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AN ELASTICITY MEASURE OF WELFARE LOSS IN SYMMETRIC OLIGOPOLY

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Abstract:

We derive a measure of welfare loss as a proportion of the value of sales under quantity-setting symmetric oligopoly in terms of the equilibrium industry price elasticity of demand, the number of firms in the industry and a conjectural variation term in the context of the standard linear model. This generalises the monopoly measure in James and McHardy (1997).

Key words: oligopoly, welfare loss, elasticity
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1. INTRODUCTION

Economists have long held concerns about the accuracy and consistency of profit reporting and the use of accounting rates of return to infer economic rates of return (e.g., see Schmalensee, 1989). However, cases in recent years such as Enron and WorldCom have further undermined confidence in reported profit figures, rendering as potentially impotent those elements of the regulator's toolkit that rely upon profit measures. One such tool is the Harberger (1954) deadweight loss triangle (henceforth DWL). Existing techniques used to measure this DWL rely upon estimates of industry profit or price-cost margins (e.g., Cowling and Mueller, 1981 and Dixon *et al.*, 2001). This paper derives an alternative to the existing profit-based measures of DWL which relies instead upon industry demand elasticities that are less subject to manipulation.

2. THE MEASURE

Consider an industry with n identical firms facing the inverse linear demand function,[1]

$$P = \alpha - \beta X, \qquad [1]$$

where P, X and x_i are, respectively, the levels of price, industry output and firm output ($X = \sum_{i=1}^{n} x_i$) and α and β are positive constants.

[1] Eq. [1] is based upon a standard quadratic quasi-linear utility function making the later welfare measure a valid one.

Given constant marginal costs, c $(0 \leq c < \alpha)$, profit maximisation yields an optimal individual output for firm *i* of

$$x_i = \frac{\alpha - c}{\beta(1 + n + \gamma_{ij}(n-1))}, \quad [2]$$

where $\gamma_{ij} = \frac{dx_j}{dx_i} \cdot \frac{x_i}{x_j}$ $(i \neq j = 1, \ldots, n)$ expresses firm *i*'s expectation of *j*'s proportionate output reaction to a change in x_i and may be interpreted as a measure of the implicit collusiveness of the industry: $\gamma_{ij} = 1$ $[\gamma_{ij} = 0]$ implies perfect collusion [Cournot].[2] Given symmetry we drop arguments on this term.

Summing Eq. [2] over all *n* firms, gives equilibrium industry output,[3]

$$X = \frac{n(\alpha - c)}{\beta(1 + n + \gamma(n-1))}. \quad [3]$$

Substituting Eq. [3] in Eq. [1] yields the equilibrium price,

$$P = \frac{\alpha\big(1 + \gamma(n-1)\big) + nc}{(1 + n + \gamma(n-1))}. \quad [4]$$

PROPOSITION 1: *Under symmetric n-firm oligopoly with linear demand and constant marginal cost, the common conjectural variation term is bounded from below according to*

$$\gamma > -\frac{1}{n-1}. \quad [5]$$

PROOF 1: *From the second order condition for a maximum, we have that,*

$$\frac{d}{dx_i}\Big(\alpha - c - \beta\{[2 + \gamma(n-1)]x_i + \textstyle\sum_{j \neq i} x_j\}\Big) < 0,$$

[2] Use of a conjectural variation term to characterise conduct in oligopoly models is commonplace (e.g., Clarke and Davis, 1982, Dixit and Stern, 1982, Cable *et al.*, 1994, and Dixon *et al.*, 2001). Whilst the approach has its critics (see Shapiro, 1989), it also has its exponents (see Bresnahan, 1989, and Fraser, 1994).

[3] As is well known, this equilibrium if only Nash in the case of $\gamma = 0$.

Hence,

$$-\beta\{2+\gamma(n-1)+\gamma(n-1)\}<0.$$

Since $\beta>0$, *we require* $\{.\}>0$. **Q.E.D.**

Corollary: *Eq. [5] ensures that the denominator of* X *is strictly positive, hence in the usual case where* $\alpha-c>0$, *it also guarantees* $X>0$.

