The University Of Sheffield. | Department Of Economics

Welfare-enhancing taxation and price discrimination

Anna D'Annunzio, Antonio Russo

Sheffield Economic Research Paper Series

SERPS no. 2023001

ISSN 1749-8368

05 Jan 2023

Welfare-enhancing taxation and price discrimination*

Anna D'Annunzio

TBS Business School (dannunzio.anna@gmail.com)

Antonio Russo

University of Sheffield and CESifo (a.russo@sheffield.ac.uk)

January 2023

Abstract

We study commodity taxation in markets where suppliers implement second-degree price discrimination, by offering different package sizes or quality-differentiated versions of a product. In these markets, suppliers distort the quantity (or quality) intended for all types of consumers, except for those with the highest marginal willingness to pay. We show that differentiated ad valorem taxes can alleviate this distortion, and thus increase government revenue as well as welfare, provided the tax rate increases with the size of the package (or quality of the version) of the good supplied.

JEL Classification: D4, H21, H22, L1

Keywords: Commodity taxation, tax incidence, price discrimination

*We thank Anna Rita Bennato, Subho Chowdhury, Paul Dobson, Luke Garrod, Matthew Rablen and Christopher Wilson for useful comments and suggestions. All errors are ours.

1 Introduction

Second-degree price discrimination is common in markets where firms have market power and serve heterogeneous consumers. This practice can take various forms, such as offering a product in different sizes (e.g., soft drinks in 0.5 and 2 litre bottles), or in versions with different quality (e.g., phones with different amounts of memory or first- and second-class tickets) and/or functionalities (e.g., a car with different engine types and features). Although these goods are typically subject to standard indirect taxation regimes (e.g., VAT, sales taxes, etc.), whereby the same tax rate applies to *all* versions and sizes of the same product, there are several examples of tax rates that depend on the characteristics of the (version of the) product sold. For instance, some countries apply tax rates on flight tickets that depend on the class of travel.[1] As another example, road taxes often depend on factors including the weight, size, engine displacement and power of the vehicle.[2] Finally, some countries impose tax rates on certain goods (such as food items) that depend on the type of packaging the goods are sold in (e.g., according to the amount of plastic in the packaging).[3] This observation raises the following questions: how do tax rates that depend on the size or version of a product affect prices, output, consumer surplus and welfare when firms apply price discrimination? Do differentiated taxes have similar effects to standard, undifferentiated tax rates? The objective of this work is to shed some light on these questions, which, quite surprisingly, have not been explored so far.

Taxes on consumer goods and services may serve multiple purposes, the most common ones being to raise revenue for the government, either at the national or local level, and to discourage the excessive consumption of some goods.[4] Conventional wisdom suggests (in line with previous literature that we review below) that taxes result in higher equilibrium prices and reduced supply, imposing a burden on consumers as well as firms. This implies that taxes aggravate the classic distortions arising with supplier market power. We show that this presumption holds for standard tax instruments, such as unit and ad valorem taxes applied uniformly to all versions of a given good. However, we reach a different conclusion if we allow the government to apply different tax rates on different package sizes and versions of the same good. More specifically, we characterize conditions

[1] See, e.g., the UK Air Passenger Duty: https://www.gov.uk/guidance/rates-and-allowances-for-air-passenger-duty.

[2] See https://en.wikipedia.org/wiki/Road_tax for several examples of road taxes throughout the world.

[3] See, e.g., the plastic packaging tax applied in the UK since April 2022: https://www.gov.uk/government/publications/introduction-of-plastic-packaging-tax-from-april-2022/introduction-of-plastic-packaging-tax-2021.

[4] Taxes may be used to discourage excessive consumption of goods that generate negative externalities. Examples include environmental taxes (e.g., on polluting fuels) or sin taxes (e.g., on tobacco and sugar). We ignore these externalities in this paper, but we briefly discuss how our results could be generalized to a setting with externalities in Section 7.

such that imposing *differentiated* ad valorem taxes can generate revenue to the government and alleviate the typical distortions imposed by a price-discriminating supplier, thereby increasing welfare.

Suppliers implementing second-degree price discrimination must ensure the appropriate self-selection by consumers. Given they cannot observe consumers' type, firms achieve this objective by appropriately designing the size/quality of different packages/versions, and pricing them accordingly. Generally, a supplier must ensure that consumers with high willingness to pay for its product do not self-select on versions or package formats intended for consumers with lower willingness to pay. These "incentive compatibility" constraints limit the amount of revenue that can be extracted from the consumers with higher willingness to pay, who receive an information rent in equilibrium (i.e., they pay less than what they would be willing to). Economic analysis (Maskin and Riley, 1984; Laffont and Martimort, 2002; Stole, 2007) has shown that the most profitable way of dealing with this problem is to (i) set the quality/quantity intended for the "high" type at the efficient level (i.e., such that marginal utility equals marginal cost), and (ii) distort the quality/quantity for all other types (so that marginal utility exceeds marginal cost). By so doing, the supplier can reduce the information rent of the high types.

For simplicity, in the baseline model we focus on a setting with a monopolist supplier and two consumer types. Our main result is that, by applying an ad valorem tax on the package/version of the good intended for the "high" type, the government can alleviate the distortion imposed on the "low" type. Therefore, this tax produces a "double dividend": it raises welfare and generates revenue for the government at the same time. By taking away a percentage of the revenue extracted from the high type, the government makes collecting revenue from the low types relatively more attractive to the supplier. As a result, the supplier's incentive to distort the allocation intended for such types diminishes. The drawback is that, by the same token, the tax introduces a distortion in the allocation intended for the high type. However, starting from the laissez-faire equilibrium, this second distortion is small in magnitude and total surplus increases. Interestingly, the tax also reduces the price of the (high-type) version of the good it applies to, and so increases the surplus of the high consumer type, while surplus for the low type remains unchanged.

The policy maximizing welfare is such that differentiated tax rates are applied to the two bundles. In our simple setting, the welfare-maximizing policy is such that the government subsidizes the package intended for the low type. Compared to the "high type" tax, the subsidy to the low type has the advantage of not distorting the allocation intended for the high type. However, in practice subsidies have the obvious downside of costing money, which the government must collect via (potentially distortionary) taxation in other markets. Therefore, an ad-valorem tax on the high bundle is preferable when the government faces a (strict) budgetary constraint.

In the final part of the analysis, we show that the "surplus enhancing" effect described above is specific to differentiated ad valorem taxes targeting the "high type" package. More conventional forms of taxation, including uniform unit and ad valorem taxes (or subsidies) on all the versions of the good provided by the supplier, as well as unit taxes that target either package, have the standard effect of aggravating the distortions imposed by the supplier. In a nutshell, these taxes all have the same effect as an increase in the cost of production. Therefore, the firm's optimal response can only be to reduce supply. Instead, differentiated ad valorem tax rates may provide the incentive of redesign the price structure so as to increase welfare. Finally, we show that our main result is robust to including competition among suppliers and more than two consumer types.

