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Abstract

Vertical linkages accounts for a large proportion of the volume of intermediate inputs used in an industry. This paper analyses the role played by vertical linkages on the effects of trade liberalization on technology adoption and their consequences on average productivity and welfare in a trade model with heterogeneous firms. We find that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare.

Keywords: trade liberalization, heterogeneity, selection, technology adoption, vertical linkages.

J.E.L. Classification: F1.

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1 Introduction

The last two decades have witnessed the birth of a rich literature that theoretically and empirically shows that firm heterogeneity is the key to understanding the impact of trade liberalization policies on an industry's average productivity. According to this literature, trade increases average productivity due to a reallocation effect that operates through a selection mechanism (trade expels the less efficient firms out of the market and reallocates production factors towards the most efficient units (Melitz, (2003)) and through a within plant effect (providing firms with incentives to upgrade their productivity (Bustos (2011), Lileeva and Trefler (2010) and Unel (2013)).[1] Both sources, selection and plant productivity growth, have been found to be empirically relevant when examining the impact of trade liberalization on an industry's average productivity (Pavnick (2002),Trefler, (2004), Bloom, Draca and Van Reenen (2011)).

While the first branch of this literature has focused exclusively on firms heterogeneity in export activities, a very recent avenue points out that this dimension is far more complex with industries heavily relying on vertical linkages and the use of intermediate inputs. In this literature importing intermediate inputs also plays an important role in determining the ultimate effect of trade on average productivity (Kasahara and Rodrigue (2008), Halpern, Koren and Szeidl (2015)). This paper contributes to the literature by examining the impact of trade liberalization policies on average productivity and ultimately welfare in an environment where firms are interconnected by vertical linkages because they use as intermediate inputs goods supplied by other domestic and foreign firms and they are allowed to technology upgrade, which is a relatively unexplored channel in the literature.

Intermediate inputs allow firms to become connected in such a way that a shock that affects the production of one of these inputs is likely to be transmitted to the production of final goods thereby generating effects that go beyond the original ones. In addition, if firms use as production factors for the intermediate input sector a share of the production of final goods, this creates a multiplier effect that could, in principle, enlarge firms' innovation profits generated by a process of trade liberalization. This could provide firms with larger incentives to engage in R&D activities enhancing innovation. Yet, this comes with a double side: the multiplier effect could intensify firms' competition for scarce production factors in such a way that their prices would be driven up, reducing operating profits and, eventually, deterring firms from adopting more productive technologies.

Our paper is motivated by the fact that vertical linkages not only bring in new channels that could modify the link between trade liberalization, productivity and welfare, but also by the recognition that they are empirically relevant.

[1] See also the seminal work by Atkeson and Burstein (2010) and a more recent contribution by Impulliti and Licandro (2012).

Indeed, intermediate inputs constitute a fundamental part of the production structure of an open economy. Recent studies find that around 50 to 60% of total gross output is accounted for intermediate inputs.[2] Intermediates are also responsible for a large share of the total volume of international trade with the World Trade Report 2014 (p. 43) stating that "the average import content of exports is around 25 per cent – and increasing over time – and almost 30 per cent of merchandise trade is now in intermediate goods or components". Moreover, Di Giovanni and Levchenko (2009) use input-output tables from the UNIDO Industrial Statistics Database for a panel of 28 manufacturing industries over 55 developed and developing countries and they find that the largest part of the volume of intermediate inputs consumed by a manufacturing industry comes from inputs produced by the same industry, although sectors differ substantially in the extent to which they use them. These findings not only reinforce the idea that industry-specific vertical linkages are relevant at the sectoral level, but also suggest that differences in the strength of vertical linkages can result in differences in export behavior or technology upgrading across industries.

To analyze the role played by vertical linkages in determining the effects of trade liberalization on innovation, productivity and welfare, we construct a trade model with heterogeneous firms that are interconnected by vertical linkages in such a way that all firms can employ, as intermediate inputs, the final goods produced by both domestic and foreign exporting firms, as in Krugman and Venables (1995). These firms are also allowed to upgrade the current state of the technology they use by reducing their marginal cost to be a fixed proportion γ of its initial value, bearing a fixed cost of adoption of f_I units of the final good, as in Bustos (2011) among others. While simple, this framework is consistent with the stylized fact that many firms within an industry do not engage in R&D activities (Klette and Kortum (2004)).

Our results suggest that the strength of vertical linkages shapes the effects that trade liberalization produces on firms' survival and technology upgrading decisions, having an impact on the average productivity of the economy and, ultimately, on welfare. The model exhibits different type of equilibria depending on the parameter configuration, which are related to different hierarchies among adopters, exporters and domestic firms. Specifically, if *innovating* is an expensive activity relative to exporting, the most productive firms engage in both innovation and exporting, the less productive ones serve the domestic market using their original technology and the firms in the middle rank export using their original technology.[3] If, instead, *exporting* is relatively more expensive, the firms in the middle rank adopt the new technology but do not export.[4] Finally, for intermediate trade costs, an equilibrium in which firms are innovators and

[2] See, for instance, Jones (2010, 2011).

[3] This equilibrium, analysed in depth by Bustos (2011), is consistent with the empirical evidence provided in the same paper.

[4] Castellani and Zanfei (2007) finds that a non-negligible proportioon of the italian manufacturing firms are domestic innovators. This equilibrium is consistent with this evidence.

exporters or only domestic firms may exist.[5] The effects of trade liberalization on technology upgrading will vary with the intensity of vertical linkages and across equilibria (when the intensity is either low or high). For intermediate levels, however, the results are common across equilibria: trade liberalization promotes technology upgrading when the intensity is intermediate-low and deters technology upgrading when it is intermediate-high.

These results on innovation and productivity have a clear impact on the effects of trade liberalization on welfare. Specifically, trade liberalization will have a positive impact on welfare when the intermediate input intensity is low or intermediate-low but it will have a negative impact when it is intermediate-high and high. In the case in which trade liberalization has a positive impact on welfare, we find that vertical linkages magnify the effects of trade liberalization on welfare through its impact on the price of intermediate goods.

This paper is related to different strands of the literature. The very first one is the incipient literature on macroeconomics that explores the consequences of vertical linkages for explaining cross-country income and productivity differences (Ciccone (2002), Jones (2011), Klenow and Rodriguez-Clare (2005)) or business cycle transmission (Di Giovanni and Levchenko (2009)). Another line of research in trade also explores the consequences of vertical linkages for aggregate trade and the impact of trade liberalization on welfare (Helpman and Krugman (1985), Hummels, Ishii and Yi (2001), Nocco (2012)).[6]

However, the potential role that these links could play on innovation or technology adoption has received so far limited attention with some recent exceptions. These include a recent strand of the literature that has investigated the complementarity of intermediate inputs and technology adoption (Bas and Berthou (2013), Boler, Moxnes and Ullveit-Moe (2015)), and the work by Eslava et al. (2015) who introduce input-output linkages in a quantitative model of trade and firm heterogeneity in which firms have the option of "quality" upgrading.[7] Specifically, in Eslava et al. (2015), trade liberalization promotes quality upgrading, not only in exporting firms, but also in the most productive non-exporting firms when most productive firms are assumed to use high quality intermediate inputs more intensively than low productive ones. However, while our paper shares with these works a common focus on the role played by intermediate inputs on the effects of trade liberalization on technology upgrading, our analysis focuses on aspects absent in their analysis and, as far as we know, in other previous works. These aspects are the role played by the strength of vertical linkages and general equilibrium effects in determining the

[5] See Lileeva and Trefler (2010) for evidence supporting this equilibrium.

[6] Moreover, Lee, Padmanabhan and Whang (1997) have analyzed the so called "bullwhip effect" that underlines how small changes in final demand can cause a big change in the demand for intermediate goods along the value chain.

[7] In Eslava et al. (2015) it is assumed that quality increases not only the marginal utility of consumers but also the productivity of the firms using these goods as intermediate inputs

ultimate impact of trade liberalization on productivity and welfare. This is the reason why in our work we get a more complex set of results, which, for instance, encompasses some that have the same flavour as those in Eslava et al. (2015) with trade liberalization promoting technology adoption for the most productive non-exporting firms, even though this happens in our framework only in a special case, that is when the strength of vertical linkages is middle-low.[8]

The rest of the paper is organized as follows. Section 2 presents the structure of the model and its different equilibria with explicit solutions. Sections 3 and 4, respectively, discuss the effects of trade liberalization on cost cutoffs and on welfare. Section 5 concludes.

2 The Model

Let us consider a world in which there are two symmetric countries H and F populated by L individuals endowed with one unit of time that is dedicated entirely to work. Individuals derive utility from the consumption of two different goods, T and M, according to the following Cobb-Douglas functional form

$$U = \frac{C_T^{1-\mu} C_M^{\mu}}{\mu^{\mu}(1-\mu)^{1-\mu}} \quad 0 < \mu < 1$$

where the parameter μ represents the proportion of expenditure dedicated to the good M. The good T is a homogenous good, produced with a linear technology (i.e. one unit of labour is required to produce one unit of output) under perfect competition and freely traded. In contrast, M is a differentiated good and individuals derive utility from a continuum of varieties (indexed by $j \in \Omega$) according to the following specification

$$C_M = \left(\int_{j \epsilon \Omega} C_{\varepsilon}(j)^{\frac{\sigma-1}{\sigma}} \, dj \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the constant elasticity of substitution and $C_{\varepsilon}(j)$ denotes the individual consumption of variety j.

