylised facts implied by the empirical model. In other words, we identify all those structural models that could replicate (to some extent) the empirical IRFs. Furthermore, the "value-added" of the structural model increases by showing that it can reproduce the observed moments for a set of parameters, as its ability to help us understand the economic transmission mechanism is not pinned down to one unique parameter vector.

This analysis reveals that the presence of investment adjustment costs, variable capital utilization and adjustment costs for labor movements across sectors are important components for generating an expansion in output in response to news about IST or TFP. The relevance of the two first components is consistent with the existing literature on TFP news in closed economies, see for example Jaimovich and Rebelo (2009). Our two-sector model with these features is successful in qualitatively capturing the dynamics of the empirical impulse responses of all macroeconomic aggregates, including the RPI, for a wide range of parameter values.

The role of investment specific and total factor productivity for macroeconomic developments has been a question of interest for a long time, yet most studies confine themselves to assessing the shocks' importance for US business cycles.[1] A growing recent literature is concerned with the open economy aspects of technology shocks. Miyamoto and Nguyen (2017b), for example, show that the spillover of persistent US TFP shocks to Canada is important for business cycle comovement between these economies. Consistent with our empirical setup, Rabanal et al. (2011) provide evidence for a common TFP component across countries in that they document that US and the rest of the world TFP processes are co-integrated. Mandelman et al. (2011) find corresponding results on the co-integration of US and international IST shocks. Guerron-Quintana (2013) estimates a small open economy DSGE model on data from a set of advanced small open economies. He finds that a common international non-stationary TFP shock is of particular importance during the Great Recession, and accounts for between 10% and 19% of the variance of output over his 1980Q1-2010Q4 sample. Dogan (2019) documents in the context of a two-country two-sector international real business cycle framework that permanent US IST shocks are important for Mexican business cycle dynamics. In her model however, non-stationary IST shocks do not compete with permanent TFP shocks in explaining aggregate fluctuations. We add to this strand of the literature in that we provide a holistic analysis of technology-induced macroeconomic fluctuations in a small open economy by jointly estimating the role of non-stationary IST and TFP shocks and by differentiating between domestic and international origins of these shocks. It is noteworthy that Ireland (2013) identifies non-stationary shocks to TFP and IST in the United States and the Euro Area in an estimated two-country dynamic stochastic general equilibrium model. This work points to important differences between these

[1] See e.g. Fisher (2006), Justiniano et al. (2011), Schmitt-Grohe and Uribe (2011) and Benati (2014).

regions regarding the effects of IST and TFP shocks in the 1970s and 1990s. Our analysis also focuses on international IST and TFP shocks, yet we consider anticipated as well as unanticipated technology shocks, and we are agnostic in that we apply a minimum of structure for their empirical identification.

Our work also builds on a series of empirical contributions in the closed-economy literature which highlight the importance of anticipated technological advances for aggregate fluctuations. This literature focuses almost exclusively on the effects of anticipated technology shocks for the US economy.[2] There are three notable exceptions which investigate the relevance of anticipated TFP shocks for small open economies. Levchenko and Pandalai-Nayar (2018) identify shocks originating in the United States and study their effect on the Canadian economy. Consistent with our findings they document that anticipated US TFP shocks are more important than their unanticipated counterparts for explaining aggregate fluctuations in Canada. Klein and Linnemann (2021) investigate the consequences of anticipated and unanticipated US TFP shocks on the G-6 economies. They highlight the strong transmission of US TFP surprise shocks to the G-6 economies and a somewhat weaker, but still considerable effect of their anticipated counterparts. In contrast, Kamber et al. (2017) study the effects of domestic TFP news shocks on four small open economies, but abstract from the effects of international technology shocks. Our work builds on these insights in that we account for anticipated and unanticipated technological advances. We add to this strand of the literature by differentiating between the effects of anticipated investment specific and total factor productivity shocks on a small open economy, and by jointly accounting for international as well as country-specific technological advances.

The paper proceeds as follows. Section 2 provides an illustrative look at unconditional productivity trends. Section 3 introduces a two-country international business cycle model that is used to inform a state space model that empirically establishes the relevance of productivity shocks for the UK economy. Section 4 provides an overview of the data and Section 5 introduces the state space model. Section 6 discusses the empirical results of this model. Section 7 develops a multi-sector model that can resemble the empirical results and Section 8 concludes.

[2]See for example Beaudry and Portier (2006), Barsky and Sims (2011), Schmitt-Grohe and Uribe (2011), Beaudry and Portier (2014), Cascaldi-Garcia and Vukotic (2020), Görtz et al. (2022b) and Benhima and Cordonier (2022) on the importance of TFP news shocks and Zeev and Khan (2015) and Görtz and Tsoukalas (2018) who also point to the relevance of anticipated IST shocks.

2 An Illustrative Look at Unconditional Productivity Trends

The motivation for our work becomes apparent from an illustrative look at US and UK time series data. The left subplot of Figure 1 shows log UK and US real consumption and the corresponding country specific log labor productivity. A large literature argues that TFP shocks are the sole contributor to long-run labor productivity movements.[3] US and UK consumption exhibit a positive long-run trend and the figure suggests this may be driven by improvements in TFP.

The right subplot in Figure 1 shows UK and US real investment as well as the corresponding inverse of the relative prices of investment (RPI). A candidate explanation for the positive long-run trend in US and UK investment, according to Greenwood et al. (1997) and Fisher (2006), is a continuous improvement in investment specific technology. The latter can be proxied for by the inverse of the relative price of investment goods. Consistent with the notion that the positive trend in US and UK investment may be driven by improvements in IST, Figure 1 shows a long-run increase in the inverse RPI for these countries.

Based on Figure 1, it remains an open question in how far the long-run trends in UK consumption and investment are driven by country-specific or international advances in TFP and IST. This illustrative discussion suggests that a holistic understanding of the anatomy of productivity trends in the United Kingdom requires a joint assessment of the role played by international and UK-specific TFP and IST shocks as the driving forces of UK macroeconomic aggregates. It is also interesting to note that when considering HP-filtered correlations of the displayed data, the highest correlation between a UK macroeconomic aggregate and UK or US productivity proxies is not the contemporaneous one. This points towards interrelations between UK macroeconomic aggregates and technological disturbances beyond the widely used surprise technology shocks — for example via the effects of anticipated technological advances or a slow diffusion of domestic and international technology shocks through the UK economy.

In the following, we will use US and UK data on labor productivity and the RPI to identify TFP and IST shocks, respectively. Informed by the discussion above, we identify international and domestic technology shocks as well as their anticipated and unanticipated components. The US and UK economies are closely integrated but very asymmetric in size. The latter implies that the identified US shocks would unlikely be contaminated by endogenous US responses to developments in the United Kingdom. It is of particular interest for the UK economy to investigate international technology shocks that originate in the United States as the latter is widely perceived as the leader in global technology trends. Our identification of domestic and international technology shocks will be guided by the restrictions imposed in a standard

[3]See for example Gali (1999) and Francis et al. (2014).

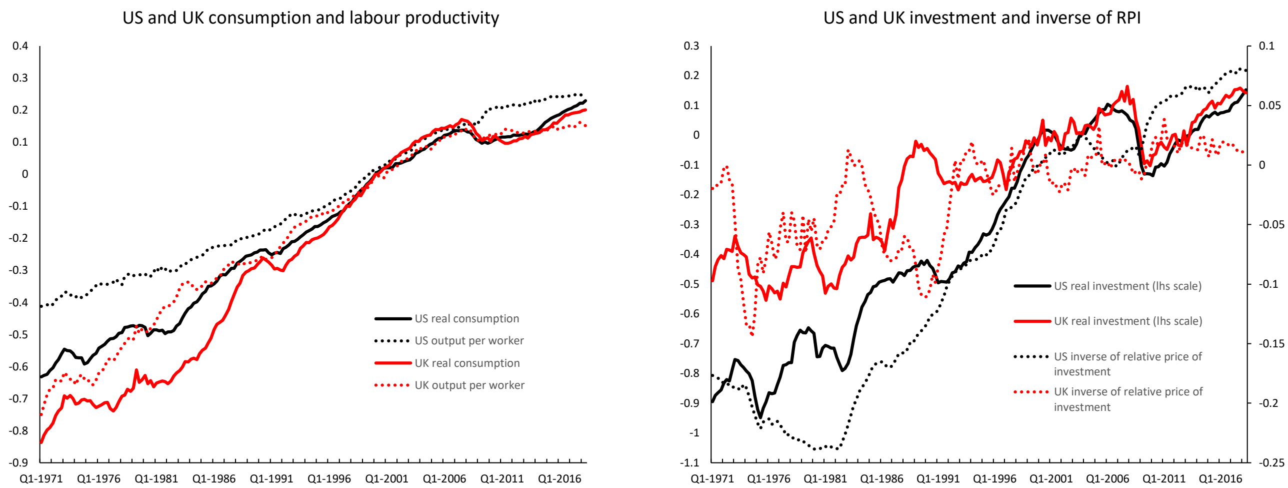

Figure 1: **UK and US macroeconomic aggregates.** All series are in natural logs and normalized to zero at 2000Q1.

two-country international business cycle model and discussed in detail in the next section.

3 Structural Model

We consider a two-country flexible price, international business cycle model similar in structure to Backus et al. (1994), with incomplete financial markets as in Heathcote and Perri (2002) and more recently Bodenstein et al. (2018) and variants of it are widely used in the literature.[4] In each of the two countries, firms produce a specialised tradable good that is used in the production of final consumption and investment goods. Households consume a final good which is a composite of home and foreign-produced goods. The basket of consumption and investment goods reflects a preference for domestically produced goods, i.e. there is home-bias in consumption and investment. Households in both countries are able to smooth consumption across time by trading in one-period non-state contingent bonds. Both economies are subject to the same non-stationary TFP and IST shocks, following the closed-economy framework by Justiniano et al. (2011).

The rather general theoretical framework developed in this section, and particularly the restrictions derived in Section 3.6, are subsequently used to inform an state space model developed in Section 5. This state space model is used to empirically establish the relevance of the two international non-stationary technology shocks.

[4]See for example Chari et al. (2002), Corsetti et al. (2008), Kamber et al. (2017).

3.1 Households

The representative household in the domestic economy derives utility from the consumption of final goods, C_t, and dis-utility from supplying labor, L_t,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \tag{1}$$

where the discount factor is denoted by $0 < \beta < 1$ and the period utility by $U(\cdot)$. Households maximize their expected utility given by equation (1) with respect the following flow budget constraint

$$C_t + p_{F,t} B_t = (1 + r_{t-1}) p_{F,t} B_{t-1} + W_t L_t + \pi_t,$$

where $p_{F,t}$ denotes the price of the foreign-produced intermediate good relative to the domestic final good, $\frac{P_{F,t}}{P_t}$, and where $W_t L_t$ denotes the representative household's wage income and π_t is the dividend income received due to ownership of firms. Households are able to smooth consumption risk by holding non-state contingent bonds, B_t, denominated in terms of the foreign-produced intermediate good, that pay a quarterly yield of r_t.[5]

3.2 Final Goods Producers

Final goods, used for consumption, C_t, and for the production of investment goods, X_t, are produced by combining home and foreign-produced intermediate goods according to a constant elasticity of substitution (CES) technology

$$C_t = \left[\eta^{\frac{1}{\theta}} \left(C_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}} \left(C_{F,t}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad X_t = \left[\eta^{\frac{1}{\theta}} \left(X_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}} \left(X_{F,t}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where $C_{H,t}$ and $X_{H,t}$ are domestic consumption and investment of home-produced intermediate goods, and $C_{F,t}$ and $X_{F,t}$ denote domestic consumption and investment of foreign intermediate goods. The parameter $0 < \eta < 1$ determines the share of home and foreign produced intermediate goods. The representative agent has a home-bias and the real exchange rate can deviate from purchasing power parity if η is greater than the relative size of the home country, N. The parameter $\theta > 0$ governs the elasticity of substitution between home and foreign-produced goods.

[5] As in Schmitt-Grohe and Uribe (2003) we allow the interest rate on domestically held foreign bonds to differ from the rate applicable to foreign agents by a small debt-elastic premium. Specifically, $(1 + r_t) = (1 + r_t^*) e^{-\phi_B B_t}$, the premium decreases with the net foreign asset position of the home country. This small bond holding cost eliminates the unit root in bond holdings and closes the model.

3.3 Investment Goods Producers

Investment goods producers in the home economy purchase final goods X from final goods producers at price P_t and transform these into investment goods I_t. The latter are then sold at price $P_{I,t}$ to domestic intermediate goods producers. Hence, investment goods producers maximize profits

$$\pi_t^I = P_{I,t} I_t - P_t X_t,$$

subject to the constraint

$$X_t = V_t X_t,$$

so that final goods X_t are turned into investment goods using the time-varying investment specific technology, V_t. Profit maximization links the relative price of investment goods to investment specific technology

$$\frac{P_{I,t}}{P_t} = V_t^{-1}. \tag{2}$$

Investment specific technology V_t is an international non-stationary process which affects the relative price of investment goods in both, the domestic and the foreign economy.

3.4 Intermediate Goods Producing Firms

Domestic intermediate goods producing firms produce country-specific output goods, Y_t, that are used in the production of final consumption and investment goods. These firms maximize cash-flow

$$\pi_t = \frac{P_{H,t}}{P_t} Y_t - W_t L_t - \frac{P_{I,t}}{P_t} I_t,$$

subject to the representative firm's production function and the capital accumulation constraint, where $\frac{P_{H,t}}{P_t}$ denotes the price of the home-produced intermediate good relative to the domestic final good. The firm produces output goods according to a standard Cobb-Douglas production function

$$Y_t = (K_{t-1})^{\alpha} (Z_t L_t)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where K_{t-1} denotes physical capital services, L_t is hours worked and Z_t is a non-stationary international productivity process that affects labor productivity in both, the home and foreign economy. The parameter α determines the share of capital in production. The capital accumulation constraint is given by

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}} - \Delta I \right)^2 \right] I_t \tag{3}$$

where ψ determines the curvature of investment adjustment costs, as in Christiano et al. (2005) and ΔI denotes the steady state growth rate of investment.

3.5 The Foreign Economy and Market Clearing

Sections 3.1 to 3.4 outline the structure of the domestic economy. An analogous set of equations applies to the foreign economy, so that the two economies are symmetric and both subject to the same international TFP and IST shocks.

The model is closed by the following market clearing conditions for home and foreign-produced intermediate goods.

$$NY_t = N[C_{H,t} + X_{H,t}] + (1 - N)[C^*_{H,t} + X^*_{H,t}]$$

and

$$(1 - N)Y^*_t = N[C_{F,t} + X_{F,t}] + (1 - N)[C^*_{F,t} + X^*_{F,t}],$$

where N denotes the relative size of the domestic economy and expressions with a star refer to variables corresponding to the foreign economy. Total production of home and foreign goods must equal total home and foreign uses of the goods. Our empirical work treats the home country as a small open economy such that $N \to 0$. The bond market clearing condition is the current account which is derived by accounting for firms' profit in their domestic households' budget constraint. Clearing of bond and goods markets then implies

$$C_t + X_t + p_{F,t}B_t = (1 + r_{t-1})p_{F,t}B_{t-1} + p_{H,t}Y_t.$$

3.6 Disentangling Trend and Cyclical Components

The model dynamics are driven by the two international non-stationary technology processes on TFP, Z_t, and IST, V_t, which hence affect both the home and domestic economy. Mandelman et al. (2011) provide evidence that IST shocks are co-integrated between the US and rest of the World and Benati (2014) shows that IST and TFP processes are not co-integrated within the US. Hence, we model the driving forces that affect both countries of the model as two international non-stationary processes. Both the home and foreign economy are hit by the same non-stationary shock to TFP and IST.

We assume the underlying processes to be first-difference stationary so that $\Gamma^Z_t = \frac{Z_t}{Z_{t-1}}$ and $\Gamma^V_t = \frac{V_t}{V_{t-1}}$ are given by

$$\Gamma^Z_t = \rho_Z \Gamma^Z_{t-1} + \epsilon_{Z,t} \qquad \text{and} \qquad \Gamma^V_t = \rho_V \Gamma^V_{t-1} + \epsilon_{V,t},$$

where $\hat{\epsilon}_{z,t}$ and $\hat{\epsilon}_{v,t}$ are i.i.d. with mean zero and constant standard deviations.

