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Jolian McHardy*, Michael Reynolds and Stephen Trotter

ON THE PROBLEM OF NETWORK MONOPOLY

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Department of Economics
University of Sheffield
9 Mappin Street
Sheffield
S1 4DT
United Kingdom
www.shef.ac.uk/economics

Abstract:

We introduce a new regulatory concept: the independent profit-maximising agent, as a model for regulating a network monopoly. The agent sets prices on cross-network goods taking either a complete, or arbitrarily small, share of the associated profit. We examine welfare and profits with and without each agent type under both network monopoly and network duopoly. We show that splitting up the network monopoly (creating network duopoly) may be inferior for both firm(s) and society compared with a network monopoly "regulated" by an agent and that society always prefers any of the four agent regimes over network monopoly and network duopoly.

JEL #s: D43, L13, R48

Keywords: Network, Monopoly, Agent

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1. Introduction

Since Cournot (1838) economists have known that equilibrium prices are generally lower (higher) and society is better-off (worse-off) where firms producing complementary (substitute) goods collude to jointly maximise profits than when they behave independently. Where firms are easily separable according to whether they produce complements or substitutes, the policy advice follows straightforwardly: encourage (discourage) collusion under complements (substitutes). The tough line generally taken by governments and regulators against collusive behaviour follows the "firms producing substitutes" argument, whilst examples such as the 'block exemption' from the relevant provisions of the 1998 Competition Act given to multi-operator public transport pricing schemes in the UK (see Office of Fair Trading, 2006) follow the reasoning of "firms producing complements".

Unfortunately, the coexistence of substitute and complementary relationships between firms' demands in networks complicates matters. A policy of allowing network monopoly (or collusion everywhere) has the advantage that externalities due to independent pricing between complements (which lead to higher prices) are internalised.[1] However, collusion everywhere undermines the potency of potentially beneficial independent pricing across substitute parts of the network. Similar converse arguments apply to splitting up a network monopoly (or discouraging collusion) to exploit competitive effects across substitutes. Indeed, Economides and Salop (1992), who explore the issue of pricing on a network, show that breaking up a network monopoly, even in the presence of substitutes, can result in higher prices. Inevitably, this result is dependent on the relative weight of complementary

[1] There are also possible co-ordination and cost benefits to network monopoly, but we are concerned in this paper with price effects associated with strategic interplay between firms and agents under different organisations of the market.

versus substitute externalities in the network. McHardy (2006) addresses a related question: on splitting a monopoly producer of complementary goods, creating complementary monopoly, how much post-split entry/competition is required into the production of each complement in order to yield an overall welfare improvement? It is shown, in the two complementary good case, at least, that relatively little post-split entry/competition in one or other of the complementary goods is required to achieve this. Nevertheless, entry may not always be an attractive policy option, especially on a network.

An alternative policy would be to allow the firms to collude along complementary lines but not along substitute lines. Such an approach is inevitably fraught with problems, not least of which is how to ensure that when the firms meet to collude on one variable they do not make agreements over the other. Indeed, the general idea that collusion can be beneficial is at the very least somewhat counter-intuitive, and there is a widespread tendency to introduce a regulator to control pricing across a network, especially where the services are all operated by a single monopolist.

Yet another approach would be to enforce a separation in the industry e.g. by inserting a (supposedly) competitive layer between the network provider and the consumer. This was done in the early days of mobile telephony in the UK, when Cellnet and Vodaphone were obliged to sell to the public through service providers, but did not prove to be a very satisfactory solution (see Cave and Williamson, 1996, Section VI, for further details). The regulator is typically a government agency, such as the Federal Communications Commission in the USA or the Office of the Rail Regulator in the UK, and in these circumstances even an active, pro-competition regulator is likely to be seen as the 'dead hand' of the state. The rising importance of network industries in modern economies coupled with the difficulties of applying traditional regulatory approaches (e.g., Gilbert and

Riordan, 1995) makes this area one of considerable interest for policy-makers.

This paper introduces a different type of regulatory agent (that we have not seen before in the literature), one who operates much more within the industry and who may even take a share of the industry's profits. We examine how the employment of an independent agent may provide a useful instrument for regulators in separating the pricing decisions on complementary and substitute aspects of a network: allowing collusive pricing on complementary elements of the network without compromising the benefits of independent pricing amongst substitutes.

The following section introduces a simple network model with differentiated demands and derives the benchmark case of the welfare-maximising social planner. In Section 3, the equilibria under three regimes are derived and compared: a network monopoly which (i) is unregulated (ii) faces an independent profit-maximising agent who sets the price on the cross-network commodity bundles, taking an arbitrarily small share of the associated profit (iii) faces an agent, but the agent sets the price and takes *all* profit on the cross-network commodity bundles. Section 4 repeats the analysis for the case of independent (non-collusive) network duopoly. Section 5 examines the rankings of the seven regimes and the relative size of equilibrium values of key variables under each regime. Section 6 is a conclusion.

2. The Model

Consider a simple demand system where consumption involves two commodities X and Y in fixed and (for simplicity but without loss of generality) equal proportions.[2] Let there be two

[2] It is easily shown that given the assumption of fixed proportions along with (i) the fact that all agents maximise objective functions over a complete bundle of commodities rather than an

distinct versions of each commodity, X_i and Y_i $(i = 1,2)$, with firm m producing the combination (X_m, Y_m) $(m = 1,2)$. Assuming that the distinct versions of each commodity are interchangeable (but not perfect substitutes), we refer to commodity bundle (X_m, Y_m) as the single-network bundle and (X_m, Y_n) $(m \neq n = 1,2)$, as the cross-network bundle. Therefore consumers of the single-network bundle use only components provided by firm m whilst cross-network consumers consume bundles with one component from each firm. For example, mobile phone service operators charge different prices for access to their own network relative to other service providers' networks, transport companies provide interchangeable tickets (tickets which can be used on other companies' services), Microsoft and Apple both produce operating systems and software which are interchangeable to some extent and there are now a few companies who offer broadband/telephone-TV hybrid deals as well as the stand-alone products. The issues examined here have similarities with those discussed in the bundling literature – however, in the present case, for simplicity and in order to allow a focus on pricing strategy effects, the forms in which consumption can take place and the degree of differentiation and compatibility are predetermined and do not feature as strategic choices (e.g., to weaken price competition, Denicolò, 2000, or deter entry, Peitz, 2008) and the modelling framework does not include consumer network externalities (e.g., Economides and Himmelberg, 1995). Also, with the number of decision-makers and the market structure given in each regime, the modelling differs from work concerned with two-sided platforms (e.g., Rochet and Tirole, 2003).