Each variety in the economy is produced by a single firm in a monopolistic competition environment. Unlike Melitz (2003), firms combine labour and a set of differentiated intermediate inputs to produce their variety using a Cobb-Douglas technology. More precisely, the firm producing the $j-th$ variety uses the following technology

$$q(j) = \frac{1}{a(j)} L(j)^{1-\alpha} M(j)^{\alpha}$$

[8] We will define this in the paper as Equilibrium B.

where $q(j)$ is the quantity produced and $a(j)$ represents the unit input requirements of firm j, obtained combining $L(j)$ units of labour and $M(j)$ units of the differentiated intermediate inputs, and it is an inverse measure of the productivity level of the firm. Since $\alpha \in (0,1)$ is the intermediate share and determines the importance of intermediate inputs in the production of the final good, it also represents the strength of vertical linkages.

Following Krugman and Venables (1995) and Nocco (2012) among others, and consistent with the empirical observation in Di Giovanni and Levchenko (2009), we assume that there are vertical linkages so firms are using part of the production of the final differentiated good M as an intermediate input. The intermediate composite good used by each firm j is therefore given by

$$M(j) = \left(\int_{l \epsilon \Omega} B_j(l)^{\frac{\sigma-1}{\sigma}} dl \right)^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1$$

where $B_j(l)$ denotes the amount of good l used as intermediate input by the firm producing variety j.

To enter into a market a firm needs to invest f_E units of labour to create a new variety (product innovation). At the moment of entry, the firm is uncertain regarding its productivity although the firm knows that its unit input requirement $a(j)$ follows a Pareto Distribution with the cumulative density function given by $G(a) = \left(\frac{a}{a_M}\right)^{\kappa}$ with $0 \leq a \leq a_M$ and shape parameter $\kappa \geq 1$. After entry, the productivity is revealed to the firm and the firm has the option to leave or stay in the market. Firms that produce must incur a per period fixed cost of operation f_D in terms of the differentiated final good.

Once the firm stays, it has the possibility to export to the foreign market. Exporting, however, entails both fixed and variable trade costs. More precisely, exporters need to incur a per period fixed cost of exporting f_X in terms of the differentiated final good and a variable trade cost of the iceberg type so that $\tau \geq 1$ units of the good have to be shipped from the production country in order to sell one unit in the foreign country. At this stage, we depart from Nocco (2012) assuming that the firm can simultaneously decide to adopt a more efficient Hicks-neutral technology which reduces the marginal cost of production to be a proportion γ of the original value $a(j)$, with $0 < \gamma \leq 1$, so that the unit input requirement of the composite good of firm j that innovates is $\gamma a(j)$. Adopting the most efficient technology bears a cost of f_I units in terms of the final differentiated good.[9]

[9] Following the notation of Baldwin and Forslid (2004), the parameters f_D, f_X and f_E are, respectively, the discounted value of the fixed cost of producing for the domestic country, for export and entry. These elements consequently are the equivalent to δf_D, δf_X and δf_E in Melitz (2003). Moreover, f_I is the discounted value of the fixed cost of innovation.

We assume that the homogenous good is the numeraire. This, together with the assumption of perfect competition, the linear technology (with unit input requirement equal to one) and zero trade cost assumed for this good, imply that the equilibrium wage is equal to one in both countries ($w = 1$). Finally, as it is common in the literature in trade and firm heterogeneity, and to keep the analysis tractable, the paper focuses on the case of symmetric economies (i.e. H and F exhibit identical parameter values, and therefore the solutions for the endogenous variables are symmetric).

2.1 Equilibrium

Solving the consumers' utility maximization problem, the demand function for variety j in the final good sector in each country is given by

$$C(j) = \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \mu I$$

where I represents the aggregate income of the country, $p(j)$ the price of variety j and $P_M = [\int\limits_0^N p(l)^{1-\sigma} dl]^{\frac{1}{1-\sigma}}$ is the price index of the set of all the N differentiated varieties bought in the country, which includes both domestically produced and imported varieties.

To obtain the total demand function that each firm faces we need to specify the different components of it. Specifically, a firm can produce to satisfy the demand of both domestic and foreign consumers and firms. In general, the quantity produced in H by firm j is given by the following expression

$$\begin{aligned} q(j) &= C_D(j) + B_{HDN}(j) + B_{HDI}(j) + B_{HXN}(j) + B_{HXI}(j) + \\ & +\epsilon \left(C_X(j) + B_{FDN}(j) + B_{FDI}(j) + B_{FXN}(j) + B_{FXI}(j) \right) \end{aligned}$$

where: $\epsilon = \tau$ if the firm exports and 0 otherwise; $C_D(j)$ and $C_X(j)$, respectively, denote the demand for variety j for domestic and foreign consumption; the variable $B_{vsm}(j)$ indicates the demand for variety j used as an intermediate input by firms producing in country $v = H, F$ for their domestic market (obtained when $s = D$) and to export to the other country (when $s = X$). Note that, within each case, the model distinguishes between the demand of the adopters of the most efficient technology (when $m = I$) and the non-adopters (when $m = N$) since the unit input requirements are different across the two cases. An analogous expression holds for the quantity produced by firm j producing in F.

A firm's demand for a variety used as an intermediate input is obtained by applying Shepard's lemma to its total cost function. In general terms, the total cost function of firm ι is given by

$$TC(\iota) = P_M^{\alpha}(f_D + \xi f_X + \zeta f_I + \gamma^{\zeta} a(\iota) q_{\iota})$$

where the variable ξ takes value one if the firm exports and zero otherwise, while ζ takes the value one if the firm innovates and zero otherwise. The demand of firm ι for each variety j is, thus, given by $B_\iota(j)$

$$\begin{aligned} B_\iota(j) &= \frac{\partial TC(\iota)}{\partial p(j)} = \alpha \frac{p(j)^{-\sigma} P_M^\alpha}{P_M^{1-\sigma}} \left(f_D + \xi f_X + \zeta f_I + \gamma^\zeta a(\iota) q_\iota \right) = \\ &= \frac{p(j)^{-\sigma}}{P_M^{1-\sigma}} \alpha TC(\iota) \end{aligned}$$

Notice that firms behave monopolistically and so the price that firm j charges in the domestic market is

$$p_{Dm}(j) = \left(\frac{\sigma}{\sigma - 1} \right) a(j) \gamma^\zeta P_M^\alpha$$

In the case in which firm j exports, the price that it sets abroad is $p_{Xm}(j) = \tau p_{Dm}(j)$. Thus, exporting firms set higher prices abroad and firms adopting the most efficient technology charge lower prices in both markets.

In each country the total value of the expenditure in the differentiated manufactured varieties E is given by the sum of the share of consumers' income, μI, and of the share of the total cost of production spent on intermediates, αTC, i.e. $E = \mu I + \alpha TC$. Equilibrium aggregate production for each variety j is, consequently, given by

$$q(j) = (1 + \xi \phi) \left(\frac{p_{Dm}(j)^{-\sigma}}{P_M^{1-\sigma}} \right) E$$

where $\phi \equiv \tau^{1-\sigma}$ denotes the freeness of trade with its value ranging from zero when iceberg trade costs are prohibitively high to one when they are null.

A firm's operating profits with unit input requirement $a(j)$ are then given by

$$\pi(j) = (1 + \xi\phi) \Delta \gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma}$$

where $\Delta \equiv \left(\frac{E}{\sigma (P_M)^{(1-\alpha)(1-\sigma)}} \right) \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma}$ captures aggregate variables and parameters affecting firms' demand and. In the case when $\alpha > 0$, this term also includes parameters affecting its production cost levels. $\gamma^{\zeta(1-\sigma)}$ denotes a measure of the efficiency gains obtained by the firm when it adopts the most efficient technology (when $\zeta = 1$), with larger gains obtained for smaller values of γ.[10]

A firm with unit input requirement $a(j)$ decides to upgrade its state of technology when the benefits of adopting the new technology expressed in terms of larger operating profits overcome the costs of doing so, that is when

$$(1 + \xi\phi) \Delta \left(\gamma^{1-\sigma} - 1 \right) a(j)^{1-\sigma} \geqslant f_I P_M^\alpha$$

[10] As smaller values of γ imply larger values of $\gamma^{(1-\sigma)}$

with the sign of equality holding if the firm is indifferent between adopting or not adopting the new technology. Its unit input requirements, a_I, will be denoted as the innovation cutoff.

