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Jolian McHardy*, Michael Reynolds and Stephen Trotter

On the Economics of Integrated Ticketing.

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* Corresponding author
Department of Economics
University of Sheffield
9 Mappin Street
Sheffield
S1 4DT
United Kingdom
www.shef.ac.uk/economics

Abstract

In this paper we explore alternative pricing and regulatory strategies within a simple transport network with Cournot duopoly and differentiated demands. We show that whilst firms always prefer to offer integrated ticketing, a social planner will not. With integrated ticketing, the firms always prefer complete collusion but there is not a uniform ranking of some of the less collusive regimes. Society generally prefers the less collusive regimes to complete collusion but prefers some collusion to independent pricing.

Keywords: Integrated ticketing, duopoly, collusion

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Michael Reynolds and Stephen Trotter are affiliated with the Centre for Economic Policy, University of Hull, UK.

1. Introduction

Over the last fifty years bus and coach use in the UK has fallen by half when measured in passenger-kilometres, and by five-sixths when measured as a share of total passenger transport. Increased road congestion and the accompanying damage to the environment, reinforced by the UK's commitment to its Kyoto (1897) targets, have led to continuing efforts to shift traffic back onto public transport, especially buses. Road-pricing, fuel taxes and other 'push' techniques are available to make car use more expensive and/or less attractive, but there have also been 'pull' attempts to make bus use more attractive. Deregulation was one of these, but unfortunately it generally did not work. It inevitably segmented the market and reduced the availability of integrated tickets[1]. Moreover, the schemes that do exist following the introduction of competition have some perverse incentives, which could be seen as increasing the average generalised cost of travelling by public transport. Deregulation also created a situation of uncertainty, and Tyson (1890) suggests that it was possible that some of the upheaval and uncertainty brought about by deregulation of the bus industry may have permanently altered individuals' travel behaviour.

Separation of operators – as seen under deregulation – can also be damaging for other reasons. For example, in a simple framework with two complementary services – such as a network in which all passengers demand travel from A to B via service provider 1 and from B to C via service provider 2 (fixed proportions) – the overall price to the traveller will be lower if the service providers are allowed to collude on price than if they set prices independently (see Cournot, 1838, for the original development of this result, and Else and James, 1995, for an application of it). The policy recommendation is thus that in such situations price collusion should be preferred to an alternative of independent pricing. However this conclusion may not carry over to a situation with more complicated demands. Whether price collusion is always a sensible recommendation in these circumstances, and – if so – how much collusion should be allowed, is the subject of the present paper.

In this paper we examine how the introduction of integrated tickets on a transport network with differentiated demands may affect welfare and profits. We examine a number of pricing regimes which allow varying degrees of collusion and consider the extent to which the adoption of such regimes would be in the social interest or the private interest or both.

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[1] By 'integrated tickets' we mean tickets which can be used on services run by different operators of the same means of transport (here buses). We follow other writers in reserving the term 'inter-available ticketing' for schemes that facilitate cross-modal transfer.

The next section of the paper sets out the basic model and applies it to the simple case of a welfare-maximising social planner; Sections 3 and 4 analyse the cases of a network monopoly and a network duopoly respectively; and Section 5 contains our conclusions.

2. Integrated Ticketing

Consider a single-route transport network which faces demands for travel which are differentiated according to the time of travel. For simplicity, let there be two distinct outward services, O_i ($i = 1, 2$), and two inward services I_j ($j = 1, 2$). Given that we are interested in the effects of integrated ticketing, which in the present system implies round-trip travel, we assume that all consumers have an outward and inward element in their demand for travel, Q_{ij}. There are no consumers who wish to travel only in one direction. The possible travel combinations over the four services are therefore as described in Figure 1.