The non-stationary technology processes Z_t and V_t govern the trends of model variables. For each variable we can determine the particular combination of these two technology processes, such that dividing all variables by their respective trend, yields a stationary model. In this sense, our theoretical model can be informative about the behavior of international stochastic trends across model variables, which in turn motivates key assumptions made for the design of the empirical model in Section 5 on the relationship between observables and technology processes. In particular, we can derive a formulation

$$Y_t = y_t Z_t V_t^{\frac{\alpha}{1-\alpha}} \tag{4}$$

which disentangles non-stationary output, Y_t, into a stationary component, y_t, and the underlying trend component driven by TFP and IST. Lower case variables are stationary and as such, we can interpret y_t as the cyclical component of output. Output, consumption and labor productivity share a common trend, so that the non-stationary and the cyclical components of consumption and labor productivity can be disentangled analogous to equation (4). For investment the relationship between trend and cyclical components is given by

$$I_t = i_t Z_t V_t^{\frac{1}{1-\alpha}} \tag{5}$$

and for the relative price of investment by

$$\frac{P_{I,t}}{P_t} = p_{I,t} V_t^{-1}. \tag{6}$$

In this version of the model, the stationary or cyclical component of the relative price of investment goods, $p_{I,t}$, is a constant. Given the symmetry of our model, the relationships shown in equations (4) to (6) hold for both, foreign and domestic variables. These relationships will be used in the in the following sections to empirically identify structural trends by informing long-run restrictions in the state space model.[6]

4 Data

The main focus in our study is on the UK and US economies. We consider quarterly data over the horizon 1971Q1-2018Q2. Output, y_t is real GDP per capita and investment, i_t, is gross fixed capital formation. These variables are in logs and real per capita units. Labor productivity, lp_t, is defined as log real per capita GDP minus log hours worked per capita. The real price of investment, rpi_t, is defined as the deflator for gross fixed capital formation over the GDP deflator. The share of real net UK exports, nx_t,

[6] It is noteworthy that the derived relationships between non-stationary variables, the corresponding cyclical component and the trend are unaffected by a number of widely used extensions to this baseline framework. This holds for example for adding price rigidities (see e.g. Benigno and Thoenissen (2003)), search and matching friction (see e.g. Bodenstein et al. (2018)), or variable capacity utilisation (see e.g. Görtz and Tsoukalas (2013)).

is defined as the difference of real exports and real imports divided by real GDP. All data mentioned previously is obtained from the OECD Quarterly National Accounts. The effective real exchange rate for the United Kingdom, ex_t, is obtained from the Bank for International Settlements and puts the national currency in relation to a broad basket of foreign currencies. The real (ex-post) long-term interest rate , r_t^L, and real equity returns, r_t^E, are derived by structuring inflation from nominal 10 year (spot) government yields and nominal equity returns, respectively. The data has been collected from the US and UK central banks.

5 The Empirical Model

In this section, we introduce a state space model that is subsequently estimated and we discuss the identification of shocks to the trend of productivity. In particular, we outline how we separately identify the contribution of anticipated and unanticipated IST and TFP trend shocks and how these can be disentangled from cyclical components in explaining the variations in macroeconomic aggregates.

The identification of several non-stationary shocks is a complicated task that determines – to a large extend – the format of the empirical model. The latter is a state-space model of the following form

$$\zeta_t = C + A\xi_t, \tag{7}$$

$$\xi_t = B\xi_{t-1} + \omega_t, \tag{8}$$

where ζ_t denotes the vector of the observable variables. We employ as observables:

- **Foreign US Block:** real GDP growth, real long-term government yields, real equity returns, real investment growth, real labor productivity growth, relative prices of investment growth
- **Domestic UK Block:** real GDP growth, real long-term government yields, real equity returns, real investment, real labor productivity growth, relative prices of investment growth, net-trade as % of GDP and the real exchange rate.

The variable ξ_t summarizes the vector of the state variables which includes:

- **Foreign US Block:** TFP growth, news (1,2,3 and 4 quarters head) about TFP growth, IST growth, news (1,2,3 and 4 quarters head) about IST growth, cyclical GDP, cyclical long-term government yields, cyclical equity returns, cyclical investment, cyclical labor productivity, cyclical relative prices of investment.

- **Domestic UK Block:** TFP growth, news (1,2,3 and 4 quarters head) about TFP growth, IST growth, news (1,2,3 and 4 quarters head) about IST growth, cyclical GDP, cyclical long-term government yields, cyclical equity returns, cyclical investment, cyclical labor productivity, cyclical relative prices of investment, net-trade as % of GDP and the UK real exchange rate.

The vector C captures a constant in the relationship between observable and state variables and the vector of errors, ω_t, is normally distributed with zero mean and constant Σ covariance matrix ($\omega_t \sim N(0, \Sigma)$).

The matrix A maps the state vector into the set of the observable variables. In our exercise, the matrix A also contributes to the identification of the structural trends by preserving the long-run and news (anticipated shocks) restrictions discussed in Section 3.6 and below (see equation (10)).

Consistent with those, we impose the restrictions

$$\begin{aligned}
\Delta \ln Y_t^* &= \Delta \ln Y^* + \hat{y}_t^* - \hat{y}_{t-1}^* + \Delta \ln Z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln V_t^* \\
\Delta \ln I_t^* &= \Delta \ln I^* + \hat{i}_t^* - \hat{i}_{t-1}^* + \Delta \ln Z_t^* + \frac{1}{1-\alpha} \Delta \ln V_t^* \\
\Delta \ln LP_t^* &= \Delta \ln LP^* + \widehat{lp}_t^* - \widehat{lp}_{t-1}^* + \Delta \ln Z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln V_t^* \\
\Delta \ln RPI_t^* &= \Delta \ln RPI^* + \widehat{rpi}_t^* - \widehat{rpi}_{t-1}^* - \Delta \ln V_t^* \\
r_t^{L,*} &= r^{L,*} + \hat{r}_t^{L,*} \\
r_t^{E,*} &= r^{E,*} + \hat{r}_t^{E,*} \\
\Delta \ln Y_t &= \Delta \ln Y + \hat{y}_t - \hat{y}_{t-1} + \Delta \ln Z_t + \frac{\alpha}{1-\alpha} \Delta \ln V_t \qquad (9) \\
\Delta \ln I_t &= \Delta \ln I + \hat{i}_t - \hat{i}_{t-1} + \Delta \ln Z_t + \frac{1}{1-\alpha} \Delta \ln V_t \\
\Delta \ln LP_t &= \Delta \ln LP + \widehat{lp}_t - \widehat{lp}_{t-1} + \Delta \ln Z_t + \frac{\alpha}{1-\alpha} \Delta \ln V_t \\
\Delta \ln RPI_t &= \Delta \ln RPI + \widehat{rpi}_t - \widehat{rpi}_{t-1} - \Delta \ln V_t \\
\ln NX_t &= \ln NX + \hat{nx}_t \\
\ln EX_t &= \ln EX + \hat{ex}_t \\
r_t^L &= r^L + \hat{r}_t^L \\
r_t^E &= r^E + \hat{r}_t^E
\end{aligned}$$

where, in our context, variables with (without) a star relate to the United States (United Kingdom). A variable without a hat indicates a non-stationary observable, while the corresponding variable with the hat denotes the stationary cyclical component. Δ indicates the first-difference of a variable so that $\Delta X_t = X_t - X_{t-1}$ and variables without a time subscript stand for historical averages. The paper focusses on how the United Kingdom, as a small open economy, is affected by international and domestic technology shocks. We use the United States to capture international technology shocks. In comparison to the structural model introduced in Section 3, we include the domestic trend shocks for the UK economy in the empirical model. In this sense, the restrictions in equations (9) are agnostic in that they give the empirical model a way out to explain movements through an additional channel.[7]

Another angle in which the state-space model is consistent with the structural framework is that we further restrict the domestic and foreign TFP to be co-integrated, and the same is true for the IST trends. While we allow the stochastic trends to be functions of both anticipated and unanticipated shocks, as apparent from equation (10). Matrix B governs the dynamics of the state vector and is also subject to small open economy block-recursive restrictions ensuring that domestic disturbances have no (lag) effects on the foreign economy. The stochastic trends in domestic and international TFP and IST are defined as

[7] To be precise, by allowing for the presence of domestic trends – in addition to foreign trends – we do not force the empirical model to explain the low frequency dynamics of the domestic UK data by using the US trends. In other words, we do not bias upwards the importance of the US TFP and IST non-stationary shocks to the UK economy. The theoretical model abstracts from the UK specific TFP and IST non-stationary shocks as they are not the key focus of this paper due to the vast attention that they have received from the literature mentioned earlier.

$$
\begin{aligned}
\Delta \ln Z_t^* &= \rho_{Z^*} \Delta \ln Z_{t-1}^* + \sigma_{Z^*} e_t^{Z^*} + \sigma_{Z^*} news_{t-1}^{Z^*,1} \\
news_t^{Z^*,1} &= news_{t-1}^{Z^*,2} + \sigma_{Z^*} e_t^{Z^* news,1} \\
news_t^{Z^*,2} &= news_{t-1}^{Z^*,3} + \sigma_{Z^*} e_t^{Z^* news,2} \\
news_t^{Z^*,3} &= news_{t-1}^{Z^*,4} + \sigma_{Z^*} e_t^{Z^* news,3} \\
news_t^{Z^*,4} &= \sigma_{Z^*} e_t^{Z^* news,4} \\
\Delta \ln V_t^* &= \rho_{V^*} \Delta \ln V_{t-1}^* + \sigma_{V^*} e_t^{V^*} + \sigma_{V^*} news_{t-1}^{V^*,1} \\
news_t^{V^*,1} &= news_{t-1}^{V^*,2} + \sigma_{V^*} e_t^{V^* news,1} \\
news_t^{V^*,2} &= news_{t-1}^{V^*,3} + \sigma_{V^*} e_t^{V^* news,2} \\
news_t^{V^*,3} &= news_{t-1}^{V^*,4} + \sigma_{V^*} e_t^{V^* news,3} \\
news_t^{V^*,4} &= \sigma_{V^*} e_t^{V^* news,4} \\
\Delta \ln Z_t &= \rho_Z \Delta \ln Z_{t-1} - \kappa_Z \left(\ln Z_{t-1} - \ln Z_{t-1}^* \right) + \sigma_Z e_t^Z + \sigma_Z news_{t-1}^{Z,1} \qquad (10) \\
news_t^{Z,1} &= news_{t-1}^{Z,2} + \sigma_Z e_t^{Znews,1} \\
news_t^{Z,2} &= news_{t-1}^{Z,3} + \sigma_Z e_t^{Znews,2} \\
news_t^{Z,3} &= news_{t-1}^{Z,4} + \sigma_Z e_t^{Znews,3} \\
news_t^{Z,4} &= \sigma_Z e_t^{Znews,4} \\
\Delta \ln V_t &= \rho_V \Delta \ln V_{t-1} - \kappa_V \left(\ln V_{t-1} - \ln V_{t-1}^* \right) + \sigma_V e_t^V + \sigma_V news_{t-1}^{V,1} \\
news_t^{V,1} &= news_{t-1}^{V,2} + \sigma_V e_t^{Vnews,1} \\
news_t^{V,2} &= news_{t-1}^{V,3} + \sigma_V e_t^{Vnews,2} \\
news_t^{V,3} &= news_{t-1}^{V,4} + \sigma_V e_t^{Vnews,3} \\
news_t^{V,4} &= \sigma_V e_t^{Vnews,4}
\end{aligned}
$$

where ρ_Z and ρ_V govern the persistence of the domestic TFP and IST processes, respectively. The parameters κ_Z and κ_V control the cointegration between international and domestic technology of either the TFP or IST type. The terms e_t^Z and e_t^V are the unanticipated innovations in domestic TFP and IST. The anticipated (or news) innovations in domestic TFP and IST at horizon $h = \{1, 2, 3, 4\}$ are $e_t^{Znews,h}$ and $e_t^{Vnews,h}$, so that news can be anticipated four quarters ahead. These innovations are i.i.d. with $N(0, \sigma_Z^2)$ and $N(0, \sigma_V^2)$. The parameters and innovations relating to the international productivity processes are defined analogously and denoted with a star. It is assumed that the anticipated and unanticipated innovations are uncorrelated across country, technology type, horizon and time. Note that the stochastic processes above allow for revisions in expectations. Information received $t-h$ periods in advance can later be revised by updated information via innovations received at $t-h+1$, ..., $t-1$, or by the unanticipated component at time t. This implies news received at any anticipation horizon may only partially (or fail to) materialize.[8]

The empirical model employed in this study shares many features with those proposed by Crump et al. (2016), Negro et al. (2017), Del Negro et al. (2019) and Johannsen and Mertens (2021). Similar to the latter studies, our procedure disentangles the trend and cyclical component of the observed series during the estimation. In our case, the trend cycle decomposition, as well as the relationships between the observable vector and the set of international stochastic trends, are pinned down by the economic theory that forms the core of the modern DSGE literature (see Fisher (2006), Justiniano et al. (2011), Schmitt-Grohe and Uribe (2011) and Ireland (2013) among others). Estimating the state space model hence allows us to establish the relevance of non-stationary UK and international TFP shocks, Z_t and Z_t^*, non-stationary UK and international IST shocks, V_t and V_t^*, and stationary cyclical shocks, ω_t, as drivers for variations in the set of UK observables.

5.1 Estimation of the State State Model

The estimation algorithm is reviewed here only briefly, with the discussion regarding all the important details and necessary steps to take place in Section B.1 of the Appendix. Our decision to use economic theory to determine the long-run relationships among the set of the observed variables allows us to "map" our empirical results to the structural model discussed earlier and identify the theoretical transmission mechanism that explain any stylized facts emerging from the empirical exercises. In addition, the reliance on the theoretically implied long-run relationships also simplifies the estimation of the state space model. Given the value for the capital share, α, the state-space model summarized by the equations (7) and (8) is a linear Gaussian model that we estimate using Gibbs sampling and simulation smoothing techniques (Carter and Kohn (1994) and Durbin and Koopman (2002)).[9] In other words, the posterior distribution is approximated by sampling parameters $\mu = (vec(B)', vec(\Sigma)')'$ and states $\xi_{1:T}$ sequentially from their

[8]This way of introducing news shocks is standard in the literature, see e.g. Schmitt-Grohe and Uribe (2012), Khan and Tsoukalas (2012) and Beaudry and Portier (2014).

[9]We set α to 0.3 which is a standard in the literature.

conditional posterior distributions

$$p\left(\xi_{1:T}|\zeta_{1:T},\mu\right) \propto p\left(\xi_{1:T}|\mu\right)p\left(\zeta_{1:T}|\xi_{1:T},\mu\right) \tag{11}$$

$$p\left(\mu|\zeta_{1:T},\xi_{1:T}\right) \propto p\left(\xi_{1:T}|\mu\right)p\left(\zeta_{1:T}|\xi_{1:T},\mu\right)\underbrace{N\left(vec(\tilde{B}),\tilde{\Sigma}_B\right)IW\left(\left(\varkappa+1+d\xi\right)\tilde{\Sigma},\varkappa\right)}_{\text{Parameters' Prior Distribution}} \tag{12}$$

respectively.[10] The sampling scheme could be summarized in two steps, namely:

- For $i = 1, ..., N_{simulations}$
  - Draw $\mu^{(i)}$ form $p\left(\mu|\zeta_{1:T},\xi_{1:T}^{(i-1)}\right)$.

    Given $\xi_{1:T}$, this step collapses to a BVAR Gibbs sampling draw (Kadiyala and Karlsson (1997a), Koop and Korobilis (2010)). However, due to the "small open economy" and "trend exogeneity" restrictions the draws are obtained using seemingly unrelated regression (SUR) schemes as in Zha (1999) and Justiniano and Preston (2010), among others.
  - Draw $\xi_{1:T}^{(i)}$ from $p\left(\xi_{1:T}|\zeta_{1:T},\mu^{(i)}\right)$.

    The unobserved state vector $\xi_{1:T}$ is derived using the smoother proposed by Durbin and Koopman (2002).

Given the large scale of the estimated model, the use of prior information as a vehicle to shrink the space of the estimated parameters is unavoidable. As a result, the use of Minnesota type prior distribution for the VAR parameter vector seems a natural choice (Sims (1980), Doan et al. (1984a)). Finally, the vector of constants C in equation (7) is set equal to the average historical value of the observable vector.

5.2 Trend Shocks Identification

Similar to the work of Crump et al. (2016), Negro et al. (2017), Del Negro et al. (2019) and Johannsen and Mertens (2021), the identification of the trend (international and domestic) shocks takes place during the estimation of the model. This paragraph provides only a high-level description of how the identification works. The vector of the observable variables, ζ_t, contains ten variables that grow over time, and two that are stationary. On the other hand, the state vector, ξ_t, and the stability restrictions imposed on B, indicate that four variables (US TFP growth, US IST growth, UK TFP growth, and UK IST growth) out of sixteen variables display a stochastic trend. These four trends are the I(1) driving forces of the observed data, while the remaining twelve variables describe the cyclical variation of ζ_t. The matrix A

[10] Information about prior and posterior moments can be found in Appendix B.1.

disciplines the exact decomposition between the trend and cyclical component of the individual data series in the observable vector. For our exercise, the matrix A is pinned down by the economic theory that is a fundamental block of the DSGE literature concerned with stochastic trends, as detailed in the discussion above.