For simplicity, we denote demand for commodity bundle (X_i, Y_j) as Q_{ij}. As the basis for the system of demands we refer to the quadratic utility function (e.g. Shubik and Levitan, 1980):

individual commodity and (ii) the symmetry of the model, the further assumption of equal proportions changes nothing within the model.

$$U\big(Q_{ij}\big) = v\sum Q_{ij} - \frac{n}{2(1+\mu)}\Big[\sum(Q_{ij})^2 + \frac{\mu}{n}\big(\sum Q_{ij}\big)^2\Big] + z\,, \qquad (i.j = 1{,}2). \tag{1}$$

where n is the number of commodity bundles (here $n = 4$), $\mu \in [0, \infty)$ is a measure of the degree of substitutability amongst the commodity bundles (with $\mu = 0$ for zero substitutability and $\mu \to \infty$ for perfect substitutes), z is a numeraire good hence $U\big(Q_{ij}(\mathbf{P})\big)$ is quasi-linear, justifying the use of a partial equilibrium analysis, v is a positive parameter and $\mathbf{P}$ is a vector of commodity bundle prices, P_{ij}. Consequently, demand for commodity bundle Q_{ij} is linear in prices:

$$Q_{ij}(\mathbf{P}) = \alpha - \beta P_{ij} + \sum_{mn \neq ij} \delta P_{mn}\,, \qquad (i, j = 1{,}2). \tag{2}$$

$$\alpha = \frac{v}{4},\ \beta = \frac{1+\mu}{4},\ \delta = \frac{\mu}{16}.$$

In this specification, β, which is related to the partial own-price elasticity of demand, is common for each commodity bundle the cross-price co-efficient, δ, is also common across all alternative commodity combinations to ij: all the alternative commodity bundles are equally good, but generally (for $\mu < \infty$) imperfect, substitutes.

Having established the demand structure for the model we now briefly turn our attention to costs. The central concern of this paper is with the relative prices, outputs, profit and welfare under different regulatory regimes. For simplicity we assume marginal cost is constant and equal to zero. Given that the structure of the model (the number of physical commodities) is a constant over all regimes, fixed costs play no part in decision-making across regimes and are also assumed to be zero. This does, however, assume away possible co-ordination costs under collusive regimes and assumes the introduction of the regulatory agent (whose role involves making only price decisions – it produces no physical outputs) is at zero additional cost.

The benchmark case of first-best social welfare maximisation is straight-forward. Welfare is the sum of profit and consumer surplus at a given set of prices. Given the utility function is quasi-linear, consumer surplus $CS(\mathbf{P})$ is a valid measure of welfare, where:

$$CS(\mathbf{P}) = U\big(\mathbf{Q}(\mathbf{P})\big) - \mathbf{Q}(\mathbf{P})\mathbf{P} - \mathrm{z}. \tag{3}$$

Welfare, $W(\mathbf{P})$, is therefore:

$$W(\mathbf{P}) = CS(\mathbf{P}) + \tilde{\pi}(\mathbf{P}), \tag{4}$$

where $\tilde{\pi}(\mathbf{P})$ is aggregate profit across the network[3]. With zero costs $\tilde{\pi}(\mathbf{P}) = \mathbf{Q}(\mathbf{P})\mathbf{P}$, hence:

$$W(\mathbf{P}) = U\big(\mathbf{Q}(\mathbf{P})\big) - z. \tag{5}$$

The first-best social planner maximises (5) with respect to $\mathbf{P}$ yielding the familiar result of price equal to marginal cost and hence in the present case a zero price regime: hence, the price of each commodity bundle under the social planner regime S is $P_{ij}^{S} = 0$ ($\forall i, j = 1{,}2$). From (2) the social planner's production of each composite bundle is: $Q_{ij}^{S} = \alpha$. Aggregate output, profit and welfare (here equal to consumer surplus) under the first-best welfare maximising case are then, respectively:

$$Q^{S} = 4\alpha, \;\; \tilde{\pi}^{S} = 0\,, \quad W^{S} = CS^{S} = 8\alpha^{2}. \tag{6}$$

With the theoretical framework of the paper and benchmark case established, we now proceed to consider the relative merits of two different regulatory regimes against the unregulated case under, first, network monopoly in which a single firm initially provides all the services on the network, and second, non-collusive network duopoly, in which two rival

[3] Tilde is used to indicate profit measured across the entire network as distinct from the case where profit is measured only across the firms (i.e. not accruing to the agent as in regimes M3 and D3 below).

firms initially offer differentiated single-network operations which can be combined into two further cross-network operations.

