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A consumer surplus, welfare and profit enhancing strategy for improving urban transport networks*

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Abstract

We show that a novel pricing system can help resolve a series of perennial problems evident in the deregulated British urban public transport market that have compromised economic growth, access equality and environmental ambitions. A two-stage pricing system, with operators setting their multi-operator service ticket prices collusively in one stage and their single-operator ticket prices independently, in the other, offers potential consumer surplus, profit and welfare gains over, what we characterise as, the 'Status Quo'. The proposed win-win pricing regime can also support a larger number of operators and services with potential additional welfare gains. The Block Exemption in the UK allowing collusive pricing on a limited basis is due to expire and is under statutory review, making this is a timely contribution. We also compare the proposed regime against a multi-operator ticketing card (MTC) scheme, permitted under the Block Exemption, and show, whilst the MTC offers higher welfare when all regimes provide the same number of services, the proposed regime supports a larger number of operators in the presence of fixed costs, which can reverse the welfare ranking in its favour. A calibration exercise indicates the market may be in the region where the proposed regime can dominate the 'Status Quo' in profit, consumer surplus and welfare terms and supports a larger network than the 'Status Quo' or MTC with further welfare gains. The resulting higher public transport patronage may also offer further indirect benefits via reduced pollution, congestion and accidents. Furthermore, by improving transport efficiency it may help improve city density, especially in Britain's second-tier cities which do not tend to benefit from extensive public transit rail and underground networks, with associated agglomeration effects contributing to the current leveling-up priority. Given the salience amongst developed countries of the private aspect of urban public transport in Britain, along with an unresolved private vs public debate, this issue is of potential interest to urban planners and policymakers beyond the UK.

Keywords: Urban Transport; Networks; Pricing; Welfare
JEL #s: D43, L13, L92, R11

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1 Introduction

A common feature of urban public transport systems around the world is the dual existence of services across different modes and/or operators which (i) have a rival or substitute function, but (ii) can also be used in combination and hence have a complementary function. It is well known that in networks with such dual characteristics, independent setting of prices can lead to inefficient equilibrium outcomes, potentially worse than monopoly (e.g., see Economides and Salop, 1992; Socorro and Viecens, 2013; van der Weijde et al., 2013; Clark et al., 2014). Examples in urban public transport arise when Origin-Destination ($O-D$) journeys can be undertaken in two or more component parts (say an x journey followed by a y journey), with different operators within a mode, and/or across modes, providing rival services for these components that can also be used interchangeably to some extent. For instance, a bus and tram operator might each provide return-trip travel between points A and B (e.g., x is the outward, A to B, part and y is the return, B to A, part). Passengers can make their entire $O-D$ journey with a Single Operator (SO), e.g., taking both x and y components using the bus. However, the outward bus x component can also be combined with a return tram y component, and vice versa, creating a Multi-Operator (MO) journey. Where such SO and MO journeys are available, theory suggests (e.g., see McHardy, 2022, henceforth $M22$) that independent and simultaneous price-setting will result, amongst other things, in elevated MO ticket prices and sub-optimal quantities. We refer to this free-market, independent price-setting, scenario as the 'Status Quo' regime (henceforth, SQ), employing it as a stylised representation of current market conditions.

The deregulated urban public transport network in Britain is an example of a market where distortions of the sort outlined above appear to be in evidence. For instance, TAS (2020) report that whilst the availability of MO tickets extends to around three quarters of services, their prices can be significantly higher than for SO tickets. Indeed, the Department for Transport (2013, p. 43) cites examples of MO prices exceeding SO levels by up to 40%. Issues around MO prices on urban public transport in the UK are also explicitly recognised in Department for Transport (2021). This report emphasises the importance for achieving environmental commitments and enhancing mobility, jobs and economic growth of an efficient, well-functioning, urban public transport system. But it also recognises ongoing large-scale failure of the market to deliver towards these ends. It focuses on the urban bus sector, with its relative flexibility (e.g., infrastructure), cost advantages and potential for innovative developments, alongside better integration and coordination with other urban transport modes, as the key channels through which the malaise in urban public transport can be addressed. Organisational change and innovation, amongst other things, are identified as the means of driving the reversal in declining urban bus patronage and elevating bus as the go-to option for urban travel.[1] But it also identifies poor integration and coordination across urban transport services and modes, in particular with regard to MO ticketing, as limiting factors in the functioning of the market.

Whilst private provision of urban public transport is not uncommon across the developing

[1] Whilst there is evidence of large gains from provision of urban public transport in terms of reducing car externalities (e.g., see Adler and van Ommeren, 2016), there is also evidence to suggest some public transport modes might be more effective than others at achieving gains (e.g., see Winston and Maheshri, 2007, who find U.S. urban rail to be potentially damaging to social welfare).

world (e.g., see Gwilliam, 2001), amongst developed economies Britain stands out with widespread private service operation. With the debate on the relative merits of private versus public provision of urban public transportation being far from one-sided (e.g., see Gagnepain et al., 2011), the case of Britain's deregulated services continues to draw interest from around the world with developments therein promising potentially rich new evidence for the pro and anti columns of either side of the debate. Given the prominence of the British bus sector in this debate, as well as among proposed solutions to ongoing urban transport problems, we briefly review the British bus deregulation story highlighting key themes and potential barriers to the development of a well-functioning market. It is against this backdrop that we will set out the contribution of this paper.

Deregulation of the British local bus sector outside of London in 1986, under The Local Transport Act (1985), split networks into 'supported' and 'commercial' services. The former introduced competition via competitive tendering which aimed at helping drive down costs, including via the replacement of less effective operators, within a framework with local authority support. Commercial services, on the other hand, were open to unsupported, free-market, competition. Competition in tendered services lasted relatively well, but despite some early intense competition within commercial networks it was not well sustained. There have been tangible benefits of deregulation in terms of innovation and competition. By 2000, across the whole sector, real-terms unit costs per bus-km had fallen by around 45% (e.g., see White, 2019). However, a number of failings have long been identified, for instance White (2010) cites, amongst these, the inability to coordinate ticketing to the detriment of the user, in particular the large-scale collapse in MO ticketing, and insufficient passenger volumes to support multiple operators coexisting in networks, limiting the scope for competition.