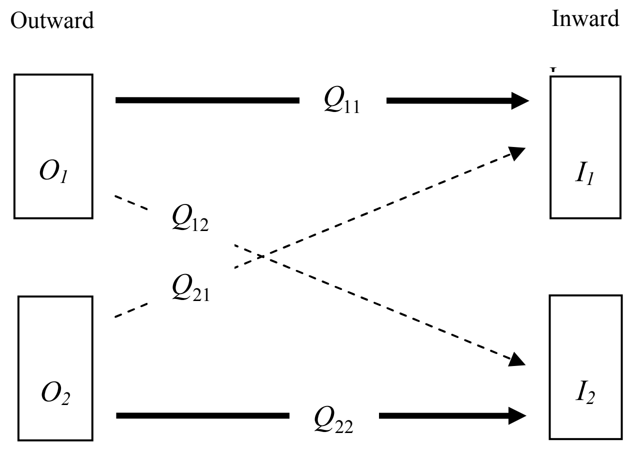

Figure 1 – A simple Transport Network

For the purpose of characterising specific demands, Q_{ij}, we refer to the round-trip price, P_{ij}. Let demand Q_{ij} be linear in its own price and also in the round-trip prices of all other possible service combinations:

$$Q_{ij} = \alpha - \beta P_{ij} + \sum_{mn \neq ij} \delta P_{mn} \,. \qquad (1)$$

In this specification the cross-price co-efficient, δ, is common across all alternative services to *ij*: all the alternative services are equally good substitutes.[2] A corollary of this symmetry is that any benefits of the integrated ticket in terms of improved flexibility of travel are ignored.

Since we are going to compare situations in which integrated ticketing is not provided against those in which it is provided, it is necessary to specify the options available to those preferring cross-service travel in the absence of an integrated ticket option. The benchmark we adopt when no integrated ticket option is available is that passengers wishing to travel across the services must purchase a round-trip ticket for each stage of the journey – one outbound and one inbound – hence:

$$P_{mn} = P_{mm} + P_{nn}\ . \qquad (2)$$

The assumption may at first seem an extreme one. However, there are many cases where a one-way ticket is indeed approximately equal in price to a round-trip ticket.

$$Q_{mn} = \alpha - \beta P_{mn} + \delta P_{nn} + \delta(P_{mm} + P_{nn})\ , \qquad (3a)$$

$$Q_{mm} = \alpha - \beta P_{mm} + \delta P_{nn} + 2\delta(P_{mm} + P_{nn})\ . \qquad (3b)$$

Given (3b), the following restriction is required to ensure a system of gross substitutes:[3]

$$\beta > 5\delta\ . \qquad (4)$$

To aid tractability, and without loss of generality, we now normalise the framework with the parameterisation:

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[2] It is important to recognise that δ indicates the degree to which services are differentiated and might realistically be expected to feature as a strategic choice variable of a firm rather then be parametric as it is here.

[3] Gross substitutes describes a situation where, as the price of one good increases, the "Marshallian" demand for the other good increases – "Marshallian" demand being the demand as a function of prices and incomes as opposed to "Hicksian" demands that are a function of prices and utility. Gross substitutes imply that an equal increase in the prices of all goods will lead to a fall in demand for each good, so total demand will also fall. In our system of demands gross substitution means that as the price of one good increase then the demands for the three other goods also increases. It also ensures that if all prices were to rise by an equal amount then total demand would fall. This ensures that the negative relationship between overall demand and price is maintained.

$$\delta = 1 . \quad (5)$$

Having established the demand structure for the model we now briefly turn our attention to costs. The central concern of this paper is with the private provision or otherwise of integrated ticketing and the degree to which collusion between the firms is in the private and social interest. Given that the structure of the model (the number of physical services) is a constant over all regimes, and assuming that the provision of integrated ticketing can be undertaken at zero additional cost, fixed costs will not play a role in the private organisation of the industry in general. We therefore set fixed costs equal to zero. Further, given the marginal passenger costs for most public transport systems are very low, for simplicity we take them to be zero. Finally, since journey distance is not a consideration in the present context, we take marginal distance costs to be zero.[4]

The case of the welfare maximising social planner is a trivial one. With zero marginal costs, a welfare maximising social planner will set the price for each round trip equal to zero. Thus in all regimes, revenue, costs and hence profit are all zero. From (1):

$$Q_{ij}^{s} = \alpha . \quad (6)$$

However, throughout the paper reference to the first-best outcome is not always possible, given the inter-relationship between demands and the consequent inability to derive a unique (path independent) measure of surplus. If, following a change in regime, all prices across the network moved in one direction whilst the quantities moved in the opposite direction, it would be straightforward to draw conclusions about the welfare superiority of one regime over another. Unfortunately, this will often not be the case. Nevertheless, given that one of the central motivations for the paper is to identify regimes which help to increase the patronage of public transport in order to reduce pollution and congestion, a regime which engenders a high total patronage across the network should be considered superior to one with a lower patronage. A decrease in the average ticket price might also be a favourable indicator for a regime in itself.[5] To some extent these are complementary objectives, but including them both in the social planner's objective function allows for instances where, for example, increased output is due to

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[4] Note that the marginal cost assumptions are especially plausible in the short run, when operators are committed to a given timetable irrespective of demand.