5.3 Identification of Anticipated and Unanticipated Changes in Technology

The identification of the anticipated shocks is achieved again during the estimation by:

- imposing news shocks to not have a contemporaneous effects on the productivity trends (equation (10));
- allowing news shocks to impact the endogenous part of the state vector ξ_t contemporaneously.

Loosely speaking, given an estimate of the news shocks (derived using the Durbin and Koopman smoother) the effects of news to ξ_t can be obtained by regressing the exogenous forces on the endogenous variables.

The restrictions applied in this context are similar to those utilized in DSGE models incorporating news. Nevertheless, the adoption of Minnesota priors in this case implies that the prior mean of the impact exerted by news shocks on the endogenous component of the state vector ξ_t is zero (unlike DSGE models). Consequently, posterior estimates will deviate from the prior mean solely if the available data contains robust information indicating that these effects are statistically significant.

By incorporating four forward-looking financial variables, namely US real long-term yields, US real equity returns, UK real long-term yields, and UK real equity returns, into the set of observables, we are able to evaluate the effectiveness of the agnostic restrictions imposed in this analysis. The subsequent section's Table 1 provides a visual representation of this assessment, demonstrating that news shocks account for over 80% of the variability observed in these forward-looking financial variables. It is noteworthy that no explicit constraint has been imposed to establish a relationship between the news and the financial variables (as described in equations (9) and (10)), yet the correlation between them is remarkably robust.

6 The Role of International Technology Shocks

In this section, we investigate the empirical relevance and transmission of domestic and international non-stationary technology shocks for the UK economy. Table 1 summarizes the median forecast error variance decompositions at business cycle fluctuations and allows for a more formal inspection of the shocks' importance for US and UK aggregate fluctuations.

Fluctuations in UK GDP are explained to a large extent by international non-stationary technology shocks (54.48%), where the role of the TFP type (30.33 %) is somewhat larger than that of IST (24.15%). Domestic non-stationary technology shocks play a somewhat more limited role (24.10%) which is only slightly larger than the contribution of cyclical shocks to fluctuations in GDP. Interestingly, for domestic and international IST and TFP shocks, it is the anticipated components that explain the bulk of fluctuations in UK GDP while the unanticipated shocks play no substantial role. The picture is very similar for the other UK variables, including the trade balance and the real exchange rate. TFP and IST shocks are both important for understanding their fluctuations. The anticipated domestic and international shocks are all highly relevant while the role of the surprise components is very limited. These results highlight the importance of jointly identifying different sources of technological progress as well as the associated surprise and news components for explaining aggregate fluctuations in small open economies.

While our focus is mainly on explaining UK macroeconomic fluctuations, Table 1 also shows forecast error variance decompositions for US aggregates. Consistent with the existing literature, technology shocks originating in the US of either kind, TFP or IST, are rather important for fluctuations in US GDP. While the TFP news shock is the most important single driving force (54.16%), the TFP surprise shock also plays a role (15.49). The anticipated US IST shock accounts for 15.91% of the fluctuations in US GDP while the importance of the surprise component is very limited (3.82%).[11] A similar picture emerges for US labor productivity and investment. Fluctuations in the relative price of investment goods are dominated by the news (49.76%), and to a somewhat lesser extent, the surprise IST shock (18.63%). This picture is similar for the UK relative price of investment good where the international and domestic IST shocks explain a large share of fluctuations (37.42%). Where the US and the UK differ is that the UK TFP news shock (29.85%), and to a somewhat lesser extent the international TFP (14.21%), and UK cyclical shocks (17.42%) also drive fluctuations in the UK relative price of investment goods. This is at odds with standard one-sector structural models where the relative price of investment is a function of IST only.

Overall, international and domestic non-stationary technology news shocks account for the vast majority of fluctuations in UK GDP, labor productivity, investment and the relative price of investment. The remaining share of variance is explained by the cyclical shocks while unanticipated technology shocks only play a very minor role.

[11] These results are broadly consistent with the notion for example in Fisher (2006), Justiniano et al. (2011), Schmitt-Grohe and Uribe (2012), Zeev and Khan (2015) and Görtz et al. (2022b).

Table 1: Forecast error variance decomposition at business cycle frequencies

	US TFP		US IST		UK TPF		UK IST		Cyclical
	surprise	news	surprise	news	surprise	news	surprise	news	shocks
UK Output	3.53	26.8	0.5	23.65	0.38	9.17	1.28	13.27	21.42
UK Labor Productivity	2.51	13.72	0.4	24.23	2.32	19.46	0.45	12.75	24.17
UK Real Long-Term Rate	1.52	10.35	5.29	38.11	3.45	18.35	0.48	7.34	15.11
UK Investment	1.25	19.22	0.57	14.39	1.17	13.75	0.35	17.82	31.47
UK Real Equity Return	0.81	15.14	0.42	48.93	1.05	5.5	1.73	19.13	7.29
UK Rel. Price of Investment	1.08	13.13	0.48	19.34	1.11	29.85	2.46	15.14	17.42
Trade Balance	1.59	16.18	1.48	16.76	2.3	10.09	1.02	8.48	42.11
Real Exchange Rate	4.63	19.14	1.02	8.55	5.06	21.06	3.69	19.93	16.92
US Output	15.49	54.16	3.82	15.91	0	0	0	0	10.62
US Labor Productivity	9.72	39.03	1.04	13.25	0	0	0	0	36.96
US Real Long-Term Rate	3.26	27.1	6.11	54.78	0	0	0	0	8.75
US Investment	14.25	49.94	3.54	18.91	0	0	0	0	13.35
US Real Equity Return	3.03	19.44	0.19	76.91	0	0	0	0	0.43
US Rel. Price of Investment	2.31	12.9	18.63	49.76	0	0	0	0	16.4

UK Productivity in a Historical Context. The illustrative discussion of unconditional time series in Section 2 raised questions on the historical role of different types of technology shocks as drivers of variation in UK macroeconomic aggregates. The discussion above provides an overview of the average importance of different types of shocks as drivers of fluctuations in macroeconomic aggregates. Next, we inspect their role in a historical context. Panels (a) and (b) of Figure 2 contrast UK GDP (black solid line) with a set of conditional paths for this variable. The red dashed line in panel (a) shows the model-implied path of UK GDP based on non-stationary technology shocks only, i.e. the anticipated and unanticipated international and domestic IST and TFP shocks, but abstracting from the cyclical disturbances. The blue dash-dotted line shows the path of GDP conditional on domestic non-stationary technology shocks. By comparing the unconditional path of GDP, the black line, to the path of GDP implied by all permanent technology shocks, the dashed red line, in panel (a) it is evident that the trend and fluctuations of UK GDP are largely dominated by the permanent technology shocks. Only at turning points around the recessions in the early 1990s and 2007 do the cyclical shocks imply substantial deviations for the path of this conditional GDP series. Cyclical shocks dampened the former recession and deepened the decline in GDP during the financial crisis. An interesting observation from panel (a) is that domestic productivity shocks supported UK GDP growth in mid 1980s and during the 2007 financial crisis. Following the recession in early 1990s, international shocks were very important to counteract the 10 year stagnation in domestic productivity advances which is evident from the flat blue dash-dotted line. Without international technology shocks UK GDP would be substantially lower at the end of our sample.

Panel (b) differentiates between TFP and IST shocks. It is evident that advances in TFP (red dashed line) have been very important for GDP growth in 1970s and 1980s. Until late the 1980s IST growth has been extremely limited, but it picks up substantially in the 1990s. Since that time, IST growth has been a highly important factor driving the positive trend growth in GDP (see blue dot-dashed line); particularly [also] in light of the much more limited contribution of TFP to UK GDP growth since the late 1980s. Around the time of the financial crisis, the path for UK GDP conditional on IST shocks exhibits a somewhat flatter trend, closer to the long-run trend of the series conditional on TFP. The slow IST and TFP growth will have played a role in the sluggish recovery after the 2007 financial crisis.

Panels (c) and (d) show unconditional and conditional paths for UK labor productivity. From the red dashed line in panel (c), it is evident that even for labor productivity cyclical shocks play only a limited role — like in panel (a) mainly around the turning points of the recessions in the early 1990s and 2007. The path for labor productivity conditional on international non-stationary technology shocks (blue dash-dotted line in panel (c)) shows strikingly that domestic TFP shocks were a large driver in labor productivity growth in the first third of the sample, while they were a drag on labor productivity particularly in the second third and to a slightly lesser extent in the aftermath of the financial crisis. International technology shocks have dampened UK labor productivity growth in the 1970s and early 1980s, but began to push it substantially since the late 1980s. Panel (d) differentiates further. It reveals that mainly domestic TFP (blue dashed line), but also international IST shocks (red dash-dotted line) were dampening forces on labor productivity growth after the 2007 financial crisis. For this episode, the series conditional on international TFP shocks (green line in panel (d)) and domestic IST shocks (magenta line in panel (d)) implies, via their steeper slope, a much faster recovery after the 2007 recession than those conditional on other technology shocks. Our analysis shows that it is mostly the domestic TFP and international IST shocks that are behind the remarkable slowdown in productivity after 2007, also coined the productivity puzzle, while particularly international TFP and domestic IST shocks supported labor productivity growth.

Panels (e) and (f) show unconditional and conditional paths of the UK relative price of investment. The difference between the unconditional path and that implied by technology shocks in panel (e) suggests that cyclical shocks where important drivers of the relative price of investment goods from the mid-1990s until after the financial crisis. The evidence from panels (e) and (f), which show paths for the UK RPI conditional on international IST, domestic IST and all of these IST shocks taken together, highlights that IST shocks alone fail to explain the path of RPI, particularly in the 1980s, mid-1990 and during the financial crisis. This suggests that UK RPI, consistent also with the findings in Table 1, is driven by more than just IST shocks, as assumed in many structural models, but also cyclical as well as domestic and international TFP shocks play a non-negligible role.

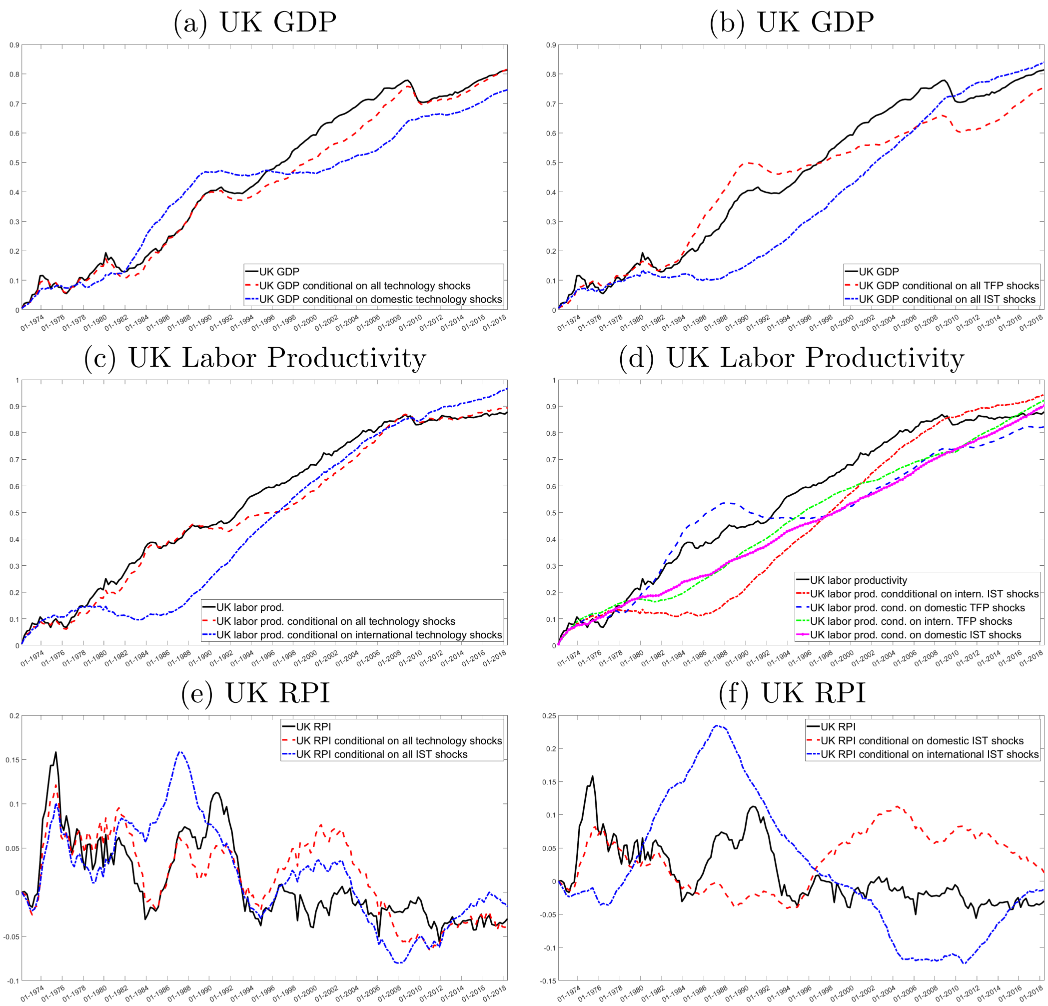

Figure 2: Unconditional UK time series and conditional on specific shocks.

Impulse Responses to Technology News. The variance decomposition exercise reveals the particular importance of international anticipated TFP and IST shocks for fluctuations in UK GDP. For this reason, we inspect the dynamics implied by the international four quarter ahead TFP and IST news shocks. In all following figures, variables with a * are associated with US responses and variables without a star correspond to the UK.

Figure 3 shows the impulse responses to an international four-quarter ahead TFP news shock. Positive news to the growth rate of TFP triggers a broad based expansion in the US, in particular output and investment rise upon arrival of the positive news and labor productivity increases. This is consistent with the findings in e.g. Jaimovich and Rebelo (2009) and Görtz et al. (2022a).

Our empirical methodology allows us to examine the response of the UK economy to this international shock. Importantly, the international news shock affects also the UK economy substantially and transmits with approximately equal speed through the two economies. Dynamics for the UK shown in Figure 3 mirror, to a large extent, the behavior seen for the US economy. The international news shock leads to a strong and persistent expansion in output and investment, consistent with the findings in Klein and Linnemann (2021) for an aggregate of the G-6 economies. Any movements in the trade balance are insignificant. This is consistent with the fact that the exogenous part of the technology employed in both countries shares a common component, meaning that a common improvement in TFP leads to an expansion of the potential supply in both economies. This feature leaves little room for "consumption risk-sharing" – i.e. goods and/or asset trade dynamics – and agents in both economies are left only with the option to consume these additional resources. This lack of consumption risk sharing associated with an international TFP shock is particularly important in light of adverse shocks as demand would need to contract sharply (i.e. not smoothing) to adjust to the permanent loss of the potential output.[12] Consistent with the lack of risk sharing, the exchange rate remains muted in the first three years, while it appreciates significantly after about four years. This seems to reflect the fact that potential supply expands relatively more in the US than in the UK, as indicated by the somewhat stronger output, labor productivity and investment responses in the US.

Market participants understand that news about TFP implies that the level of the natural interest rate will be higher in the future (due to the expectations about potential supply to expand), causing government yields to increase today (in both countries) as investors adjust their portfolios. News about higher profitability translates to higher equity prices in the UK as dividends are expected to increase. UK equity prices rise, despite the heavier discounting from higher rates. On the other hand, US equity returns decrease during the anticipation period and they increase only when TFP actually increases, suggesting that the negative contributions from higher interest rates exceed the increase in the dividends. A stronger response to US real long-term interest rates (relative to UK) after a US TFP shock, appears consistent with: i) the role of the US as a "safe asset provider", the share of the US government debt in portfolios of international investors and iii) risk characteristics of the two (debt versus equity) assets. In other words, the substitution from equities to government is expected to be stronger in the US.

The shock triggers a rise in the US and UK relative price of investment, indicating that investment

[12]This is consistent with the notion in e.g. Glick and Rogoff (1995) and Bussière et al. (2010) who argue that international shocks should not substantially affect current accounts.