Limited interest amongst operators for MO ticketing had been identified from early on after deregulation (e.g., see Office of Fair Trading, 2009) with the Competition Commission (2011) explicitly recommending the need to make greater use of MO tickets. What use of MO ticketing there had been following the original deregulation was compromised by the terms of 1998 Competition Act with operators fearing falling foul of competition law in undertaking coordination arrangements on MO tickets. Whilst efforts were made to stimulate MO ticket use, notably the Public Transport Ticketing Scheme Block Exemption (Competition Commission, 2001), henceforth Block Exemption, and Competition Commission (2011) recommendations, MO ticket prices have remained relatively expensive.[2] Despite more recent policy changes supporting greater opportunities for coordination (e.g., the Bus Services Act, 2017), MO ticket use has not adequately improved (e.g., see Department for Transport, 2021).

Central to the plans for a well-functioning urban transport market are tackling a set of perennial problems that have broadly frustrated the industry in Britain (outside of London)

[2]Note, alternative explanations for observations of high prices and ineffective levels of price competition in the urban British bus market include the relative attraction of frequency or timetabling competition (e.g., see Mackie et al., 1995; Ellis and Silva, 1998; Gomez-Lobo, 2007). These studies tend to abstract away from the network aspect of the market and multi-stage $O-D$ travel and, in assuming first-takes-all, do not account for price competition in the context of more advanced information systems alerting passengers to potential short waits for a known lower fare. In this paper we demonstrate the importance of the network aspect of the market in driving up prices especially on MO tickets and with profit-harming impacts of SQ pricing which provides incentives to avoid direct competition (e.g., see $M22$).

since deregulation. Three of particular interest are: (i) the large-scale failure to establish effective use of MO ticketing including relatively high MO prices, (ii) the failure of incentives to stimulate provision of more extensive network coverage (geographic and temporal), (iii) inadequate profit incentives to sustain rival operators, and, likely related to these, (iv) the failure to sustain significant advances in passenger usage.

In this paper we develop a pricing structure which we show can help resolve all four of these issues across urban public transport networks. In particular, we show that under the novel pricing system proposed here, lower equilibrium prices, especially MO ones, can obtain relative to SQ as defined above, making it more attractive towards resolving problem (i).[3] However, it is also shown that, with prices under SQ potentially above even monopoly levels on MO services, the reduction in prices under the proposed system can drive, not only consumer surplus gains, but also profit, and, therefore, unambiguous welfare gains. Higher service profitability, of course, makes services, including marginal ones, more viable and incentivises wider and/or denser network provision helping resolve problem (ii) as well as helping support the co-existence of more competitors towards mitigating problem (iii). Of course, lower prices and wider and/or denser network provision should provide impetus for increased patronage, addressing problem (iv). In addition, the analysis reveals the potential merits of trying to achieve better connectivity of existing services including across modes under the SQ pricing regime. However, by raising operator profitability the new pricing system may well incentivise better integration between existing operators who might be deterred from integrating under SQ.[4]

The novel pricing system involves a two-stage price-setting process with operators colluding in one stage on their MO component prices whilst setting their SO prices independently in another stage.[5] The reader might be concerned that such a scheme would face immediate opposition on anti-competitive grounds, yet, in the UK, such a scheme would potentially be allowed under the Block Exemption. Indeed, this is a timely piece as the Block Exemption is due to expire in 2026 and is currently under statutory review. Under this legislation collusion amongst operators is permitted on Multi-operator Ticket Cards (MTCs) and additionally under certain conditions, that we will introduce formally later, which we argue might encompass the proposed new pricing system. We employ the n-operator transport network model due to $M22$, which allows us to capture both substitute and complementary service strategic interaction effects in a differentiated, multi-operator market setting. The $M22$ model provides an n-operator extension of an Economides and Salop (1992)-type network model, where the latter restricts n to two.[6] The Economides and Salop (1992) model has had

[3] The pricing structure is first proposed in a two-operator setting in the unpublished work McHardy et al. (2012).

[4] It is well known, and is also evident in this paper, that in a transport network, better connecting services can damage profit under independent price setting, potentially making some services nonviable (e.g., see Bataille and Steinmetz, 2013).

[5] Lin (2004) and McHardy and Trotter (2006), amongst others, also consider network effects in the airline sector under a two-stage pricing system, however, in the former case, the leader-follower roles are taken by one or other of two airlines, and in the latter, airport pricing is the leader with airlines playing follower. In contrast, here all operators are setting prices in each stage, with the stages separating the pricing of different ticket types.

[6] Economides and Salop (1991) solve an n-firm version of the Economides and Salop (1992) 2-firm model of parallel vertical integration. However, it is important to note that the pricing structure is not directly

widespread application in the transport literature, either directly or in studies which can be nested within it (e.g., see Shy, 1996; Lin, 2004; Mantin, 2012; Bataille and Steinmetz, 2013; Silva and Verhoef, 2013; van den Berg, 2013; Clark et al., 2014; D'Alfonso et al., 2016; van den Berg et al., 2022). We adopt the $M22$ *Independence* regime as our stylised model of the SQ free-market outcome. In this regime all operators set all their prices (their SO price and the price for their component of any MO services which they are involved with providing) independently and simultaneously. We explore pricing and service provision incentives under SQ and the proposed new pricing system, examining the equilibrium outcomes and implications for profit, consumer surplus and welfare. We also study the relative performance of the new pricing system against the MTC and explore the potential for the former to support a larger/denser network than SQ and the MTC with associated potential further welfare benefits.

The following Section introduces the theoretical model. Section 3 introduces the stylised SQ model and Monopoly and sets out the basis for the poor performance of the former regime based on $M22$. A stylised MTC is also introduced. Section 4 introduces the proposed pricing system and explores how this can help alter incentive structures in the market to resolve the problems identified above. Section 5 analyses the potential for different regimes to support the co-existence of different numbers of rival operators with associated welfare effects. Section 6 concludes.