Overall, the results provide novel insights for the design of commodity taxes in presence of second-degree price discrimination. Our findings indicate that the government can increase welfare while generating revenue by adopting ad valorem taxes and imposing a higher tax rate on the larger package or top quality version of the product. In more practical terms, our results suggest implementing (ad valorem) taxes that target, e.g., larger packages of household products, full-optional versions of goods such as smartphones, and first-class travel tickets.[5]

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 describes the model, and Section 4 characterizes the effects of differentiated ad valorem taxes on quantities, prices, consumer surplus and welfare. Section 5 compares the effects of differentiated ad valorem taxes with uniform ad valorem taxes and (uniform and differentiated) unit taxes. Section 6 extends the analysis to more than two types (Section 6.1) and duopoly (Section 6.2). Section 7 concludes.

2 Literature

The literature on price discrimination is very extensive. See, e.g., Tirole (1988), Laffont and Martimort (2002) and Stole (2007) for comprehensive overviews of the topic, and Bergemann et al. (2015) and Haghpanah and Siegel (2022) for more recent contributions. Despite the widespread adoption of second-degree price discrimination in real-world markets, only a handful of studies have investigated the effects of taxation in such setting. Laffont (1987) studies (unit) taxation of a monopolist that applies nonlinear pricing. Cheung (1998) shows, in a similar setting, that ad valorem taxes dominate unit taxes from a welfare perspective. Both papers consider only tax rates that apply

[5]Clearly, if consumers' type is correlated with their income, the kind of differentiated ad valorem taxes we propose may also have redistributive effects. A discussion of this point is beyond the objective of this paper.

uniformly to all sizes (or versions) of the good supplied, and obtain that taxes cannot alleviate the distortions imposed by the monopolist. Focusing on a specific form of second-degree discrimination, i.e. menus of multi-part tariffs, Jensen and Schjelderup (2011) find that taxation (either ad valorem or unit) increases the usage fee for all consumers, and so reduces consumption at the margin, although the effect on the access fee is ambiguous. More recently, D'Annunzio et al. (2020) show that ad valorem taxes can correct the tendency by suppliers to restrict consumption by "low" consumer types if differentiated tax rates are applied to the usage and access parts the tariffs. In this paper, we ignore multi-part tariffs and focus on firms that adopt pricing strategies such as quantity discounts and versioning. We obtain novel and counterintuitive effects of taxation by allowing for different tax rates on different versions of the good supplied. McCalman (2010) considers a similar type of differentiation when analyzing the effects of unit (but not ad valorem) trade tariffs in presence of a foreign monopolist that adopts price discrimination. Tariffs have conventional effects on equilibrium quantities and prices in his model, although they can increase domestic welfare (that is, when the surplus of the supplier is ignored).

The incidence of indirect taxes in business-to-consumer markets is a classical topic in economics (see, e.g., Fullerton and Metcalf, 2002). Many previous studies have looked at tax incidence in imperfectly competitive markets, focusing in large part on firms that supply a single product and adopt uniform (linear) pricing (Delipalla and Keen, 1992; Anderson et al., 2001; Auerbach and Hines, 2002). Weyl and Fabinger (2013) provide general principles for the pass-through of production costs (akin to unit taxes) in that context. A fundamental result in this literature is that taxes result in lower supply and higher prices, with very few exceptions. These include Cremer and Thisse (1994), who show that taxation can increase welfare in a vertically differentiated oligopoly with endogenous quality, and Carbonnier (2014), who shows that price reductions can be obtained with price-dependent tax schedules. Although we focus on a price-discriminating supplier, there is a similarity with these two papers, because our results call for differentiating ad valorem tax rates according to the size (or version) of the product supplied.

Our study also relates to a much smaller, but growing, branch of the indirect taxation literature that studies multi-product firms (with linear pricing). This literature was initiated by Edgeworth (1925), who provided an example where a monopolist supplying two substitute goods responds to a unit tax on one good by reducing the price of both, though supply of the taxed good goes down. This finding is known as Edgeworth's paradox of taxation, and was later re-elaborated by other authors, including Hotelling (1932), Coase (1946) and Salinger (1991).Hamilton (2009) considers an oligopoly finding that ad valorem taxes can increase output in the long run (with endogenous entry and product breadth). More recently, Armstrong and Vickers (2022) provide general conditions for the Edgeworth's paradox to occur, focusing on unit taxes. D'Annunzio and Russo (2022) show that ad valorem taxes can not only reduce prices, but also increase supply of all goods provided by a multi-product firm. Though the results of the present paper are somewhat similar, the logic behind them is different since we focus on markets with single-product suppliers that adopt price discrimination.

3 The model

We consider a monopolist providing a single good to two types of consumers, indexed by $i = H, L$, where H stands for "high" and L for "low." We normalize the total number of consumers to one, denoting the share of type H consumers by $v \in (0,1)$. The utility function of a consumer is

$$u(q, \theta_i) - p,\ i = H, L,$$

where q is the quantity of the good in the bundle supplied by the monopolist, p is the price paid and $\theta_H > \theta_L$ is the parameter capturing the intensity of preferences for such good, assumed to be private information of the consumer (Maskin and Riley, 1984). We assume that $\frac{\partial u}{\partial q} > 0, \frac{\partial^2 u}{\partial q^2} < 0, \frac{\partial u}{\partial \theta} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. The cost of providing one unit of the good to the supplier is c. Note that, although we refer to q as the quantity of the good for concreteness, this variable can also be interpreted as quality (Mussa and Rosen, 1978).

The supplier offers to consumers two "bundles", (q_i, p_i), to choose from, each intended for one type. These bundles can be interpreted as two packages of different sizes (e.g., a "regular" and a "supersize" package), each with its own price, or as two versions of the good, one of which is offered with extra features (e.g., a phone with more memory or processing power). To concentrate on the effects of interest, we let the government impose different ad valorem tax rates on each bundle, denoted by $t_i,\ i = H, L$ (we consider uniform ad valorem taxes and unit taxes in Section 5). We assume without loss that taxes are paid by the supplier, whose problem can thus be written as

$$\max_{q_H, p_H, q_L, p_L} \quad \pi = v\left((1-t_H)\, p_H - c q_H\right) + (1-v)\left((1-t_L)\, p_L - c q_L\right), \tag{1}$$

$$s.t. \quad u(q_H, \theta_H) - p_H \geq u(q_L, \theta_H) - p_L, \tag{2}$$

$$u(q_L, \theta_L) - p_L \geq u(q_H, \theta_L) - p_H, \tag{3}$$

$$u(q_H, \theta_H) - p_H \geq 0, \tag{4}$$

$$u(q_L, \theta_L) - p_L \geq 0, \tag{5}$$

where (2) and (3) are the incentive compatibility constraints, while (4) and (5) are the participation constraints for H and L-type consumers, respectively (we normalize the utility from no consumption to zero).