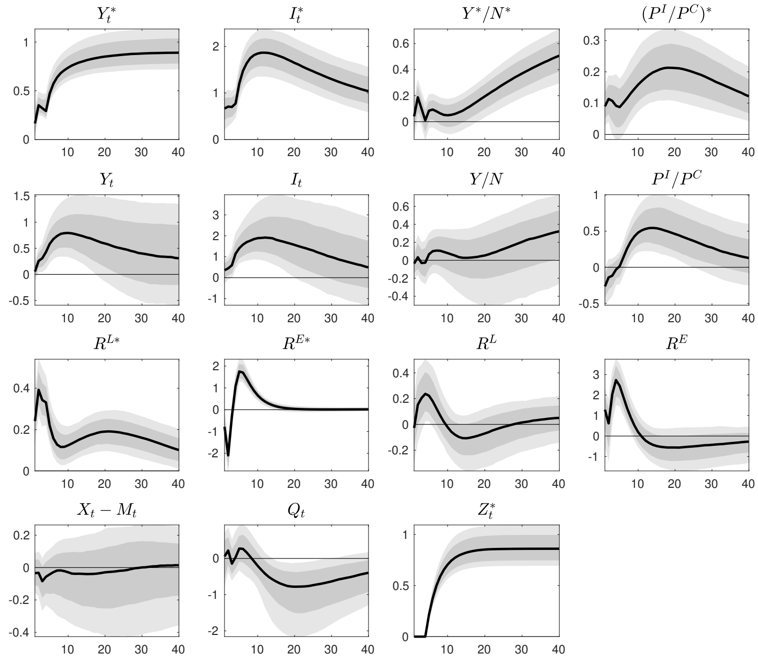

Figure 3: **Responses to a non-stationary international TFP shock which is anticipated four quarters ahead.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

goods become more expensive relative to consumption goods. This is is a notable finding as the standard workhorse model introduced in Section 3 implies that the relative price of investment remains constant in response to a TFP shock (see equation (2)). The empirically documented movements of the relative price of investment can be driven by structural differences between the consumption and investment goods producing sectors – e.g. by different dynamic patterns of capital to labor ratios across sectors. While the standard model of Section 3 has been useful to derive identification restrictions for our empirical investigation, its assumptions confining it to a one-sector framework appear to be too restrictive to allow it to resemble the empirical movements of the relative price of investment. In Section 7, we build on this observation and develop a two-sector model that can help us to resemble and interpret the empirical responses of the relative price of investment.

A strength of our empirical setup is that it allows us to jointly identify international anticipated and unanticipated TFP and IST shocks. Jointly identifying these shocks gives a broader picture of the effects of international productivity shocks, including the relative importance of TFP and IST trends. Figure 4 shows responses to an international IST shock which is anticipated four quarters ahead.[13] The shock

[13]We show responses to the four quarter ahead TFP and IST news shocks as these are the empirically most relevant amongst the anticipated and unanticipated technology shock components. Impulse responses to the surprise international

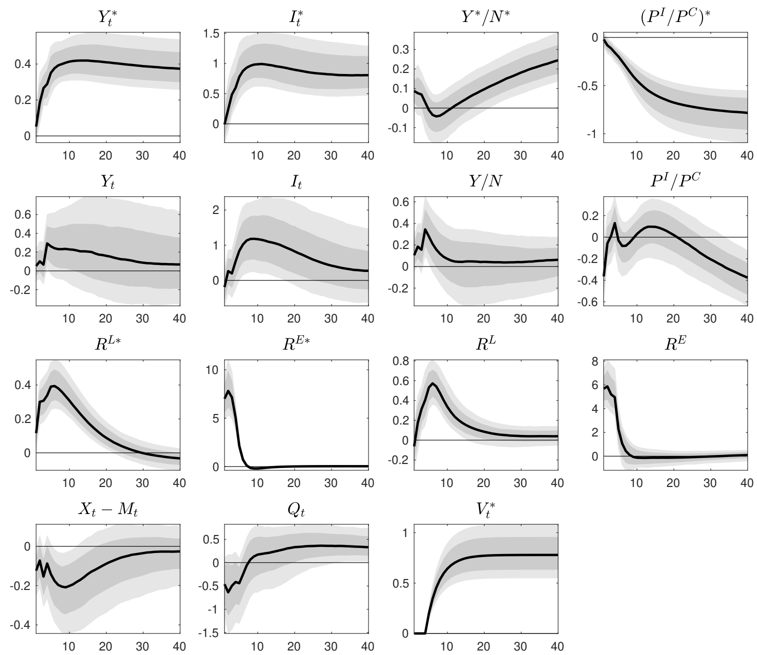

Figure 4: **Responses to a non-stationary international IST shock which is anticipated four quarters ahead.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands.

leads to an expansion in output and investment in the US and the UK. Qualitatively, the responses of the macroeconomic aggregates are very similar to those to an international TFP news shock. A notable difference is that the boom is driven by a strong and persistent decline in the US and UK relative price of investment. This is consistent with the evidence for US IST shocks e.g. in Fisher (2006) and Zeev and Khan (2015) and the decline in the relative price of investment clearly distinguishes this shock from the response to the TFP news shock. The international IST news shock transmits strongly through the UK economy. As in the case of the TFP shock, international IST is a part of the UK production technology and increases the potential supply in the UK and US economies. The short-run decline in the UK trade balance is consistent with the strong initial increase in US and UK investment, which is of approximately equal size, while UK output expands substantially less than its UK counterpart. The real exchange rate is unresponsive at 86% confidence bands.

7 Reconciling the Structural Model with the Data

Having shown the importance of anticipated international TFP and IST trend shocks in the data, we now return to the standard international business cycle model of Section 3 that we used to motivate our shock

TFP and IST shocks and news shocks at different horizons are shown in Appendix A.

identification. In particular, we ask if a structural model of this type can match the impulse responses from the state space model shown in Figures 3 and 4.

The model has difficulties to match the empirical responses in at least two dimensions. First, the response of the state-space model to an anticipated four quarter ahead international TFP growth shock shows that the relative price of investment goods increases in the US and the UK. This dynamic is inconsistent with the standard model presented in Section 3. In this model, profit maximisation by investment goods producers implies that the relative price of investment goods is simply an inverse function of the investment specific technology shock,

$$\frac{P_{i,t}}{P_t} = V_t^{-1},$$

and as such not affected by a total factor productivity shock.

Second, the standard model is also at odds with our empirical evidence on the effects of an anticipated shock to the trend of international IST. Figure 4 suggest that this shock has a larger and more immediate effect on the relative price of investment goods in the US than in the UK. The model however implies an identical path for US and UK RPI when modelling the international shock to IST as a common shock, and near identical when modelling the diffusion of foreign IST on domestic IST using an error correction mechanism, as is the case in the empirical model. In other words, the dynamics in RPI only reflect the dynamics of the stochastic IST process.

For the two reasons mentioned above, the standard model shown in Section 3 implies tight restrictions on the dynamics of US and UK RPI that are not supported by the data. This has strong implications for exercises that take this class of models to the data to assess the absolute and relative importance of IST and TFP shocks for aggregate fluctuations. In Section 7.1 we will develop a two-sector model that relaxes this restriction by making the relative price of investment goods an endogenous variable which can in principle be affected by all shocks. We will show that this model can resemble the dynamics in the data, shown in Figures 3 and 4.

7.1 A two-sector model

In this section, we extend the standard model presented in Section 3 by distinguishing explicitly between consumption and investment goods producing sectors. Importantly, this two-sector model yields the same set of identification restrictions as in equation (9). Consumption and investment goods are assumed to be produced in two distinct sectors as e.g in Huffman and Wynne (1999) and Görtz and Tsoukalas (2017). Firms in each sector employ their own labor and accumulate their own capital stock. This ensures that the relative price of investment goods is an endogenous relative price and not just the inverse of investment specific technology. We follow Miyamoto and Nguyen (2017a) who illustrate that Jaimovich and Rebelo (2009) type preferences and variable capacity utilization along with Christiano et al. (2005) type investment adjustment costs can amplify the transmission mechanism of non-stationary international productivity shocks in small open economy models. To account for the fact in the data that workers cannot instantaneously move between sectors, we allow for adjustment costs to the sectoral reallocation of labor, as e.g. in Petrella et al. (2019). Consistent with our empirical identification strategy, we model the home economy as a small open economy whose actions or shocks origination from it do not affect the foreign economy. A complete set of model equations are available in Appendix C.

7.1.1 Households

This section gives a brief description of the household side of the model. We confine the exposition to the home economy, but the foreign economy is modelled in the same manner. The specification of preferences for the representative household follows Jaimovich and Rebelo (2009) where utility is derived from the consumption of final goods, C_t, and dis-utility from supplying labor, L_t to firms in the consumption and investment goods producing sectors:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - X_t \frac{\phi_0}{1+\phi} L_t^{1+\phi}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{13}$$

where

$$X_t = C_t^{\varsigma} X_{t-1}^{1-\varsigma} \left(\frac{Z_t V_t^{\frac{\alpha}{1-\alpha}}}{Z_{t-1} V_{t-1}^{\frac{\alpha}{1-\alpha}}} \right)^{1-\varsigma}. \tag{14}$$

The discount factor is denoted by $0 < \beta < 1$. The parameters $\sigma \geq 1$ and $\phi > 0$ are coefficient of relative risk aversion and the inverse of the Frisch elasticity, respectively. The parameter $\phi_0 > 0$ determines the dis-utility of labour. Households internalize the dynamics of X_t when making optimal choices. The shift factor X_t, adjusted by the growth rate of consumption, introduces non-time separability between consumption and hours whilst nesting both King et al. (1988) (when $\varsigma = 1$) and Greenwood et al. (1988) (when ς=0) type preferences. Importantly, as long as $\varsigma > 0$, these preferences are consistent with balanced growth.

Labor is supplied to either the consumption goods or the investment goods producing sectors according to a CES aggregator as in Petrella et al. (2019):

$$L_t = \left(\chi_c^{-\frac{1}{\xi}} L_{c,t}^{\frac{1+\xi}{\xi}} + \chi_i^{-\frac{1}{\xi}} L_{i,t}^{\frac{1+\xi}{\xi}} \right)^{\frac{\xi}{1+\xi}}, \tag{15}$$

where $\chi_c = L_c/L$ and $\chi_i = L_i/L$ are the steady state shares of sectoral labor supplies and ξ denotes the elasticity of substitution in labor supply. This specification allows for different degrees of labor mobility between sectors. When $\xi = 0$, labor is immobile across sectors and sectoral labor supplied is determined by their steady state values. When $\xi = \infty$, workers movement across sectors is frictionless. In this case, sectoral wages are fully equalised.[14]

Households maximize their expected utility given by equations (13) and (14) with respect to (15) and the following flow budget constraint

$$C_t + \frac{P^*_{c,t}}{P_{c,t}} B_t = (1 + r_{t-1}) \frac{P^*_{c,t}}{P_{c,t}} B_{t-1} + W_{c,t} L_{c,t} + W_{i,t} L_{i,t} + \pi_{c,t} + \pi_{i,t} \tag{16}$$

where $\pi_{c,t}$ and $\pi_{i,t}$ are profits accrued to the household from owing the firms in the domestic consumption and investment goods producing sectors. $W_{c,t}L_{c,t}$ and $W_{i,t}L_{i,t}$ denote the household's labor income from supplying hours of work to the consumption and investment goods producing sector. Note that with labor adjustment costs real wages, in terms of the final consumption good, can differ across sectors in the short run.

7.1.2 Consumption goods producing sector

The production of consumption goods is modelled using a standard Cobb-Douglas production function, similar to the one used in the one-sector model of Section 3:

$$Y_{c,t} = \left(u_{c,t} K_{c,t-1}\right)^{\alpha} \left(Z_t L_{c,t}\right)^{1-\alpha}, \tag{17}$$

where $K_{c,t-1}$ denotes physical capital services used in the production of consumption goods, $u_{c,t}$ is the rate at which physical capital is utilised, $L_{c,t}$ is hours worked and Z_t is a non-stationary productivity process that affects TFP in both, the home and foreign economy.

7.1.3 Final Consumption Goods Producers

Final goods, used for consumption, C_t, are produced by combining home, $C_{H,t}$, and foreign-produced, $C_{F,t}$, intermediate consumption goods according to a constant elasticity of substitution (CES) technology where $\theta > 0$ determines the elasticity of substitution between home and foreign produced consumption goods

$$C_t = \left[\eta_c^{\frac{1}{\theta}} \left(C_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\eta_c)^{\frac{1}{\theta}} \left(C_{F,t}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

[14] Petrella et al. (2019) and Davis and Haltiwanger (2001) discuss evidence that suggests that labour supply is imperfectly mobile across sectors.

7.1.4 Investment goods producing sector

The production of investment goods is modelled in a similar fashion, with the one exception that output is affected by both the non-stationary productivity process Z_t that is common to both sectors as well as a sector specific non-stationary productivity process, V_t. This formulation of the shock processes implies that Z_t and V_t in the two-sector model are equivalent to their counterparts in the one-sector model in Section 3. Production in the investment sector is undertaken according to

$$Y_{i,t} = V_t \left(u_{i,t} K_{i,t-1}\right)^{\alpha} \left(Z_t L_{i,t}\right)^{1-\alpha}. \tag{18}$$

The parameter $0 < \alpha < 1$, determines the share of capital in production and to be consistent with our empirical model is assumed to be common across both sectors.

7.1.5 Capital accumulation

Firms accumulate sector specific capital. The accumulation constraint for firms in both sectors $j = i, c$ is given by

$$K_{j,t} = (1 - \delta_{j,t}) K_{j,t-1} + \left[1 - \frac{\psi}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - \Delta I\right)^2\right] I_{j,t} \tag{19}$$

where $\psi > 0$ captures investment adjustment costs as in Christiano et al. (2005) and ΔI denotes the steady state growth rate of investment. Firms in both sectors can vary the degree to which they utilize their capital stock. The depreciation rate $\delta(u_{j,t})$ varies with the intensity of capital use as e.g. in Görtz and Tsoukalas (2013)

$$\delta(u_{j,t}) = \bar{\delta} + \nu_0 \left(u_{j,t}^{\nu} - 1\right)).$$

Since $\delta(u_{j,t})$ is strictly increasing and convex, more intensive use of capital accelerates depreciation exponentially. The parameter $\nu > 0$ measures the costliness of varying the capital utilization and the elasticity of marginal capital utilization equals $\nu - 1$. The steady state depreciation rate is given by $\bar{\delta} > 0$ and $\nu_0 > 0$ allows calibrating steady state utilization and depreciation independently of each other.

7.1.6 Market clearing

The labor market clears according to equation (15). We assume that investment goods are only purchased from domestic firms, so that $Y_{i,t} = I_t$ and the market for investment goods clears according to

$$I_t = I_{c,t} + I_{i,t}.$$

Economy wide output is given by

$$Y_t = \frac{P_{c,t}^H}{P_{c,t}} Y_{c,t} + \frac{P_{i,t}}{P_{c,t}^H} \frac{P_{c,t}^H}{P_{c,t}} Y_{i,t},$$

where $\frac{P_{c,t}^H}{P_{c,t}}$ is the price of home-produced consumption goods relative to the price of the domestic final consumption good which is a CES function of home and foreign-produced consumption goods. $\frac{P_{i,t}}{P_{c,t}^H} \frac{P_{c,t}^H}{P_{c,t}}$ is the price of investment goods in terms of the final consumption good. The market clearing condition for home-produced consumption goods is:

$$NY_{c,t} = NC_{H,t} + (1-N)C_{H,t}^*$$

where N denotes normalized size of the home country and $C_{H,t}$ and $C_{H,t}^*$ denote home and foreign demand for home-produced consumption goods, respectively.

7.1.7 Relative price of investment goods

An important distinguishing feature of this model in comparison to the standard one-sector model of Section 3, is that the relative price of investment goods, $\frac{P_{i,t}}{P_{c,t}}$ is an endogenous variable. We can use the first-order conditions for labor input in the two sectors to derive an expression for the relative price of investment goods:

$$\frac{P_{i,t}}{P_{c,t}^H} = \frac{1}{V_t} \left(\frac{u_{c,t}}{u_{i,t}} \frac{K_{c,t-1}}{K_{i,t-1}} \frac{L_{i,t}}{L_{c,t}} \right)^{\alpha} \frac{W_{i,t}}{W_{c,t}}. \tag{20}$$

As in the one-sector model, the relative price of investment goods is an inverse function of our measure of investment specific technological progress, V_t, but it is now also determined by relative inputs of labor and capital utilisation rates as well as differences in sectoral wages. In principle, the RPI can hence be affected by any exogenous shock and not only those of the investment specific type.

7.1.8 Foreign economy

The foreign economy, whose variables are denoted with a (*), is symmetric to the home economy. Where home and foreign differ is in their relative size. We treat home as a small open economy, such that foreign shocks affect the home economy, but not vice versa.