2 Network Model

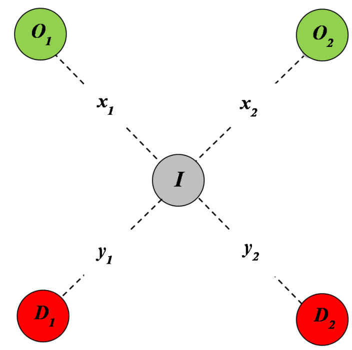
We envisage an urban public transport network with n operators providing SO and MO services based on the n-operator framework in $M22$. Each operator i ($i = 1, ..., n$) provides two services, x_i and y_i, which are differentiated across operators by space and/or time. Combinations of x_i and y_j take passengers from origin, O_i, to destination, D_j, ($i, j = 1, ..., n$). Figure 1 depicts the simple case with two operators ($n = 2$).

For instance, in the case of round-trip travel, transport origins (O_1 and O_2) and destinations (D_1 and D_2) might be the same geographical location but x_1 and y_1 represent outward and return journeys via some interim destination, I, using operator 1 and, x_1 and y_2, the same geographical journey but with the return journey provided by operator 2 at a different time to y_1. Alternatively, urban transport origins and destinations might be geographically distinct. Suppose a passenger, whose home is situated at a point between O_1 and O_2, wishes to travel to a destination located between D_1 and D_2 via the city centre, I. All else being equal, if their home is close to O_1 and the final destination close to D_2, then of the four alternative (substitute) journey plans available via the city centre, they might prefer to use combination $\{x_1, y_2\}$.

Clearly, the x and y journeys are complementary components of a composite $O - D$ service. Piecing together all x and y components gives rise to $N \equiv n^2$ differentiated, sub-

comparable to that used here and in $M22$. In particular, in Economides and Salop (1991), SO prices are made up of a firms' two MO component prices, whereas, in transport settings, it is often the case that an operator's SO price is set independently of its component MO prices. Indeed, the latter has tended to be the convention in the transport literature (e.g., see Flores-Fillol and Moner-Colonques, 2011; van den Berg et al., 2022). Hence, pricing here is akin to that in Economides (1993).

Figure 1: Two Operator Origin-Destination Urban Transport Network



stitute, $O-D$ services. For later convenience it is useful to identify each substitute service combination $\{x_i, y_j\}$, where $(i, j = 1, ..., n)$, with a distinct number $t = 1, ..., N$. Let P_{ij} be the total ticket price for $O-D$ travel using service combination $\{x_i, y_j\}$. Hence, the ticket price for SO travel under operator i is P_{ii}, and for the MO case, $P_{ij} = p_{ij} + q_{ij}$ $(i \neq j)$, where p_{ij} (q_{ij}) is the ticket price of operator i's x-component (operator j's y-component) of the $\{x_i, y_j\}$ journey. Where convenient, and interpretation is clear from context, we use P and P_x (Q and Q_x) to indicate SO and MO prices (quantities) respectively.[7]

For simplicity we assume away costs for the most part. First, with most interest resting on price, profit, consumer surplus and welfare ratios across regimes, with the focus on whether these are above, below or equal to unity (but with little interest in the size of the of deviation from unity), results can easily be seen to be entirely neutral to the introduction of constant marginal costs, a standard assumption for urban public transport (e.g., see Clark et al., 2014), and which has empirical support (e.g., see Jørgensen and Preston, 2003). Second, it is straightforward to show that in the case of non-zero constant marginal costs that are common across all operators, but differ across components x and y, equilibrium $O-D$ prices are unaffected by changes in the distribution of costs between x and y (where the composite cost remains constant). Hence, zero constant marginal cost follows with no further loss of generality. Third, with the structure of an $O-D$ journey taken as common and fixed across all journeys (i.e., interchange and route lengths are common for all services and under all scenarios), including these will not affect where ratios of variables across regimes are equal to, below or above unity, permitting their exclusion without loss of generality. However, in Section 5, where different regimes are compared under different numbers of service operators, we employ a non-zero fixed cost, which, in analysis elsewhere, is zero.

Following $M22$, let differentiation be captured by the following quasi-linear utility function which has the property of consumer surplus increasing in the size of network (at given

[7]Hence, $P = P_{ii}$ and $P_x = P_{ij}$ where $i \neq j$.

prices) such that it captures the added benefits of variety as the network grows with n:[8]

$$U(\boldsymbol{Q}, M_0) = \alpha \sum_t^N Q_t - \frac{1}{2}\left[\sum_t^N Q_t^2 + 2\gamma \sum_t^N \sum_{r>t} Q_t Q_r\right] + M_0 \quad (r \neq t = 1, ..., N) \tag{1}$$

where Q_t is the demand for $O-D$ service combination with index, t, $\boldsymbol{Q}$ is the N-vector of these service quantities, $\gamma \in [0,1]$ represents the degree of substitution between the differentiated services with perfect substitutes (independent services) under $\gamma = 1$ ($\gamma = 0$) and M_0 is expenditure on other goods.[9] Selecting a utility function which allows additional differentiated services to increase consumer surplus at a given price is likely of importance in the current urban transport modelling setting where a more extensive network would be expected to raise consumer surplus not redistribute a fixed surplus. Hence, increasing n in this network will increase consumer surplus directly through the utility function, which values variety, but may also do so indirectly by stimulating competition and reducing prices.