3. Perfect Collusion or Network Monopoly

In this section we consider the equilibrium prices, outputs, profits and welfare in a situation of network monopoly where all commodity bundles $(X_i,\ Y_j)$ $(i,j = 1{,}2)$ are provided by a single profit-maximising firm or by two perfectly collusive firms with firm m producing m components X_m, Y_m $(m = 1{,}2)$.[4]

We are interested in examining three regimes. The first regime (M1) is the unregulated case in which the monopolist sets all prices to maximise profit across the network. In the context of this paper, this is equivalent to two single-network providers being allowed to collude on all prices. The advantage of such a regime is that it internalises the cross-network externalities in the model which put upward pressure on independently set prices: private incentives are, to some extent, aligned with social incentives in that by colluding on prices on this part of the network firms decrease price, raising profit and also social welfare. Such benefits would not be achieved if the social planner were to insist on separating the network monopoly into two single-network operations or to eliminate all collusion on pricing between two single-network operators (this is the situation in regime D1). Clearly this policy does not come without its drawbacks: by allowing collusion on the network or by not splitting up the network monopolist, the potential gains in terms of decreased prices through competition between single-network commodity bundles is lost. In the second regime (M2), the social planner employs an independent agent who is responsible for

[4] There is a possible distinction to be made between the network monopoly and perfectly collusive network duopoly in terms of the optimal choices of the number of commodity variants and the degree of differentiation between them. However, in this paper we treat them as a constant across all regimes.

setting the price on the cross-network commodity bundles (X_i, Y_j) $(i \neq j = 1,2)$ so as to maximise the associated profit of which it takes a share. An independent, profit maximising agent is again employed in the third regime (M3). However, in this case the agent keeps all the cross-network profit. As in the case of M1, it is assumed that the social planner is unable or unwilling to either prevent collusion between a network duopoly or split up a network monopoly. In examining regimes M2 and M3, we are asking whether the employment of an agent can yield an improvement upon a situation where firms are simply allowed to collude on price across all commodity bundles.

Beginning with regime M1, the network monopolist's profit, in general terms, is given by:

$$\Pi^{M1}(\mathbf{P}) = \sum_{m=1,2} P_{mm}\, Q_{mm}(\mathbf{P}) + P_x \sum_{m \neq n=1,2} Q_{mn}(\mathbf{P}). \tag{7}$$

Where, for ease of reference and given the symmetry of the model, $P_x (= P_{mn} = P_{nm}; m \neq n = 1,2)$ is the common price of the cross-network commodity bundles. The choice of this simplifying notation on cross-network price becomes apparent in Section 4.[5]

Using (2) and maximising (7) with respect to both P_{mm} and P_x yields the following equilibrium single- and cross-network prices[6]:

$$P_{mm}^{M1} = P_x^{M1} = \frac{8\alpha}{4+\mu}. \qquad (m = 1,2) \tag{8}$$

Hence, the network monopolist does not discriminate on price across the different commodity bundles. This result has to do with the symmetry of the model. Substituting (8)

[5] It is important to note that the imposition of this symmetry on P_x at this point has no bearing on the solutions given the general symmetry of the model and the assumption of monopoly, and later a single agent concerned with setting both P_{mn} *and* P_{nm}.

[6] Note, all the second order conditions in the paper are met for the relevant parameter values and are not stated.

into (2) yields the following equilibrium expressions for quantity demanded of single- and cross-network commodity bundles:

$$Q_{mm}^{M1} = Q_{mn}^{M1} = \frac{\alpha}{2}. \qquad (m \neq n = 1,2) \tag{9}$$

Aggregate output, profit and welfare under regime M1 are then, respectively:

$$Q^{M1} = 2\alpha, \;\; \tilde{\Pi}^{M1} = \frac{16\alpha^2}{4+\mu}, \quad W^{M1} = 6\alpha^2. \tag{10}$$

Remark. *It follows straightforwardly from comparison of (6) and (10) that the welfare loss due to the network monopoly regime (M1) against the first-best regime (S) is 25%, the usual value for a single-product monopoly under constant marginal costs and linear demand.*

We now consider the second monopoly regime, M2, in which an agent is employed by the social planner to set the price on the cross-network commodity bundle. Pricing is simultaneous and the agent seeks to maximise profit over cross-network demands, from which it gets an arbitrarily small fixed proportion, θ. The monopolist now sets P_{mm} and the agent sets $P_x = P_{mn}$ $(m \neq n = 1,2)$, in order to maximise their respective profit functions:

$$\Pi_{\mathrm{M}}(\mathbf{P}) = \textstyle\sum_{m=1,2} P_{mm}\, Q_{mm}(\mathbf{P}) + (1-\theta)P_x \sum_{m\neq n=1,2} Q_{mn}(\mathbf{P}), \tag{11}$$

$$\Pi_{\mathrm{A}}(\mathbf{P}) = \theta P_x \textstyle\sum_{m\neq n=1,2} Q_{mn}(\mathbf{P}).$$

Recognising symmetry, the relevant first-order conditions are, respectively:

$$P_{mm} = \frac{\alpha+2\delta(2-\theta)P_x}{2(\beta-\delta)}, \quad P_x = \frac{\alpha+2\delta P_{mm}}{2(\beta-\delta)}. \tag{12}$$

Note, that whilst the monopolists reaction function is a function of θ, the agent's is not. Solving (12) simultaneously yields:

$$P_{mm} = \frac{\alpha\big(\beta+\delta(1-\theta)\big)}{2(\beta(\beta-2\delta)-\delta^2(1-\theta))}, \quad P_x = \frac{\alpha\beta}{2(\beta(\beta-2\delta)-\delta^2(1-\theta))}. \tag{13}$$

Taking limits yields the following equilibrium single- and cross-network prices under regime M2:

$$P_{mm}^{\mathrm{M2}} = \lim_{\theta\to 0} P_{mm} = \frac{8\alpha(4+5\mu)}{7\mu^2+24\mu+16}, \quad P_{\mathrm{x}}^{\mathrm{M2}} = \lim_{\theta\to 0} P_{\mathrm{x}} = \frac{32\alpha(1+\mu)}{7\mu^2+24\mu+16}. \tag{14}$$

It is important to note that this regime introduces strategic interaction between the firm(s) and the agent, making the case distinct from one in which $\theta = 0$ is imposed in the general profit functions (11). The latter simply returns the network monopoly case, M1. Note also, from inspection of (14) and given $\mu > 0$, the following is always true: $P_{mm}^{\mathrm{M2}} > P_{\mathrm{x}}^{\mathrm{M2}}$.