7.1.9 Shock processes

Define the growth rate of TFP as $\Gamma_t^Z = \frac{Z_t}{Z_{t-1}}$ and that of IST as $\Gamma_t^V = \frac{V_t}{V_{t-1}}$ in the home country and as $\Gamma_t^{Z*} = \frac{Z_t^*}{Z_{t-1}^*}$ and $\Gamma_t^{V*} = \frac{V_t^*}{V_{t-1}^*}$ in the foreign country. Consistent with our empirical model, the growth rates of TFP and IST follow an AR(1) process. The growth rates of domestic TFP and IST are affected by domestic sector specific shocks as well as by the the level of foreign TFP and IST via an error correction mechanism whose size is determined by κ. We assume that the home economy is *small*, such that the growth rates of foreign TFP and IST are not affected by the level of home TFP and IST.

$$\log \Gamma_t^Z = \rho_Z \log \Gamma_{t-1}^Z - \kappa_Z \log \left(\frac{Z_{t-1}}{Z_{t-1}^*} \right) + \epsilon_{t,0}^Z + \epsilon_{t-1,1}^Z + \epsilon_{t-2,2}^Z + \epsilon_{t-3,3}^Z + \epsilon_{t-4,4}^Z \tag{21}$$

$$\log \Gamma_t^V = \rho_V \log \Gamma_{t-1}^V - \kappa_V \log \left(\frac{V_{t-1}}{V_{t-1}^*} \right) + \epsilon_{t,0}^V + \epsilon_{t-1,1}^V + \epsilon_{t-2,2}^V + \epsilon_{t-3,3}^V + \epsilon_{t-4,4}^V \tag{22}$$

$$\log \Gamma_t^{Z*} = \rho_{Z*} \log \Gamma_{t-1}^{Z*} + \epsilon_{t,0}^{Z*} + \epsilon_{t-1,1}^{Z*} + \epsilon_{t-2,2}^{Z*} + \epsilon_{t-3,3}^{Z*} + \epsilon_{t-4,4}^{Z*} \tag{23}$$

$$\log \Gamma_t^{V*} = \rho_{V*} \log \Gamma_{t-1}^{V*} + \epsilon_{t-1,1}^{V*} + \epsilon_{t-2,2}^{V*} + \epsilon_{t-3,3}^{V*} + \epsilon_{t-4,4}^{V*} \tag{24}$$

The growth rates of home and foreign productivity growth rates are subject to shocks. The first component of the shock, $\epsilon_{t,0}^X$, is unanticipated (with $X = Z, V, Z*, V*$) whereas the components $\epsilon_{t-1,1}^X$ to $\epsilon_{t-4,4}^X$ are anticipated and represent news about period t that arrives between one and four quarters ahead, respectively. The innovations are uncorrelated across horizon, technology type and country.

7.2 Anticipated increases in the trend of TFP and IST in the DSGE model

The aim of this part of the paper is to show that, and under which conditions, our two-sector DSGE model can generate data-congruent news-driven business cycles.

7.2.1 Prior predicitve analysis

We calibrate a subset of parameters and establish plausible ranges for the remaining parameters via a prior predictive analysis. Panel A of Table 2 contains parameters that are directly determined by the data or are used to tie down the steady state of the model. We treat the UK as a small open economy with a relative size of 0, and an openness to trade, γ of 0.24, which corresponds to share of imports in GDP in our sample. The discount rate, β is set at 0.995 which corresponds to an annualised real interest rate of 2%. We set σ to 1, which determines the coefficient of relative risk aversion. ϕ_b is a small bond holding cost that is used ensure stationarity in models with incomplete financial markets, as suggested by Schmitt-Grohe and Uribe (2003). This parameter is usually set to a small value. As in Kamber et al. (2017), we set this parameter to 0.001. The share of capital in the production function, α, is assumed to be the same in both sectors. This assumption in necessary for the two-sector model to be consistent with the set of identifying restrictions, (9), that are based on the standard one-sector framework, but can in principle be relaxed in other circumstances. The parameters determining the properties of the anticipated TFP and IST shocks, ρ_z, σ_z and ρ_v, σ_v are chosen so that the model implied paths matches their empirical dynamics.

For the remaining parameters, we carry out a prior predictive analysis. This type of analysis also used by, amongst others, Leeper et al. (2017) helps us to understand the distribution of model parameters for which our two-sector DSGE model can generate news-driven business cycles. Specifically, we specify a mean as well as a lower and upper bound for each parameter, detailed in Panel B of Table 2. We then take a sequence of random parameter draws, within a specified parameter range, and solve the model for each draw under the assumption that the model is hit by 4-quarter ahead shocks to the trend of foreign TFP and IST. We keep only those draws that generate impulse response functions that satisfy the condition that domestic GDP rises in each of the four anticipation periods, for both TFP and IST shocks. In other words, we keep only parameter draws that allow the model to generate news-driven business cycle responses of domestic output.

We choose the mean values and parameter ranges for each parameter as follows. The inverse of the Frisch elasticity of labor supply, ϕ, is set to 1 for both countries, with a range of 0.25 to 5. Lambertini and Proebsting (2023), Petrella et al. (2019) and Horvath (2000) all suggest a value for the elasticity of substitution of labor supply across sectors, ξ, of 1. We examine a range for this parameter between 0.01, which corresponds labor that is perfectly immobile between sectors, and 100, which corresponds to labor being perfectly flexible. We set the trade elasticity, θ, to 1 with a range from 0.6 to 10. The mean value of the capital utilisation rate parameter, ν takes a relatively small value of 1.15 as in Görtz and Tsoukalas (2013) and Smets and Wouters (2003). A low value helps the model generate a positive output response to news. As equation (20) suggests, matching the differences in the initial response of RPI to an anticipated increase in the trend of TFP across countries, suggests that a less responsive utilisation rate is required in the US investment goods sector. For this reason, we set the mean value of $\nu_i^* = 10$. For all sectoral values of ν we chose a range between 1.01 and 25, encompassing the largest possible as well as a very small degree of responsiveness of the the utilisation rate. Investment adjustment costs work in conjunction with variable capacity utilisation to generate an uplift in output and labor supply in response to a positive news shock. We choose a mean of 25 and a range of 0 to 50 for this parameter. Finally, the parameter ς, from the utility function takes a mean of 0.5 with a range, corresponding to the theoretical setup, between 0.01 and 0.99.

Table 2: Model parameters — two-sector model

Parameter	Description	Value	
Panel A: calibrated parameters			
β	discount rate	0.995	
N	relative size of UK to the US	0	
γ	openness	0.24	
α	share of capital in GDP	0.3	
δ	depreciation rate	0.02	
ϕ_b	bond holding cost	0.001	
ϕ_0	utility function parameter	to yield L=1/3	
ϕ_0^*	utility function parameter	to yield L=1/3	
σ	Constant relative risk aversion parameter	1	
ρ_{Z*}	Persistence of international TFP shock	0.7503	
σ_{Z*}	Standard deviation of international TFP shock	0.2149	
ρ_{V*}	Persistence of international IST shock	0.7480	
σ_{V*}	Standard deviation of international IST shock	0.1964	
κ_Z, κ_V	Error correction terms	0.1	
Panel B: specifications for prior predictive analysis			
		Mean	Range
ς	Jaimovich and Rebelo (2009) parameter	0.5	(0.01 - 0.99)
ϕ	UK Inverse of Frisch elasticity	1	(0.25 - 5)
ϕ^*	US Inverse of Frisch elasticity	1	(0.25 - 5)
θ	Trade elasticity	1	(0.6 - 10)
ψ_c	UK investment adjustment cost in c-sector	25	(0 - 50)
ψ_i	UK investment adjustment cost in i-sector	25	(0 - 50)
ψ_c^*	US investment adjustment cost in c-sector	25	(0 - 50)
ψ_i^*	US investment adjustment cost in i-sector	25	(0 - 50)
ν_c	UK capital utilisation parameter in c-sector	1.15	(1.01 - 25)
ν_i	UK capital utilisation parameter in i-sector	1.15	(1.01 - 25)
ν_c^*	US capital utilisation parameter in c-sector	1.15	(1.01 - 25)
ν_i^*	US capital utilisation parameter in i-sector	10	(1.01 - 25)
ξ	UK elasticity of substitution in labor supply	1	(0.01 - 100)
ξ^*	US elasticity of substitution in labor supply	1	(0.01 - 100)

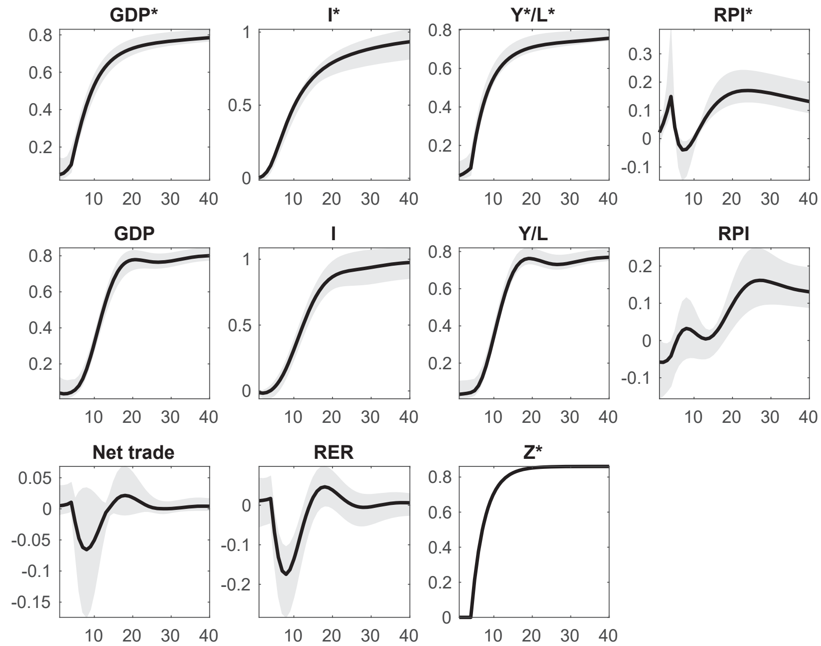

Figure 5: **Responses to a non-stationary international TFP shock in the two sector DSGE model** The chart shows real GDP, total investment, GDP per hour worked and the relative price of investment goods for the foreign (*) and the home economy, as well as home-country net trade, real exchange rate. Z^* denotes the international TFP process. The black solid lines are the median responses. The grey shaded area corresponds to the 5th and 95th percentile of parameter draws.

7.2.2 DSGE model impulse responses

Figure 5 shows impulse responses to an anticipated 4-quarter ahead shock to the trend of foreign economy-wide TFP, Γ_t^{Z*}. This shock affects the trend of domestic TFP, Γ_t^Z via an error correction mechanism as specified in equation (21). The black solid lines show the 50th percentile impulse responses and the grey shaded area is the range between the 5th and the 95th percentile of the (prior) distribution of the IRFs consistent with the empirical model predictions. Since we reject all parameter draws that do not show domestic GDP rising in the first four quarters, by construction, the DSGE model generates an expansion in domestic GDP. It is remarkable, that it indeed matches most of the qualitative features of the data from Figure 3. GDP, both at home and abroad, rises in anticipation of higher future TFP, as does investment. Output per hour also rises, on impact and continues to rise once the trend in TFP rises. The real appreciation observed in data is also present in the model. Since an increase in foreign TFP affects home TFP with a lag via an error correction model, one would expect the home real exchange rate to appreciate once the shock is realised. The impact of net trade in the model is small in magnitude consistent with the insignificant response in the data. Importantly, the model also manages to match the qualitative response of the relative price of investment goods. Equation (20) suggests that one way for the relative price of investment goods to increase in response to a positive TPF shock is for the utilisation rate in the consumption goods sector to increase by more than in the investment goods sector. This can be achieved for values of ν_i^* and ν_c^* such that the former exceeds the latter in size, making the utilization rate in the consumption goods sector more elastic than its counterpart in the investment goods sector.

Figure 6 shows impulse responses for an anticipated 4-quarter ahead shock to the trend of foreign investment specific technology, Γ_t^{V*}. This shock affects the trend of domestic IST, Γ_t^V, via an error correction mechanism as specified in equation (22). As in the empirical responses, output rises throughout the anticipation period, both in the home and the foreign economy. Investment also starts to rise before the shock to the trend of IST is realised. A realised shock to investment sector TFP lowers the relative price of investment goods, both in the home and in the foreign economy. The median response of the real exchange is to depreciate, however, the range between the 5th and the 95th percentile contains parameter draws consistent with both a real appreciation and a real depreciation. A similar conjecture applies to the response of net trade. An increase in domestic GDP in anticipation of an increase in the trend of foreign IST is for realistic parameter values consistent with an increase in net trade or a deterioration.

Figure 7 shows the histogram of the parameters in our prior predictive analysis. We took 100,000 draws of the parameters listed in the bottom half of Table 2 and solved the model for each draw. 89,749 of the draws satisfied the conditions that domestic GDP increase during the anticipation period for both TFP and IST trend shocks. The accepted parameter draws appear to be clustered around their prior values. The prior predictive analysis clearly rejects low, as well as very high values of the adjustment cost parameters. It also clearly rejects very high values ν, which determines the responsiveness of capacity utilisation. Jaimovich and Rebelo (2009) show that one modelling feature that allows GDP in our model to rise in response to positive TFP news is variable capacity utilization.[15] If utilization rises in response to positive TFP news, output will increase on impact. A rise in the utilization rate also raises the demand for labor. The response of the utilization rate itself is linked to the dynamics of Tobin's Q, which becomes clearer when considering the representative firm's optimality condition for the utilization rate in, for example, the consumption goods sector:

$$\alpha \frac{P_{c,t}^H}{P_{c,t}} Y_{c,t} = Q_{c,t} K_{c,t-1} \nu_0 \nu u_{c,t}^{\nu}$$

where ν and ν_0 are parameters of the function determining the costs of utilization. For the utilization rate to rise, $Q_{c,t}$ has to fall, furthermore, the bigger the decline in $Q_{c,t}$ the larger will be the response of

[15] Jaimovich and Rebelo (2009) actually show that variable capacity utilisation and investment adjustment costs can help a closed economy model generate news driven business cycles. This combination of modelling features will also work in a two-country model such as ours when considering international shocks.

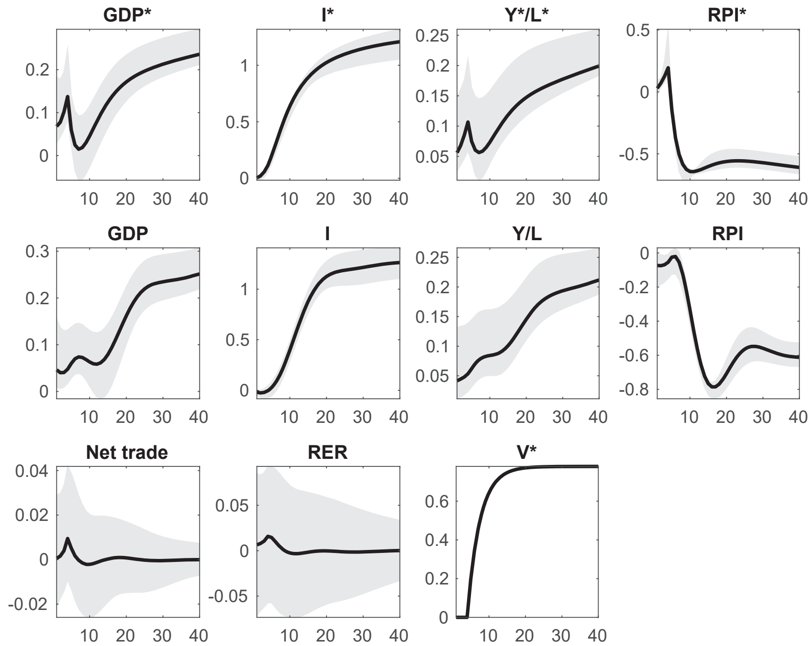

Figure 6: **Responses to a non-stationary international IST shock in the two sector DSGE model** The vertical red line denotes the period in which total factor productivity is expected to rise. The chart shows GDP, total investment, GDP per hour worked and the relative price of investment goods for the foreign (*) and the home economy, as well as home-country net trade, real exchange rate. V denotes the international IST process.

the utilisation rate. The investment adjustment cost parameter, ψ, determines the volatility of Q. When $\psi = 0$, Q_t is always equal to 1. This explains why the the parameter draws from the prior predictive analysis are centred around large values of the investment adjustment cost parameter and low values of ν.

The parameters ξ and ξ^* denote the elasticity of substitution in labour supply. Despite taking draws from a near zero to 100, the bulk of the accepted draws are for values less than 2. Since there a no accepted draws of 10 or above, in order to generate news-driven business cycles, the model requires a degree of imperfect labor mobility across sectors. The parameter ς, which determines whether the utility function is GHH or KPR, is equally distributed between 0.01 (GHH preferences) and 0.99 (KPR preferences) which suggests that model's ability to generate news-driven business cycles is independent of this parameter.

In summary, the two-sector model is successful in qualitatively matching the empirical impulse responses to TFP and IST news shocks. The two-sector setup is key to resembling the empirical movements of the RPI which distinguish the responses to the different type of technology shocks. Adjustment costs in capital and sectoral labor reallocation, as well as a low value for the capital utilization rate parameter, ν, enable to model to resemble the empirically observed broad based macroeconomic expansions.