A firm decides to export if the operating profits obtained from the foreign market overcome the fixed cost of exporting, that is if

$$\phi \Delta \gamma^{\zeta(1-\sigma)} a(j)^{1-\sigma} \geqslant f_X P_M^\alpha$$

with the sign of equality holding if the firm is indifferent between exporting or not. The indifferent exporter is called the marginal exporter and its unit input requirement is denoted by a_X.

The firm that is indifferent between staying or leaving the market must satisfy the following condition

$$\Delta \gamma^{\zeta(1-\sigma)} a_D^{1-\sigma} = f_D P_M^\alpha$$

where a_D denotes its unit input requirements.

Note that if a firm finds it profitable to adopt the new technology just considering the domestic market ($\Delta(\gamma^{(1-\sigma)} - 1)a^{1-\sigma} \geqslant f_I P_M^\alpha$), then this firm must adopt the same technology if it decides to export. This implies that there cannot be simultaneously in an equilibrium domestic firms adopting the most efficient technology and exporters relying on their original technology.

Depending on the parameter configurations, this model exhibits three different types of equilibria.[11] Moreover, different types of equilibria are associated with different firm hierarchies regarding export and innovation activities.

If *innovating* is an expensive activity relative to exporting, that is if the following condition holds

$$\frac{f_I}{(\gamma^{1-\sigma} - 1)} \left(\frac{\phi}{1 + \phi} \right) > f_X > \phi f_D,$$

firms will be sorted according to the following status: innovating exporters (the most productive ones), non innovating exporters with intermediate productivity levels and domestic firms. In this equilibrium, technology adoption is a relatively expensive activity (compared to exporting) and only a subset of the most productive exporters are willing to innovate. We denote this equilibrium by A.

If, instead, *exporting* is a relatively expensive activity the following condition holds

$$f_X > \frac{f_I}{(\gamma^{1-\sigma} - 1)} \phi \gamma^{1-\sigma} > \phi \gamma^{1-\sigma} f_D,$$

[11] See Navas and Sala (2015) for a complete characterization of all types of equilibria in a model without vertical linkages. The existence of vertical linkages does not alter the conditions determining the parameter configuration associated with each type of equilibria. This comes from the fact that external linkages affect both export and innovation activities in a similar way as they affect the production for the domestic market given that all fixed costs are using intermediates with the same intensity.

and firms are grouped according to the following categories: exporting innovators, non exporting innovators and domestic firms. We denote this equilibrium by B.

Finally, there is another case for intermediate trade costs in which firms do not find it profitable to innovate if they do not export and viceversa. More precisely, this equilibrium is sustained when the following parameter configuration holds

$$\frac{f_I}{(\gamma^{1-\sigma}-1)}\frac{\phi}{(1+\phi)} \leq f_X \leq \frac{f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}\phi.$$

In this equilibrium firms are sorted according to two different types: innovators that export or just domestic firms. We denote this equilibrium by C.

In the following subsections we characterize the different equilibrium conditions for each type of equilibrium. We denote the respective cost cutoffs associated with each equilibrium with the superscript $i = \{A, B, C\}$.

2.1.1 Equilibrium A

In equilibrium A every firm that innovates is an exporter but only a subset of the most productive exporters innovate. Consequently the conditions defining the three cost cutoffs ranked as $a_I^A < a_X^A < a_D^A$, explained above, are given by

$$(1+\phi)\Delta^A(\gamma^{1-\sigma}-1)\left(a_I^A\right)^{1-\sigma} = f_I\left(P_M^A\right)^\alpha \tag{1}$$

$$\phi\Delta^A\left(a_X^A\right)^{1-\sigma} = f_X\left(P_M^A\right)^\alpha \tag{2}$$

$$\Delta^A\left(a_D^A\right)^{1-\sigma} = f_D\left(P_M^A\right)^\alpha \tag{3}$$

A firm's decision to adopt the new technology, is affected by the presence of vertical linkages through two channels. The first one affects clearly a firm's demand through Δ^A. In a model without vertical linkages, total expenditure includes only domestic and foreign consumer expenditure. In this model, however, total expenditure is determined not only by consumer expenditure but also by each firm's demand for varieties used as intermediates. In principle, firms can rely on more incentives to adopt the most efficient technology since market size is larger. The existence of vertical linkages also has an impact on the firm's marginal costs of production, having an impact on global sales and operating profits through the price index P_M affecting Δ^A when $\alpha > 0$. This second channel, which is absent in a model without vertical linkages, also affects the cost of adoption. As the fixed costs of adoption involve the use of intermediates, any change in the cost of intermediates affects the adoption cost.

Using (1) and (3), the proportion of surviving firms which adopt the most efficient technology is obtained as

$$\frac{N_I^A}{N_D^A} = \left(\frac{a_I^A}{a_D^A}\right)^{\kappa} = \left[\frac{(1+\phi)(\gamma^{1-\sigma}-1)f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{4}$$

where N_I^A and N_D^A are, respectively, the number of innovating and surviving firms.

Using (2) and (3) the proportion of surviving firms exporting is

$$\frac{N_X^A}{N_D^A} = \left(\frac{a_X^A}{a_D^A}\right)^{\kappa} = \left(\frac{\phi f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{5}$$

with N_X^A denoting the number of exporting firms.

2.1.2 Equilibrium B

In equilibrium B, the firm that is indifferent between innovating or staying with the original technology is a domestic firm and all exporters are in fact innovators, with the three cost cutoffs ranked as $a_X^B < a_I^B < a_D^B$. In this case the following conditions hold

$$\phi\gamma^{1-\sigma}\Delta^B \left(a_X^B\right)^{1-\sigma} = f_X \left(P_M^B\right)^{\alpha}$$

$$(\gamma^{1-\sigma}-1)\Delta^B \left(a_I^B\right)^{1-\sigma} = f_I \left(P_M^B\right)^{\alpha}$$

$$\Delta^B \left(a_D^B\right)^{1-\sigma} = f_D \left(P_M^B\right)^{\alpha}$$

Similar conclusions relative to the impact of vertical linkages on the decision of adopting the most efficient technology can be extracted. Analogous to the previous equilibrium, the proportion of surviving firms exporting or adopting the most efficient technology are, respectively, given by

$$\frac{N_X^B}{N_D^B} = \left(\frac{a_X^B}{a_D^B}\right)^{\kappa} = \left(\frac{\phi\gamma^{1-\sigma} f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{6}$$

and

$$\frac{N_I^B}{N_D^B} = \left(\frac{a_I^B}{a_D^B}\right)^{\kappa} = \left[\frac{\left(\gamma^{1-\sigma}-1\right) f_D}{f_I}\right]^{\frac{\kappa}{\sigma-1}} \tag{7}$$

2.1.3 Equilibrium C

In equilibrium C, the set of innovators and exporters coincide. The firm which is indifferent between innovating or relying in the original technology knows that if it does not innovate, it won't be able to export and viceversa. Consequently, that firm evaluates the benefits of jointly innovate and export instead of relying with the original technology and remain domestic. The two resulting cutoffs are ranked as follows $a_I^C < a_D^C$, where a_I^C denotes the unit input requirements associated with that firm. The conditions associated with this equilibrium are given by the following expressions

$$\Delta^C \left[(1+\phi)\gamma^{(1-\sigma)} - 1\right] \left(a_I^C\right)^{1-\sigma} = (f_I + f_X) \left(P_M^C\right)^{\alpha}.$$

$$\Delta^C a_D^{1-\sigma} = f_D \left(P_M^C\right)^{\alpha}$$

Dividing them and rearranging terms, the proportion of incumbents which innovate and export is given by:

$$\frac{N_X^C}{N_D^C} = \frac{N_I^C}{N_D^C} = \left(\frac{a_I^C}{a_D^C}\right)^{\kappa} = \left\{ \frac{\left[(1+\phi)\gamma^{1-\sigma} - 1\right] f_D}{f_I + f_X} \right\}^{\frac{\kappa}{\sigma-1}} \tag{8}$$

Note that the presence of vertical linkages does not have an impact on the productivity distribution conditional on entry in any of the three equilibria, (i.e. the proportion of surviving firms adopting the most efficient technology or exporting is unchanged by the presence of vertical linkages). This is the result of both channels affecting symmetrically the variable profits of innovating, exporting and staying in the market. This can be observed by looking at conditions (4) and (5) in equilibrium A, (6) and (7) in equilibrium B and (8) in equilibrium C and noting that neither Δ^i nor $\left(P_M^i\right)^{\alpha}$ are affecting these proportions.[12] However, the presence of vertical linkages has interesting implications for the survival productivity cut-off having a clear impact on the technology adoption cut-off and, ultimately, the average industry productivity and the welfare of the economy.

2.2 Solution

An equilibrium in this economy is characterized by a vector of productivity cut-offs (a_I^i, a_X^i, a_D^i), and a vector of aggregate variables $(P_M^i, E^i, N_D^i, N_X^i, N_I^i)$ that satisfy the specific equations associated with each equilibrium described above, the market clearing conditions for each good and the Free Entry condition.

[12] Given that the expression for P_M^i is different across equilibria because it depends on θ^i defined in the following subsection, Δ^i changes across equilibria and it should therefore be changed accordingly.