Constrained optimisation yields the linear demands for SO and MO services, respectively:

$$Q_{ii} = a - bP_{ii} \; + d\left[(p_{ij} + q_{ij}) + \sum_{k\neq i} P_{kk} + \sum_{k\neq i,j} (p_{ik} + q_{ik}) + \sum_{k\neq i} (p_{ki} + q_{ki}) + \sum_{k\neq m,i} \sum_{m\neq i} (p_{mk} + q_{mk})\right] \tag{2}$$

$$Q_{ij} = a - b\,(p_{ij} + q_{ij}) + d\left[P_{ii} + \sum_{k\neq i} P_{kk} + \sum_{k\neq i,j} (p_{ik} + q_{ik}) + \sum_{k\neq i} (p_{ki} + q_{ki}) + \sum_{k\neq m,i} \sum_{m\neq i} (p_{mk} + q_{mk})\right]$$

where:

$$a \equiv \frac{\alpha}{(1+\gamma(N-1))}, \quad b \equiv \frac{1+\gamma(N-2)}{(1-\gamma)(1+\gamma(N-1))}, \quad d \equiv \frac{\gamma}{(1-\gamma)(1+\gamma(N-1))} \tag{3}$$

Calculating utility in this N-service setting is aided by the symmetry of the model, and follows from the use of combinatorics, as set out in $M22$, such that utility under regime R can be written:

$$U^R(\boldsymbol{Q}, M_0) = \alpha(nQ^R + n(n-1)Q_x^R) - \frac{1}{2}\left[n(Q^R)^2 + n(n-1)(Q_x^R)^2\right] - \gamma\left[X(Q^R)^2 + Y(Q_x^R)^2 + ZQ^RQ_x^R\right] + M_0 \tag{4}$$

where $X = \frac{n(n-1)}{2}$, $Y = \frac{n(n-1)[n(n-1)-1]}{2}$, and, $Z = n^2(n-1)$.

Let π^R denote total profit across the network under regime R, hence consumer surplus, S, and welfare, W, under regime R are given by:

$$S^R = U^R - M_0 - M^R, \quad W^R = S^R + \pi^R \tag{5}$$

where $M^R = \sum_t P_t^R Q_t^R$ $(t = 1, ..., N)$ is the total expenditure across the urban transport network.

For simplicity we employ the following assumption defining the relevant parameter set for the analysis which follows.

[8] According to Choné and Linnemer (2020) this as a Spence (1976)-type utility function, as employed in a range of industrial and transport studies (e.g., see Hackner, 2000; Silva and Verhoef, 2013), and given it is specified here in an N-dimensional version, recommend citing Shubik and Levitan (1980).

[9] As we capture complementary interrelationships in the network explicitly in the design of $O-D$ journeys, for simplicity we rule out negative values of γ which imply complementarity between $O-D$ services.

Assumption 1. *(i) $\gamma \in (0,1)$, (ii) $n \geq 2$, and (iii) gross substitutes: $b > (n^2-1)d$.*

Assumption 1(i) eliminates the uninteresting case of fully independent services, avoids some discontinuity of equilibria at $\gamma = 1$, and, facilitates simpler statements of results and proofs. Assumption 1(ii) recognises that the model loses essential aspects of interaction in the network making it uninteresting for analysis for $n < 2$ whilst offering substantially greater analytical complexity in some scenarios. Assumption 1(iii) requires that under an equal increase in the prices of all $O-D$ services, the demand for each service is reduced. Henceforth, for all stated results we should interpret the relevant range as determined under Assumption 1.

3 The 'Status Quo' (SQ) Problem & the $\boldsymbol{MTC}$

In this Section we draw on $M22$ to introduce two benchmark regimes: Monopoly and SQ. We envisage the SQ model as a stylised representation of the current conditions in the free-market urban transport setting in Britain (outside London) under deregulation. Analysis of these regimes helps illustrate, not only the potential for the SQ regime to perform poorly in welfare terms, but also indicate the drivers of this performance. On the one hand, this helps motivate the case for policy alternatives but also provides the SQ as a benchmark against which we can measure the performance of our novel pricing system, in particular the extent to which it might help resolve performance issues under SQ, shortcomings which appear to match those identified in the Introduction afflicting the British deregulated urban transport sector.

Under Monopoly, a single operator runs all N services and solves the problem:[10]

$$\max_{\{\boldsymbol{P}\}} \pi = P_t Q_t + \sum_{r \neq t} P_r Q_r - nF, \quad (r \neq t = 1, ..., N) \tag{6}$$

where $\boldsymbol{P}$ is an N-vector of all $O-D$ fares across the N services. The resulting equilibrium has a single, common, $O-D$ price across all services:

$$P^M = \frac{\alpha}{2} \tag{7}$$

Under SQ (equivalent to the 'Independence' regime in $M22$) each operator i solves the following problem:

$$\max_{\{P_{ii}, \boldsymbol{p_{ij}}, \boldsymbol{q_{ji}}\}} \pi_i = P_{ii}Q_{ii} + \sum_{j \neq i} p_{ij}Q_{ij} + \sum_{j \neq i} q_{ji}Q_{ji} - F \quad (j \neq i = 1, ..., n) \tag{8}$$

where $\boldsymbol{p_{ij}}$ and $\boldsymbol{q_{ji}}$ are $(n-1)$-vectors of operator i's MO component prices. The resulting equilibrium SO and MO prices are, respectively:

$$P^{SQ} = \frac{3\alpha(1-\gamma)}{6+\gamma(2n^2-3n-5)}, \quad P_x^{SQ} = \frac{4\alpha(1-\gamma)}{6+\gamma(2n^2-3n-5)} \tag{9}$$

[10]Whilst there are N services across the network, there are only n fixed costs - one associated with the provision of each $\{x, y\}$ pair.