To complete the setup, we write the social welfare function, obtained as the sum of consumer surplus, profit and tax revenue. This expression boils down to welfare in this market

$$W = v\left(u_H - cq_H\right) + (1-v)\left(u_L - cq_L\right). \tag{6}$$

4 Analysis

Using standard procedures (Laffont and Martimort, 2002), one can show that only (2) and (5) are binding at the allocation that solves the monopolist's problem. Therefore, we shall ignore (3) and (4), and we have

$$p_H = u\left(q_H, \theta_H\right) - u\left(q_L, \theta_H\right) + u\left(q_L, \theta_L\right), \tag{7}$$

$$p_L = u\left(q_L, \theta_L\right). \tag{8}$$

Hence, we can rewrite the monopolist's problem in (1) as

$$\max_{q_H, q_L} \quad \pi = v\left(\left(1-t_H\right)\left(u_H - u_{HL} + u_L\right) - cq_H\right) + (1-v)\left(\left(1-t_L\right)u_L - cq_L\right), \tag{9}$$

where we have used the shorthand notation $u_i \equiv u\left(q_i, \theta_i\right)$, $i = H, L$, and $u_{HL} \equiv u\left(q_L, \theta_H\right)$. In the above expression, $u_{HL} - u_L$ represents the high type's information rent, which the supplier must accept to prevent consumers of this type choosing the bundle intended for the low types. Therefore, the supplier cannot extract the entire surplus from consumption of the high types. There is, instead, no such rent left to the low types, meaning that the supplier extracts all of their surplus.

The equilibrium quantities included in the two bundles solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H}\left(1-t_H\right) - c = 0, \tag{10}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)\left(1-t_H\right) + (1-v)\left(\frac{\partial u_L}{\partial q_L}\left(1-t_L\right) - c\right) = 0. \tag{11}$$

Setting taxes aside ($t_H = t_L = 0$), these equations indicate that the supplier offers an "efficient" bundle to the high types, in the sense that their marginal utility from an additional unit of the good equals its marginal cost. The bundle intended for the low types, however, is distorted downwards (since $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$ by assumption), in order to reduce the information rent.

4.1 Effect of taxation on quantities, prices and consumer surplus

It is interesting to analyze the effects of the tax rates on the quantities q_L and q_H. As we show in Appendix A.1, we have

$$\frac{\partial q_L}{\partial t_L} < 0, \quad \frac{\partial q_L}{\partial t_H} > 0, \quad \frac{\partial q_H}{\partial t_L} = 0, \quad \frac{\partial q_H}{\partial t_H} < 0. \tag{12}$$

The sign of the last derivative is counterintuitive: the quantity intended for the low type increases with the tax rate that targets the high bundle. To see why, recall that the supplier tends to distort q_L downwards because it wants to extract more revenue from the high types by reducing the information rent. The ad valorem tax t_H takes away part of the revenue earned from selling to the high types (without affecting the revenue from the L bundle directly), so it makes it relatively less profitable to implement this distortion, as can be seen from (11). By the same token, though, the tax induces a downward distortion in q_H. However, as we shall see, this latter distortion is of second order magnitude, and thus less relevant from the perspective of welfare (at least starting from the laissez-faire equilibrium).

It is also interesting to look at the effect of taxes on prices, (7) and (8). These effects mirror, of course, the effects on quantities. Given (12) and recalling that $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$, we have

$$\frac{\partial p_L}{\partial t_L} < 0, \quad \frac{\partial p_L}{\partial t_H} > 0, \quad \frac{\partial p_H}{\partial t_L} > 0, \quad \frac{\partial p_H}{\partial t_H} < 0. \tag{13}$$

Notably, the tax on the H bundle reduces the price of that bundle. To see why, consider equation (7) and note that q_H decreases, whereas q_L increases, which raises the high type's information rent, given $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$. On the other hand, the tax increases the price of the other, low-type, bundle. This is because the quantity included in the L bundle increases with t_H (see (8)), implying that the L bundle is sold at a higher price. Hence, quite interestingly, the high types benefit from the tax applied to "their" bundle, while the net surplus of the low types remains equal to zero in equilibrium. It follows that, overall, the tax t_H increases consumer surplus.

The tax on the L bundle has a very different effect. It induces a reduction in q_L, as one would expect, but no change in q_H. Accordingly, p_L goes down, but p_H increases, because the information rent left to the high type goes down. Therefore, this tax makes no consumer better off.

4.2 Effect of taxation on social welfare

Consider now the effects of taxation on welfare. Differentiating (6) and given the first-order conditions of the supplier's problem, we obtain

$$\frac{\partial W}{\partial t_H} := v \frac{\partial q_H}{\partial t_H} \frac{\partial u_H}{\partial t_H} t_H + (1-v) \frac{\partial q_L}{\partial t_H} \left(\frac{v}{1-v} \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L} \right) (1-t_H) + \frac{\partial u_L}{\partial q_L} t_L \right), \tag{14}$$

$$\frac{\partial W}{\partial t_L} := (1-v) \frac{\partial q_L}{\partial t_L} \left(\frac{v}{1-v} \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L} \right) (1-t_H) + \frac{\partial u_L}{\partial q_L} t_L \right). \tag{15}$$

As a first step, we consider the above derivatives at the laissez-faire equilibrium, where $t_H = t_L = 0$. Given (12), we obtain

$$\frac{\partial W}{\partial t_H} := v \frac{\partial q_L}{\partial t_H} \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L} \right) > 0, \qquad \frac{\partial W}{\partial t_L} := v \frac{\partial q_L}{\partial t_L} \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L} \right) < 0. \tag{16}$$

These expressions show that welfare increases with the introduction of a subsidy on the L bundle or a *tax* on the H bundle. As shown above, this tax reduces the distortion that the supplier imposes on the L bundle. Although the tax also creates a distortion on the H bundle, this distortion is of second order when starting from $t_H = t_L = 0$. Therefore, introducing an ad valorem tax on the H bundle produces a *double dividend*: welfare increases and, at the same time, the tax generates revenue to the government (which can, in principle, be used to reduce other forms of distortionary taxation).