The conditions derived in each of the equilibria above reveals that a_I^i, a_X^i can be expressed as a function of a_D^i. The value of a_D^i can be obtained as in the Melitz (2003) model using the Zero Profit condition (ZP) and the Free Entry condition (FE). The ZP condition is given by

$$\Delta^i \left(a_D^i\right)^{1-\sigma} = f_D \left(P_M^i\right)^{\alpha} \tag{9}$$

and the FE is given by

$$\bar{\pi}^i = P_M^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)}, \quad i = A, B, C.$$

where $\bar{\pi}^i$ represents average operating profits, $\theta^i \equiv \left[1 + \frac{G(a_I^i)}{G(a_D^i)} \frac{f_I}{f_D} + \frac{G(a_X^i)}{G(a_D^i)} \frac{f_X}{f_D}\right]$ and, in the specific case of equilibrium C, the cutoffs are such that $a_X^C = a_I^C$. The free entry condition states that average expected operating profit of active producers must be equal to their expected fixed cost $P_M^{\alpha} f_D \theta^i$ - which is given by the sum of $P_M^{\alpha} f_D$, plus $P_M^{\alpha} f_I$ times the probability of being an innovator (conditional on it being a producer), plus $P_M^{\alpha} f_X$ times the probability of being an exporter (again, conditional on it being a producer) - plus the expected cost of developing a successful entrant, that is $f_E/G(a_D^i)$.

Specifically, the term θ^i represents the expected total fixed cost of producing, exporting and innovating relative to the expected fixed costs of a surviving firm that just serves the domestic market. This variable will help us to characterize the survival productivity cut-off and the expression for the relevant aggregate variables across the different types of equilibria.

Given that $\bar{\pi}^i = \bar{r}^i/\sigma$, where $\bar{r}^i$ represents the average revenues, substituting $\bar{r}^i = E^i/N_D$, we find that the free entry condition can be rewritten as

$$\frac{E^i}{\sigma N_D} = \left(P_M^i\right)^{\alpha} f_D \theta^i + \frac{f_E}{G(a_D^i)} \tag{10}$$

To solve the model, an expression for the aggregate price index P_M^i and one for total expenditure E^i must be obtained. Using their definitions we obtain:

$$P_M^i = \left(\frac{\sigma}{\sigma-1} a_D^i\right)^{\frac{1}{(1-\alpha)}} \left(\frac{\beta}{\beta-1}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(N_D^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\theta^i\right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{11}$$

$$E^i = \mu L + \alpha \left(P_M^i\right)^{\alpha} N_D^i f_D \left[1 + \frac{(\sigma-1)\kappa}{\kappa - (\sigma-1)}\right] \theta^i \tag{12}$$

where $\beta \equiv \frac{\kappa}{\sigma-1} > 1$.[13] Conditions (9)-(12) characterize a system of equations in 4 endogenous variables (a_D^i, P_M^i, N_D^i and E^i). Solving the system we find that:

$$a_D^i = \left[\frac{\kappa \mu L}{\delta_0 f_D} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}} \left[\frac{a_M^{\kappa} f_E}{f_D} \frac{(\beta-1)}{\theta^i}\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} \tag{13}$$

[13] This condition is required to have the price index P_M^i converging to a positive value.

$$P_M^i = \left[\frac{\kappa \mu L}{\delta_0 f_D} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \right]^{-\frac{\kappa}{\chi}} \left[\frac{a_M^\kappa f_E}{f_D} \frac{(\beta - 1)}{\theta^i} \right]^{\frac{\sigma-1}{\chi}}$$

$$N_D^i = \frac{\left(\frac{\beta-1}{\beta} \right) \left(\frac{\sigma}{\sigma-1} \right)^{(\sigma-1)} \left[\frac{\kappa \mu L}{\delta_0 f_D} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \right]^{(\sigma-1)\frac{\alpha+\kappa(1-\alpha)}{\chi}}}{\left[\frac{f_E}{f_D} (\beta - 1) a_M^\kappa \right]^{\alpha \frac{\sigma-1}{\chi}} \left[\theta^i \right]^{\kappa \frac{\sigma - \alpha\sigma - 1}{\chi}}}$$

where $\chi \equiv (\sigma - 1)(\alpha + \kappa) - \alpha\kappa\sigma$ and $\delta_0 \equiv \alpha(\sigma - 1) + \kappa\sigma(1 - \alpha) > 0$. Note that χ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$, with $\alpha_1 \equiv \kappa/(\beta\sigma - 1) < 1$, while θ^i is given by:

$$\theta^i = \begin{cases} 1 + \phi^\beta \left(\frac{f_X}{f_D} \right)^{1-\beta} + \left(\gamma^{1-\sigma} - 1 \right)^\beta (1 + \phi)^\beta \left(\frac{f_I}{f_D} \right)^{1-\beta} & \text{if } i = A \\ 1 + \left(\phi\gamma^{1-\sigma} \right)^\beta \left(\frac{f_X}{f_D} \right)^{1-\beta} + \left(\gamma^{1-\sigma} - 1 \right)^\beta \left(\frac{f_I}{f_D} \right)^{1-\beta} & \text{if } i = B \\ 1 + \left[(1 + \phi)\gamma^{1-\sigma} - 1 \right]^\beta \left(\frac{f_X + f_I}{f_D} \right)^{1-\beta} & \text{if } i = C \end{cases} \tag{14}$$

Note that with any change in trade costs (variable or fixed), the efficiency gains from innovation, γ, and its fixed cost have an impact on the survival cost cut-off through the variable θ^i. Observing the equilibrium expression above one can conclude that θ^i can be interpreted as an indicator of trade openness and innovation opportunities generated by the levels of international integration and the conditions characterizing the innovative environment in the industry. Higher θ^i (i.e. lower variable trade costs, larger efficiency gains from innovation and/or lower fixed costs of exporting or innovating) has contrasting effects: from one side it expands the business opportunities of the most productive firms *(near the highest marginal cutoffs)*, but from the other side it endures competition for every firm toughening the conditions for surviving in the market. Eventually, this implies a more selective industry.

The analysis of (14) reveals that θ^i is not affected by the existence of vertical linkages. Vertical linkages, however, play an interesting role on the effects on the survival productivity cut-off and on welfare through the price index P_M^i.

3 The impact of an increase in economic integration on cost cutoffs.

The table below summarizes and represents with the arrows the effects of a trade liberalization policy, that consists of a reduction in variable trade costs (and thus increases ϕ), on the different productivity cutoffs.

	Equilibrium A		**Equilibrium C**		**Equilibrium B**
	a_D^A	a_I^A	a_D^C	a_I^C	a_D^B and a_I^B
$0 < \alpha < \alpha_0$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\alpha_0 < \alpha < \alpha_1$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\alpha_1 < \alpha < \alpha_2^i$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\alpha_2^i < \alpha < 1$	$\downarrow$	Case 1 $\downarrow$ Case 2 $\uparrow$	$\downarrow$	Case 1 $\downarrow$ Case 2 $\uparrow$	$\downarrow$

Table I: The effects of increases in ϕ on the cutoffs a_D^i and a_I^i.

In the table: $\alpha_0 \equiv \frac{\sigma-1}{\sigma}$, $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$, α_2^i is equal to $\alpha_2^A \equiv \frac{\kappa(\theta^A-(1+\phi))}{\phi(1-\beta\sigma)+(\phi+\beta\sigma)(\theta^A-1)}$ for Equilibrium A and it is equal to $\alpha_2^C \equiv \frac{\kappa}{\beta\sigma-\theta^C}$ for Equilibrium C. We show how the values of α_2^i can be determined in the Appendix where we derive also the signs of the derivatives $\frac{\partial a_D^i}{\partial\phi}$ and $\frac{\partial a_I^i}{\partial\phi}$ used to establish the effects of increases in ϕ on the cutoffs. Notice that for Equilibria A and C different effects are produced by trade liberalization only on a_I^A and a_I^C when vertical linkages are very large, that is when $\alpha_2^i < \alpha < 1$, and that for Equilibrium B there is no threshold α_2^B as a unique case occurs for $\alpha > \alpha_1$.