[5] Maximising passenger-miles was adopted as a target by London Transport (see Glaister and Collings (1878) and the references therein). It was also put forward by Sir Peter Parker, when Chairman of British Rail, in his 1978 Haldane Lecture. An "output-related profits levy" which would reward faster growth of output was one regulatory mechanism considered when British Telecom was privatised in 1984, and of course a (weighted) average price is the focus of the 'RPI–X' regulation that was actually introduced.

general economic growth and not to any action of transport operators. This function can thus be summarised by:

$$S(\tilde{Q}, \tilde{P}) \qquad\qquad S_{\tilde{Q}} > 0,\ S_{\tilde{Q}\tilde{Q}} < 0,\ S_{\tilde{P}} < 0,\ S_{\tilde{P}\tilde{P}} < 0\,. \qquad (7)$$

where $\tilde{Q}$ is the total patronage on the network and $\tilde{P}$ is the average (per passenger) fare. Subscripts denote partial derivatives.

Given the above discussion, we suggest that the weight on the former term would be strictly greater than that on the latter. We refer to a regime which improves both terms, $S(+, -)$, as *strictly* superior, whilst one regime is *weakly* superior to another regime if $S(+, +)$, i.e. total patronage increases but (despite this) there is a rise in the average passenger cost. Conversely, a decrease in the average passenger cost should not dominate a decrease in total patronage, and we therefore describe such a regime, $S(-, -)$, as *weakly* inferior. Finally a regime which has lower patronage and higher average passenger cost, $S(-, +)$ is *strictly* inferior.

3. Network Monopoly

In this section we consider the equilibrium prices and outputs in a situation of network monopoly where all services are provided by a single profit-maximising firm. We examine two regimes: the first (M1) in which the network monopolist does not provide integrated ticketing and the second (M2) in which a cross-service ticket is provided. Beginning with regime M1, the network monopolist's profit in general terms is given by:

$$\Pi^{M1} = \sum_{m=1,2} P_{mm} Q_{mm} + \sum_{m \neq n = 1,2} P_{mm} Q_{mn}\ . \qquad (8)$$

Substituting (3) in (8) and maximising with respect to P_{11} and P_{22} yields the following equilibrium prices for the single and cross services, respectively:

$$P_{mm}^{M1} = \frac{3\alpha}{2(5\beta - 13)}\,, \qquad (9a)$$

$$P_{mn}^{M1} = 2P_{mm}^{M1} = \frac{3\alpha}{(5\beta - 13)}\,. \qquad (9b)$$

Substituting (9) into the relevant demand functions (3) yields the equilibrium quantities of single-service and cross-service journeys, respectively:

$$Q_{mm}^{M1} = \frac{\alpha(7\beta - 11)}{2(5\beta - 13)}, \tag{10a}$$

$$Q_{mn}^{M1} = \frac{\alpha(2\beta - 7)}{5\beta - 13}. \tag{10b}$$

Inspection of (10a) and (10b) shows that $Q_{mm}^{M1} > Q_{mn}^{M1}$, as we would expect given the cross-service pricing rule (2). Finally, using (9) and (10) in (8):

$$\Pi^{M1} = \frac{9\alpha^2}{2(5\beta - 13)}. \tag{11}$$

We now consider how the monopoly equilibrium changes when integrated tickets are introduced, allowing cross-service travel without the need to purchase two separate round-trip tickets. Let P_x be the price for the integrated ticket. The relevant demand functions are now:

$$Q_{mm} = \alpha - \beta P_{mm} + P_{nn} + 2P_x, \tag{12a}$$

$$Q_{mn} = \alpha - \beta P_x + P_x + P_{mm} + P_{nn}. \tag{12b}$$

The network monopolist's profit, in general terms, is now given by:

$$\Pi^{M2} = \sum_{m=1,2} P_{mm} Q_{mm} + P_x \sum_{m \neq n = 1,2} Q_{mn}. \tag{13}$$