Substituting (14) into (2), the equilibrium quantities in regime M2 are given by:

$$Q_{mm}^{\mathrm{M2}} = \frac{\alpha}{2}, \quad Q_x^{\mathrm{M2}} = \frac{2\alpha(3\mu^2+7\mu+4)}{7\mu^2+24\mu+16}. \tag{15}$$

Hence, on the single-network bundle the combined prices yield the monopoly level of output of single-network bundles but a higher level of output of cross-network bundles..

Aggregate output and profit, firm profit and welfare under regime M2 are then, respectively:

$$Q^{M2} = \frac{\alpha(19\mu^2+52\mu+32)}{7\mu^2+24\mu+16}, \quad \widetilde{\Pi}^{M2} = \Pi^{M2} = \frac{8\alpha^2(83\mu^3+308\mu^2+352\mu+128)}{(7\mu^2+24\mu+16)^2},$$

$$W^{M2} = \frac{\alpha^2(703\mu^5+5262\mu^4+14832\mu^3+19744\mu^2+12544\mu+3072)}{2(1+\mu)(7\mu^2+24\mu+16)^2}. \tag{16}$$

Proposition 1.[7] *(i) The firm(s) (weakly) prefer regime M1 over M2 (ii) The social planner (weakly) prefers regime M2 over M1:*

$$\begin{Bmatrix} \tilde{\Pi}^{M1} = \tilde{\Pi}^{M2} \\ \tilde{\Pi}^{M1} > \tilde{\Pi}^{M2} \end{Bmatrix} \ and \begin{Bmatrix} W^{M2} = W^{M1} \\ W^{M2} > W^{M1} \end{Bmatrix} \ if \ \begin{Bmatrix} \mu = 0 \\ \mu > 0 \end{Bmatrix}.$$

The monopolist and social planner, not surprisingly, have opposing rankings of the two regimes.

The third monopoly regime, M3, involves an agent who sets the price on the cross-network commodity bundles, but keeps the entire share of profit on this part of the network, i.e. $\theta = 1$. Imposing $\theta = 1$ in (13), yields the following equilibrium prices and quantities:

$$P_{mm}^{M3} = P_{mn}^{M3} = \frac{4\alpha}{2+\mu}, \quad Q_{mm}^{M3} = Q_{mn}^{M3} = \frac{\alpha(4+3\mu)}{4(2+\mu)}. \qquad (m \neq n = 1,2) \qquad (17)$$

Like M1, under M3 there is symmetry in single- and cross-network prices and outputs. This is a consequence of the demand symmetry of the model: both the firms(s) and the agent are faced with equivalent profit maximisation problems.

Aggregate output and profit, firm profit and welfare under regime M3 are then, respectively:

$$Q^{M3} = \frac{\alpha(4+3\mu)}{(2+\mu)}, \ \tilde{\Pi}^{M3} = 2\Pi^{M3} = \frac{8\alpha^2(4+3\mu)}{(2+\mu)^2}, \quad W^{M3} = \frac{4\alpha^2(12+5\mu)(4+3\mu)}{2(2+\mu)^2}. \qquad (18)$$

Note, Π^{M3} captures the total profit accruing to the firm(s), excluding the agent, which is the relevant measure of profit for comparison of firm well-being with other regimes. However,

[7] All proofs either follow straightforwardly or are detailed in the Appendix.

our measure of society's well-being, W^{M3}, includes all profit on the network. In cases M1 and M2 profit to the firms and profit across the industry are the same.

Proposition 2. *(i) The firm(s) strictly prefer regime M2 over M3:* $\Pi^{\mathrm{M2}} > \Pi^{\mathrm{M3}}$; *(ii) The social planner (weakly) prefers regime M3 over M2:*

$$\begin{Bmatrix} W^{\mathrm{M3}} = W^{\mathrm{M2}} \\ W^{\mathrm{M3}} > W^{\mathrm{M2}} \end{Bmatrix} \ if \ \begin{Bmatrix} \mu = 0 \\ \mu > 0 \end{Bmatrix}.$$

To summarise, so far, the preference ranking of the network monopolist over the three regimes is the reversal of the ranking for the social planner with the case of the agent taking all cross-network profit being the least favourable to the monopolist and best for the social planner, and the free-market case being best for the monopolist and least good for the social planner. M2 is a compromise regime for both firm(s) and the social planner.

4. Non-Collusive Network Duopoly

In this section, we examine the effects of ruling out collusion or splitting a network monopoly. We now have a situation in which two separate non-collusive firms provide the substitute single-network operations (X_i, Y_i), and also contribute to two cross-network commodities: firm m provides the m component of (i) Q_{mm}, (ii) Q_{mn}, and, (iii) Q_{nm} $(m \neq n = 1{,}2)$. Essentially, this section seeks to address the question: Could the monopoly or perfectly collusive regimes in Section 3, with and without an agent, be improved upon by splitting up a network monopoly or deterring collusion between network duopolists? Note, when comparing regimes in terms of attractiveness to the firms it is now important to treat the monopoly case as a perfectly collusive duopoly where each firm takes half of the overall profit. Thus, in each case half the non-agent profit goes to each of the firms under each regime and the firms' preferences can be determined by referring to aggregate firm profit (which is distinct from industry-wide profit).

We begin, as in section 3, by considering a regime, D1, in which the firms operate without an agent but now set their single-network price and component of the cross-network commodity bundle prices independently and simultaneously. Regime D2 involves the employment of an agent in the setting of the price for the cross-network commodity bundles in such a way as to maximise its arbitrarily small share of profit on the cross-network operation, whilst regime D3 involves employment of an agent who retains all the profit on the cross-network commodity bundles.