The baseline case, that is the one without vertical linkages ($\alpha = 0$), has the same qualitative effects as those described by the first row in the table above with low intensity of vertical linkages. Specifically, in the case without vertical linkages, trade liberalization increases θ^i and, in the equilibria A and C, it increases the proportion of surviving firms that undertake technology adoption through an expansion in their business opportunities due to better access to the foreign market. However, the increase in θ^i produced by trade liberalization reduces the aggregate price index P_M^i, and this has a negative impact in the global sales of the firm through a competition effect as the reduction in variable trade costs reduces the price of imported varieties and facilitates the access of foreign firms to the domestic market. As *the less productive firms* are not exporters and do not benefit from the expansion in their business opportunities abroad, these effects contribute to a reduction in the firm's market share implying a reduction in the firm's survival cutoff. This is captured by the term $\left(P_M^i\right)^{(1-\sigma)}$ (that increases as P_M^i decreases), when we substitute Δ^i into condition (9) and we evaluate it for the case $\alpha = 0$,

$$\left(\frac{E^i}{\sigma\left(P_M^i\right)^{(1-\sigma)}}\right)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(a_D^i\right)^{1-\sigma} = f_D$$

Thus, as the increase in θ^i reduces the price index P_M^i, it implies a reduction in a_D^i and a reduction in the proportion of entrants that survive in the market. Overall, an increase in the freeness of trade ϕ, increases a_I^A and a_I^C , so a trade liberalization policy increases the innovation cost cut-off and the proportion of entrants undertaking technology adoption. On the contrary, given that in equilibrium B, the firm that is indifferent between adopting or relying in the original technology does not see its business opportunities expanded with trade, trade liberalization affects this firm only through its reduction in domestic sales. Since this firm's market size is shrunk, innovation/technology adoption is less profitable and a trade liberalization policy in this case must necessarily decrease the innovation cost cut-off and the proportion of entrants adopting the most efficient technology.

Similar results hold when the intensity of vertical linkages is small ($0 < \alpha < \alpha_0$). In this case, substituting Δ^i into (9) gives

$$\frac{E^i \left(P_M^i\right)^{\alpha(1-\sigma)}}{\sigma \left(P_M^i\right)^{(1-\sigma)}} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(a_D^i\right)^{1-\sigma} = f_D \left(P_M^i\right)^{\alpha}$$

As stated above, through the existence of vertical linkages, the reduction in the aggregate price index reduces both production costs, the variable ones (captured by the term $\left(P_M^i\right)^{\alpha(1-\sigma)}$ in the left hand side of the equation above) and the fixed operational ones (captured by the element $\left(P_M^i\right)^{\alpha}$ in the right hand side of the equation). These two effects have a positive effect on the survival cost cut-off a_D^i but this effect is clearly shaped by α, the degree of vertical linkages. In addition the degree of vertical linkages also has a positive effect on each variety's demand which also increases survival. (This effect is included in the element E^i in the model). Since α is low, the negative effect on domestic sales captured by the element $\left(P_M^i\right)^{(1-\sigma)}$ in the denominator dominates and overall survival is lower.

However, these results are challenged when the intensity of vertical linkages is medium ($\alpha_0 < \alpha < \alpha_1$). When the intensity of vertical linkages is middle-low, trade liberalization still reduces the aggregate price index. Nevertheless, in this case, vertical linkages are strong enough to have a positive impact on survival: the initial reduction in firms' domestic sales is dominated by the effect that the reduction in the aggregate price index has on the firm's production costs and the demand effect. Because production becomes cheaper, the marginal firm finds it easier to survive. This clearly increases the survival cost cut-off and it has a double impact on innovation: on the one hand a larger proportion of surviving firms become innovators (only for equilibria A and C) and, on the other hand, trade liberalization increases the cutoff of firms surviving in the market (common to all the three type of equilibria). Overall the proportion of entrants adopting the most efficient technology increases after trade liberalization in all equilibria. This result was not present in a model without vertical linkages.

When the degree of vertical linkages is relatively high $\alpha_1 < \alpha < \alpha_2$, the reverse happens. Not only does an increase in θ^i have a negative impact in the cost cut-off (which makes firms' survival more difficult), but also the effect is strong enough to reduce the cutoff of firms that adopt the most efficient technology. In this case, trade liberalization is increasing the aggregate price index. Trade liberalization has an initial negative impact on the price index. Expression (11) reveals that, holding constant a_D^i and N_D^i, an increase in θ^i reduces the level of the aggregate price index. This reduces production costs increasing the demand for intermediate inputs. The degree of vertical linkages is so strong in this case, that this rise in demand increases the equilibrium price index and the less productive firms can no longer compensate the increase in the production costs with an increase in sales since they only serve the domestic market and consequently they must leave the market. The industry and the technology adoption cost cut-offs decrease, decreasing both the proportion of entrants that survive and the proportion of entrants that innovate in all equilibria. This result is not present in a model without vertical linkages.

In the extreme cases in which α is very high $\alpha_2 < \alpha < 1$, the impact of trade liberalization on survival and technology adoption depends clearly on the parameters of the model. When the economy is in equilibrium A or in equilibrium C we can distinguish two main cases: one in which trade liberalization deters technology adoption (Case 1) and one in which trade liberalization promotes technology adoption (Case 2). Case 1 arises when the initial level of international integration is already relatively high (either because the fixed cost of exporting is relatively small or because trade costs are relatively low, or both of them are small) or when the initial level of international integration is small but associated with a small fixed cost of adoption of a new technology and/or associated with large gains in productivity levels associated with the new technology adopted. In both situations the pressures on demand for goods and intermediates are already *large* and a reduction in trade costs puts a lot of pressure on demand that increases the price index of goods and deters technology adoption. Case 2, on the contrary, arises when the initial level of international integration is relatively low (either because the fixed cost of exporting is relatively large or because trade costs are relatively high, or both of them are large) and this is associated with a high fixed cost of adoption of a new technology and/or with small gains in productivity levels associated with new technology. In this case the pressures on the demand for goods and intermediates are smaller as *initially* fewer firms innovate due to the large cost of technology adoption and/or the small potential productivity gain. Thus, a larger proportion of very productive firms would profit from trade liberalization, adopting the new technology even though the price index of intermediates is rising.

Moreover, the next proposition states interesting parameter configurations in which the economy is in either case 1 or case 2 independently of the degree of trade integration:

Proposition 1 *In Equilibrium A, if* $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{\beta+1}{\beta-1}$ *and* $\left(\gamma^{1-\sigma}-1\right) < \beta - 1$

or $\frac{f_I}{f_D} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta} 2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$ *the economy is in case 2 independently of the value of* ϕ.

Proof. See Appendix. ■

Moreover, we can state what follows.

Proposition 2 *Consider that the economy is in equilibrium C, then:*[14]

$$\text{if} \begin{cases} \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the economy is in case 2} \\ \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} < \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the equilibrium of the economy depends on the value } \phi \\ \left(\frac{f_I+f_X}{f_D}\right)^{\beta-1} < \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1} & \text{the economy is in case 1} \end{cases}$$

Proof. See Appendix. ■

The previous propositions conclude that for cases in which the fixed costs of exporting or innovation are relatively high, trade liberalization will increase the proportion of firms undertaking technology upgrading when the degree of vertical linkages are very strong in equilibria A and C. Consequently, the results suggest that while in case 1, trade liberalization induces tougher selection, and reduces the proportion of entrants undertaking technology adoption, in case 2 the latter is not so strong and the overall positive effect dominates. This is consistent with the fact that in case 2 the requirements for innovating and exporting are so large that very few firms decide to undertake these activities. This moderates the rise in the demand of intermediate inputs and consequently the rise in the relative cost of intermediate inputs. The latter has a moderate impact on the survival productivity threshold and the proportion of technology upgrading firms.

4 Effects on Welfare

The previous section suggests that vertical linkages shape the effect that trade liberalization has on innovation. In this section we analyze the main implications of this for welfare. To do so, we first compare the changes in welfare produced by trade liberalization in a model in which there are vertical linkages with and without technology upgrading, and then those in a model in which technology upgrading is possible with and without vertical linkages. This strategy allows us to better capture the specific contribution of each channel to the effect of trade liberalization on welfare. Our main conclusion is that both channels exacerbate the impact of trade liberalization on welfare and for the case of vertical linkages

[14] All of the conditions considered are consistent with the economy being in equilibrium C.

the ultimate effect of trade liberalization on welfare depends crucially on the intensity of vertical linkages (α).

Substituting the optimal values for C_T and C_M, the indirect utility function can be expressed as

$$U = \frac{L}{\left(P_M^i\right)^{\mu}}$$

Thus, as it is standard in this literature, to evaluate the impact of trade liberalization on welfare it is useful to see how the aggregate price index changes with respect to changes in trade policy.

The following proposition states that trade liberalization has a positive effect on welfare if and only if the degree of vertical linkages are not too strong.

Proposition 3 *Trade liberalization has a positive impact on welfare when* $0 \leq \alpha < \alpha_1$.
Proof. *See Appendix.* ■

Nocco (2012) establishes that trade liberalization has a negative impact on welfare when the degree of vertical linkages is relatively strong as trade liberalization increases the price index of the composite good. This is due to the positive effect that trade liberalization has on the demand for the composite good, because firms' willingness to become exporters increases. This increases the costs of surviving and entry, reducing the mass of varieties produced in equilibrium. In the particular context of the present model, the effect could be stronger for some cases since, apart from the effect on the mass of varieties, the increase in the cost of intermediates can have, in certain cases, a negative impact on technology upgrading.

