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Smooth transitions, asymmetric adjustment and unit roots

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Abstract

The aim of this paper is to develop a unit root test that takes into account two sources of nonlinearites in data, i.e. asymmetric speed of mean reversion and structural changes. The asymmetric speed of mean reversion is modelled by means of a exponential smooth transition autoregression (ESTAR) function for the autoregressive parameter, whereas structural changes are approximated by a smooth transition in the deterministic components. We find that the proposed test performs well in terms of size and power, in particular when the autoregressive parameter is close to one.

J.E.L. Classification : C12, C32.

Key words: Unit Roots, nonlinear trends, exponential smooth transition autoregressive model, structural change.

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1 Introduction

During the last decades there has been an increasing number of studies which have developed tests to analyse the order of integration of variables. Within them, particular attention has been paid to unit root tests that take into account structural changes and nonlinearities.

On one hand, it is well known that the existence of structural changes in the series, might affect the power of the augmented Dickey-Fuller (ADF) test. Some authors such as Campbell and Perron (1991), amongst others, suggest that the ADF test tends to suffer from power problems when the deterministic components are incorrectly specified. Therefore, traditional unit root tests might not be able to distinguish a stationary process with an autoregressive parameter close to 1, from a unit root. This problem is even more important when there are structural changes in the series, since in this case, the behaviour of an I(1) process can be very similar to that of a stationary process with breaks. Rappoport and Reichlin (1989) and Perron (1989, 1990) show that traditional unit roots might incorrectly conclude that the series have a unit root when in fact they are stationary with structural changes. The point is that in the latter case, breaks in slope or intercept have permanent effect on the variable, similar to a stochastic shock, i.e. the case of an unit root process. The difference however is that in the former, the shocks occur periodically and in the latter the changes occur only at certain points of time. In order to overcome this issue, several authors have developed unit root tests in order to take into account structural changes (see Perron, 1989, 1990; Zivot and Andrews, 1992; Perron and Vogelsang, 1992a, 1992b; Lumsdaine and Papell, 1997; Perron and Rodríguez, 2003; and Bai and Perron, 2003).

Nevertheless, most of these tests take into account a sudden change rather than smooth. This behaviour of the modelled variable might be inappropriate though, since at the aggregate level, changed may be smooth when the individuals suddenly change their behaviour in close but different moments of time (Granger and Teräsvirta, 1993). Some authors have proposed different ways to address this point. For instance, Bierens (1997) proposes a unit root test versus the alternative hypothesis of stationarity about a nonlinear deterministic trend. The nonlinear trends are approximated by means of Chebishev polynomials. Nevertheless, this is not the only way that has been considered in the literature to approximate nonlinear trends; Leybourne et al. (1998) propose a unit root test, whereby the series are detrended before performing the ADF test.

On the other hand, nonlinearities can be present in the series as an asymmetric speed of mean reversion, i.e. the autoregressive parameter varies depending upon values of a variable. This nonlinear behaviour implies that there is a central regime where the series behave as a unit root whereas for values outside the central regime, the variable tends to revert to the equilibrium. These type of nonlinearities can be modelled as a threshold autoregressive model (TAR) (see Enders and Granger, 1998; Caner and Hansen, 2001). However, it might be plausible to assume that the change between regimes is smooth rather than sudden. Following the latter, Kapetanios et al. (2003) (KSS) develop a unit root test within the exponential smooth transition autoregressions framework (ESTAR). The null hypothesis is tested against the alternative of globally stationary ESTAR process. Nevertheless, KSS though controlling for an asymmetric speed of mean reversion, do not take into account nonlinearities in the deterministic components. Chong et al. (2008), propose a modification of the KSS auxiliary regression by including deterministic trends, which can be linear or quadratic. Recently, Christopoulos and León-Ledesma (2010) have proposed a unit root test that takes into account asymmetric speed of mean reversion, as well as structural changes in the intercept, approximated by means of a Fourier function. This allows the intercept to vary along the sample but is restricted to be the same at the beginning and at the end of the sample. This testing procedure is specially appropriate to test for purchasing power parity since real exchange rate should be stationary around a constant in order to accept such a theory.

In this paper we aim at contributing to the literature on unit roots and nonlinearities. Following the approach by Christopoulos and León-Ledesma (2010), in the next section we propose a unit root test that takes into account both sources of nonlinearities, i.e. in the deterministic components, approximated by a logistic smooth transition function not only in the intercept, but also in the slope, and asymmetric adjustment of mean reversion. In the third section we compare the power and size distortions of the proposed test with KSS' test. The last section concludes.