We assume each firm m sets the price of its own component, P_{xm} $(m = 1,2)$, of the cross-network commodity bundle price. Whilst the usefulness of the notation P_x should now be apparent it requires further explanation here. As it stands, the notation P_{xm} does not distinguish between firm m's price component on the cross-network commodity mn and cross-network commodity nm. However, given the symmetry of the model, this restriction turns out to be unimportant. It also follows that the cross-network commodity bundle price, P_x, is the sum of these two component prices for both cross-network combinations:

$$P_x = P_{mn} = \sum_{m=1,2} P_{xm} , \qquad (m \neq n = 1,2). \qquad (19)$$

Given (17) the general expression for the profit of firm m is given by, Π_m^{D1}:

$$\Pi_m^{D1}(\mathbf{P}) = P_{mm}\, Q_{mm}(\mathbf{P}) + P_{xm}\,(Q_{mn}(\mathbf{P}) + Q_{nm}(\mathbf{P})), \qquad (m \neq n = 1,2). \qquad (20)$$

Using (2) and (19) in (20) and maximising with respect to P_{mm} and P_{xm} for $m = 1,2$, yields the following first order conditions, respectively:

$$\alpha - 2\beta P_{mm} + \delta P_{nn} + 4\delta P_{xm} + 2\delta P_{xn} = 0, \qquad (m \neq n = 1,2), \qquad (21a)$$

$$2\alpha - (2\beta - \delta)2P_{xm} - 2(\beta - \delta)P_{xn} + 4\delta P_{mm} + 2\delta P_{nn} = 0. \tag{21b}$$

Given the symmetry of the problem, in equilibrium, $P_{xm} = P_{xn}$ and $P_{mm} = P_{nn}$, and given (19), solving (20) simultaneously yields the equilibrium expressions for the cross- and single-network commodity bundle prices, respectively:

$$P_x^{D1} = P_{x1}^{D1} + P_{x2}^{D1} = \frac{64\alpha}{3(3\mu+8)}, \quad P_{mm}^{D1} = \frac{16\alpha}{3\mu+8}. \tag{22}$$

Clearly, $P_{mm}^{D1} < P_{mm}^{M1}$ and $P_x^{D1} > P_{mm}^{D1}$ and the cross-network bundle is more expensive than the single-network bundle. We will see later that this is the only regime where this is true. Using (22) in (2) yields the following equilibrium expressions for cross- and single-network demands, respectively:

$$Q_{mn}^{D1} = \frac{\alpha}{3}, \quad Q_{mm}^{D1} = \frac{4\alpha(2\mu+3)}{3(3\mu+8)}.$$

Aggregate output, profit and welfare under regime D1 are then, respectively:

$$Q^{D1} = \frac{2\alpha(11\mu+20)}{3(3\mu+8)}, \ \widetilde{\Pi}^{D1} = \Pi^{D1} \frac{128\alpha^2(9\mu+17)}{9(3\mu+8)^2}, W^{D1} = \frac{2\alpha^2(275\mu^3+1586\mu^2+1504\mu+1504)}{9(1+\mu)(3\mu+8)^2}. \tag{23}$$

Proposition 3. *(i) The firm(s) strictly prefer regime M1 over D1 and D1 over M3:* $\widetilde{\Pi}^{M1} > \widetilde{\Pi}^{D1} > \Pi^{M3}$*; (ii) The preferences of the firms over regimes M2 and D1 depend upon the degree of substitutability according to:*

$$\begin{Bmatrix} \widetilde{\Pi}^{M2} > \widetilde{\Pi}^{D1} \\ \widetilde{\Pi}^{M2} = \widetilde{\Pi}^{D1} \\ \widetilde{\Pi}^{D1} > \widetilde{\Pi}^{M2} \end{Bmatrix} \ if \ \begin{Bmatrix} 0 \le \mu < \mu^* \\ \mu = \mu^* \\ \mu > \mu^* \end{Bmatrix} \ where \ \mu^* = 3.490 \ (3 \ d.p.);$$

(iii) The social planner strictly prefers regime M2 over D1: $W^{M2} > W^{D1}$*; (iv) The social planner's preferences over regimes M1 and D1 vary with the degree of substitutability according to:*

$$\begin{Bmatrix} W^{M1} > W^{D1} \\ W^{M1} = W^{D1} \\ W^{M1} < W^{D1} \end{Bmatrix} \; if \; \begin{Bmatrix} 0 \leq \mu < \mu^{**} \\ \mu = \mu^{**} \\ \mu > \mu^{**} \end{Bmatrix} \; where\ \mu^{**} = 2.418\ (3\ d.p.).$$

This is an important result. First, network monopoly (M1) may be preferred by society to unregulated duopoly (D1) if μ is sufficiently small: hence the positive effects through competition on substitute services created by splitting up a monopoly network are outweighed by the negative complementary externalities. Second, for society, the monopoly under regulatory agent M2 is always preferable to unregulated duopoly (D1). McHardy (2006) concludes that social planners wishing to make improvements upon monopoly (perfectly collusive complementary monopoly) should not split up the monopoly and create a situation of complementary monopoly if there is little prospect of separation leading to entry and competition in the production of the complementary goods. Proposition 3 suggests that even where such post-separation entry is unlikely, employment of a regulatory agent may provide a partial solution to this problem.