Although the tax on the H bundle has an unexpected, welfare-enhancing effect, it is not the best instrument if the government's only goal is to maximize welfare in this market. This is easily seen by comparing the first-order derivatives (14) and (15): unlike t_H, the (negative) tax rate t_L does not distort the quantity q_H. Accordingly, one can easily show that the first-best policy, i.e. the unique solution to the system of first-order conditions $\frac{\partial W}{\partial t_L} = \frac{\partial W}{\partial t_H} = 0$, is such that the L-bundle is subsidized:

$$t_L^{FB} = -\frac{v}{1-v} \frac{\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L} \right)}{\frac{\partial u_L}{\partial q_L}} < 0, \qquad t_H^{FB} = 0. \tag{17}$$

Nevertheless, in practice government budgets are constrained. Furthermore, the money to finance subsidies is generated by imposing other, typically distortionary, taxes. It is therefore interesting to characterize the second-best optimal policy that maximizes (6) under the constraint that the government pays no subsidy. Under this condition, we find that the optimal policy is such that

$$t_L^{SB} = 0, \qquad t_H^{SB} = \frac{\frac{\partial q_L}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)}{\frac{\partial q_L}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t_H}\frac{\partial u_H}{\partial q_H}} > 0. \tag{18}$$

This result confirms that, by targeting the bundle intended for the high types with an ad valorem tax, the government can increase welfare while generating additional revenue.[6]

Our results provide novel insights for the design of commodity taxes with a price-discriminating supplier that offers its product in packages of different sizes or versions that differ in quality (e.g., the number of enabled functionalities). Specifically, by adopting ad valorem taxes with a higher rate on the larger package (or top quality version), the government can increase output and welfare, as well as generate revenue. In the next section, we show that the same results cannot be obtained with other, conventional, forms of taxation.[7]

5 Other tax instruments

It is natural to wonder whether the government can obtain an increase in output and welfare by adopting other tax instruments than the differentiated ad valorem taxes we considered above. To answer this question, in this section we consider a uniform ad valorem tax rate applied to all bundles. We also consider unit taxes. We show that none of these instruments can produce the same welfare-enhancing effects described above.

[6]We could augment the model by including a cost of public funds (i.e., a weight higher than one for government revenue) in the welfare function, and solve for the optimal policy conditional on this cost. We do not engage in such exercise to keep things short and simple. Still, we expect that, when the cost of public funds is high enough, the result of such exercise would be that the optimal policy involves no subsidy and a higher tax rate on the H bundle.

[7]It should be noted that, given it induces a reduction in q_H and an increase in q_L, a sufficiently high tax rate t_H might result in the two quantities being equal. In this case, the supplier could not implement bundles such that $p_H > p_L$, because the constraint (2) would be violated. Hence, the supplier might stop serving the low-type consumers and offer a single bundle that targets the high-type consumers. We have ignored this possibility in the above analysis because establishing a threshold on t_H such that this scenario occurs is cumbersome. However, this possibility does not change our main result, at least qualitatively. There would always exist a strictly positive t_H (possibly smaller than t_H^{SB}), such that the supplier serves both types and welfare is strictly higher than in the laissez-faire.

5.1 Uniform ad valorem tax

Suppose the government imposes a uniform ad valorem tax on the two bundles, i.e. $t_H = t_L = t$. As we show in Appendix A.2, such tax can only reduce the quantity in both bundles, i.e.

$$\frac{\partial q_L}{\partial t} < 0, \qquad \frac{\partial q_H}{\partial t} < 0. \tag{19}$$

To grasp the intuition, replace $t_H = t_L = t$ in the first-order conditions of the supplier's problem, (10) and (11), and divide both expressions by $1-t$, to obtain

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H} - \frac{c}{1-t} = 0, \tag{20}$$

$$\frac{\partial \pi}{\partial q_L} := \left(v \left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L} \right) + (1-v) \frac{\partial u_L}{\partial q_L} \right) - (1-v) \frac{c}{1-t} = 0. \tag{21}$$

A higher uniform tax rate simply reduces the marginal revenue from increasing the quantity in either bundle. Therefore, the tax has the same effect as an increase in the unit cost of production. It follows that the supplier's optimal response is to reduce the quantity in both bundles. The effect on welfare is thus negative.

5.2 Unit taxes

Suppose now that the supplier is subject to unit taxes, denoted by τ_i, on bundle $i = H, L$. The profit function is therefore

$$\pi = v \left(p_H - (c + \tau_H) q_H \right) + (1-v) \left(p_L - (c + \tau_L) q_L \right). \tag{22}$$

The supplier maximizes this function subject to the same constraints as above, i.e. (2)-(5).

We solve this problem following the same steps as in Section 4: constraints (2) and (5) are binding, meaning that equilibrium prices are as given in (7) and (8). Replacing these prices in (22), and using again the shorthand notation $u_i \equiv u(q_i, \theta_i)$, $i = H, L$, and $u_{HL} \equiv u(q_L, \theta_H)$, we get

$$\pi = v \left(u_H + u_{HL} - u_L - (c + \tau_H) q_H \right) + (1-v) \left(u_L - (c + \tau_L) q_L \right). \tag{23}$$

The equilibrium quantities are the solution to the following system of equations

$$\frac{\partial \pi}{\partial q_H} := v \left(\frac{\partial u_H}{\partial q_H} - c - \tau_H \right) = 0, \tag{24}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{HL}}{\partial q_L}\right) + (1-v)\left(\frac{\partial u_L}{\partial q_L} - c - \tau_L\right) = 0. \tag{25}$$

These expressions suggest that the effect of either tax rate is similar to that of an increase in the cost of production of either bundle. Therefore, as we show in Appendix A.3, each tax rate reduces the quantity in the bundle it applies to. Hence, we have

$$\frac{\partial q_L}{\partial \tau_L} < 0, \qquad \frac{\partial q_H}{\partial \tau_L} = 0, \qquad \frac{\partial q_H}{\partial \tau_H} < 0, \qquad \frac{\partial q_L}{\partial \tau_H} = 0. \tag{26}$$

It follows that welfare decreases with either tax rate. Intuitively, similar results apply with a uniform unit tax rate, i.e. $\tau_L = \tau_H = \tau$.

6 Robustness checks

We now present two extensions to the baseline model. First, we consider a setting with more than two types of consumers. Second, we consider a duopoly of suppliers. The purpose of these extensions is to show that our findings concerning the effect of an ad valorem tax targeting the H bundle continue to hold. For reasons of space, we provide a summary description here and relegate the analysis to the Online Appendix.

6.1 More than two types

We consider a setting with three types of consumers in Appendix B.1. Specifically, we assume the preference parameter θ can take three different values $\theta \in \{\theta_H, \theta_M, \theta_L\}$, with $\theta_H > \theta_M > \theta_L$. Let v_H, v_M and v_L be the shares of consumers of type H, M and L, respectively, with $v_L + v_M + v_H = 1$. Furthermore, we assume that $\frac{v_L}{v_M} < \frac{v_L + v_M}{v_H}$, i.e. that the distribution of types satisfies the monotone hazard rate property.[8] The model is otherwise identical to our baseline setup.