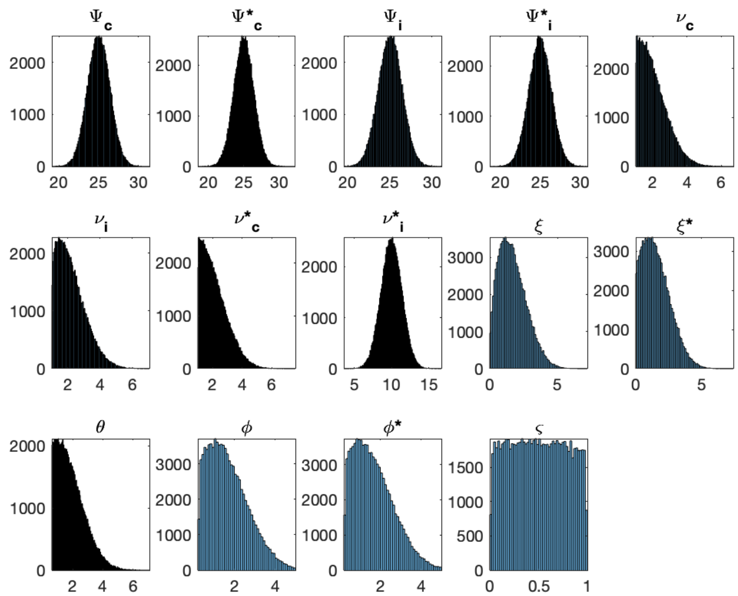

Figure 7: **Distribution of parameter values that allow the model to generate news-driven business cycles.** This figure shows the histogram for each of the parameters subject to the prior predictive analysis. We took 100,000 parameter draws of which 89,749 draws allowed the model to generate an increase in domestic GDP in the first four quarters following the arrival of positive news about the trend in TFP and IST.

8 Conclusion

This paper studies the anatomy of productivity trend shocks for the UK economy. We employ a novel econometric strategy to jointly identify international and domestic, anticipated and unanticipated investment specific (IST) and total factor productivity (TFP) trend shocks. In our state space model, these disturbances compete with cyclical shocks in explaining UK aggregate fluctuations. The long-run restrictions used to jointly identify these types of disturbances in the data are informed by a standard two-country structural model. This approach allows for a holistic picture of productivity trends in a small open economy. We find that anticipated international non-stationary TFP and IST shocks and, to a somewhat lesser extent, anticipated UK-specific non-stationary TFP and IST shocks are important for explaining fluctuations in UK macroeconomic aggregates. In contrast, the role of their unanticipated counterparts, which often play a prominent part in structural models, is extremely limited. We show that a standard international business cycle model implies restrictions that are inconsistent with the empirically observed dynamics of the relative price of investment. This has strong implications for exercises that take these models to the data to assess the absolute and relative importance of IST and TFP shocks for aggregate fluctuations. We show that a two-sector international business cycle model can overcome the limitations of the standard framework. In presence of investment adjustment cost and adjustment costs for sectoral labor reallocation, this model can resemble the empirically observed dynamic responses to international anticipated TFP and IST trend shocks.

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Appendix (for online publication)

A Additional Empirical Evidence

This section shows additional impulse response functions in response to unanticipated and anticipated international technology shocks.

A.1 Unanticipated International Technology Shocks

Figures 8 and 9 show the responses of the US and UK economies to international non-stationary TFP and IST surprise shocks, respectively. Both shocks trigger a rise in macroeconomic aggregates in the US economy. The international TFP surprise shock also generates an expansion in UK GDP, investment and labor productivity, while IRFs of these variables in response to the IST surprise shock are not significantly different from zero. As for their anticipated counterparts the surprise TFP shock triggers a rise in the US and UK relative price of investment while these variables decline in response to a surprise IST shock.

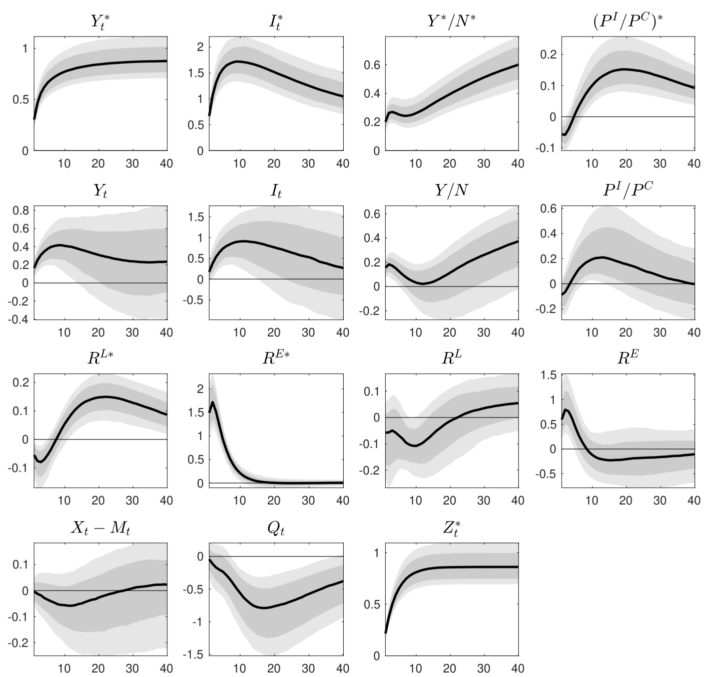

Figure 8: **Responses to an unanticipated non-stationary international TFP shock.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

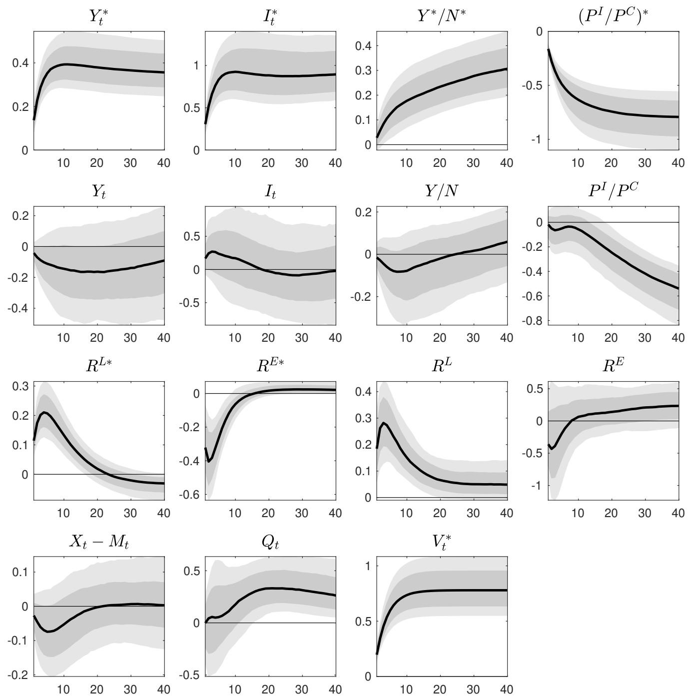

Figure 9: **Responses to an unanticipated non-stationary international IST shock.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

A.2 Anticipated International Technology Shocks

The four quarter ahead international technology news shock components account for most of the forecast error variance in US and UK GDP. Responses to the one, two and three quarter ahead news components are qualitatively very similar to those of the four quarter ahead news shocks shown in Section 6. Figures 10 and 11 exemplify this by depicting the IRFs to three quarter ahead international TFP and IST news shocks, respectively. IRFs to further news components are available upon request.

B Estimation of the Empirical Model

This appendix provides information about the prior moments or the VAR parameters and the Gibbs sampler steps.

B.1 Prior & Posterior VAR Parameter Moments

As proposed by Zha (1999) we exploit the block recursive structure of the VAR model – due to the small open economy restrictions and homogeneity of trends – and break the likelihood into $m = 6$ (two for

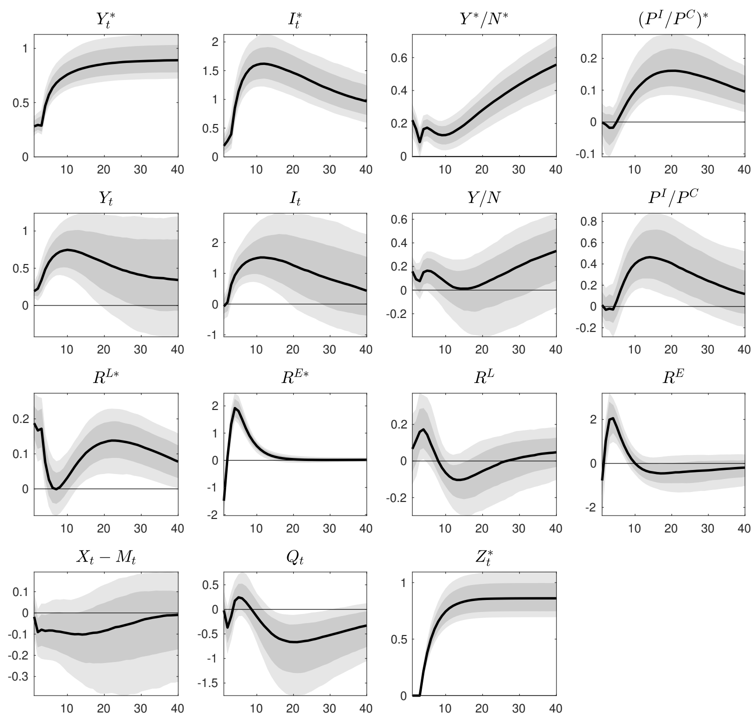

Figure 10: **Responses to a non-stationary international TFP shock which is anticipated three quarters ahead.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

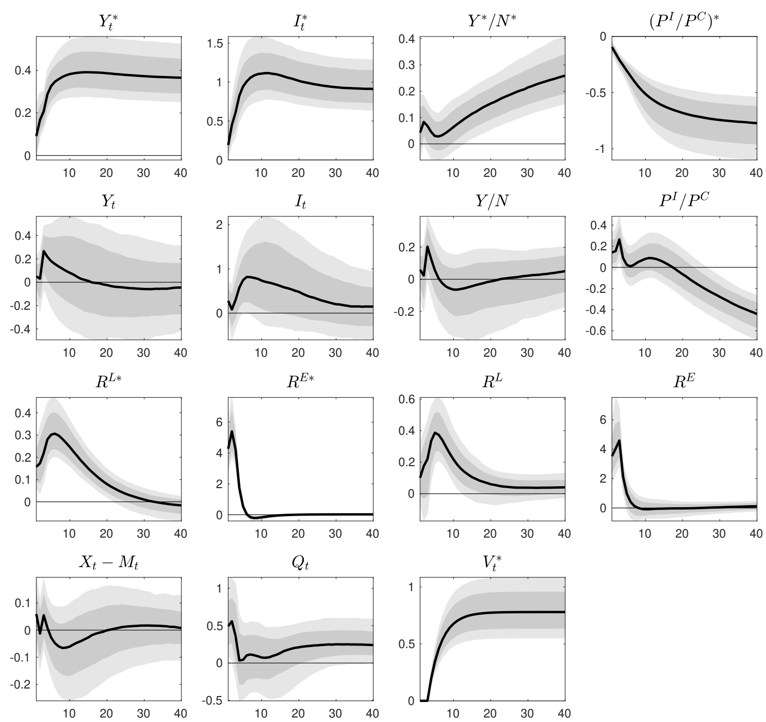

Figure 11: **Responses to a non-stationary international IST shock which is anticipated three quarters ahead.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

the foreign trends, one for the foreign economy, two for the domestic trends and one for the domestic economy) blocks. As explained in Zha (1999), this is extremely convenient property as it allows to draw parameters block by block and preserving the zero restrictions. The VAR model can take the following form

$$\xi_t = \begin{bmatrix} \Delta \ln z_t^* \\ \Delta \ln v_t^* \\ \xi_t^* \\ \Delta \ln z_t \\ \Delta \ln v_t \\ \xi_t^{domestic} \end{bmatrix} = \begin{bmatrix} \psi_t^1 & & & & & \\ & \psi_t^2 & & & & \\ & & \psi_t^3 & & & \\ & & & \psi_t^4 & & \\ & & & & \psi_t^5 & \\ & & & & & \psi_t^6 \end{bmatrix} \begin{bmatrix} B^1 \\ B^2 \\ B^3 \\ B^4 \\ B^5 \\ B^1 \end{bmatrix} + \begin{bmatrix} \omega_t^1 \\ \omega_t^2 \\ \omega_t^3 \\ \omega_t^4 \\ \omega_t^5 \\ \omega_t^6 \end{bmatrix}$$

Each block can be expressed as

$$\xi_t^j = B^j \psi_t^j + \omega_t^j, \;\; \omega_t^j \sim \mathcal{N}(\mathbf{0}, \Sigma^j)$$

where ψ_t^j contains lags of ξ_t^j and it can also contains contemporenaous and lag values of ξ_t^{j-1}, ω_t^j is a $d\omega^j \times 1$ vector of reduced-form errors that is normally distributed with zero mean and covariance matrix Σ^j. The regression-equation representation of this system is

$$\Xi^j = \Psi^j B^j + U^j \tag{25}$$

where $\Xi = [\xi_{h+1}^j, ..., \xi_T^j]$ is a $d\xi^j \times T$ matrix, $\Psi^j = \begin{bmatrix} \Xi_{-h}^j & \Xi^{j-1} & \Xi_{-h}^{j-1} \end{bmatrix}$ is a $(Hd\xi^j + (H+1)\, d\xi^{j-1}) \times T$ matrix containing the h-th lag of Ξ^j, contemporaneous and lag values Ξ^{j-1}, $B^j = [B_1^j, ..., B_H^j]$ is a $d\xi^j \times (Hd\xi^j + (H+1)\, d\xi^{j-1})$ matrix, and $\Omega^j = [\omega_{h+1}^j, ..., \omega_T^j]$ is a $d\xi^j \times T$ matrix of disturbances.

The Bayesian estimation of VAR models has become standard in empirical macroeconomics. Specifically, we use a Minnesota-type prior (Doan et al., 1984b; Litterman, 1986). It is assumed that the prior distribution of the VAR parameters has a Normal-Wishart conjugate form

$$\beta^j | \Sigma^j \sim \mathcal{N}(\tilde{\beta}^j, \Sigma^j \otimes \tilde{F}^j), \;\; \Sigma^j \sim \mathcal{IW}(\kappa, \tilde{\Sigma}^j), \tag{26}$$

where β^j is obtained by stacking the columns of B^j, $\tilde{\beta}^j = vec\left(\tilde{B}^j\right)$ and $\tilde{\Sigma}_B^j = \Sigma^j \otimes \tilde{F}^j$. In contrast to Litterman (1986), the covariance matrix Σ^j in the prior described in Equation (26) is not replaced by an estimated and thus known (diagonal) counterpart. Therefore, sampling from the conditional posterior distributions described below requires Gibbs sampling (see also Mumtaz and Zanetti, 2012). The

(Minnesota) prior moments of β^j are given by

$$\mathbb{E}[(B_h^j), i, k] = \begin{cases} \delta_i^j & i = k, h = 1 \\ 0 & \text{otherwise} \end{cases} \quad , \quad Var[(B_h), i, h] = \lambda \left(\sigma_i^j\right)^2 / \left(\sigma_k^j\right)^2,$$

and, as outlined in Bańbura et al. (2010), they can be constructed using the following T_D^j dummy observations

$$Y_D^j = \begin{pmatrix} \frac{diag(\delta_1\sigma_1,...,\delta_{d\xi^j}\sigma_{d\xi^j})}{\lambda} \\ 0_{d\xi^j \times \left(Hd\xi^j + (H+1)d\xi^{j-1}\right)} \\ .... \\ diag(\sigma_1, ..., \sigma_{d\xi^j}) \\ .... \\ 0_{1\times d\xi^j} \end{pmatrix} \text{and } X_D^j = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1,...,\sigma_{d\xi^j})}{\lambda} \\ 0_{d\xi^j \times \left(Hd\xi^j + (H+1)d\xi^{j-1}\right)} \\ .... \\ 0_{1\times Hd\xi^j} \end{pmatrix},$$

where $J_P = diag(1, 2, ..., H)$ and $diag$ denotes the diagonal matrix. The prior moments in Equation (26) are functions of Y_D^j and X_D^j, $\tilde{B}^j = Y_D^j X_D^{j\prime} \left(X_D^j X_D^{j\prime}\right)^{-1}$, $\tilde{F}^j = (X_D X_D')^{-1}$, $\tilde{\Sigma}^j = (Y_D^j - \tilde{B}^j X_D^j)(Y_D^j - \tilde{B}^j X_D^j)'$ and $\kappa^j = T_D^j - d\xi^j H$. Finally, the hyper-parameter λ controls the tightness of the prior and our baseline choice is $\lambda = 2$.