While the effects of trade liberalization on welfare are qualitatively the same as in Nocco (2012), this cannot be said from a quantitative point of view. As a matter of fact, the welfare effects of trade liberalization are exacerbated in this framework due to the possibility of firms to technology upgrade. Notice that in Nocco (2012), the equivalent to θ^i is $\tilde{\theta} = 1 + \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Since the effects of trade liberalization are determined by the elasticity of θ^i to τ the following can be concluded:

Proposition 4 *When* $0 < \alpha < \alpha_1$*, the increasing welfare effects of trade liberalization are larger when technology upgrading is possible.*
Proof. *See Appendix* ■

This can be easily seen by noticing that the effect of trade liberalization on welfare clearly depends on θ^i.Note that θ^i is larger in the technology adoption case. The main reason behind this result lies in the fact that trade liberalization promotes technology upgrading in equilibria A and C and this subsequently reduces the price index, having a larger impact on welfare. In equilibrium

B, trade liberalization, while deterring technology upgrading in certain cases (i.e. $0 < \alpha < \alpha_0$), facilitates the replacement of domestic varieties by more productive foreign ones. This increases welfare.

Technology upgrading, therefore, exacerbates the impact of trade liberalization on welfare in the presence of vertical linkages. The next proposition suggests that vertical linkages not only does shape the effects of trade liberalization on welfare, as seen above, but in the case in which they are positive, they also enlarge the impact of trade liberalization on welfare.

Proposition 5 *When technology upgrading is possible, the presence of moderate vertical linkages* $(0 < \alpha < \alpha_1)$ *strenghtens the impact that trade liberalization has on welfare.*
Proof. *See Appendix.* ■

The existence of vertical linkages generates a multiplier effect on welfare. This multiplier effect comes clearly through the general effect of trade liberalization on the price index, which declines with *moderate vertical linkages* $(0 < \alpha < \alpha_1)$, and the specific effects on selection and technology upgrading that vary according to the strength of vertical linkages and type of equilibria. Trade liberalization gives access to more productive foreign varieties. This reduces the cost of the composite good M having a positive impact on welfare and a reduction in the costs of production (through the vertical linkages) having a further reduction in the aggregate price index. In addition, trade liberalization intensifies competition expelling the less efficient varieties out of the market when vertical linkages are weak $(0 < \alpha < \alpha_0)$ increasing also the average productivity of the economy $(\tilde{a})^{1-\sigma}$, even though the effect of technology upgrading is different across equilibria.[15] The higher level of productivity required to survive in the economy reduces the price of the composite good through the concentration of production across the most efficient units and, through vertical linkages, the costs of production. This will produce a further reduction in the aggregate price index. When vertical linkages are intermediate-low $(\alpha_0 < \alpha < \alpha_1)$, the larger general effect of the reduction in the price index is sustained by technology adoption even though contrasted by a softening in the survival conditions of firms. In this case, the price index decreases by more either because average productivity increases by more (when $\alpha_0 < \alpha < \alpha^* = \frac{(\sigma-1)^2+(\sigma-1)\kappa}{(\sigma-1)^2+\sigma\kappa}$), or because the number of varieties increases by more when the softer conditions required to survive reduce the average productivity of firms (that is when $\alpha^* < \alpha < \alpha_1$).

Finally, let us conclude pointing out that when vertical linkages are sufficiently strong to increase the price index, even if the welfare level decreases due to a drastic reduction in the mass of varieties, the average productivity of the economy increases which is mainly driven by tougher selection.

[15] Computations regarding the weighted average productivity $(\tilde{a})^{1-\sigma}$ can be found in the appendix.

5 Conclusion

In this paper we have analyzed the impact of trade liberalization on technology adoption, average productivity and welfare in a framework in which heterogeneous firms that can decide to upgrade the technology they use are interconnected by vertical linkages, a relatively unexplored dimension in the literature. Our analysis is motivated by the prominent role of intermediate inputs in world trade and GDP and the importance of industry-specific vertical linkages as documented in Di Giovanni and Levchenko (2009).

We have shown that the inclusion of vertical linkages changes the effects of trade liberalization on innovation, average productivity and welfare although its final impact depends on the strength of vertical linkages, generating a very rich set of results. When the strength of vertical linkages is low, vertical linkages augment the effects of trade liberalization on technology upgrading. When the strength of vertical linkages are middle-low, instead, trade liberalization increases technology upgrading independently of the parameter configuration. In this case trade liberalization reduces the survival and innovation costs allowing more firms to stay and to adopt the new technology, which is not present in a model without vertical linkages. However, when the strength of vertical linkages is middle-high trade liberalization reduces both technology upgrading and the industry cutoff independently of the parameter configuration. For very high levels of vertical linkages the effect on innovation depends on the level of economic integration and other parameters.

These results have a clear impact on average productivity and welfare. Specifically, trade liberalization increases average productivity, except when $\alpha^* < \alpha < \alpha_1$. In this case, trade liberalisation promotes technology adoption but it softens selection. The latter effect dominates contributing to a decrease in average productivity. We also show that trade liberalisation has a positive impact on welfare only when vertical linkages are moderate (i.e. $0 < \alpha < \alpha_0$). When vertical linkages are strong the reduction in the number of varieties decreases utility even if there is an increase in average productivity due to a stronger selection effect.

Finally, our framework also has implications for the *magnitude* of the effects of trade on both welfare and average productivity. More precisely, we have shown that in the scenarios in which trade liberalization promotes technology upgrading, the effects on welfare are larger with vertical linkages than in a case without vertical linkages. We have also shown that in the case in which vertical linkages are moderate the impact of trade liberalization on average productivity and welfare is larger when firms are allowed to technology upgrade than in a case without the opportunity to adopt more productive technologies.

Thus, we conclude that our paper helps to shed some light on the analysis of the relationships that exist among trade liberalization, technology adoption, productivity and welfare changes, showing how its multifaceted nature can become even more complex when firms producing in open economies are interconnected by vertical linkages.

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6 Appendix

6.1 Results in Table I.

This part of the Appendix contains the derivation of the signs of the derivatives of the cutoffs with respect to ϕ reported in Table I.

In general, from (13) and (14) it can be derived that

$$\frac{\partial a_D^i}{\partial \phi} = -\frac{\sigma - 1 - \alpha\sigma}{\gamma}\left[\frac{\kappa\mu L}{\delta_0 f_D}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left[\frac{a_M^\kappa f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}}\left(\theta^i\right)^{-\frac{\sigma-1-\alpha\sigma}{\chi}-1}\frac{\partial\theta^i}{\partial\phi}$$

Given that $\frac{\partial\theta^i}{\partial\phi} > 0$, the sign of $\frac{\partial a_D^i}{\partial\phi}$ depends on the sign of $-\frac{\sigma-1-\alpha\sigma}{\chi} = \frac{\alpha\sigma-(\sigma-1)}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ as in Nocco (2012). Hence, $\frac{\partial a_D^i}{\partial\phi} < 0$ for $\alpha \in (0, \alpha_0)$ and $\alpha \in (\alpha_1, 1)$, and viceversa $\frac{\partial a_D^i}{\partial\phi} > 0$ for $\alpha \in (\alpha_0, \alpha_1)$ with $\alpha_0 \equiv \frac{\sigma-1}{\sigma}$, $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$ and $0 < \alpha_0 < \alpha_1 < 1$.

Turning to the sign of $\frac{\partial a_I^i}{\partial\phi}$, it has to be analyzed in detail for each case.

Equilibrium A In equilibrium A, substituting a_D^A into (4) yields

$$a_I^A = z\left[\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\right]$$

where $z \equiv \left[\frac{\mu\kappa}{\delta_0 f_D}L\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left\{\left[\frac{f_I}{f_D}\frac{1}{(\gamma^{1-\sigma}-1)}\right]^{\frac{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}{(1-\sigma)(\sigma-1-\alpha\sigma)}}\frac{f_E}{f_D}(\beta-1)\,a_M^\kappa\right\}^{\frac{\sigma-1-\alpha\sigma}{\chi}} > 0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^A}{\partial\phi} = -z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}(h-l)$$

and given that $z\left(\theta^A\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left(\frac{1}{1+\phi}\right)^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A} > 0$, the sign of $\frac{\partial a_I^A}{\partial\phi}$ depends on the sign of $h-l$, where $h \equiv \frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}$ and $l \equiv \frac{1}{(\sigma-1)}\frac{1}{1+\phi}/(\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A})$. It can be readily verified that l does not depend on α and can be represented by a horizontal line in the cartesian plane where α is represented on the axis of abscissas (See Figures 1-2 that follow). Moreover, when the independent variable is α, h is a hyperbola, whose expression can be rewritten as follows $h = \frac{\sigma\alpha-(\sigma-1)}{[\kappa\sigma-(\sigma-1)]\alpha-(\sigma-1)\kappa}$ with center in $O\left(\frac{\kappa}{\beta\sigma-1}, \frac{1}{\kappa-\frac{(\sigma-1)}{\sigma}}\right)$, and its two asymptotes having, respectively, expressions $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1}$ in the case of the vertical asymptote and $h = \frac{1}{\kappa-(\sigma-1)/\sigma}$ in that of the horizontal asymptote. Moreover, it can be verified that: 1) $h = \frac{1}{\kappa}$ when $\alpha = 0$; 2) $h = \frac{1}{\kappa-(\sigma-1)} > 0$ (because $\kappa > \sigma-1$) when $\alpha = 1$; 3) and, finally, $h = 0$ when $\alpha = \frac{\sigma-1}{\sigma} \equiv \alpha_0$.