We now introduce regime D2 in which the independent agent sets the cross-network commodity bundle prices so as to maximise its own profit: an arbitrarily small fixed proportion, θ, of the profit on the cross-network operation. The general expression for profit on the cross-network operation is given by:

$$\Pi_x^{D2}(\mathbf{P}) = P_x(Q_{mn}(\mathbf{P}) + Q_{nm}(\mathbf{P})), \qquad (m \neq n = 1,2).$$

of which the share to firm *m* and to the agent are respectively:

$$\Pi_{xm}^{D2}(\mathbf{P}) = \tfrac{1}{2}(1-\theta)P_x(Q_{mn}(\mathbf{P}) + Q_{nm}(\mathbf{P})), \tag{24a}$$

$$\Pi_A^{D2}(\mathbf{P}) = \theta P_x(Q_{mn}(\mathbf{P}) + Q_{nm}(\mathbf{P})), \qquad (m \neq n = 1,2). \tag{24b}$$

In the simultaneous price-setting scenario, the agent sets P_x to maximise its profit (24b), whilst firm m sets P_{mm} to maximise:

$$\Pi_m^{D2}(\mathbf{P}) = P_{mm}\,Q_{mm}(\mathbf{P}) + \tfrac{1}{2}(1-\theta)P_x(Q_{mn}(\mathbf{P}) + Q_{nm}(\mathbf{P})), \qquad (m \neq n = 1{,}2). \qquad (25)$$

Substituting (2) into (25) and maximising with respect to P_{mm}, yields the following first order condition:

$$\alpha - 2\beta P_{mm} + \delta(2P_x + P_{nn}) + 2\delta P_x(1-\theta) = 0\,, \qquad (m \neq n = 1{,}2). \qquad (26)$$

Similarly, substituting (2) into (24b) and maximising with respect to P_x:

$$\alpha - 2\beta P_x + \delta(P_{mm} + P_{nn} + 2P_x) = 0. \qquad (27)$$

Recognising symmetry ($P_{mm} = P_{nn}$) and solving (26) and (27) simultaneously, yields:

$$P_{mm} = P_{nn} = \frac{16\alpha(4+\mu(5-\theta))}{(17+2\theta)\mu^2+52\mu+32}\,, \quad P_{\mathrm{x}} = \frac{8\alpha(8+9\mu)}{(17+2\theta)\mu^2+52\mu+32}\,, \qquad (m = 1{,}2). \qquad (28)$$

Taking limits yields the following expressions for the equilibrium single- and cross-network commodity bundle prices, respectively:

$$P_{mm}^{\mathrm{D2}} = \lim_{\theta\to 0} P_{mm} = \frac{16\alpha(4+5\mu)}{17\mu^2+52\mu+32}\,, \quad P_x^{\mathrm{D2}} = \lim_{\theta\to 0} P_x = \frac{8\alpha(8+9\mu)}{17\mu^2+52\mu+32}\,, \ (m \neq n = 1{,}2). \qquad (29)$$

Clearly $P_{mm}^{\mathrm{D2}} > P_x^{\mathrm{D2}}$: the firms set prices above the agent. Substituting (29) in (2) yields the following equilibrium commodity bundle demands:

$$Q_{mm}^{D2} = \frac{\alpha(11\mu^2+28\mu+16)}{17\mu^2+52\mu+32}\,, \quad Q_{mn}^{D2} = \frac{\alpha(27\mu^2+60\mu+32)}{2(17\mu^2+52\mu+32)}\,, \qquad (m \neq n = 1{,}2).$$

Aggregate output, profit and welfare under regime D2 are then, respectively:

$$Q^{\mathrm{D2}} = \frac{\alpha(49\mu^2+116\mu+64)}{(17\mu^2+52\mu+32)}, \quad \widetilde{\Pi}^{D2} = \frac{8\alpha^2(463\mu^3+1492\mu^2+1536\mu+512)}{(17\mu^2+52\mu+32)^2},$$

$$W^{D2} = \frac{\alpha^2(4263\mu^5+29030\mu^4+74528\mu^3+91232\mu^2+53760\mu+12288)}{2(1+\mu)(17\mu^2+52\mu+32)^2}. \tag{30}$$

Proposition 4. *(i) The firm(s) strictly prefer regime D2 over M3:* $\widetilde{\Pi}^{D2} > \widetilde{\Pi}^{M3}$*; (ii) The firm(s) (weakly) prefers regime M2 over D2;*

$$\begin{Bmatrix} \widetilde{\Pi}^{\mathrm{M2}} = \widetilde{\Pi}^{\mathrm{D2}} \\ \widetilde{\Pi}^{\mathrm{M2}} > \widetilde{\Pi}^{\mathrm{D2}} \end{Bmatrix} \; if \; \begin{Bmatrix} \mu = 0 \\ \mu > 0 \end{Bmatrix};$$

(iii) The preferences of the firms over regimes D2 and D1 depend upon the degree of substitutability according to:

$$\begin{Bmatrix} \widetilde{\Pi}^{D2} > \widetilde{\Pi}^{D1} \\ \widetilde{\Pi}^{D2} = \widetilde{\Pi}^{D1} \\ \widetilde{\Pi}^{D2} < \widetilde{\Pi}^{D1} \end{Bmatrix} \; if \; \begin{Bmatrix} 0 \le \mu < \mu^{***} \\ \mu = \mu^{***} \\ \mu > \mu^{***} \end{Bmatrix} \; where \; \mu^{***} = 2.175 \; (3 \; d.p.);$$

(iv) The social planner (weakly) prefers regime M3 over D2 and D2 over M2:

$$\begin{Bmatrix} W^{\mathrm{M3}} = W^{\mathrm{D2}} = W^{\mathrm{M2}} \\ W^{\mathrm{M3}} > W^{\mathrm{D2}} > W^{\mathrm{M2}} \end{Bmatrix} \; if \; \begin{Bmatrix} \mu = 0 \\ \mu > 0 \end{Bmatrix}.$$

Finally, we introduce regime D3, in which the independent agent keeps all profit on the cross-network bundle. Setting $\theta = 1$ in (28) yields the equilibrium single- and cross-network prices, respectively:

$$P_{mm}^{\mathrm{D3}} = P_{nn}^{\mathrm{D3}} = \frac{64\alpha(1+\mu)}{19\mu^2+52\mu+32}, \quad P_{\mathrm{x}}^{\mathrm{D3}} = \frac{8\alpha(8+9\mu)}{19\mu^2+52\mu+32} \qquad (m = 1,2) \tag{31}$$