Maximising (13) with respect to P_{11}, P_{22} and P_x yields the following equilibrium prices for the single service and integrated ticket, respectively:

$$P_{mm}^{M2} = P_x^{M2} = \frac{\alpha}{2(\beta - 3)}. \tag{14}$$

Note, the network monopolist does not discriminate on price across the different ticket types. This has to do with the symmetry of the model. Were the integrated ticketing to be extended across all round-trips, this result would change with a premium on integrated tickets if there is a utility gain from increased flexibility.

Substituting (14) into (12), yields the following equilibrium expression for quantity demanded of each ticket type:

$$Q_{mm}^{M2} = Q_{mn}^{M2} = \alpha / 2 \,. \tag{15}$$

Finally, using (15) and (14) in (13), we have the equilibrium profit under regime M2:

$$\Pi^{M2} = \frac{\alpha^2}{\beta - 3} \,. \tag{16}$$

Proposition 1. [6] *(i) The network monopolist always prefers the integrated ticketing regime M2 over regime M1:* $\Pi^{M2} > \Pi^{M1}$. *(ii) The social planner strictly prefers regime M1 over regime M2:* $S(\tilde{Q}^{M1},\, \tilde{P}^{M1}) \succ S(\tilde{Q}^{M2},\, \tilde{P}^{M2})$.

The rationale for this proposition is straightforward: in the absence of an integrated ticket and given the "double price" cross-service penalty the network monopolist is forced to charge a very low fare on the single-service round trip in order to make profit. The "double price" effect penalises the network monopolist harshly against increasing the single-service price.

4. Network Duopoly

In this section, we examine the effects of introducing strategic interaction in the model by assuming a duopoly in which two separate firms provide substitute single-service operations: firm *m* provides Q_{mm} ($m \neq n = 1, 2$). We begin, as in section 3, by considering a regime, D1, in which the duopolists do not provide cross-service tickets. In regime D2, the duopolists are allowed to collude on a "price rule" for the integrated ticket price (but not the actual ticket price), but no other collusion is allowed. In regime D3, the duopolists provide an integrated ticket and are required to set the price for their component of the integrated ticket independently: no collusion is allowed. Finally, in regime D4 the duopolists provide the integrated ticket and are allowed to collude on the price of the integrated ticket but not on any other price.

The relevant demands for regime D1 follow from (3), with firm I setting P_{mm} ($m \neq n = 1, 2$). Profit for firm *m* is given in general terms by:

$$\Pi_m^{D1} = P_{mm}Q_{mm} + (P_{mm} + P_{nn})Q_{mn} \qquad (m \neq n = 1, 2) \,. \tag{17}$$

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[6] Proofs available upon request from the corresponding author.

Maximising (13) with respect to P_{mm}, yields the following expression for the equilibrium duopoly price:

$$P_{mm}^{D1} = \frac{3\alpha}{8\beta - 19} \qquad (m \neq n = 1,\ 2). \qquad (18)$$

Substituting (18) into (3), yields, respectively, the equilibrium quantities demanded of the single and cross-services:

$$Q_{mm}^{D1} = \frac{\alpha(5\beta - 4)}{8\beta - 19}, \qquad (19a)$$

$$Q_{mn}^{D1} = \frac{\alpha(2\beta - 7)}{8\beta - 19} \qquad (m \neq n = 1,\ 2). \qquad (19b)$$

Again, as would be expected, $Q_{mm}^{D1} > Q_{mn}^{D1}$. Finally, substituting (19) and (18) into (17) and summing over both firms, aggregate profit in regime D1 is:

$$\tilde{\Pi}^{D1} = \frac{54\alpha^2(\beta - 2)}{(8\beta - 19)^2}.$$

Proposition 2. *(i) Both firms prefer regime M1 (joint profit maximisation) over regime D1:* $\Pi^{M1} > \Pi^{D1}$.

(ii) The social planner strictly prefers regime M1 over regime D1: $S(\tilde{Q}^{M1},\ \tilde{P}^{M1}) \succ S(\tilde{Q}^{D1},\ \tilde{P}^{D1})$.