The solution follows the same steps as in 4. The main difference is that, in this setting, there are two incentive compatibility constraints to be satisfied for each type, since each type has the option of buying two bundles not intended for her or him. Using standard steps (Laffont and Martimort, 2002), it can be shown that, at equilibrium, only the incentive compatibility constraints ensuring that a higher type does not mimic the type immediately below are binding. Specifically, in equilibrium the H (respectively, M) type is indifferent between buying the bundle intended for her or him and the bundle intended for the M (respectively, L) type. Finally, the participation constraint is binding

[8] This assumption ensures that there is no "bunching" of types in the laissez-faire (Laffont and Martimort, 2002, p.90).

for the L type. As a result, the supplier distorts the quantity (or quality) of the bundle intended for all types except the highest one. Types H and M receive an information rent, while the supplier extracts all the surplus from the L-type consumers.

We find that an ad valorem tax applied to the H bundle has similar effects as in the baseline model. That is, the tax reduces the distortion on the M and L bundles, by inducing the supplier to increase the quantity for both types. Furthermore, the tax on the H bundle reduces the price of the that bundle, while raising the price of the L bundle (the effect on the price of the M bundle is ambiguous). The logic behind this result is the same as above: a tax on the H bundle makes collecting revenue from the H types less attractive to the supplier, and so reduces the incentive to relax their incentive compatibility constraint. It is interesting to note that there is a knock-on effect on the L bundle as well: its price increases, but the quantity increases too. The reason is that an increase in the quantity of good in the M bundle, q_M, relaxes the incentive compatibility constraint that applies to the M type and, hence, reduces the need to distort the quantity in the L bundle in order to satisfy such constraint. By the same token, also the tax applied to the M bundle increases the quantity in the L bundle and induces the monopolist to decrease the price of the M bundle.

To sum up, the analysis with more than two types confirms our baseline findings and suggests that a tax applied on the largest-sized package of a certain good should bring to an increase in the size of the other packages, with a positive effect on welfare.

6.2 Duopoly

In Appendix B.2, we consider a setting with two horizontally differentiated, symmetric suppliers. We assume for simplicity that there is perfect correlation between a consumers' preference for a supplier and the marginal utility from consuming the good. Specifically, we consider two "high" consumer types, each with a strict preference for one of the two suppliers (i.e., these consumers get zero utility from consuming the good supplied by the other). Moreover, we consider two "low" types that have a weak preference for one of the suppliers (i.e., they get a smaller, but positive, utility from consuming from the least preferred one). Moreover, compared to the high types, the low types get a smaller marginal utility from consuming the good from their preferred vendor.

In accordance with previous literature (Spulber, 1989; Stole, 2007), we show that the bundles offered by the duopolists in equilibrium are similar to those offered by a monopolist. Specifically, each supplier serves only the two types (high and low) that have a preference for its good, and offers a distorted bundle to the low types to relax the incentive compatibility constraint of the high types. The main difference with respect to the monopoly case is that each supplier must leave the low types with some surplus to avoid such types switching to the rival. As a result, the price charged to the high types must decrease as well. Intuitively, therefore, competition implies that the suppliers can extract less surplus from consumers overall. However, the first-order conditions of the maximization problem of each firm have the same shape as in (10) and (11). Thus, the effects of taxation in this setting are the same as in the monopoly setting considered above.

7 Conclusions

We have studied the effects of taxation in markets with second-degree price discrimination. We have shown that, by imposing an ad valorem tax on the bundle intended for the high types, the government can have a double dividend, increasing welfare and generating revenue. Moreover, the tax increases consumer surplus (weakly) for all types. By contrast, uniform ad valorem and (uniform and differentiated) unit taxes always decrease quantities and welfare. In the baseline model we consider a monopolist supplier and two types, but the results are robust to considering more than two types and competing suppliers. In practical terms, our results call for (ad valorem) taxes targeting, e.g., larger packages of beverages, and first-class travel tickets.

Some of the markets where firms apply the kind of price discrimination we consider in this paper are subject to specific excise taxes, intended to control the consumption of unhealthy products. Examples include sugary food and beverages, alcohol and tobacco products. Although we have not considered the implications of health-related consumption externalities in the paper, we note that, by inducing suppliers to reduce the size of the largest packages, the differentiated taxation we analyze here may result in an additional social benefit if we consider the largest package to be more harmful. However, these differentiated taxes also increase the size of the package intended for the low types. Hence, an analysis of the effects of differentiated taxation in these specific markets should consider the characteristics of different groups of consumers to capture the effects of these relative changes. This analysis is an interesting topic for further research.

References

Anderson, S. P., de Palma, A., and Kreider, B. (2001). The efficiency of indirect taxes under imperfect competition. *Journal of Public Economics*, 81(2):231–251.

Armstrong, M. and Vickers, J. (2022). Multiproduct cost passthrough: Edgeworth's paradox revisited. *CEPR Discussion Paper*, (DP17202).

Auerbach, A. J. and Hines, J. (2002). Taxation and economic efficiency. In Auerbach, A. J. and Feldstein, M., editors, *Handbook of Public Economics*. Elsevier, Amsterdam.

Bergemann, D., Brooks, B., and Morris, S. (2015). The limits of price discrimination. *American Economic Review*, 105(3):921–57.

Carbonnier, C. (2014). The incidence of non-linear price-dependent consumption taxes. *Journal of Public Economics*, 118:111–119.

Cheung, F. K. (1998). Excise taxes on a non-uniform pricing monopoly: Ad valorem and unit taxes compared. *The Canadian Journal of Economics / Revue canadienne d'Economique*, 31(5):1192–1203.

Coase, R. H. (1946). Monopoly pricing with interrelated costs and demands. *Economica*, 13(52):278–294.

Cremer, H. and Thisse, J.-F. (1994). Commodity taxation in a differentiated oligopoly. *International Economic Review*, 35(3):613–633.

D'Annunzio, A., Mardan, M., and Russo, A. (2020). Multi-part tariffs and differentiated commodity taxation. *The RAND Journal of Economics*, 51(3):786–804.

D'Annunzio, A. and Russo, A. (2022). Negative Tax Incidence with Multiproduct Firms. CESifo Working Paper Series 9881, CESifo.

Delipalla, S. and Keen, M. (1992). The comparison between ad valorem and specific taxation under imperfect competition. *Journal of Public Economics*, 49(3):351–367.

Edgeworth, F. Y. (1925). The pure theory of monopoly. In *Papers Relating to Political Economy*. Burt Franklin, New York.

Fullerton, D. and Metcalf, G. E. (2002). Tax incidence. In Auerbach, A. J. and Feldstein, M., editors, *Handbook of Public Economics*, pages 1787–1872. Elsevier, Amsterdam.

Haghpanah, N. and Siegel, R. (2022). The limits of multi-product price discrimination. *American Economic Review: Insights*.

Hamilton, S. F. (2009). Excise taxes with multiproduct transactions. *The American Economic Review*, 99(1):458–471.

Hotelling, H. (1932). Edgeworth's taxation paradox and the nature of demand and supply functions. *Journal of Political Economy*, 40(5):577–616.