Analysis of these equilibria in $M22$ reveals the following insights. First, Monopoly, as expected, is strictly more profitable, but also dominates the SQ regime in terms of both consumer surplus and welfare below some critical thresholds of substitutability. These thresholds are falling, reducing the parameter set over which Monopoly dominates in consumer surplus and welfare terms, as the number of services increases. To understand what is driving this, note that under the SQ regime, neither of the strategic externalities present in the network are internalised. That is, externalities associated with substitute aspects of the network are at play, which impact prices with downward pressure: the relationship between $O-D$ prices is strategic complements with increasing differences. However, the externalities associated with the complementary aspects are also present under SQ, and these act on the MO prices in an upward direction: the relationship between prices of component x_i and y_j ($i \neq j$) is strategic substitutes with decreasing differences. Under Monopoly, both externality types are internalised. The result is that under SQ, SO prices, via the rival externalities, are lower than the Monopoly level, whilst the MO prices, which exhibit the complementary service externalities, are above Monopoly levels if the services are not sufficiently differentiated (substitutability is not too high).[11]

$M22$ also considers the relative performance of a multi-operator ticketing scheme permitted under the Block Exemption. This exercise derives an equilibrium for a stylised representation of one of the ticketing options allowed under the Block Exemption, the MTC, as introduced earlier, in line with the Competition Commission's recommended pricing framework (see Department for Transport, 2013, pp.22):

$$P^{MTC} = \text{“Average or median single fares x Estimated [typical] ticket usage x Passenger discount for purchasing a multi-journey ticket”} \tag{10}$$

In brief, this involves the operators being able to collude on MO prices, setting them at some discounted level of the weighted average of the SO prices on the network.[12] Note however, this pricing framework is a recommendation, rather than compulsory, as stressed in the Block Exemption guidance (see Competition and Markets Authority, 2016, footnote 33 p. 30), and indeed, engagement with an MTC, is itself, also optional.

We later undertake comparisons between the new pricing system and a stylised MTC, which we now outline here. Operators are envisaged to set their SO prices knowing that the MO price will be set at a discount δ of the weighted average of the SO prices across the market. For simplicity, we consider the case where the discount is zero, and hence the MO price will be the weighted average of the SO prices, which, given symmetry will mean SO

[11] Note, whilst $O-D$ services (service components) are substitutes (complements) their prices are related as strategic complements (substitutes) with increasing (decreasing) differences.

[12] The stylised MTC in $M22$ assumes away any benefits arising from reduced transactions costs under the MTC e.g., only one ticket is needed to be bought instead of multiple tickets over time. It also takes the limiting case where the estimated average number of journeys is one, whilst the Block Exemption requires at least three. However, as the model does not accommodate associated flexibility and transactions cost advantages of the multi-journey aspect of the ticket, this restriction is without further loss of generality.

and MO prices are the same at equilibrium.[13] Operator i solves the problem:[14]

$$\max_{\{P_{ii}\}} \pi_i = P_{ii}Q_{ii} + (n-1)P_xQ_x - F$$

where the x subscript denotes an MO variable, as set out earlier. From $M22$, imposing a zero discount, the equilibrium SO and MO prices under the MTC are the same, hence there is a single price:

$$P^{MTC} = \frac{\alpha(1-\gamma)(2n-1)}{2(2n-1)(1-\gamma)+n^2\gamma(n-1)} \tag{11}$$

from which it is immediately apparent that P^{MTC} lies strictly below the Monopoly price in the relevant range.

Given the above discussion about undesirable upward pressure, for operators and passengers, on MO prices under SQ, this MTC has a clear potential attraction. The MTC pricing framework puts controls on the MO price, which under SQ can exceed the Monopoly level to the detriment of profit and consumer surplus. However, in their analysis $M22$ show that whilst the MTC helps improve consumer surplus and welfare relative to SQ, it *can* be damaging in terms of profitability. This potentially helps to explain why it is that in practice MO ticketing has not enjoyed more widespread use, at least under the recommended pricing framework of Eq. (10). If Eq. (10) held then this would not fit the evidence, cited earlier, of MO prices exceeding SO levels.

Finally, $M22$ undertake a calibration exercise and demonstrate that applying long-run own price elasticity of demand estimates to the SQ model results in it indicating required levels of substitutability around the region where potentially, (i) SQ under-performs Monopoly, and, (ii) the MTC is less attractive in profit terms than SQ, again perhaps helping to explain its apparent limited take-up in the form of Eq. (10). The remainder of the paper applies the $M22$ framework to the new pricing system which we solve and analyse to see the extent to which it might help resolve the issues around the poor performance of the SQ regime but might also help improve on the MTC.

4 Two-Stage Pricing with Collusion

We now introduce a two-stage model of price setting in which operators are permitted to collude in setting prices on their MO tickets in one stage but independently set their SO ticket prices in the other stage. We consider two alternative interpretations of the pricing system and denote them according to the period in which prices are set collusively. Hence, regime $C1$ ($C2$), has operators colluding to setting MO prices in period 1 (2).[15] As we have noted,

[13]It is straightforward to show that profit is decreasing in the size of the discount in the MTC calculation. Given the issue of interest around the MTC here is a paucity of evidence supporting the willingness of operators to join an MTC which sets prices in line with Competition Commission recommendations, we focus attention on the most profitable interpretation, a zero discount.

[14]Under symmetry, with each operator sharing equally the revenue from each of the $2(n-1)$ MO services they contribute toward, total revenue for an operator from their MO patronage is $\frac{1}{2}[2(n-1)]P_xQ_x$.

[15]Note, it is straightforward to see (and prove) that the equilibrium under $C2$ is identical mathematically to the outcome under which all prices are set simultaneously, with MO prices set collusively and SO prices independently. In practice, however, the latter is not likely to be an appealing prospect as maintaining the integrity of independent SO pricing would appear problematic.

in Britain transport operators are already allowed to discuss and agree prices on MTCs under the Block Exemption (e.g., see Competition and Markets Authority, 2016, point 4.32 pp. 32), but also, potentially more widely, if in so doing, they satisfy four conditions of the Competition Act (1998) (Competition Commission, 1998) Section 9(1), which may well apply here:[16] (i) efficiency gains, e.g., through reduced transactions costs or provision of additional services, (ii) no elimination of competition, (iii) a fair share for consumers, envisaged under the MTC, for instance, as the agreed MTC price being discounted relative to SO prices, and, (iv) that outcomes must not create restrictions beyond what is needed to bring the associated benefits of ticketing arrangements. Hence, collusion required to achieve 'good' MO outcomes must not leak anti-competitiveness into other aspects of decision-making.[17] We will later review the new regimes in terms of their outcomes against each of the above conditions.