Note, $P_{mm}^{\mathrm{D3}} < P_{mm}^{\mathrm{D2}}$ and $P_{\mathrm{x}}^{\mathrm{D3}} < P_{\mathrm{x}}^{\mathrm{D2}}$. Substituting (31) in (2) yields the following equilibrium commodity bundle demands:

$$Q_{mm}^{D3} = \frac{16\alpha(\mu^2+2\mu+1)}{19\mu^2+52\mu+32}, \quad Q_{mn}^{D3} = \frac{\alpha(27\mu^2+60\mu+32)}{2(19\mu^2+52\mu+32)}, \qquad (m \neq n = 1,2).$$

Aggregate output, profit and welfare under regime D3 are then, respectively:

$$Q^{D3} = \frac{\alpha(59\mu^2+124\mu+64)}{(19\mu^2+52\mu+32)}, \quad \Pi^{D3} = \frac{8\alpha^2(499\mu^3+1524\mu^2+1536\mu+512)}{(19\mu^2+52\mu+32)^2},$$

$$W^{D3} = \frac{\alpha^2(5487\mu^5+34222\mu^4+82208\mu^3+95968\mu^2+54784\mu+12288)}{2(1+\mu)(19\mu^2+52\mu+32)^2} \tag{32}$$

Proposition 5. *(i) The firms strictly prefer regime D3 over S (ii) The firms (weakly) prefer regime M3 over D3 (iii) The social planner (weakly) prefers regime D3 over M3:*

$$\begin{Bmatrix} \widetilde{\Pi}^{\mathrm{M3}} = \widetilde{\Pi}^{\mathrm{D3}} \\ \widetilde{\Pi}^{\mathrm{M3}} > \widetilde{\Pi}^{\mathrm{D3}} \end{Bmatrix} \textit{ and } \begin{Bmatrix} W^{\mathrm{D3}} = W^{\mathrm{M3}} \\ W^{\mathrm{D3}} > W^{\mathrm{M3}} \end{Bmatrix} \textit{ if } \begin{Bmatrix} \mu = 0 \\ \mu > 0 \end{Bmatrix};$$

(iv) The social planner strictly prefers regime S over D3: $W^S > W^{D3}$.

Corollary. *Propositions 1 - 5 give a complete ranking of the seven regimes in firm (non-agent) profit and welfare over the domain* $\mu \in [0, \infty)$.

5. DISCUSSION

The propositions of Sections 3 and 4 have provided information on rankings over the seven regimes for the profit-maximising firms and for the social planner. The positions of certain regimes in the rankings are dependent on the level of μ. These conditional rankings are reproduced in Table 1, below.

Table 1

Firm (Non-Agent) Profit and Welfare Rankings by Regime (from highest to lowest)

μ = 0		0 < μ < 2.175		2.175 < μ < 2.418		2.418 < μ < 3.490		3.490 < μ	
π	*W*	π	*W*	π	*W*	π	*W*	π	*W*
M1*	S	M1	S	M1	S	M1	S	M1	S
M2*	D3†	M2	D3	M2	D3	M2	D3	D1	D3
D2*	M3†	D2	M3	D2	M3	D1	M3	M2	M3
D1	D2†	D1	D2	D1	D2	D2	D2	D2	D2
M3**	M2†	M3	M2	M3	M2	M3	M2	M3	M2
D3**	M1†	D3	M1	D3	M1	D3	D1	D3	D1
S	D1†	S	D1	S	D1	S	M1	S	M1

Regimes of equal rank are indicated by matching *, **, †.

First, for the firms, not surprisingly, the unregulated monopoly regime M1 is always (at least weakly) preferred. However, it is least preferred for the social planner for sufficiently high levels of μ. If μ is sufficiently low, then regime D1 is actually worse than M1 for the social planner: splitting up the profit-maximising network monopolist into independent network duopoly reduces welfare. This is a variant of the result that complementary monopoly is worse than monopoly. For high levels of μ regime D1 becomes the second best regime for the firm(s). Regimes M3 and D3, are always high-ranking for the social planner but understandably not for the firms – all profit on the cross-network bundles go to the agent. Regimes M2 and D2 are in the top four for the firms and D2 is ranked fourth for the social planner. Regime D2 has potential as a compromise for both firms and the social planner.

In order to draw more insightful judgements about the merits of the various regimes, it is necessary to have some indication about the *relative* payoffs for each party (excluding the agent) in each regime. Table 2 reports simulations for the percentage loss in firm (non agent) profit under each regime relative to the network monopoly case (M1) and the percentage loss in welfare, consumer surplus and aggregate quantity under each regime relative to the first best case, S under differing assumptions about the degree of substitutability, μ.

Table 2

Firm (Non-Agent) Profit Simulations of % Loss Relative to Regime M1 and Welfare, Consumer Surplus and Aggregate Output Loss Relative to Regime S