We now introduce integrated ticketing into the duopoly model. In regime D2 the firms first collude to maximise joint profit on the cross-service demands and then independently set their respective single-service prices. The general expression for profit on the cross-service operation is given by:[7]

[7] Note, given the equilibrium prices for the integrated ticket always exceed those for the single-service ticket, only passengers wishing to travel cross-service will purchase the integrated ticket: the two are synonymous in this model.

$$\Pi_x^{D2} = P_x(Q_{mn} + Q_{nm}) \qquad (m \neq n). \tag{20}$$

Substituting (12b) into (20) and maximising with respect to $P_x (= P_{nm} = P_{mn}; m \neq n = 1, 2)$ yields the following expression for the integrated ticket price in terms of the single-service prices, P_{mm} ($m = 1, 2$):

$$P_x = \frac{a(P_{mm} + P_{nn})}{2(\beta - 1)}. \tag{21}$$

Given that the firms have agreed a rule for maximising joint profit on the cross-service travel using P_x (given the single-service ticket prices), each firm now chooses its single-service price by maximising its own profit independently taking (21) as given. Assuming each firm takes an equal share of the profits from the integrated ticket, the general expression for the profit of firm *m* is given by:

$$\Pi_m^{D2} = P_{mm}Q_{mm} + \frac{1}{2}P_x(Q_{mn} + Q_{nm}) \qquad (m \neq n = 1,2). \tag{22}$$

Maximising (22) with respect to P_{mm} and solving using (21) gives the following equilibrium prices:

$$P_x^{D2} = \frac{\alpha(2\beta^2 - \beta - 2)}{2(2\beta^3 - 5\beta^2 + 3)}, \tag{23a}$$

$$P_{mm}^{D2} = \frac{\alpha(2\beta + 1)}{2(2\beta^2 - 3\beta - 3)}. \tag{23b}$$

Substituting (23) in (12), yields the equilibrium demands for single-service and cross-service, respectively:

$$Q_{mm}^{D2} = \frac{\alpha(2\beta^3 - 3\beta^2 - 2\beta + 1)}{2(2\beta^3 - 5\beta^2 + 3)}, \tag{24a}$$

$$Q_{mn}^{D2} = \frac{\alpha(2\beta^2 - \beta - 2)}{2(2\beta^2 - 3\beta - 3)}. \tag{24b}$$

Using (24) and (23) in (22) and summing over both firms, aggregate profit across the network is:

$$\tilde{\Pi}^{D2} = \frac{\alpha^2 (8\beta^4 + 5 + 4\beta - 8\beta^3 - 14\beta^2)}{2(4\beta^5 + 21\beta^2 + 9\beta^3 - 16\beta^4 - 9\beta - 9)} . \tag{25}$$

Proposition 3. *(i) The firms prefer regime D2 over regime D1:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D1}$. *(ii) The social planner weakly prefers regime D2 over regime D1:* $S(\tilde{Q}^{D2},\ \tilde{P}^{D2}) \succeq S(\tilde{Q}^{D1},\ \tilde{P}^{D1})$.

Proposition 4. *(i) The firms prefer regime M2 over regime D2:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D2}$. *(ii) The social planner strictly prefers regime D2 over regime M2:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succeq S(\tilde{Q}^{M2}, \tilde{P}^{M2})$.

We now introduce regime D3 in which the duopolists are not allowed to collude on any aspect of pricing in the network. What this amounts to is a situation of independent pricing on components of the cross-service ticket: each firm *m* sets the price of its component, P_{xm}, of the integrated ticket price. The integrated ticket price is the sum of these two component prices:

$$P_m = \sum P_{xm} \qquad (m = 1,2). \tag{26}$$

Given (26) the general expression for the profit of firm *m* is given by, $\Pi_m{}^{D3}$:

$$\Pi_m{}^{D3} = P_{mm}Q_{mm} + P(Q_{mn} + Q_{nm}) \qquad (m \neq n = 1,2) . \tag{27}$$