2 Nonlinear unit root test

In this section we propose a unit root test that takes into account two types of nonlinearities that might be present in the data and may affect the power properties of traditional unit root tests, i.e. smooth transitions in the deterministic components and asymmetric adjustment towards equilibrium. This test can be considered as an alternative to Leybourne et al. (1998), KSS and Christopoulos et al. (2010). Note that the test proposed in this section is a more general version of these authors' tests, since we do not restrict the intercept to be equal at the beginning and at the end of the sample.

In order to develop the test we, first, consider the following model

$$y_t = g(t) + \epsilon_t \tag{1}$$

where $\epsilon \sim NIID(0, \sigma^2)$ and $g(t)$ is a non-constant function of time. In order to model $g(t)$ we use a logistic smooth transition regression,

$$g(t) = g_1 + g_2 t + g_3 L_t(\gamma) + g_4 t L_t(\gamma) \tag{2}$$

where $L_t(\gamma)$ is a logistic smooth transition function defined as

$$L_t(\gamma) = \frac{1}{1 + e^{-\gamma t}} \tag{3}$$

where $\gamma > 0$. Note that this function allows the changes in intercept/slope to be smooth rather than sudden, that is between g_1 and $g_1 + g_3$ for the case of the intercept and between g_2

and $g_2 + g_4$ for the slope. The speed of transition adjustment in controlled by the parameter γ, i.e. the larger the parameter is, the faster is the adjustment. For instance if $\gamma = 0$, $L_t = 0.5$ and there is no structural change. On the other hand, for larger values of this parameter the change is nearly instantaneous[1].

Under this set up of the deterministic components, we aim to test the following unit root hypothesis

$$H_0 \equiv \epsilon_t = \mu_t, \;\; \mu_t = \mu_{t-1} + \varepsilon_t \tag{4}$$

where ε_t is assumed to be an I(0) process with zero mean.

In this paper we follow Christopoulos and León-Ledesma (2010) who propose to apply the KSS unit root test to the residuals of equation (1) in order to take into account the possibility of asymmetric adjustment. Therefore, this test involves a two-step procedure; first, estimate equation (1) by nonlinear least squares (NLLS). Second, apply the KSS unit root test to the residuals

$$\hat{\epsilon} = y_t - \hat{g}(t). \tag{5}$$

KSS tests the unit root null hypothesis versus the alternative of globally stationary ESTAR process, i.e.

$$\Delta\hat{\epsilon}_t = \alpha\hat{\epsilon}_{t-1} + \vartheta\hat{\epsilon}_{t-1}(1 - e^{-\theta\hat{\epsilon}^2_{t-1}}) + \varepsilon_t \tag{6}$$

KSS impose $\alpha = 0$, implying that $\hat{\epsilon}_t$ is an I(1) in the central regime. In order to test the unit root null hypothesis, KSS propose a Taylor approximation for equation (6)

$$\Delta\hat{\epsilon}_t = \delta\hat{\epsilon}^3_{t-1} + \eta_t \tag{7}$$

where η_t is an error term. Note that equations (6) and (7) can also include lags of the dependent variable to avoid autocorrelation in the error term. Now, it is possible to test $H_0 \equiv \delta = 0$ against $H_1 \equiv \delta < 0$. The proposed test is called $\hat{t}_{SNL}$.

[1]Note that the $L_t(\gamma)$ function takes values between 0 and 1. In the limiting case with $\gamma = +\infty$, L_t changes almost instantaneously from 0 to 1.

Since this test does not follow the standard t distribution, the tabulated values are not valid in order to perform the test. Although KSS and Christopoulos et al. (2010) propose the critical values for their tests, they are not valid in our case, since the approximation of the deterministic components is different from those used by KSS[2] and Christopoulos et al. (2010). Therefore, in table 1 we report the critical values for the test based on 50,000 replications for different sample sizes[3].

3 Size distortions and power comparisons

In this section we carry out a Monte Carlo investigation of the small sample size and power of the test proposed in the previous section. This Monte Carlo experiment is based on 2,000 replications.