		Degree of Substitutability (μ)						
	Regime	**0**	**0.5**	**1**	**5**	**10**	**50**	**∞**
% reduction in profit relative to π^{M1}	**M2**	0.0	0.5	1.4	7.1	10.0	14.0	15.3
	M3	50.0	50.5	51.4	56.4	58.7	61.6	62.5
	D1	5.6	4.7	4.5	6.2	7.8	10.2	11.1
	D2	0.0	0.7	1.9	9.5	13.3	18.2	19.9
	D3	50.0	50.6	51.7	57.7	60.3	63.5	64.5
% reduction in welfare relative to W^S	**M1**	25.0	25.0	25.0	25.0	25.0	25.0	25.0
	M2	25.0	22.4	20.6	14.9	13.1	11.0	10.3
	M3	25.0	20.3	17.4	10.3	8.5	6.7	6.3
	D1	34.7	31.4	29.1	21.6	19.0	16.0	15.1
	D2	25.0	21.2	18.8	12.2	10.3	8.4	7.8
	D3	25.0	19.3	16.1	8.8	7.1	5.4	5.0
% reduction in consumer surplus relative to CS^S	**M1**	75.0	69.4	65.0	47.2	39.3	28.7	25.0
	M2	75.0	66.6	60.0	35.6	25.9	14.2	10.3
	M3	75.0	64.3	56.3	29.7	20.3	9.6	6.3
	D1	81.9	73.8	67.3	42.4	32.2	19.4	15.1
	D2	75.0	65.4	58.0	32.3	22.7	11.4	7.8
	D3	75.0	63.1	54.4	27.3	18.2	8.1	5.0
% reduction in aggregate quantity relative to Q^S	**M1**	50.0	50.0	50.0	50.0	50.0	50.0	50.0
	M2	50.0	47.5	45.7	40.4	38.5	36.4	35.7
	M3	50.0	45.0	41.7	32.1	29.2	26.0	25.0
	D1	58.3	55.3	53.0	45.7	43.0	39.9	38.9
	D2	50.0	46.1	43.3	34.8	32.0	28.9	27.9
	D3	50.0	43.9	40.0	29.6	26.5	23.3	22.4

Having identified regime D2 as a case of potential interest, we note that percentage loss in firm (non-agent) profit relative to regime M1 increases from 0% (6%) to 20% (11%) under regime D2 (D1) from $\mu = 0$ to ∞. At the same time, welfare loss under regime D2 (D1) relative to regime S varies between 25% (35%) to 8% (15%). Substantial reductions in welfare loss may be obtained pursuing D2 relative to M1 or D1 at relatively small cost to the firms.

6. CONCLUSIONS

In this paper we have explored how the employment of an independent profit-maximising agent as a price-setter on cross-network commodity bundles in a network can be used to address the problem of network monopoly. We showed that, whilst the welfare-maximising social planner may prefer network monopoly over splitting up the network (creating non-collusive network duopoly) for sufficiently low levels of μ (the degree of substitutability between commodity bundles), the reverse holds for relatively high levels of μ. However, both of these scenarios are welfare-inferior to regulation using any one of the agent models: so in answer to the basic question of what to do with a network monopoly, it is important to note society prefers all four of the agent regimes over "doing nothing" (network monopoly) or splitting up the network monopoly into non-collusive network duopoly. The relative welfare advantage of the agent-based regimes over the non-agent regimes is also increasing in μ.

Predictably, some of the regimes that rank highly with the social planner are very harmful to the firms: such regimes might suffer from significant efforts by firms to avoid such regulation (i.e., rent seeking) or to distort the information available for make policy-maker/regulator interventions. Interestingly, two of the agent regimes, where the agent takes an arbitrarily small share of cross-network profit faces and faces a network monopoly or non-collusive network duopoly, have attractions for both firms and the social planner relative to alternative regimes making them potentially important compromise cases for both parties.

Useful, non-trivial, developments of the existing work would examine the effectiveness of the agent solution in the context of consumption externalities and introducing bundling and compatibility as strategic choices.

APPENDIX

Proof of Proposition 3. (i) Let $\sigma^{\pi}_{M2,D1} = \frac{\widetilde{\Pi}^{M2}}{\widetilde{\Pi}^{D1}}$. First, note that $\lim_{\mu \to 0} \sigma^{\pi}_{M2,D1} = \frac{1}{2} < 1$ and, using *L'Hôpital's Rule,* $\lim_{\mu \to \infty} \sigma^{\pi}_{M2,D1} = \frac{784}{747} > 1$. Finally, $\frac{\partial}{\partial \mu}(\sigma^{\pi}_{M2,D1}) > 0 \ for\ \mu \geq 0$, hence $\sigma^{\pi}_{M2,D1}$ is a positive monotonic function of μ in the relevant range, and, $\lim_{\mu \to 3.490} \sigma^{\pi}_{M2,D1} \approx 1$.

Q.E.D.

Proof of Proposition 3. (iii) Let $\sigma^{W}_{D1,M1} = \frac{W^{D1}}{W^{M1}}$. First, note that $\lim_{\mu \to 0} \sigma^{W}_{D1,M1} = \frac{47}{54} < 1$ and, using *L'Hôpital's Rule,* $\lim_{\mu \to \infty} \sigma^{W}_{D1,M1} = \frac{275}{243} > 1$. Finally, $\frac{\partial}{\partial \mu}(\sigma^{W}_{D1,M1}) < 0 \ for\ \mu \geq 0$, hence $\sigma^{W}_{D1,M1}$ is a positive monotonic function of μ in the relevant range and $\lim_{\mu \to 2.418} \sigma^{W}_{D1,M1} \approx 1$.

Q.E.D.

Proof of Proposition 4. (ii) Let $\sigma^{\pi}_{D1,D2} = \frac{\widetilde{\Pi}^{D1}}{\widetilde{\Pi}^{D2}}$. First, note that $\lim_{\mu \to 0} \sigma^{\pi}_{D1,D2} = \frac{17}{18} < 1$ and, using *L'Hôpital's Rule,* $\lim_{\mu \to \infty} \sigma^{\pi}_{D1,D2} = \frac{4624}{4167} > 1$. Finally, $\frac{\partial}{\partial \mu}(\sigma^{\pi}_{D1,D2}) > 0 \ for\ \mu \geq 0$, hence $\sigma^{\pi}_{D1,D2}$ is a positive monotonic function of μ in the relevant range, and, $\lim_{\mu \to 2.175} \sigma^{\pi}_{D1,D2} \approx 1$.

Q.E.D.

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