The motivation for the proposed pricing structure comes from the earlier recognition that the elevation of MO prices under SQ is driven by decreasing differences across operators' MO price components: collusive pricing on these components would result in lower, not higher prices. This strategic externality is internalised under Monopoly, but, of course, Monopoly also suffers from internalising the strategic complements (increasing differences) externality across the rival service prices which would result in SO prices being lower without collusion. It will be important to understand how increasing the number of operators in this context works in terms of whether it emphasises one effect over the other (strategic substitute versus strategic complement) and hence increasing n favours the performance of SQ or the collusive models. In particular, note that whilst an increase in n under SQ increases the number of rival $O-D$ services by $2n-1$, it also raises the number of complementary interactions within MO price setting by $2(n-1)$. The former adds to downwards pressure on prices and the latter, upward pressure. Under the collusive models, with all the MO prices being set jointly, the addition of an operator adds nothing to the complementary service externality (and associated upward pressure on prices), but has a significantly muted impact on rival strategic interaction, relative to SQ, adding just one new rival service where price is set independently. A priori, it is not obvious how the zero externality across complementary services and muted rival externality under collusion will play against the large increases in both rival and complementary service strategic interactions under SQ, and hence, how changes in n may alter the relative prices under the different regimes.

4.1 Stage 1 Collusion

Beginning with regime $C1$, the operators at stage one of the game will collude to set the MO price for the network, P_x.[18] At stage two, each operator will independently set their SO price taking P_x as given. Solving by backward induction involves finding the profit maximising selection of SO fares that operators will set at stage two taking P_x as given. Hence, operator

[16] See Competition and Markets Authority (2016), which provides guidance on these conditions.

[17] The guidance also states that routes covered by operators participating in an MTC should not be too similar (see Competition and Markets Authority, 2016, point 3.16 pp. 14).

[18] Given the symmetry of the model this is a single price, common across all MO services.

i solves the problem:[19]

$$\max_{\{P_{ii}\}} \pi_i = P_{ii}Q_{ii} + (n-1)P_xQ_x - F \tag{12}$$

yielding the following first-order condition:

$$Q_{ii} - bP_{ii} + (n-1)dP_x = 0 \tag{13}$$

The operators, jointly optimising profit at stage one, will identify the equilibrium SO price as a function of P_x associated with Eq. (13), which given symmetry ($P_{ii} = P_{jj} \quad \forall i, j = 1, ..., n$) is:[20]

$$P(P_x) = \frac{a + d(n^2-1)P_x}{2b - d(n-1)} \tag{14}$$

The associated optimising stage-two response of operators in their setting of SO prices, to a change in P_x, is then:

$$A \equiv \frac{\partial P(P_x)}{\partial P_x} = \frac{d(n^2-1)}{2b - d(n-1)} \tag{15}$$

The problem the operators collectively solve at stage one is then:[21]

$$\max_{\{P_x\}} \pi = nP(P_x)Q + n(n-1)P_xQ_x - nF \tag{16}$$

Deriving the first-order condition, employing Eq. (15), and then recognising symmetry (i.e. $P_{ii} = P_{jj}$ $\forall i, j = 1, ..., n$), we have:

$$AQ + [d(n-1)(n+A) - bA]P + (n-1)Q_x + (n-1)[d(nA + n(n-1) - 1) - b]P_x = 0 \tag{17}$$

Solving Eqs. (14) and (17) simultaneously results in the following equilibrium SO and MO prices in the model with collusion at stage one:

$$P^{C1} = \frac{1}{\nabla}\alpha(1-\gamma)(2 + \gamma(n^2+n-4)), \quad P_x^{C1} = \frac{1}{\nabla}2\alpha(1-\gamma)(1+\gamma(n^2-2)) \tag{18}$$

where $\nabla \equiv 4 + \gamma(4n^2-12) + \gamma^2(n+1)(n^2-7n+8)$.

Lemma 1. *The MO price under regime $C1$ is strictly lower than the Monopoly level in the relevant range: $P_x^{C1} < P^M$.*

Hence, trivially, we have established the positive observation, that unlike the case of SQ, incidences of prices on MO services exceeding monopoly levels do not arise under $C1$.

Interior solutions for Q_x^{C1} require γ to not be too high as, with very close substitutes, P^{C1} slightly below P_x^{C1} drives Q_x^{C1} to zero. Interior solutions require $\gamma < \tilde{\gamma}(n) \equiv \frac{n^2-3+\sqrt{n^4-2n+1}}{3n^2-n-4}$, where $Q_x^{C1} = 0$ at $\tilde{\gamma}(n)$. All Figures illustrating hypothetical realisations of $C1$ variables are right-truncated at the point MO quantities reach zero. Note, we will later see, under a calibration exercise, that real-world elasticity estimates suggest the market may be operating at levels of substitutability (well) below $\tilde{\gamma}(n)$.

[19]As before, given symmetry, an individual operator receives half the fare revenue for each of the $2(n-1)$ MO services it provides components for.

[20]This gives rise to the closed-loop solution to the two-stage game (e.g., see Fudenberg and Tirole, 1991, pp. 132). Note, the same outcome ensues if we don't recognise symmetry at this stage, but is more cumbersome.

[21]Again, we assume operators share MO revenues on a pro rata basis, which given symmetry means equal shares.

4.2 Stage 2 Collusion

We now consider the reverse scenario where, in the first stage operators independently select their SO prices knowing they will jointly maximise profit, setting MO prices, P_x, collusively in the second stage, taking as given the first-stage SO prices.