Jensen, S. and Schjelderup, G. (2011). Indirect taxation and tax incidence under nonlinear pricing. *International Tax and Public Finance*, 18(5):519–532.

Laffont, J.-J. (1987). Optimal taxation of a non-linear pricing monopolist. *Journal of Public Economics*, 33(2):137–155.

Laffont, J.-J. and Martimort, D. (2002). *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press.

Maskin, E. and Riley, J. (1984). Monopoly with incomplete information. *RAND Journal of Economics*, 15(2):171–196.

McCalman, P. (2010). Trade policy in a supersize me world. *Journal of International Economics*, 81(2):206–218.

Mussa, M. and Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory*, 18(2):301–317.

Salinger, M. A. (1991). Vertical mergers in multi-product industries and Edgeworth's paradox of taxation. *The Journal of Industrial Economics*, 39(5):545–556.

Spulber, D. F. (1989). Product variety and competitive discounts. *Journal of Economic Theory*, 48(2):510–525.

Stole, L. (2007). Price discrimination and competition. In Armstrong, M. and Porter, R., editors, *Handbook of Industrial Organization*, volume 3, chapter 34, pages 2221–2299. Elsevier, 1 edition.

Tirole, J. (1988). *The Theory of Industrial Organization*, volume 1 of *MIT Press Books*. The MIT Press.

Weyl, E. G. and Fabinger, M. (2013). Pass through as an economic tool: Principles of incidence under imperfect competition. *Journal of Political Economy*, 121(3):528–583.

A Proofs

A.1 Establishing the signs of the derivatives in (12) and (13)

By totally differentiating the first-order conditions of the monopolist's problem in(10) and (11), we find that

$$\frac{\partial q_i}{\partial t_i} = -\frac{\frac{\partial^2\pi}{\partial q_j^2}\frac{\partial^2\pi}{\partial q_i \partial t_i} - \frac{\partial^2\pi}{\partial q_i \partial t_j}\frac{\partial^2\pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial t_i} = -\frac{\frac{\partial^2\pi}{\partial q_i^2}\frac{\partial^2\pi}{\partial q_j \partial t_i} - \frac{\partial^2\pi}{\partial q_i \partial t_i}\frac{\partial^2\pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $H \equiv \frac{\partial^2\pi}{\partial q_L^2}\frac{\partial^2\pi}{\partial q_H^2} - \left(\frac{\partial^2\pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2\pi}{\partial q_j^2} < 0$, $\frac{\partial^2\pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2\pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2\pi}{\partial q_H \partial t_L} = 0$ and $\frac{\partial^2\pi}{\partial q_L \partial t_H} = \frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0$, $\frac{\partial^2\pi}{\partial q_H \partial t_H} = \frac{\partial u_H}{\partial q_H} > 0$ and $\frac{\partial^2\pi}{\partial q_L \partial t_L} = -\frac{\partial u_L}{\partial q_H} < 0$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t_H}\right) = \operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_L^2}\frac{\partial u_H}{\partial q_H}\right) < 0$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t_H}\right) = \operatorname{sgn}\left(-\frac{\partial^2\pi}{\partial q_H^2}\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\right) > 0$$

$$\frac{\partial q_H}{\partial t_L} = 0$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t_L}\right) = \operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_H^2}\frac{\partial u_L}{\partial q_L}\right) < 0.$$

Let us now compute the derivatives of the equilibrium prices $p_H = u_H - u_{HL} + u_L$ and $p_L = u_L$ with respect to t_i, $i = H,L$. Taking into account that $\frac{\partial u}{\partial q} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$, we have

$$\frac{\partial p_H}{\partial t_H} = \frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t_H} < 0, \qquad \frac{\partial p_L}{\partial t_H} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_H} > 0.$$

$$\frac{\partial p_H}{\partial t_L} = -\frac{\partial q_L}{\partial t_L}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0, \qquad \frac{\partial p_L}{\partial t_L} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_L} < 0.$$

A.2 Uniform ad valorem tax

Consider profits in (1) with $t_L = t_H = t$. The first-order conditions of the monopolist's problem are

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H}(1-t_H) - c = 0, \tag{27}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + (1-v)\left(\frac{\partial u_L}{\partial q_L}(1-t_L) - c\right) = 0. \tag{28}$$

By totally deriving the above first-order conditions of the monopolist's problem with respect to a uniform tax t, we find that

$$\frac{\partial q_i}{\partial t} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t} - \frac{\partial^2 \pi}{\partial q_i \partial t}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H},$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0, \quad \frac{\partial^2 \pi}{\partial q_j^2} < 0, \quad \frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t} = -v\frac{\partial u_H}{\partial q_H} < 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t} = -\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right) < 0$. Hence,

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H \partial t}\right) = \operatorname{sgn}\left(-v\frac{\partial u_H}{\partial q_H}\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial^2 \pi}{\partial q_L \partial t}\right) = \operatorname{sgn}\left(-\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right)\right) < 0.$$

Furthermore, the introduction of a small ad valorem tax has negative effects on welfare

$$\left.\frac{\partial W}{\partial t}\right|_{t=0} = \frac{\partial q_H}{\partial t}v\left(\frac{\partial u_H}{\partial q_H} - c\right) + \frac{\partial q_L}{\partial t}(1-v)\left(\frac{\partial u_L}{\partial q_L} - c\right) < 0.$$

A.3 Unit taxes

By totally differentiating the first-order conditions of the monopolist's problem in(24) and (25), we find that

$$\frac{\partial q_i}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_j}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \qquad \frac{\partial q_j}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0, \quad \frac{\partial^2 \pi}{\partial q_j^2} < 0, \quad \frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_H} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_H} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_L} = -(1-v) < 0$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau_H}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2}v\right) < 0$$

$$\frac{\partial q_L}{\partial \tau_H}=0,\frac{\partial q_H}{\partial \tau_L}=0$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial \tau_L}\right)=\operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right)<0.$$

Consider now a uniform unit tax $\tau_L=\tau_H=\tau$. By totally differentiating the first-order conditions of the monopolist's problem in(24) and (25), we find that

$$\frac{\partial q_i}{\partial \tau}=-\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau}-\frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau}=-\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau}-\frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j=H,L, \; j\neq i.$$

where $\frac{\partial^2 \pi}{\partial q_H \partial q_L}=0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau}=-v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau}=-(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau}\right)=\operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2}v\right)<0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial \tau}\right)=\operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right)<0.$$

B Robustness checks (online)

B.1 More than two types

We assume there are three types of consumers, characterized by the preference parameter $\theta \in \{\theta_H,\theta_M,\theta_L\}$, with $\theta_H>\theta_M>\theta_L$. Let v_H, v_M and v_L be the shares of consumers of type H, M and L, respectively, with $v_H+v_M+v_L=1$. Furthermore, to avoid "bunching" of types we assume that $\frac{v_L}{v_M}<\frac{v_L+v_M}{v_H}$, i.e. that the distribution of types satisfies the monotone hazard rate property (Laffont and Martimort, 2002, p.90). The model is otherwise identical to our baseline setup.