Using (12) and (26) in (27) and maximising with respect to P_{xm} and P_{mm} for *m* = 1,2 yields the following equilibrium expressions for the cross-service and single-service ticket prices, respectively:

$$P_x^{D2} = P_{x1}^{D2} + P_{x2}^{D2} = \frac{4\alpha}{3(2\beta - 5)} , \tag{28a}$$

$$P_{mm}^{D2} = \frac{\alpha}{2\beta - 5} . \tag{28b}$$

Using (28) in (12) yields the following equilibrium expressions for cross-service and single-service ticket prices, respectively:

$$Q_{mn}^{D2} = \alpha / 3 , \tag{29a}$$

$$Q_{mm}^{D2} = \frac{\alpha(3\beta - 4)}{3(2\beta - 5)} . \tag{29b}$$

Profit across the network then follows from substituting (29) and (28) into (27) and summing across the two firms:

$$\tilde{\Pi}^{D3} = \frac{2\alpha^2 (17\beta - 32)}{9(2\beta - 5)^2} . \tag{30}$$

Proposition 5. *(i) The firms prefer regime D3 over regime D1:* $\tilde{\Pi}^{D3} > \tilde{\Pi}^{D1}$. *(ii) The social planner strictly prefers regime D3 over regime D1:* $S(\tilde{Q}^{D3}, \tilde{P}^{D3}) \succ S(\tilde{Q}^{D1}, \tilde{P}^{D1})$.

Proposition 6. *(i) The firms prefer regime M2 over regime D3:* $\tilde{\Pi}^{M2} > \tilde{\Pi}^{D3}$. *(ii) The social planner strictly (weakly) prefers regime M2 over regime D3 if β> 7.25 (β≤ 7.25):* $S(\tilde{Q}^{M2}, \tilde{P}^{M2}) \underset{(-)}{\succ} S(\tilde{Q}^{D3}, \tilde{P}^{D3}) \quad S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

Proposition 7. *(i) The firms prefer regime D2 over regime D3 if* $\beta > 6.4$, *otherwise regime D3 is preferred to D2:* $\tilde{\Pi}^{D2} > \tilde{\Pi}^{D3}$ $(\tilde{\Pi}^{D2} \leq \tilde{\Pi}^{D3})$ if $\beta > 6.4$ $(\beta \leq 6.4)$. *(ii) The social planner strictly prefers regime D2 over regime D3:* $S(\tilde{Q}^{D2}, \tilde{P}^{D2}) \succ S(\tilde{Q}^{D3}, \tilde{P}^{D3})$.

Finally, we introduce regime D4 where we again allow the duopolists to collude on the integrated ticket price. However, in contrast with regime D2, the integrated ticket price itself can now be set in advance of the firms making their choices about their own respective single-service ticket prices. Regime D4 is clearly less restrictive than D2. Most importantly, regime D4 allows the firms to impose greater constraints on their own second-period behaviour. If in the first period the firms set a price for the integrated ticket which maximises total profit given their expectations of their own (independent) behaviour on the pricing of the single service tickets in period 2, we get the following problem. In stage 2 the firms will each attempt to maximise their own profit by setting their own single-service price taking the integrated price as given. The relevant general expression for the profit of firm *m* is given by (22). Substituting (12) and maximising with respect to P_{mm} gives the following expression relating firm *m*'s optimal choice of P_{mm} in terms of P_x and P_{nn}:

$$P_{mm} = \frac{\alpha + P_{nn} + 3P_x}{2\beta}. \tag{31}$$

Solving (31) simultaneously across the two firms, we have:

$$P_{mm} = P_{nn} = \frac{\alpha + 3P_x}{2\beta - 1}. \tag{32}$$

The equilibrium expression (32) is the reactions function of the firms indicating their profit maximising choice of P_{mm} in terms of P_x. Differentiating (32) with respect to P_x we arrive at the following expression for the slope of the reaction function, γ:

$$\gamma = \frac{3}{2\beta - 1}. \tag{33}$$

The firms can now exploit their knowledge of their second-stage reaction to the first-stage price agreement, P_x, in order to commit themselves to a more 'collusive' second-stage price game via strategic pre-commitment though P_x. The first-stage problem is not to identify the level of P_x which maximises joint profit across the network given (33). Profit across the network in general terms is given by:

$$\tilde{\Pi}^{D4} = P_{mm}Q_{mm} + P_{nn}Q_{nn} + P_x(Q_{mn} + Q_{nm}) \qquad (m \neq n = 1,\ 2). \tag{34}$$