Solving by backward induction, at stage two the operators collectively solve the following problem:

$$\max_{\{P_x\}} \pi = P_{ii}Q_{ii} + \sum_{j \neq i} P_{jj}Q_{jj} + n(n-1)P_xQ_x - nF \quad (i \neq j = 1, ..., n) \tag{19}$$

giving rise to the first-order condition:

$$dn(n-1)\left(P_{ii} + \sum_{j \neq i} P_{jj}\right) + n(n-1)[Q_x + P_x(d(n^2-1) - b)] = 0 \tag{20}$$

Hence, operator i at stage one will factor in that a change in its SO price will have the following impact:

$$B \equiv \frac{\partial P_x}{\partial P_{ii}} = \frac{d}{b - d(n(n-1) - 1)} \tag{21}$$

At stage one the operators independently solve the following problem, taking B as given:

$$\max_{\{P_{ii}\}} \pi_i = P_{ii}Q_{ii} + (n-1)P_xQ_x - F \tag{22}$$

The first-order condition for operator i, using Eq. (21), yields;

$$Q_{ii} - P_{ii}[b - dB(n(n-1))] - (n-1)P_x[bB - d - dB(n(n-1) - 1)] + (n-1)BQ_x = 0 \tag{23}$$

Recognising symmetry (i.e., $P_{ii} = P_{jj}\ \forall i, j = 1, ..., n$) and solving Eqs. (20) and (23) simultaneously, the equilibrium SO and MO ticket prices under regime $C2$ are, respectively:

$$P^{C2} = \frac{1}{\Delta}\alpha(1-\gamma)(2 + \gamma(n^2 + 2n - 3)), \quad P_x^{C2} = \frac{1}{\Delta}\alpha(1-\gamma)(2 + \gamma(2n^2 + n - 3)) \tag{24}$$

where $\Delta \equiv 2(1 + \gamma(n^2 - 1))(2 + \gamma(n - 3))$.

4.3 Analysis and Findings

Having recognised a key source of the problem under SQ is via the MO price distortion, we now explore the pricing outcomes under the collusive regimes. For the remainder of this subsection we assume fixed costs are zero. We begin by comparing the performance of the two new regimes using Eqs. (18) and (24).

Proposition 1. *(i) $C2$ is a strictly lower price regime than $C1$: $P^{C1} > P^{C2}$, $P_x^{C1} > P_x^{C2}$. (ii) Within each collusive price regime MO prices are strictly greater than their equivalent SO prices: $P_x^{C1} > P^{C1}$, $P_x^{C2} > P^{C2}$.*

Proposition 1 leads to the following observation.

Corollary 1. *The collusive regimes do not produce price outcomes in line with the UK Competition Commission's recommended MTC pricing framework which would require* $P^{MTC} \equiv P_x^R < P^R$ $(R \in \{C1, C2\})$.

Hence, given the freedom to collude on MO prices under the MTC, firms might not be incentivised to adopt the recommended pricing structure which explicitly has the MO price at a discount of the SO prices. Further, from Lemma 1 and Proposition 1, it follows that:

Corollary 2. *All equilibrium prices under regimes* $C1$ *and* $C2$ *are strictly below the monopoly level in the relevant range:* $P^{C1}, P_x^{C1}, P^{C2}, P_x^{C2} \in (0, P^M)$.

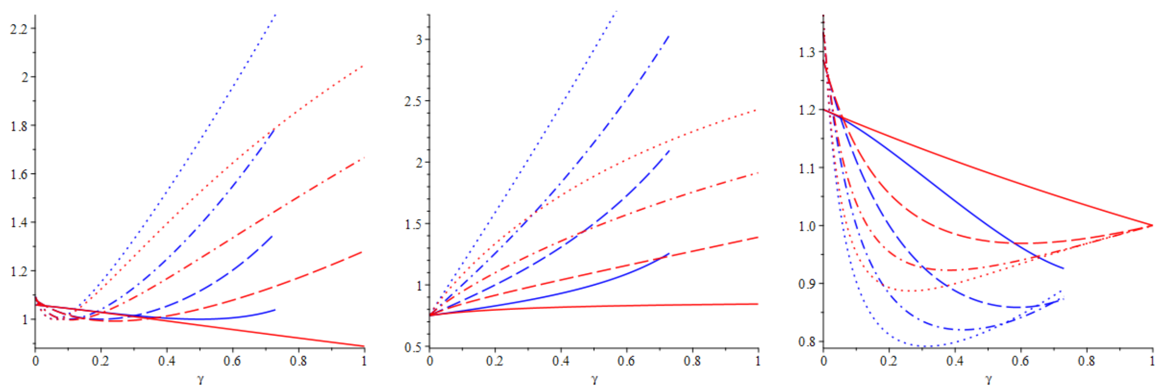
We are now ready to discuss Proposition 1, where both MO and SO prices are unambiguously lower under stage 2 collusion than under stage 1 collusion. A priori, one might have expected this result. Given that the two-stage regime $C2$ is mathematically equivalent to simultaneous (i) independent SO pricing with (ii) collusive MO pricing, it can't be the case that under this regime operators are able to exploit any potential for the two-stage process to combat downward pressures on prices via externalities across substitute services. Given both collusive regimes internalise externalities across complementary services, then we would not expect any prices above the Monopoly level (as confirmed in Corollary 2), though these are the driver of the above Monopoly MO prices under SQ. Hence, if there is any leverage in the two-stage set-up to combat competitive downward forces on prices, we would expect this to play out in regime $C1$ with both prices higher, and therefore closer to Monopoly levels, than $C2$ equivalents. What is happening here is that when the operators collude at stage one, they are able to exploit that the prices they set are taken as given by the independent SO price setters at stage two, and can therefore control the environment in which this happens. They do this by minimising the damage of the independent stage-two decision-making via exploitation of the operators' stage-two best response functions, akin to von Stackelberg (1934). Of course, whichever the stage in which the MO prices are set collusively, those prices are higher than corresponding SO prices. This means that, given the opportunity to collude on MO prices, with collusion at stage one or stage two, the operators would not choose to set MO prices at or below SO prices (Lemma 1), as recommended by the Competition Commission in Eq. (10). This perhaps offers some insight into why the evidence appears to suggest operators are not opting to adopt Eq. (10) when setting MO prices.

Comparison of Eqs (9), (18) and (24) gives rise to the following Proposition about equilibrium prices under the different regimes.