The supplier offers to consumers three bundles, (q_i,p_i), each intended for one type. These bundles must satisfy six incentive constraints (two for each type)

$$u(q_i,\theta_i)-p_i\geq u(q_j,\theta_i)-p_j, \quad i,j=L,M,H \quad i\neq j,$$

and three participation constraints (one per each type)

$$u(q_i,\theta_i)-p_i\geq 0, \quad i=L,M,H.$$

Following standard steps (Laffont and Martimort, 2002), one can show that, in equilibrium, there are two binding incentives constraints (the ones such that a higher type want to mimic a lower type) and one binding participation constraint (the one of low types). From these binding constraints we derive the equilibrium prices. Hence, the supplier maximizes the following problem

$$\max_{(q_i,p_i)} \quad \pi = \sum_{i=L,M,H} v_i\left((1-t_i)\,p_i - cq_i\right), \tag{29}$$

$$s.t. \quad p_H = u_H + u_M + u_L - u_{ML} - u_{HM}, \tag{30}$$

$$p_M = u_M + u_L - u_{ML}, \tag{31}$$

$$p_L = u_L, \tag{32}$$

where $u_i \equiv u(q_i, \theta_i)$ for each $i = L,M,H$, and $u_{ij} \equiv u\left(q_j, \theta_i\right)$ for each $i, j = L,M,H$ with $i \neq j$. Hence, we derive the following first-order conditions

$$\frac{\partial \pi}{\partial q_H} := v_H\left(\frac{\partial u_H}{\partial q_H}(1-t_H) - c\right) = 0, \tag{33}$$

$$\frac{\partial \pi}{\partial q_M} := v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)(1-t_H) + v_M\left(\frac{\partial u_M}{\partial q_M}(1-t_M) - c\right) = 0, \tag{34}$$

$$\frac{\partial \pi}{\partial q_L} := v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)(1-t_H) + v_M\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)(1-t_M) + v_L\left(\frac{\partial u_L}{\partial q_L}(1-t_L) - c\right) = 0. \tag{35}$$

Totally differentiating the above equations and taking into account that cross-profits derivatives are zero ($\frac{\partial^2 \pi}{\partial q_i \partial q_j} = 0$ for $i, j = L,M,H$ with $i \neq j$), we find that

$$\frac{\partial q_H}{\partial t_H} = -\frac{\begin{vmatrix} \frac{\partial^2 \pi}{\partial q_L^2} & \frac{\partial^2 \pi}{\partial q_L \partial q_M} & \frac{\partial^2 \pi}{\partial q_L \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_M \partial q_L} & \frac{\partial^2 \pi}{\partial q_M^2} & \frac{\partial^2 \pi}{\partial q_M \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_H \partial q_L} & \frac{\partial^2 \pi}{\partial q_H \partial q_M} & \frac{\partial^2 \pi}{\partial q_H \partial t_H} \end{vmatrix}}{H} = -\frac{\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_M^2}\frac{\partial^2 \pi}{\partial q_H \partial t_H}}{H} \leq 0,$$

where H is the determinant of the Hessian matrix, which is negative by second order conditions, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ for $i = L,M,H$ also by second order conditions, and $\frac{\partial^2 \pi}{\partial q_H \partial t_H} = -v_H \frac{\partial u_H}{\partial q_H} < 0$. Following similar steps, we find that the derivatives of q_M and q_L with respect to t_H are, respectively, such that

$$\text{sgn}\left(\frac{\partial q_M}{\partial t_H}\right) = \text{sgn}\left(-v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)\right) \geq 0, \qquad \text{sgn}\left(\frac{\partial q_L}{\partial t_H}\right) = \text{sgn}\left(-v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\right) \geq 0\ .$$

These signs follow from the assumption that $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. This establishes that the effect of the ad valorem tax applied to the H-bundle is such that the quantity of the other two bundles increases, reducing the distortion applied by the supplier.

Similarly, the derivatives of the equilibrium quantities with respect to t_M and t_L are such that

$$\frac{\partial q_H}{\partial t_M} = 0, \qquad \operatorname{sgn}\left(\frac{\partial q_M}{\partial t_M}\right) = \operatorname{sgn}\left(-v_M \frac{\partial u_M}{\partial q_M}\right) \leq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_M}\right) = \operatorname{sgn}\left(-v_M\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\right) \geq 0,$$

$$\frac{\partial q_H}{\partial t_L} = 0, \qquad \frac{\partial q_M}{\partial t_L} = 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_L}\right) = \operatorname{sgn}\left(-v_L \frac{\partial u_L}{\partial q_L}\right) \leq 0.$$

B.2 Duopoly

We consider two symmetric suppliers, indexed by $s \in \{1,2\}$ and four consumer types, indexed by $i \in \{H_1, L_1, H_2, L_2\}$, differing in (i) their intensity of preferences for the good and (ii) their preference for the two suppliers. The utility when buying from supplier s is $u_s(q, \theta_i) - p$, where p is the price and θ_i is the preference parameter. Let v_i be the share of consumers of type i, with $\sum_{i=H_1,L_1,H_2,L_2} v_i = 1$, and assume that each consumer buys from at most one supplier. We assume the utility function satisfies the following conditions:

$$u_1(q, \theta_{H_1}) > u_1(q, \theta_{L_1}) > u_1(q, \theta_{L_2}) > u_1(q, \theta_{H_2}) = 0, \quad \forall q > 0,$$

$$u_2(q, \theta_{H_2}) > u_2(q, \theta_{L_2}) > u_1(q, \theta_{L_1}) > u_1(q, \theta_{H_1}) = 0, \quad \forall q > 0,$$

$$\frac{\partial u_1}{\partial q}(q, \theta_{H_1}) > \frac{\partial u_1}{\partial q}(q, \theta_{L_1}) > \frac{\partial u_1}{\partial q}(q, \theta_{L_2}) > \frac{\partial u_1}{\partial q}(q, \theta_{H_2}) = 0, \quad \forall q > 0,$$

$$\frac{\partial u_2}{\partial q}(q, \theta_{H_2}) > \frac{\partial u_2}{\partial q}(q, \theta_{L_2}) > \frac{\partial u_2}{\partial q}(q, \theta_{L_1}) > \frac{\partial u_2}{\partial q}(q, \theta_{H_1}) = 0, \quad \forall q > 0.$$

These conditions imply a perfect correlation between the preference for one supplier and the intensity of preference for the good it supplies (Spulber, 1989). For simplicity, we assume only the "low" types are willing to buy from either supplier, whereas the "high" types do not get any utility from buying from their least preferred supplier.