Then, substituting $\frac{\frac{\partial\theta^A}{\partial\phi}}{\theta^A}$ into l yields

$$l = \frac{1}{\kappa}\frac{\phi}{1+\phi}\frac{1+\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\left(\frac{f_I}{f_D}\right)^{1-\beta}}{\phi^\beta\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^\beta(1+\phi)^\beta\frac{\phi}{1+\phi}\left(\frac{f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l < \frac{1}{\kappa-(\sigma-1)}$ for sufficiently high levels of international integration, that is with $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$. Otherwise, for low levels of international integration, that is with $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}f_D^{\beta-1}$, we find that: $l < \frac{1}{\kappa-(\sigma-1)}$

if $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$ with $\Omega \equiv \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$; and $l > \frac{1}{\kappa-(\sigma-1)}$ if $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$.

Therefore, there are two potential cases:

Case 6 (1) *$l < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} < \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ or when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} > \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$. In this case represented in Figure 1, $\frac{\partial a_I^A}{\partial\phi} > 0$ when $\alpha \leq \alpha < \alpha_1$ as $h < l$; and $\frac{\partial a_I^A}{\partial\phi} < 0$ when $\alpha_1 < \alpha < 1$ as $h > l$.*

Insert Figure 1 about here

Case 7 (2) *$l > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X/\phi)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)} f_D^{\beta-1}$ and $\left(\frac{1}{f_I}\right)^{\beta-1} < \frac{\phi(\beta-1)-\Omega(\beta+\phi)}{(\gamma^{1-\sigma}-1)^{\beta}(1+\phi)^{\beta}\phi f_D^{\beta-1}}$. In this case, $\frac{\partial a_I^A}{\partial\phi} > 0$ when $\alpha \leq \alpha < \alpha_1$ and when $\alpha_2^A < \alpha < 1$ as $h < l$; and $\frac{\partial a_I^A}{\partial\phi} < 0$ when $\alpha_1 < \alpha < \alpha_2^A$ as $h > l$.*

Insert Figure 2 about here

Equilibrium C In equilibrium C, substituting a_D^C into (8) yields

$$a_I^C = y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}$$

where $y \equiv \left[\frac{\kappa\mu L}{f_D\delta_0}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\right]^{\frac{\alpha}{\chi}}\left(\frac{f_X+f_I}{f_D}\right)^{\frac{1}{1-\sigma}}\left[\frac{a_M^{\kappa} f_E}{f_D}(\beta-1)\right]^{\frac{\sigma-1-\alpha\sigma}{\chi}} > 0$ does not depend on ϕ.

Then,

$$\frac{\partial a_I^C}{\partial\phi} = -y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C}(h-l_C)$$

and given that $y\left(\theta^C\right)^{-\frac{\sigma-1-\alpha\sigma}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma}}\left[\frac{1}{(1+\phi)\gamma^{1-\sigma}-1}\right]^{\frac{1}{(1-\sigma)}}\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C} > 0$, the sign of $\frac{\partial a_I^C}{\partial\phi}$ depends on the sign of $h-l_C$, where $l_C \equiv \frac{1}{(\sigma-1)}\frac{\gamma^{1-\sigma}}{(1+\phi)\gamma^{1-\sigma}-1}/(\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C})$ does not depend on α and can be represented by an horizontal line in the cartesian plane where α is represented on the axis of abscissas.

Then, substituting $\frac{\frac{\partial\theta^C}{\partial\phi}}{\theta^C}$ into l_C yields

$$l_C = \frac{1}{\kappa}\frac{1+\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}}{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}} > \frac{1}{\kappa}$$

with $l_C < \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)} f_D^{\beta-1}$ (Case 1), and $l_C > \frac{1}{\kappa-(\sigma-1)}$ when $(f_X+f_I)^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{(\beta-1)} f_D^{\beta-1}$ (Case 2). In Case 1, that is for relatively low (f_X+f_I) and relatively high levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial \phi} > 0$ when $0 < \alpha < \alpha_1$ as $h < l_C$, while $\frac{\partial a_I^C}{\partial \phi} < 0$ when $\alpha_1 < \alpha < 1$ because $h > l_C$. In Case 2, that is for relatively high (f_X+f_I) and relatively low levels of economic integration ϕ, $\frac{\partial a_I^C}{\partial \phi} > 0$ when $0 < \alpha < \alpha_1$ and $\alpha_2^C < \alpha < 1$ as $h < l_C$, while $\frac{\partial a_I^C}{\partial \phi} < 0$ when $\alpha_1 < 1 < \alpha_2^C$, because $h > l_C$. Notice that the two cases can be represented by similar figures as those respectively used in Figure 1 and Figure 2, even if the expression of l_C should be used instead of that of l and the value of α_2^C should be used instead of that of α_2^A.

Equilibrium B For the case of equilibrium B, it is clear from (7) that $sign(\frac{\partial a_I^B}{\partial \phi}) = sign(\frac{\partial a_D^B}{\partial \phi})$.

6.2 Proof of propositions.

Proof of Proposition 1 In equilibrium A the economy is in case number 2 if $\left(\frac{f_X}{\phi f_D}\right)^{\beta-1} > \frac{(\beta+\phi)}{(\beta-1)}$ and $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > \frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))}$ with $\Omega = \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Otherwise the economy is in case number 1 (which represents the continuity from the previous equilibria).

Rearranging terms in the first of the previous conditions we have that $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \phi^{\beta-1}\frac{(\beta+\phi)}{(\beta-1)}$. The right hand side is increasing in ϕ. Since ϕ is upper bounded by 1 we have that the first condition is always satisfied when $\left(\frac{f_X}{f_D}\right)^{\beta-1} > \frac{(\beta+1)}{(\beta-1)}$. Rearranging the second equation and replacing the value of Ω, we have that $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$. This expression is also increasing in ϕ. Since ϕ is upper bounded by 1 we have that the second condition is always satisfied when $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}2^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+1)}$. Notice that from (4) $\left(\frac{f_I}{f_D(\gamma^{1-\sigma}-1)(1+\phi)}\right)^{\beta-1} > 1$ *must be satisfied* for equilibrium A to hold. This implies that if $\frac{\phi\left(\gamma^{1-\sigma}-1\right)(1+\phi)}{(\phi(\beta-1)-\Omega(\beta+\phi))} < 1$ for any value of ϕ this condition will be always sustained independently of the value of ϕ. Making use of the expression for Ω and rearranging terms we have that for the previous condition to hold $\left(\gamma^{1-\sigma}-1\right) < \frac{\left((\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}\phi^{\beta-1}(\beta+\phi)\right)}{(1+\phi)}$. Notice that this expression is decreasing in ϕ. Evaluating the rhs under $\phi=0$ we have that: $\left(\gamma^{1-\sigma}-1\right) < (\beta-1)$.

If this condition holds, then $\left(\frac{f_I}{f_D}\right)^{\beta-1} > \frac{\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}}{(\beta-1)-\left(\frac{f_X}{f_D}\right)^{1-\beta}(\beta+\phi)\phi^{\beta-1}}$ for any value of ϕ.

Proof of Proposition 2 The equation governing the conditions under which the economy will be in either case 1 or 2 is the following:

If $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > \frac{(1+\phi)\gamma^{1-\sigma}-1}{\beta-1}$, the economy is in case 2, otherwise the economy is in case 1 (provided that $\left\{\frac{(f_I+f_X)}{[(1+\phi)\gamma^{1-\sigma}-1]f_D}\right\}^{\beta-1} > 1$ which is required from (8) for being in equilibrium C).

Rearranging terms we find that for the former condition to hold the following must be satisfied: $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$.Notice that the right hand side of this condition is increasing in ϕ. Because ϕ is bounded above and below, evaluating the rhs of that condition when $\phi = 1$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left(2\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$, then $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} > \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ for any ϕ (we are in case 2 independently of ϕ). Consider the reverse constraint $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta}}{\beta-1}$ (this is the condition for being in case 1). Evaluating the right hand side when $\phi = 0$ we have that if $\left[\frac{(f_I+f_X)}{f_D}\right]^{\beta-1} < \frac{(\gamma^{1-\sigma}-1)^{\beta}}{\beta-1}$ then $\left(\frac{(f_I+f_X)}{f_D}\right)^{\beta-1} < \frac{\left((1+\phi)\gamma^{1-\sigma}-1\right)^{\beta}}{\beta-1}$ for any ϕ. The economy is in case 1 independently of the value of ϕ.