Since the normal-inverse Wishart prior is conjugate, the conditional posterior distribution of this model is also normal-inverse Wishart (Kadiyala and Karlsson, 1997b)

$$\beta^j|\Sigma^j, \Xi^j, Z^j \sim \mathcal{N}(\bar{\beta}^j, \Sigma^j \otimes \bar{F}^j), \;\; \Sigma^j|\Xi^j, Z^j \sim \mathcal{IW}(\bar{v}^j, \bar{S}^j),$$

where variables with a bar denote the parameters of the posterior distribution. Defining $\hat{B}^j$ and $\hat{U}^j$ as the OLS estimates from Equation (25), the parameters of the conditional posterior distribution can be computed as $\bar{B}^j = \left((\tilde{F}^j)^{-1}\tilde{S}^j + \Xi^j\Psi^{j\prime}\right)\left((\tilde{F}^j)^{-1} + (\Psi^j\Psi^{j\prime})^{-1}\right)^{-1}$, $\bar{F}^j = \left((\tilde{F}^j)^{-1} + (\Psi^j\Psi^{j\prime})^{-1}\right)^{-1}$, $\bar{\kappa}^j = \kappa^j + T$, and $\bar{S}^j = \hat{B}^j + \tilde{B}^j\left(\tilde{F}^j\right)^{-1}\left(\tilde{B}^j\right)' + S_0 + \hat{U}^j\left(\hat{U}^j\right)' - \bar{B}^j\left(\bar{F}^j\right)^{-1}\left(\bar{B}^j\right)'$. Lastly, as in Mumtaz and Zanetti (2012), the values of the persistence parameter δ_i^j and the error standard deviation σ_i^j of the AR(1) model are obtained from its OLS estimation.

B.2 Gibbs Sampler Steps

Given C, A and starting values for B and Σ

For $i = 1$,...,Nsimulations:

1. Use Durbin and Koopman (2002) algorithm to draw ξ_t^i conditional on ζ_t, C, A, B^{i-1} and Σ^{i-1}

2. For $j = 1$,...,m

- Draw $\left(B^j\right)^i$ and $\left(\Sigma^j\right)^i$ from Normal-Inverse-Wishart distribution using the moments discussed in the previous section conditional on ζ_t, ξ_t^i, C, A, $\left(B^{j-1}\right)^i$ and $\left(\Sigma^{j-1}\right)^i$

3. If the maximum absolute value of B is less than one move to step 1 otherwise repeat step 2

Discard the Nburn first draws and use the remaining Nsimulations-Nburn draws to infer the statistics.

C Two-Sector Model

This appendix lays out the home country equations of the two-sector DSGE model. The set pertaining to the foreign economy are symmetric to ones presented here.

C.1 Non-stationary model equations

C.1.1 Household's optimality conditions

$$\left(C_t - X_t\frac{\phi_0}{1+\phi}L_t^{1+\phi}\right)^{-\sigma} + \Omega_t\varsigma\frac{X_t}{C_t} = \Lambda_t \tag{27}$$

$$-\left(C_t - X_t\frac{\phi_0}{1+\phi}L_t^{1+\phi}\right)^{-\sigma}\frac{\phi_0}{1+\phi}L_t^{1+\phi} + \beta\Omega_{t-1}(1-\varsigma)\frac{X_{t+1}}{X_t} = \Omega_t \tag{28}$$

$$\left(C_t - X_t\frac{\phi_0}{1+\phi}L_t^{1+\phi}\right)^{-\sigma}X_t\phi_0L_t^{\phi}\left(\frac{L_{c,t}}{L_t}\right)^{\frac{1}{\xi}}\left(\frac{1}{\chi_c}\right)^{\frac{1}{\xi}} = \Lambda_tW_{c,t} \tag{29}$$

$$\left(C_t - X_t\frac{\phi_0}{1+\phi}L_t^{1+\phi}\right)^{-\sigma}X_t\phi_0L_t^{\phi}\left(\frac{L_{i,t}}{L_t}\right)^{\frac{1}{\xi}}\left(\frac{1}{\chi_i}\right)^{\frac{1}{\xi}} = \Lambda_tW_{i,t} \tag{30}$$

$$\frac{P_{c,t}^*}{P_{c,t}}\Lambda_t = \beta(1+r_t)\frac{P_{c,t+1}^*}{P_{c,t+1}}\Lambda_{t+1} \tag{31}$$

$$X_t = C_t^{\varsigma}X_{t-1}^{1-\varsigma}\left(\frac{Z_tV_t^{\frac{\alpha}{1-\alpha}}}{Z_{t-1}V_{t-1}^{\frac{\alpha}{1-\alpha}}}\right)^{1-\varsigma} \tag{32}$$

C.1.2 Consumption good sector

$$Y_{c,t} = \left(u_{c,t}K_{c,t-1}\right)^{\alpha}\left(Z_tL_{c,t}\right)^{1-\alpha} \tag{33}$$

$$K_{c,t} = (1-\delta_{c,t})K_{c,t-1} + \left[1 - \frac{\psi}{2}\left(\frac{I_{c,t}}{I_{c,t-1}} - \Delta I\right)^2\right]I_{c,t} \tag{34}$$

$$(1-\alpha)\frac{P_{c,t}^H}{P_{c,t}}\frac{Y_{c,t}}{L_{c,t}} = W_{c,t} \tag{35}$$

$$\alpha \frac{P^H_{c,t}}{P_{c,t}} \frac{Y_{c,t}}{u_{c,t}} = Q_{c,t} K_{c,t-1} \nu_0 \nu u_{c,t}^{\nu-1} \tag{36}$$

$$Q_{c,t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[Q_{c,t+1}(1-\delta_{c,t+1}) + \frac{P^H_{c,t+1}}{P_{c,t+1}} \alpha \frac{Y_{c,t+1}}{K_{c,t}} \right] \tag{37}$$

$$\begin{aligned} \frac{P_{i,t}}{P_{c,t}} &= Q_{c,t} \left[\left(1 - \frac{\psi}{2} \left(\frac{I_{c,t}}{I_{c,t-1}} - \Delta I \right)^2 \right) - \psi \left(\frac{I_{c,t}}{I_{c,t-1}} - \Delta I \right) \frac{I_{c,t}}{I_{c,t-1}} \right] \\ &+ \beta E_t Q_{c,t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{I_{c,t+1}}{I_{c,t}} - \Delta I \right) \left(\frac{I_{c,t+1}}{I_{c,t}} \right)^2 \end{aligned} \tag{38}$$

C.1.3 Investment goods sector

$$Y_{i,t} = V_t \left(u_{i,t} K_{i,t-1} \right)^\alpha \left(Z_t L_{i,t} \right)^{1-\alpha} \tag{39}$$

$$K_{i,t} = (1-\delta_{i,t}) K_{i,t-1} + \left[1 - \frac{\psi}{2} \left(\frac{I_{i,t}}{I_{i,t-1}} - \Delta I \right)^2 \right] I_{i,t} \tag{40}$$

$$(1-\alpha) \frac{P_{i,t}}{P_{c,t}} \frac{Y_{i,t}}{L_{i,t}} = W_{i,t}$$

$$\alpha \frac{P_{i,t}}{P_{c,t}} \frac{Y_{i,t}}{u_{i,t}} = Q_{i,t} K_{i,t-1} \nu_0 \nu u_{i,t}^{\nu-1}$$

$$Q_{i,t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[Q_{i,t+1}(1-\delta_{i,t+1}) + \frac{P_{i,t+1}}{P_{c,t+1}} \alpha \frac{Y_{i,t+1}}{K_{i,t}} \right]$$

$$\begin{aligned} \frac{P_{i,t}}{P_{c,t}} &= Q_{i,t} \left[\left(1 - \frac{\psi}{2} \left(\frac{I_{i,t}}{I_{i,t-1}} - \Delta I \right)^2 \right) - \psi \left(\frac{I_{i,t}}{I_{i,t-1}} - \Delta I \right) \frac{I_{i,t}}{I_{i,t-1}} \right] \\ &+ \beta E_t Q_{i,t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{I_{i,t+1}}{I_{i,t}} - \Delta I \right) \left(\frac{I_{i,t+1}}{I_{i,t}} \right)^2 \end{aligned}$$

C.1.4 Market clearing

$$I_{c,t} + I_{i,t} = Y_{i,t} \tag{41}$$

$$N Y_{c,t} = N C_{H,t} + (1-N) C^*_{H,t} \tag{42}$$

C.1.5 Current account

$$C_t + \frac{P_{i,t}}{P_{c,t}} I_t + \frac{P^*_{c,t}}{P_{c,t}} B_t = (1+\hat{r}_{t-1}) \frac{P^*_{c,t}}{P_{c,t}} B_{t-1} + \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} + \frac{P_{i,t}}{P_{c,t}} Y_{i,t}$$

C.1.6 Labor CES aggregate

$$L_t = \left(\chi_c^{-\frac{1}{\xi}} L_{c,t}^{\frac{1+\xi}{\xi}} + \chi_i^{-\frac{1}{\xi}} L_{i,t}^{\frac{1+\xi}{\xi}} \right)^{\frac{\xi}{1+\xi}} \tag{43}$$

C.1.7 Economy-wide GDP

$$Y_t = \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} + \frac{P_{i,t}}{P^H_{c,t}} \frac{P^H_{c,t}}{P_{c,t}} Y_{i,t} \tag{44}$$

C.1.8 Relative prices

Relative prices are defined using the price index that corresponds to the CES consumption aggregate. θ denotes the elasticity of substitution between home and foreign-produced intermediate consumption goods. γ denotes openness to trade and is defined as $(1-\eta_c) = (1-N)\gamma$ for the domestic economy and $\eta^*_c = N\gamma$, where N is relative size of the home country and η_c and η^*_c are the shares of the home-produced intermediate consumption good in home and foreign consumption, respectively. The term $\frac{P^F_{c,t}}{P^H_{c,t}}$ corresponds to the terms of trade and $\frac{P^*_{c,t}}{P_{c,t}}$ to the consumption based real exchange rate.

$$\left(\frac{P^H_{c,t}}{P_{c,t}}\right)^{\theta-1} = (1-(1-N)\gamma) + (1-N)\gamma\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta} \tag{45}$$

$$\left(\frac{P^F_{c,t}}{P_{c,t}}\right)^{\theta-1} = (1-(1-N)\gamma)\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta} + (1-N)\gamma \tag{46}$$

$$\left(\frac{P^{*H}_{c,t}}{P^*_{c,t}}\right)^{\theta-1} = N\gamma + (1-N\gamma)\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta} \tag{47}$$

$$\left(\frac{P^{*F}_{c,t}}{P^*_{c,t}}\right)^{\theta-1} = N\gamma\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta} + (1-N\gamma) \tag{48}$$

$$\left(\frac{P^*_{c,t}}{P_{c,t}}\right)^{1-\theta} = \frac{N\gamma + (1-N\gamma)\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta}}{(1-(1-N)\gamma) + (1-N)\gamma\left(\frac{P^F_{c,t}}{P^H_{c,t}}\right)^{1-\theta}} \tag{49}$$

We close the model by introducing a small bond holding cost as in Schmitt-Grohe and Uribe (2003).

$$(1+r_t) = (1+r^*_t)e^{-\phi_B B_t} \tag{50}$$

C.2 Stationary equilibrium

To solve the model, turn what is a non-stationary model into a stationary one. We start by taking logs of the production function in the investment goods sector:

$$\ln Y_{i,t} = \ln V_t + \alpha \ln K_{i,t-1} + (1-\alpha)\ln L_{i,t} + (1-\alpha)\ln Z_t$$

$$\ln Y_{i,t} - \ln Y_{i,t-1} = \ln V_t - \ln V_{t-1} + \alpha \left(\ln K_{i,t-1} - \ln K_{i,t-2}\right) + (1-\alpha)\left(\ln L_{i,t} - \ln L_{i,t-1}\right) + (1-\alpha)\left(\ln Z_t - \ln Z_{t-1}\right)$$

We know that along the balanced growth path labour effort is constant and that the capital stock grows at the same rate as output.

$$\ln Y_{i,t} - \ln Y_{i,t-1} = \ln V_t - \ln V_{t-1} + \alpha \left(\ln Y_{i,t} - \ln Y_{i,t-1}\right) + (1-\alpha)\left(\ln Z_t - \ln Z_{t-1}\right)$$

$$\ln Y_{i,t} - \ln Y_{i,t-1} = \frac{1}{1-\alpha}\left(\ln V_t - \ln V_{t-1}\right) + \left(\ln Z_t - \ln Z_{t-1}\right)$$

$$\frac{Y_{i,t}}{Y_{i,t-1}} = \frac{Z_t}{Z_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{Y_{i,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} = \frac{Y_{i,t-1}}{Z_{t-1} V_{t-1}^{\frac{1}{1-\alpha}}}$$

Now that we know the growth rate of investment and capital, we can work out the growth rate of output in the goods producing sector

$$\ln Y_{c,t} = \alpha \ln K_{c,t-1} + (1-\alpha)\ln L_{c,t} + (1-\alpha)\ln Z_t$$

$$\ln Y_{c,t} - \ln Y_{c,t-1} = \alpha\left(\ln K_{c,t-1} - \ln K_{c,t-2}\right) + (1-\alpha)\left(\ln L_{c,t} - \ln L_{c,t-1}\right) + (1-\alpha)\left(\ln Z_t - \ln Z_{t-1}\right)$$

Along the BGP capital grows at the same rate as investment and hours are constant:

$$\ln Y_{c,t} - \ln Y_{c,t-1} = (1-\alpha)\left(\ln Z_t - \ln Z_{t-1}\right) + \alpha\left[\frac{1}{1-\alpha}\left(\ln V_t - \ln V_{t-1}\right) + \left(\ln Z_t - \ln Z_{t-1}\right)\right]$$

$$\ln Y_{c,t} - \ln Y_{c,t-1} = \left(\ln Z_t - \ln Z_{t-1}\right) + \alpha\left[\frac{1}{1-\alpha}\left(\ln V_t - \ln V_{t-1}\right)\right]$$

$$\frac{Y_{c,t}}{Y_{c,t-1}} = \frac{Z_t}{Z_{t-1}}\left(\frac{V_t}{V_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\frac{Y_{c,t}}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} = \frac{Y_{c,t-1}}{Z_{t-1} V_{t-1}^{\frac{\alpha}{1-\alpha}}}$$

Now that we know at which rate output in the two sectors grows along the BGP, we can define the stationary variables of the model. Variables normalized by to be stationary are dented by lower case letters.

$$y_{c,t} = \frac{Y_{c,t}}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{51}$$

$$k_{c,t} = \frac{K_{c,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} \tag{52}$$

$$i_{c,t} = \frac{I_{c,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} \tag{53}$$

$$y_{i,t} = \frac{Y_{i,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} \tag{54}$$

$$k_{i,t} = \frac{K_{i,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} \tag{55}$$

$$i_{i,t} = \frac{I_{i,t}}{Z_t V_t^{\frac{1}{1-\alpha}}} \tag{56}$$

$$c_t = \frac{C_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{57}$$

$$x_t = \frac{X_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{58}$$

$$\frac{X_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} = \left(\frac{C_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}}\right)^{\varsigma} \left(\frac{X_{t-1}}{Z_{t-1} V_{t-1}^{\frac{\alpha}{1-\alpha}}} \frac{Z_{t-1} V_{t-1}^{\frac{\alpha}{1-\alpha}}}{Z_t V_t^{\frac{\alpha}{1-\alpha}}}\right)^{1-\varsigma} \left(\frac{Z_t V_t^{\frac{\alpha}{1-\alpha}}}{Z_{t-1} V_{t-1}^{\frac{\alpha}{1-\alpha}}}\right)^{1-\varsigma}$$

$$x_t = c^{\varsigma} x_{t-1}^{1-\varsigma} \tag{59}$$

$$b_t = \frac{B_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{60}$$

What is the stationary real wage in both sectors? First, we need to determine that real wages grow at the same rate across the sectors. To do this, just look at equations (29) and (30). Given that the overall labour supply is stationary, as are the sectoral components of the supply of labour, the real wage has to grow at the same rate in the two sectors. In the consumption goods producing sector, the real wage has to grow at the same rate as output, hence the stationary real wage is:

$$w_t = \frac{W_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{61}$$

Given that we have just shown that the real wages have to grow by the same rate in both sectors, even if output in the two production sectors grows at different rates. What makes up the difference is the relative price of investment goods:

$$p_{i,t} = \frac{P_{i,t}}{P_{c,t}^H} V_t^{\frac{1-\alpha}{1-\alpha}} \tag{62}$$

This shows that along the BGP, the relative price of investment goods declines along with increases in