Let (q_i, p_i) denote the bundle that a supplier proposes to consumers of type i. Given the condition that consumers self-select on the intended bundle, there is no loss in proceeding under the assumption that supplier 1 only offers bundles intended for the couple of consumer types that prefer its product, i.e. H_1 and L_1, whereas supplier 2 only serves H_2 and L_2. We are now going to state the constraints that the suppliers face regarding each type of consumer. Considering a supplier s, we have the following incentives and participation constraints that apply to the H_s-bundle:

$$u_s(q_{H_s}, \theta_{H_s}) - p_{H_s} \geq u_s(q_{L_s}, \theta_{H_s}) - p_{L_s},\ s = 1,2, \tag{36}$$

$$u_s(q_{H_s}, \theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{L_{s'}}, \theta_{H_s}\right) - p_{L_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{37}$$

$$u_s(q_{H_s}, \theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{H_s}\right) - p_{H_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{38}$$

$$u_s(q_{H_s}, \theta_{H_s}) - p_{H_s} \geq 0,\ s = 1,2. \tag{39}$$

Constraint (36) must hold in order for H_s types not to choose the bundle offered to L_s consumers by the same supplier . The next two constraints, (37) and (38), must hold to avoid that H_s types buy any of the bundles offered by the other supplier, s'. Finally, (39) must hold for H_s types to prefer the bundle intended for them to not participating in the market at all.

Symmetrically, the constraints that apply to the L_s-bundle are as follows

$$u_s(q_{L_s}, \theta_{L_s}) - p_{L_s} \geq u_s(q_{H_s}, \theta_{L_s}) - p_{H_s},\ s = 1,2, \tag{40}$$

$$u_s(q_{L_s}, \theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{41}$$

$$u_s(q_{L_s}, \theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{L_s}\right) - p_{H_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{42}$$

$$u_s(q_{L_s}, \theta_{L_s}) - p_{L_s} \geq 0,\ s = 1,2. \tag{43}$$

Constraint (40) must hold in order for L_s types not to choose the bundle offered to H_s consumers by supplier s. The next two constraints, (37) and (38), must hold to avoid that L_s types buy from the other supplier. Finally, (39) must hold for L_s types to prefer the bundle intended for them to not participating in the market at all.

Given the differentiated ad valorem tax rates we consider in the baseline setting, the problem of supplier s is

$$\max_{q_{H_s}, p_{H_s}, q_{L_s}, p_{L_s}} \quad \pi = v_{H_s}\left[(1-t_H)\, p_{H_s} - c q_{H_s}\right] + v_{L_s}\left[(1-t_L)\, p_{L_s} - c q_{L_s}\right],\ s = 1,2, \tag{44}$$

subject to constraints (36)-(43).

We are now going to solve supplier s's problem characterized above, focusing on symmetric equilibria. Our first step is to establish which constraints are going to be binding in equilibrium to determine equilibrium prices. Given $u_{s'}\left(q_{L_{s'}}, \theta_{H_s}\right) = u_{s'}\left(q_{H_{s'}}, \theta_{H_s}\right) = 0$, constraints (37) and (38) cannot be binding, because of the participation constraints in (39). Furthermore, given (43), and that $u_s(q_{L_s}, \theta_{H_s}) > u_s(q_{L_s}, \theta_{L_s})$, constraint (39) cannot be binding either. Hence, the equilibrium must be such that (36) is binding. We have

$$p_{H_s} = p_{L_s} + u_s(q_{H_s}, \theta_{H_s}) - u_s(q_{L_s}, \theta_{H_s}),\ s = 1, 2. \tag{45}$$

Given (45), we can write the constraints (40), after some rearrangements, as

$$u_s(q_{H_s}, \theta_{H_s}) - u_s(q_{L_s}, \theta_{H_s}) \geq u_s(q_{H_s}, \theta_{L_s}) - u_s(q_{L_s}, \theta_{L_s}),\ s = 1, 2,$$

which must hold strictly by the assumption that $\frac{\partial u_s}{\partial q}(q, \theta_{H_s}) > \frac{\partial u_s}{\partial q}(q, \theta_{L_s})$. Hence, these constraints cannot be binding. Consider now the constraints (42). These can be rewritten, using (45) and after a few rearrangements as

$$u_{s'}\left(q_{H_{s'}}, \theta_{H_{s'}}\right) - u_{s'}\left(q_{L_{s'}}, \theta_{H_{s'}}\right) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{L_s}\right) - u_s(q_{L_s}, \theta_{L_s}) - p_{L_{s'}},\ s = 1, 2.$$

In a symmetric equilibrium (where $p_{L_s} = p_{L_{s'}}$ and $q_{L_s} = q_{L_{s'}}$), this inequality must hold strictly by the assumption that $\frac{\partial u_{s'}}{\partial q}\left(q, \theta_{H_{s'}}\right) > \frac{\partial u_s}{\partial q}(q, \theta_{L_s}) > \frac{\partial u_{s'}}{\partial q}(q, \theta_{L_s})$. Therefore, the only constraints that can be binding are (41) and (43). We have

$$p_{L_s} = u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right)\ s, s' = 1, 2, s' \neq s. \tag{46}$$

Given (45) and (46), we can therefore write the the problem of supplier s as

$$\begin{aligned} \max_{q_{H_s}, q_{L_s}} \quad \pi_s = {} & v_{H_s}\left[(1 - t_H)\left(u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right) + u_s(q_{H_s}, \theta_{H_s}) - (q_{L_s}, \theta_{H_s})\right) - cq_{H_s}\right] \\ & + v_{L_s}\left[(1 - t_L)\left(u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right)\right) - cq_{L_s}\right],\ \ s, s' = 1, 2, s' \neq s. \end{aligned} \tag{47}$$

Observe that $u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}$ does not depend on q_{H_s} nor on q_{L_s}. The first-order conditions of this problem are

$$\frac{\partial \pi}{\partial q_{H_s}} := \frac{\partial u_s(q_{H_s}, \theta_{H_s})}{\partial q_{H_s}}(1 - t_H) - c = 0\ s = 1, 2, \tag{48}$$

$$\frac{\partial \pi}{\partial q_{L_s}} := v_{H_s}\left(-\frac{\partial u_s(q_{L_s}, \theta_{H_s})}{\partial q_{L_s}} + \frac{\partial u_s(q_{L_s}, \theta_{L_s})}{\partial q_{L_s}}\right)(1 - t_H) + v_{L_s}\left(\frac{\partial u_s(q_{L_s}, \theta_{L_s})}{\partial q_{L_s}}(1 - t_L) - c\right) = 0\ s = 1, 2. \tag{49}$$

The key observation is that these equations have the same form as (10) and (11), which implies that the effects of taxation must be also be the same, and so are the implications for optimal policy.