Proof of Proposition 3 Deriving expression (11) with respect to τ we notice that:

$$sign\frac{\partial P_M^i}{\partial \tau} = sign\left(\left(\frac{1-\sigma}{\chi}\right)\left(\frac{\partial \theta^i}{\partial \tau}\right)\right)$$

where we know that $\frac{\partial \theta^i}{\partial \tau} < 0$ since $\frac{\partial \theta^i}{\partial \tau} = \underbrace{\frac{\partial \theta^i}{\partial \phi}}_{+}\underbrace{\frac{\partial \phi}{\partial \tau}}_{-} < 0$. The sign of the derivative of the price index is clearly depending on the sign of $\frac{1-\sigma}{\chi}$, and given that expression $(1-\sigma)$ is always negative, the sign of $\frac{\partial P_M^i}{\partial \tau}$ ultimately depends on the sign of χ that depends on the value of α. More precisely, if $\alpha < \alpha_1$ then χ is positive, while χ is negative if $\alpha > \alpha_1$. Consequently, we obtain that, no matter the equilibrium in which we are, $\frac{\partial P_M^i}{\partial \tau} < 0$ if $\alpha < \alpha_1$ and $\frac{\partial P_M^i}{\partial \tau} > 0$ if $\alpha > \alpha_1$.

Proof of Proposition 4 To show this, first notice that in a model with technology adoption the welfare effect of trade liberalization is given by: $\frac{\partial P_M^i}{\partial \tau} = \frac{1-\sigma}{\chi}\frac{\partial \theta^i}{\partial \phi}\frac{\partial \phi}{\partial \tau}$. In a model without technology upgrading we have that the effect of technology adoption on welfare is given by: $\frac{\partial P_M^i}{\partial \tau} = \frac{1-\sigma}{\chi}\frac{\partial \tilde{\theta}}{\partial \phi}\frac{\partial \phi}{\partial \tau}$ with $\tilde{\theta} =$

$1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$. Since we have that $\frac{\partial\theta^i}{\partial\phi}>0$, $\frac{\partial\tilde{\theta}}{\partial\phi}>0$, and the other two factors $\frac{1-\sigma}{\chi}$ and $\frac{\partial\phi}{\partial\tau}$ are identical in both models, the effect will be larger in a model with technology adoption when $\frac{\partial(\theta^i-\tilde{\theta})}{\partial\phi}>0$. Then, let us analyze the three specific equilibria.

Equilibrium A. In equilibrium A, $\theta^i=1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}\left(\frac{f_I}{f_D}\right)^{1-\beta}$. Since $\tilde{\theta}=1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$, then we have that $\theta^i-\tilde{\theta}=\left(\gamma^{1-\sigma}-1\right)^{\beta}(1+\phi)^{\beta}\left(\frac{f_I}{f_D}\right)^{1-\beta}>0$ increases when ϕ increases. Consequently the impact on welfare is larger in the model with technology upgrading.

Equilibrium B. In equilibrium B, $\theta^i=1+\left[\phi\gamma^{1-\sigma}\right]^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left[\gamma^{1-\sigma}-1\right]^{\beta}\left(\frac{f_I}{f_D}\right)^{1-\beta}$. In this case, we have that: $\theta^i-\tilde{\theta}=\left[\gamma^{(1-\sigma)\beta}-1\right]\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}+\left(\gamma^{1-\sigma}-1\right)^{\beta}\left(\frac{f_I}{f_D}\right)^{1-\beta}>0$ increases when ϕ increases since $(1-\sigma)\beta=-\kappa$ is negative and $\gamma\in[0,1]$.

Equilibrium C. In equilibrium C, $\theta^i=1+\left((1+\phi)\gamma^{1-\sigma}-1\right)^{\beta}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}$. Differentiating with respect to ϕ we have that $\frac{\partial\theta^i}{\partial\phi}=\beta\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta-1}\gamma^{1-\sigma}\left(\frac{f_X+f_I}{f_D}\right)^{1-\beta}>0$, and $\frac{\partial\tilde{\theta}}{\partial\phi}=\beta\phi^{\beta-1}\left(\frac{f_X}{f_D}\right)^{1-\beta}>0$. The third term of the first derivative is larger than one and the fourth term is larger than the third term of the second derivative. To ensure that $\frac{\partial\theta^i}{\partial\phi}$ is larger than $\frac{\partial\tilde{\theta}}{\partial\phi}$, so that $\theta^i-\tilde{\theta}$ increases when ϕ increases, we need to compare $\left[(1+\phi)\gamma^{1-\sigma}-1\right]^{\beta-1}$ and $\phi^{\beta-1}$. Given that $\beta>1$, both these two elements have a positive exponent. Then we have that: $(1+\phi)\gamma^{1-\sigma}-1>\phi$ if $(1+\phi)\left(\gamma^{1-\sigma}-1\right)>0$ which is the case, also ensuring that $\theta^i-\tilde{\theta}>0$.

Proof of Proposition 5 Consider first the case with no technology adoption and compare the two models with and without vertical linkages. The effect of trade on welfare in the model without vertical linkages is determined by the effect on the aggregate price index which is given by: $\frac{\partial P_M^*}{\partial\tau}=-\frac{1}{\kappa}\left(\frac{\partial\tilde{\theta}}{\partial\tau}\right)$. $\tilde{\theta}$ is not affected by vertical linkages and, consequently, it will take the same value in the model with vertical linkages. The effect of trade liberalization on welfare in this case is larger in the model with vertical linkages if the following inequality holds $\frac{\sigma-1}{\chi}>\frac{1}{\kappa}\Rightarrow\kappa(\sigma-1)>\chi\Rightarrow 0>(\sigma-1)\alpha-\alpha\kappa\sigma\Rightarrow\kappa\sigma>\sigma-1\Rightarrow\beta>\frac{1}{\sigma}$, which is the case since $\beta>1$ and $\sigma>1$.

The same line of reasoning applies in a case in which we consider that firms have the possibility to technology upgrade in both models but vertical linkages are only present in one of them. In that case θ^i instead of $\tilde{\theta}$ will be common across both models. But the difference in the effect of trade liberalization on welfare clearly depends on the same inequality as in the previous case which clearly holds.

6.3 Average productivity

The aggregate price index can be expressed as

$$P_M^i = \left(N_D^i\right)^{\frac{1}{1-\sigma}} p(\tilde{a}^i) = \left(N_D^i\right)^{\frac{1}{1-\sigma}} \left(\frac{\sigma}{\sigma-1}\right) \left(P_M^i\right)^{\alpha} \tilde{a}^i$$

that can be rewritten as

$$P_M^i = \left(N_D^i\right)^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{1-\alpha}} \left(\tilde{a}^i\right)^{\frac{1}{1-\alpha}}$$

The previous expression together with equation (13) yields average productivity as follows

$$\left(\tilde{a}^i\right)^{1-\sigma} = \left(\frac{\beta}{\beta-1}\right) \theta^i \left(a_D^i\right)^{1-\sigma}$$

The impact of trade liberalization can be assessed analyzing how a change in variable trade barriers affect average productivity. Notice that a reduction in trade barriers will have a positive effect on θ^i . However it has an ambiguous effect on a_D^i depending on the value of α. More precisely we have that

$$\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \tau} = \frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} \frac{\partial \theta^i}{\partial \tau}$$

where $\frac{\partial \theta^i}{\partial \tau} < 0$. The first term is given by:

$$\begin{aligned} \frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} &= \left(\frac{\beta}{\beta-1}\right) \left(a_D^i\right)^{1-\sigma} \left[1 + \frac{\partial \left(a_D^i\right)^{1-\sigma}}{\partial \theta^i} \frac{\theta^i}{\left(a_D^i\right)^{1-\sigma}}\right] = \\ &= \left(\frac{\beta}{\beta-1}\right) \left(a_D^i\right)^{1-\sigma} \left[1 - (1-\sigma) \frac{(\sigma - 1 - \alpha\sigma)}{\chi}\right] \end{aligned}$$

Notice that the sign of this derivative depends on the last factor in the squared brackets. More precisely the sign of $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i}$ is positive when

$$\frac{(\sigma-1)(\sigma-1-\alpha\sigma)}{(\sigma-1)(\alpha+\kappa)-\alpha\kappa\sigma} > -1 \tag{15}$$

and negative otherwise. Notice that the denominator $\chi \equiv (\sigma-1)(\alpha+\kappa) - \alpha\kappa\sigma$ is positive when $\alpha \in [0, \alpha_1)$ and negative when $\alpha \in (\alpha_1, 1)$. Hence, it can be shown that if $\alpha \in [0, \alpha_1)$, $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} > 0$ if $\alpha \in [0, \alpha^*)$ with $\alpha^* \equiv \frac{(\sigma-1)^2 + (\sigma-1)\kappa}{(\sigma-1)^2 + \sigma\kappa}$, and $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i} < 0$ if $\alpha \in (\alpha^*, \alpha_1)$. If, instead, $\alpha \in (\alpha_1, 1)$, χ is negative and given that both the numerator and the denominator of (15) are negative, the inequality in (15) is always true and hence the sign of $\frac{\partial \left(\tilde{a}^i\right)^{1-\sigma}}{\partial \theta^i}$ is always positive.