IST. This version of the two sector model looks exactly like the one sector model in terms of its growth properties. Also, along the BGP, Q has to grow at the same rate as $\frac{P_{i,t}}{P_{c,t}}$, hence:

$$q_{c,i,t} = \frac{Q_{c,i,t}}{V_t^{\frac{-(1-\alpha)}{1-\alpha}}} \tag{63}$$

GDP along the BGP – when trade is balanced:

$$Y_t = \frac{P^H_{c,t}}{P_{c,t}} Y_{c,t} + \frac{P_{i,t}}{P^H_{c,t}} \frac{P^H_{c,t}}{P_{c,t}} Y_{i,t}$$

At what rate does GDP grow? Define the growth rate of GDP as ζ_t

$$y_t \zeta_t = \frac{P^H_{c,t}}{P_{c,t}} y_{c,t} Z_t V_t^{\frac{\alpha}{1-\alpha}} + p_{i,t} V_t^{-\frac{(1-\alpha)}{1-\alpha}} \frac{P^H_{c,t}}{P_{c,t}} y_{i,t} Z_t V_t^{\frac{1}{1-\alpha}}$$

$$y_t \zeta_t = \frac{P^H_{c,t}}{P_{c,t}} y_{c,t} Z_t V_t^{\frac{\alpha}{1-\alpha}} + p_{i,t} Z_t V_t^{-\frac{(1-\alpha)}{1-\alpha}} \frac{P^H_{c,t}}{P_{c,t}} y_{i,t} V_t^{\frac{1}{1-\alpha}}$$

$$y_t \zeta_t = \frac{P^H_{c,t}}{P_{c,t}} y_{c,t} Z_t V_t^{\frac{\alpha}{1-\alpha}} + p_{i,t} y_{i,t} \frac{P^H_{c,t}}{P_{c,t}} Z_t V_t^{\frac{\alpha}{1-\alpha}}$$

$$\zeta_t = \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{c,t}}{y_t} Z_t V_t^{\frac{\alpha}{1-\alpha}} + \frac{p_{i,t} y_{i,t}}{gdp_t} \frac{P^H_{c,t}}{P_{c,t}} Z_t V_t^{\frac{\alpha}{1-\alpha}}$$

Since the fractions on the RHS are shares that sum to 1, we can see that the overall growth rate of GDP, which we observe in the data will be equal to:

$$\zeta_t = Z_t V_t^{\frac{\alpha}{1-\alpha}} \tag{64}$$

or expressed differently:

$$y_t = \frac{Y_t}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \tag{65}$$

C.3 Stationary model - Household's optimality conditions

$$x_t = c_t^{\varsigma} x_{t-1}^{1-\varsigma} \tag{66}$$

$$-\left(c_t - x_t \frac{\phi_0}{1+\phi} L_t^{1+\phi}\right)^{-\sigma} \frac{\phi_0}{1+\phi} L_t^{1+\phi} + \beta \omega_{t+1} (1-\varsigma) \frac{x_{t+1}}{x_t} \left(\frac{Z_{t+1} V_{t+1}^{\frac{\alpha}{1-\alpha}}}{Z_t V_t^{\frac{\alpha}{1-\alpha}}} \right)^{1-\sigma} = \omega_t \tag{67}$$

$$\left(c_t - x_t \frac{\phi_0}{1+\phi} L_t^{1+\phi}\right)^{-\sigma} + \omega_t \varsigma \frac{x_t}{c_t} = \lambda_t \tag{68}$$

$$\left(c_t - x_t \frac{\phi_0}{1+\phi} L_t^{1+\phi}\right)^{-\sigma} x_t \phi_0 L_t^{\phi} \left(\frac{L_{c,t}}{L_t}\right)^{\frac{1}{\xi}} \left(\frac{1}{\chi_c}\right)^{\frac{1}{\xi}} = \lambda_t w_{c,t} \tag{69}$$

$$\left(c_t - x_t \frac{\phi_0}{1+\phi} L_t^{1+\phi}\right)^{-\sigma} x_t \phi_0 L_t^{\phi} \left(\frac{L_{i,t}}{L_t}\right)^{\frac{1}{\xi}} \left(\frac{1}{\chi_i}\right)^{\frac{1}{\xi}} = \lambda_t w_{i,t} \tag{70}$$

$$L_t = \left(\chi_c^{-\frac{1}{\xi}} L_{c,t}^{\frac{1+\xi}{\xi}} + \chi_i^{-\frac{1}{\xi}} L_{i,t}^{\frac{1+\xi}{\xi}} \right)^{\frac{\xi}{1+\xi}} \tag{71}$$

Define $\frac{1}{\Gamma^y_{t+1}} = \frac{Z_t V_t^{\frac{\alpha}{1-\alpha}}}{Z_{t+1} V_{t+1}^{\frac{\alpha}{1-\alpha}}}$

$$\lambda_t = \left(\frac{1}{\Gamma^y_{t+1}} \right)^{\sigma} \beta(1+r_t) \frac{P^*_{c,t+1}}{P_{c,t+1}} \frac{P_{c,t}}{P^*_{c,t}} \lambda_{t+1} \tag{72}$$

C.4 Stationary model - supply side

Define the the growth rate of capital as: $\Gamma^k_t = \frac{Z_t V_t^{\frac{1}{1-\alpha}}}{Z_{t-1} V_{t-1}^{\frac{1}{1-\alpha}}}$

$$y_{c,t} = \left(\frac{1}{\Gamma^k_t} \right)^{\alpha} k^{\alpha}_{c,t-1} L^{1-\alpha}_{c,t} \tag{73}$$

$$y_{i,t} = \left(\frac{1}{\Gamma^k_t} \right)^{\alpha} k^{\alpha}_{i,t-1} L^{1-\alpha}_{i,t} \tag{74}$$

$$(1-\alpha) \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{c,t}}{L_{c,t}} = w_t \tag{75}$$

$$(1-\alpha) p_{i,t} \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{i,t}}{L_{i,t}} = w_t \tag{76}$$

$$\alpha \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{c,t}}{u_{c,t}} = q_{c,t} k_{c,t-1} \nu_0 \nu u^{\nu-1}_{c,t} \frac{1}{\Gamma^k_t} \tag{77}$$

$$\alpha p_{i,t} \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{i,t}}{u_{i,t}} = q_{i,t} k_{i,t-1} \nu_0 \nu u^{\nu-1}_{i,t} \frac{1}{\Gamma^k_t} \tag{78}$$

$$\delta_{c,t} = \bar{\delta} + \nu_0 \left(u^{\nu-1}_{c,t} \right) \tag{79}$$

$$\delta_{i,t} = \bar{\delta} + \nu_0 \left(u^{\nu-1}_{i,t} \right) \tag{80}$$

$$q_{c,t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1}{\Gamma^y_{t+1}} \right)^{\sigma-1} \left[q_{c,t+1}(1-\delta_{c,t+1}) \frac{1}{\Gamma^k_{t+1}} + \alpha \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{c,t+1}}{k_{c,t}} \right] \tag{81}$$

$$q_{i,t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1}{\Gamma^y_{t+1}} \right)^{\sigma-1} \left[q_{i,t+1}(1-\delta_{i,t+1}) \frac{1}{\Gamma^k_{t+1}} + \alpha p_{i,t} \frac{P^H_{c,t}}{P_{c,t}} \frac{y_{i,t+1}}{k_{i,t}} \right] \tag{82}$$

$$\begin{aligned} p_{i,t} \frac{P^H_{c,t}}{P_{c,t}} &= q_{c,t} \left[\left(1 - \frac{\psi}{2} \left(\frac{i_{c,t}}{i_{c,t-1}} \Gamma^k_t - 1 \right)^2 \right) - \psi \left(\frac{i_{c,t}}{i_{c,t-1}} \Gamma^k_t - 1 \right) \frac{i_{c,t}}{i_{c,t-1}} \Gamma^k_t \right] \\ &+ \beta E_t q_{c,t+1} \frac{1}{\Gamma^k_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1}{\Gamma^y_{t+1}} \right)^{\sigma-1} \psi \left(\frac{i_{c,t+1}}{i_{c,t}} \Gamma^k_{t+1} - 1 \right) \left(\frac{i_{c,t+1}}{i_{c,t}} \Gamma^k_{t+1} \right)^2 \end{aligned} \tag{83}$$

$$
\begin{aligned}
p_{i,t}\frac{P^H_{c,t}}{P_{c,t}} &= q_{i,t}\left[\left(1-\frac{\psi}{2}\left(\frac{i_{i,t}}{i_{i,t-1}}\Gamma^k_t-1\right)^2\right)-\psi\left(\frac{i_{i,t}}{i_{i,t-1}}\Gamma^k_t-1\right)\frac{i_{i,t}}{i_{i,t-1}}\Gamma^k_t\right] \\
&+ \beta E_t q_{i,t+1}\frac{1}{\Gamma^k_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1}{\Gamma^y_{t+1}}\right)^{\sigma-1}\psi\left(\frac{i_{i,t+1}}{i_{i,t}}\Gamma^k_{t+1}-1\right)\left(\frac{i_{i,t+1}}{i_{i,t}}\Gamma^k_{t+1}\right)^2
\end{aligned}
\tag{84}
$$

$$
k_{c,t} = (1-\delta_{c,t})k_{c,t-1}\frac{1}{\Gamma^k_t} + \left(1-\frac{\psi}{2}\left(\frac{i_{c,t}}{i_{c,t-1}}\Gamma^k_t-1\right)^2\right)i_{c,t} \tag{85}
$$

$$
k_{i,t} = (1-\delta_{i,t})k_{i,t-1}\frac{1}{\Gamma^k_t} + \left(1-\frac{\psi}{2}\left(\frac{i_{i,t}}{i_{i,t-1}}\Gamma^k_t-1\right)^2\right)i_{i,t} \tag{86}
$$

$$
y_{c,t} = (1-\gamma)\left(\frac{P^H_{c,t}}{P_{c,t}}\right)^{-\theta}c_t + \gamma\left(\frac{P^{*H}_{c,t}}{P^*_{c,t}}\right)^{-\theta}c^*_t\frac{\zeta^*_t}{\zeta_t} \tag{87}
$$

$$
y_{i,t} = i_{c,t} + i_{i,t} \tag{88}
$$

$$
c_t + b_t = (1+r_{t-1})\frac{P^*_{c,t}}{P_{c,t}}\frac{P_{c,t-1}}{P^*_{c,t-1}}\frac{1}{\Gamma^y_t}b_{t-1} + y_{c,t} \tag{89}
$$

Note that because we have assumed that investment goods are essentially non-traded, the current account equation is only affected by goods output and consumption.

C.5 Steady State

In the steady state, the households' optimality conditions determine r^*, x, ω, and λ.

$$
1/\beta = \left(\frac{1}{\Gamma^y}\right)^\sigma(1+r^*) \tag{90}
$$

$$
x = c \tag{91}
$$

$$
\omega = -\frac{\left[c\left(1-\frac{\phi_0}{1+\phi}L^{1+\phi}\right)\right]^{-\sigma}\frac{\phi_0}{1+\phi}L^{1+\phi}}{\left(1-\beta(1-\varsigma)\left(\Gamma^y\right)^{1-\sigma}\right)} \tag{92}
$$

$$
\lambda = \frac{\left[\left(1-\beta(1-\varsigma)\left(\Gamma^y\right)^{1-\sigma}\right)-\varsigma\frac{\phi_0}{1+\phi}L^{1+\phi}\right]\left[c\left(1-\frac{\phi_0}{1+\phi}L^{1+\phi}\right)\right]^{-\sigma}}{\left(1-\beta(1-\varsigma)\left(\Gamma^y\right)^{1-\sigma}\right)} \tag{93}
$$

Next, we determine the value of ϕ_0, determining the steady state value of hours worked; the steady state value of q and of ν_0, determining the steady state level of capacity utilisation.

$$
\phi_0 = \frac{w\left(1-\beta(1-\varsigma)\left(\Gamma^y\right)^{1-\sigma}\right)}{\left[\left(1-\beta(1-\varsigma)\left(\Gamma^y\right)^{1-\sigma}\right)cL^\phi + w\varsigma\frac{1}{1+\phi}L^{1+\phi}\right]} \tag{94}
$$

$$q = p_i \tag{95}$$

$$\frac{\Gamma^k \alpha y/k}{q\nu} = \nu_0 \tag{96}$$

This determines ν_0 to ensure that $u = 1$. When $u = 1$,

$$\delta_{j,t} = \delta \tag{97}$$

Next, we can tie down the output-capital ratios as a function of p_i

$$p_i = \beta \left(\frac{1}{\Gamma^y}\right)^{\sigma-1} \left[p_i(1-\delta)\frac{1}{\Gamma^k} + \alpha\frac{y_c}{k_c}\right]$$

$$\frac{p_i\left[1/\beta\left(\frac{1}{\Gamma^y}\right)^{1-\sigma} - (1-\delta)\frac{1}{\Gamma^k}\right]}{\alpha} = \frac{y_c}{k_c} \tag{98}$$

$$p_i = \beta \left(\frac{1}{\Gamma^y}\right)^{\sigma-1} \left[p_i(1-\delta)\frac{1}{\Gamma^k} + pi_i\alpha\frac{y_i}{k_i}\right]$$

$$\frac{\left[1/\beta\left(\frac{1}{\Gamma^y}\right)^{1-\sigma} - (1-\delta)\frac{1}{\Gamma^k}\right]}{\alpha} = \frac{y_i}{k_i} \tag{99}$$

Now that we have $\frac{y}{k}$ we can work out other sector specific ratios:

$$\frac{y_c}{k_c} = \left(\frac{1}{\Gamma^k}\right)^{\alpha} \left(\frac{k_c}{L_c}\right)^{\alpha-1}$$

$$\frac{y_c}{L_c} = \left(\frac{1}{\Gamma^k}\right)^{\alpha} \left(\frac{k_c}{L_c}\right)^{\alpha} \tag{100}$$

$$\frac{k_c}{L_c} = \left[\frac{y_c}{k_c}\left(\Gamma^k\right)^{\alpha}\right]^{\frac{1}{\alpha-1}}$$

$$\frac{y_i}{k_i} = \left(\frac{1}{\Gamma^k}\right)^{\alpha} \left(\frac{k_i}{L_i}\right)^{\alpha-1}$$

$$\frac{y_i}{L_i} = \left(\frac{1}{\Gamma^k}\right)^{\alpha} \left(\frac{k_i}{L_i}\right)^{\alpha} \tag{101}$$

$$\frac{k_i}{L_i} = \left[\frac{y_i}{k_i}\left(\Gamma^k\right)^{\alpha}\right]^{\frac{1}{\alpha-1}}$$

If we now take the equations for optimal labour choice, recalling that in the steady state the RER is 1 so that $\frac{P^H_{c,t}}{P_{c,t}} = 1$ we get:

$$\frac{y_c}{L_c}\frac{L_i}{y_i} = p_i \tag{102}$$

Replace $\frac{y_c}{L_c}$

$$\left(\frac{1}{\Gamma^k}\right)^{\alpha}\left[\frac{y_c}{k_c}\left(\Gamma^k\right)^{\alpha}\right]^{\frac{\alpha}{\alpha-1}}\frac{L_i}{y_i}=p_i$$

$$\left[\left(\frac{1}{\Gamma^k}\right)^{\alpha}\left[\frac{\left[1/\beta\left(\frac{1}{\Gamma^y}\right)^{1-\sigma}-(1-\delta)\frac{1}{\Gamma^k}\right]}{\alpha}\left(\Gamma^k\right)^{\alpha}\right]^{\frac{\alpha}{\alpha-1}}\frac{L_i}{y_i}\right]^{1-\alpha}=p_i \tag{103}$$

Note that $\frac{L_i}{y_i}$ is not a function of p_i so we can just leave it as a parameter. Next, let us work out the cross-sector steady state ratios. Start with market clearing in the investment goods sector:

$$y_i=i_c+i_i \tag{104}$$

$$L=L_c+L_i \tag{105}$$

$$\frac{i_c}{k_c}=\left(1-(1-\delta)\frac{1}{\Gamma^k}\right) \tag{106}$$

$$1=\frac{i_i}{y_i}+\frac{i_c}{y_c}\frac{y_c}{y_i} \tag{107}$$

$$\frac{i_i}{y_i}=\frac{i_i}{k_i}\frac{k_i}{y_i} \tag{108}$$

$$y_i=\frac{y_i}{L_i}L_i \tag{109}$$

$$y_c=\frac{y_c}{L_c}L_c \tag{110}$$

Now, set the total labour supply to 1

$$\frac{\left(1-\frac{i_i}{y_i}\right)\frac{y_c}{i_c}\left(\frac{L_c}{y_c}\right)\left(\frac{y_i}{L_i}\right)}{1+\left(1-\frac{i_i}{y_i}\right)\frac{y_c}{i_c}\left(\frac{L_c}{y_c}\right)\left(\frac{y_i}{L_i}\right)}=L_c$$

$$L_i=1-L_c \tag{111}$$