Proposition 2: *Under symmetric n-firm oligopoly with linear demand and constant marginal cost, equilibrium deadweight loss as a proportion of sales value, W, can be expressed as follows (where* η *is the industry point price elasticity of demand),*

$$W=-\frac{(1+\gamma(n-1))^2}{2\eta n^2}. \qquad [6]$$

Proof 2: *The general expression for welfare loss as a percentage of sales value with linear demand and constant marginal cost is given by*

$$W=\frac{(P-c)(\tilde{X}-X)}{2PX}=\frac{(P-c)^2}{2\beta PX}, \qquad [7]$$

where $\tilde{X}=\frac{\alpha-c}{\beta}$ *is the competitive industry output.*

Manipulating Eq. [1] we have,

$$\frac{1}{\eta}=-\beta\frac{X}{P}, \qquad [8]$$

where $=\frac{dX}{dP}.\frac{P}{X}$. *Using Eq. [8] in Eq. [7],*

$$W=-\frac{\eta(P-c)^2}{2P^2}. \qquad [9]$$

Using Eq. [3], Eq. [4] and Eq. [8], we have,

$$\frac{P-c}{P}=-\frac{1+\gamma(n-1)}{\eta n}. \qquad [10]$$

Using Eq. [10] in Eq. [9] completes the proof. **Q.E.D.**

COROLLARY: *Under $n = 1$ or $\gamma = 1$, Eq. [6] reproduces the monopoly measure in James and McHardy (1997),*

$$W^m = -\frac{1}{2\eta}.$$

In order to complete the discussion, we consider the properties of the term γ and the implications for our measure, Eq. [6].

DEFINITION 1: *Let*

$$\gamma^* = \inf\left[\gamma : \gamma > -\frac{1}{n-1}\right]. \quad [11]$$

DEFINITION 2: *The set of values of n for which the equilibrium can be affected by γ (i.e. positive integers excluding $n = 1$), is*

$$N = [n : n \in \mathbb{Z}^+, n \neq 1].$$

PROPOSITION 3:[4]

(i) *For a given $n \in N$, as $\gamma \to \gamma^*$, $W \to 0$: welfare approaches the competitive outcome.*

(ii) *$W \to 0$ as $\gamma \to k$ where $\gamma \in (-1,0)$ and k is strictly monotonic in n.*

PROOF 3: *It follows from Definition 1 that $\inf\{N\} = 2$ and $\sup\{N\} = \infty$. From Eq. [11] we have $\gamma^*(n = 2) = -1$, $\lim_{n\to\infty} \gamma^* \to 0^-$ and γ^* is strictly monotonic in $n \in N$. For a given level of $n \in N$,*

$$\lim_{\gamma\to\gamma^*} \big(1 + \gamma(n-1)\big) = 0. \quad [12]$$

[4] The case for $n = 2$ is well known.

It remains to show that the denominator of Eq. [6] does not approach zero as $\gamma \rightarrow \gamma^*$. Using Eq. [3] and Eq. [4] in Eq. [9], we have that the equilibrium level of η is

$$\eta = -\frac{\alpha\big(1+\gamma(n-1)\big)+nc}{n(\alpha-c)}.$$

Given Eq. [12],

$$\lim_{\gamma\rightarrow\gamma^*}\eta = -\frac{c}{(\alpha-c)}, \qquad [13]$$

which is invariant with respect to γ. Hence, using Eq. [12] and Eq. [13] in Eq. [6],

$$\lim_{\gamma\rightarrow\gamma^*}W = 0.$$ **Q.E.D.**

3. Concluding Remarks

Eq. [6] offers a relatively simple empirical measure of *DWL* as a proportion of sales value based upon observables n and η and a parameter γ, thus avoiding the need to rely upon reported profit figures or price-cost margins. Use of the parameter γ has its critics and exponents. However, as Vives (1999, p.186) notes, "..the conjectural variation approach has proved useful in applied work because it parameterizes the degree of competition in a market…", which is exactly its purpose here.

Theoretical studies of *DWL* often refer to welfare losses as a proportion of first-best welfare rather than sales value (e.g., Corchón, 2008, and Anderson and Renault, 2003). Empirically, this is problematic as it requires assumptions about the form of demand (underlying utility) to be *global* rather than *local*. Our measure, by referring to sales value, eliminates this problem. Furthermore, in the context of applied policy work reference to sales value (an indicator of the size/commercial importance of the industry) is likely to be more easily communicated and understood.

Finally, it is important to note that the assumed symmetry of our model will tend to bias *DWL* downwards (e.g., Corchón, 2008).

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