First, we analyse the finite sample size characteristics of the proposed test. The empirical size is analysed for different sample sizes, i.e. $T = 100, 250$, and for $\gamma = 0.5, 1, 5$ with a nominal size of $\alpha = 0.05$. We simulated the following null data generation process (DGP)

$$y_t = 1 + 10t + \frac{10t}{1 + e^{-\gamma t}} + \frac{10}{1 + e^{-\gamma t}} + v_t \tag{8}$$

$$v_t = v_{t-1} + \varepsilon_t \tag{9}$$

where $\varepsilon_t \sim N(0, 1)$.

The results are displayed in table 2. In general, we can conclude that the empirical size of the test is quite close to the nominal one, 5%. Only some significant distortions are found for $T = 100$ and $\gamma = 5$. Nevertheless, the problem reduces for $T = 250$.

Next, we investigate the power of the proposed test based on the following model

$$y_t = 1 + 10t + \frac{10t}{1 + e^{-\gamma t}} + \frac{10}{1 + e^{-\gamma t}} + v_t \tag{10}$$

[2]These authors only consider the cases of an intercept and a linear trend.
[3]The RATS code to obtain the critical values for other sample sizes is available upon request.

$$v_t = v_{t-1} + \rho v_{t-1}(1 - e^{-\theta v_{t-1}^2}) + \varepsilon_t \tag{11}$$

Table 3 displays the results of the power analysis for different values of the parameters γ, ρ and θ and T=100. For comparison purposes we also display the power analysis for the KSS test. In general it is possible to highlight that the $\hat{t}_{SNL}$ test performs better in terms of power when compared with the KSS test, in particular for higher values of ρ, i.e. closer to the unit root. Therefore, the proposed test here tends to confuse less often the unit root hypothesis with a globally stationary process with nonlinear deterministic components.

4 Conclusions

In this paper we have proposed a unit root test that accounts for nonlinear deterministic trends and asymmetric adjustment. This new test can be applied to test empirically the order of integration for a number of variables, which are believed to contain structural breaks and nonlinear trends, such as exchange rates or unemployment rates. The empirical size of the test is quite close to the nominal one and, in terms of power, the test appears to perform better than the KSS, in particular when the autoregressive parameter is closer to unity.

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Table 1: Asymptotic critical values

T	1%	5%	10%
100	-3.947	-3.380	-3.087
250	-3.928	-3.386	-3.110
500	-3.957	-3.401	-3.118

Note: Monte Carlo experiment based on 50,000 replications.

Table 2: Size distortions

T	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 5$
100	0.0560	0.0495	0.0615
250	0.0525	0.0500	0.0530

Note: Monte Carlo experiment based on 2,000 replications.

Table 3: Power comparison

	$\gamma =$	0.5		1		5	
ρ	θ	$\hat{t}_{SNL}$	KSS	$\hat{t}_{SNL}$	KSS	$\hat{t}_{SNL}$	KSS
-1.5	0.01	0.468	**0.484**	0.457	**0.469**	0.472	**0.473**
-1.5	0.05	**0.989**	0.968	**0.985**	0.963	**0.983**	0.965
-1.5	0.1	**1.000**	0.997	**1.000**	0.998	1.000	1.000
-1.5	1	1.000	1.000	1.000	1.000	1.000	1.000
-1	0.01	0.31	**0.312**	0.313	**0.314**	0.295	**0.316**
-1	0.05	**0.919**	0.873	**0.919**	0.861	**0.911**	0.872
-1	0.1	**0.994**	0.984	**0.993**	0.975	**0.992**	0.969
-1	1	1.000	1.000	1.000	1.000	1.000	1.000
-0.5	0.01	**0.161**	0.121	**0.161**	0.11	**0.162**	0.125
-0.5	0.05	**0.575**	0.573	0.558	**0.584**	0.557	**0.578**
-0.5	0.1	**0.811**	0.773	**0.807**	0.791	**0.805**	0.766
-0.5	1	0.969	**0.99**	0.971	**0.99**	0.971	**0.99**
-0.1	0.01	**0.069**	0.029	**0.071**	0.026	**0.077**	0.031
-0.1	0.05	**0.094**	0.065	**0.105**	0.073	**0.1**	0.067
-0.1	0.1	**0.117**	0.101	**0.127**	0.115	**0.14**	0.103
-0.1	1	0.162	**0.165**	0.171	0.171	0.164	**0.178**

Note: Monte Carlo experiment based on 2,000 replications and T=100.