Substituting (12) in (31) and maximising with respect to P_x, recognising that P_{mm} is a function of P_x, through (33), we have:

$$P_x = \frac{\alpha(2\beta + 3)}{4\beta^2 - 6\beta - 13}. \tag{35a}$$

Substituting (35a) into (32) gives the equilibrium second-stage single-service price:

$$P_{mm} = \frac{4\alpha(\beta^2 - 1)}{(4\beta^2 - 6\beta - 13)(2\beta - 1)}. \tag{35b}$$

Using (35) in (12) yields the equilibrium levels of demand for the single-service and cross-service tickets in regime D4, respectively:

$$Q_{mm} = \frac{\alpha(4\beta^3 - 4\beta^2 - 8\beta + 3)}{(4\beta^2 - 6\beta - 13)(2\beta - 1)}, \tag{36a}$$

$$Q_{mn} = \frac{\alpha(4\beta^3 - 4\beta^2 - 13\beta + 2)}{(4\beta^2 - 6\beta - 13)(2\beta - 1)}. \tag{36b}$$

Aggregate profit across the network under this regime then follows from substitution of (35) and (36) into (34):

$$\tilde{\Pi}^{D4} = \frac{2\alpha^2(-32\beta^4 + 8\beta^2 - 144\beta^3 + 79\beta + 32\beta^5 - 18)}{(4\beta^2 - 6\beta - 13)^2(2\beta - 1)^2}. \tag{37}$$

Proposition 8. *(i) The firms prefer regime D4 over all other regimes except M2::* $\tilde{\Pi}^{D4} < \tilde{\Pi}^{M2}$, $\tilde{\Pi}^{D4} > \tilde{\Pi}^{Dm}$ ($m = 1, 2, 3$). *(ii) The social planner strictly (weakly) prefers regime D4 over regimes M2, D2 and D3 (D1):* $S(\tilde{Q}^{D4}, \tilde{P}^{D4}) \underset{(-)}{\succ} S(\tilde{Q}^{k}, \tilde{P}^{k})$, ($k = M2, D2, D3$).

5. Conclusions

In this paper we have explored a simple model of integrated ticketing for both monopoly and duopoly markets, and from the point of view of both the firm(s) involved and a welfare-maximising social planner. As noted in the Introduction there are several cases where the preferences of the two sides coincide, and there is therefore no need for regulatory action. However in other circumstances the firms will not act in the best interests of society and intervention will be justified. We summarise the position here.

First, firms always prefer to offer an integrated ticket. Society prefers the monopolist not to introduce integrated ticketing, but otherwise at least weakly prefers integrated ticketing when the market is a duopoly.

Secondly, and not surprisingly, the firms generally prefer to be able to collude to some extent. The exception is if β is quite low – that is, when travel is relatively inelastic to (own) price – when the independent pricing regime D3 is preferred to the collusion regime D2. Society prefers some or complete collusion over independent pricing, and the firms agree. However, society prefers limited collusion to perfect collusion: D2 and D4 are both better than M2. In both cases, the firms disagree. Society also prefers some collusion to independent pricing: D4 and D2 are strictly preferred to D3. The firms agree in both cases. Of the partial collusion alternatives, D4 is strictly preferred to D2 by both the firms and society.

Overall, society's best choice is D4. The firms' best choice is M2, followed by D4. If the social planner acts to ensure that the firms play the two-stage game then D4 is ensured as an outcome by the prisoners' dilemma. If D4 cannot be a guaranteed outcome (i.e. firms manage to use the opportunity to collude and maximise prices in both stages), the social planner's next best strategy is to impose regime D2 – here the firms establish a pricing rule (rather than an actual fixed price in stage 1). Again, if the firms cannot be trusted, the social planner may ensure that D2 occurs by allowing an independent agent to set price on cross-service tickets to maximise profit – this has the same effect as D2 played without illegal collusion.

The importance of the value of β in some cases and the variety of possible rankings of the outcomes by the different parties involved means that it is not sensible to propose a general policy rule. It is a case of 'horses for courses', and a planner or regulator will need to examine the circumstances of a particular market before deciding what to do – and indeed whether to do anything at all.

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