Proposition 2. *(i) For* $n = 2$, $C2$ *prices are everywhere lower than their SQ equivalents:* $P^{C2}|_{n=2} < P^{SQ}|_{n=2}$ *and* $P_x^{C2}|_{n=2} < P_x^{SQ}|_{n=2}$*, otherwise, (ii) collusive regime prices are lower than their SQ equivalents below critical thresholds of substitutability which are strictly decreasing in* n*:* $P^R < P^{SQ}$ *and* $P_x^R < P_x^{SQ}$*, for* $\gamma < \gamma_1^R$*, where* $\partial\gamma_1^R/\partial n < 0$ $(R \in \{C1, C2\})$*, and, (iii) for any given* n*, the substitutability threshold for* $C1$ *is strictly lower than for* $C2$*:* $\gamma_1^{C1} < \gamma_1^{C2}$.

Hence, outside the special case under $C2$ with $n = 2$, where prices are everywhere strictly lower than under SQ, both collusive pricing regimes do offer lower SO and MO prices than SQ for sufficiently low levels of substitutability. The critical levels of substitutability are strictly more restrictive for the regime with collusion at stage one than at stage two. Both thresholds become more constraining with higher numbers of operators, lowering the available levels of product differentiation which support collusive prices being lower than their SQ equivalents. Relating the role of substitutability to our earlier discussion, recall, under collusion, the externality across complementary services is internalised but the number of rival service prices being set is muted, relative to SQ. The lower prices under collusion result when (i) the benefits of colluding on MO prices, internalising the damaging externality across complementary services, outweighs (ii) the anti-competitive effects of collusion, through the smaller number of rival prices being set independently. For sufficiently low levels of substitutability the rival substitute effects, placing downward pressure across the n^2 prices under SQ, (ii) , which are muted under collusion, are of relatively less importance than the complementary effects that are eliminated under collusion, (i), resulting in lower prices under the latter. Similar reasoning applies to the case of an increase in the number of operators for a given level of substitutability. The pro-competitive effects of adding an extra operator under SQ with $2n-1$ more independently priced substitutes relative to only one more under collusion, need to be dampened by lower levels of substitutability for prices under collusion to remain below SQ levels.

Figure 2: Prices and Total Quantities under $C1$ and $C2$ relative to SQ



(a) SO Prices (b) MO Prices (c) Aggregate Quantities

—— $n=2$, - - - $n=3$, -·-·- $n=4$, ······ $n=5$, Blue=C1, Red=C2

Figure 2 illustrates price and aggregate quantity ratios under collusion relative to SQ for $n \in \{2,3,4,5\}$. Amongst other things, this reveals the broad range of substitutability values for which MO prices are improved by the two-stage pricing system, relative to SQ, especially for low n and under second-stage collusion. Indeed, the collusive regimes yield lower prices and higher quantities than SQ at low levels of substitutability in the region in which the latter under performs relative to Monopoly (see $M22$). It is also important to note the role of increasing n in making SQ relatively attractive, especially in the case of comparisons with the first-stage collusion regime. However, in the Introduction, we reported the argument that a lack of operators in the British deregulated bus market is due, in part, to the inability for the market to sustain larger operator numbers. Hence, if it is not possible to incentivise an increased number of operators from inadequately low levels under SQ, then there is an argument for employing one of the collusive regimes, an issue we return to in the following section.

Having observed, in Corollary 1, that profit-maximising operators with the option to collude on MO prices do not appear to select prices according to the Competition Commissions recommendations, Eq. (10), we now turn to an explicit analysis of price comparisons across these regimes. Whilst it would appear likely that collusive MO prices are higher than the MTC levels, it is not clear where relative SO prices sit across the regimes. This is because constraints placed on the MO price under the MTC may incentivise higher SO prices.

Proposition 3. *Under the collusive regimes $R \in \{C1, C2\}$: (i) MO prices are strictly greater than the MTC price in the relevant range: $P_x^R > P^{MTC}$, and, (ii) SO prices are strictly lower than the MTC price below a threshold contour γ_2^R, with $\gamma_2^{C1} < \gamma_2^{C2}$, and the thresholds are strictly decreasing in n: $\partial\gamma_2^R/\partial n < 0$.*

The reasoning here is similar to that discussed above in relation to the comparison between the collusive regimes and SQ, if a little more straightforward. In this case there are no externalities across complementary aspects of services in any regime, and the number of independently set prices is the same in each case, n. Indeed, prices across these regimes are also all strictly below the Monopoly level in the relevant range. Given this, firms are going to want to push prices withing this system higher, closer to P^M. From Proposition 3, it is clear that the collusive regimes are better at achieving this on MO prices than under the MTC, as these are the prices they are allowed to collude on. The only mechanism for upward pressure on prices under the MTC is via higher SO prices. Hence, across the regimes, the role of SO prices in achieving higher network prices lies much more with the MTC than the collusive regimes. However, whilst an individual operator raising their SO price under the MTC has an upward impact on the MO price and profit on their MO component services, it penalises their SO profits given rival operators' SO prices are now lower. At higher levels of substitutability or with higher numbers of firms the latter effect is relatively dominant and the SO price channel for raising prices under the MTC is weak: SO prices are driven below levels on the collusive regimes. On the other hand, if substitutability and the number of rival operators is sufficiently low, the incentives support higher SO prices under the MTC.

We now turn to how the price differences across the regimes manifest in terms of profit, consumer surplus and welfare outcomes.

Proposition 4. *Whilst collusive pricing regime $C1$ is (i) strictly superior to $C2$ in terms of profit: $\pi^{C1} > \pi^{C2}$, it is strictly inferior to $C2$ in terms of (ii) consumer surplus: $S^{C2} > S^{C1}$, and (iii) welfare: $W^{C2} > W^{C1}$.*

Hence, as we anticipated, based on the ability of operators to influence SO prices under stage-one collusion, regime $C1$ is a higher profit and lower welfare regime than $C2$. However, note that higher profitability might facilitate supporting a larger/denser network with potential associated welfare gains. We return to this point in the following Section.

Turning to profit comparisons across SQ and the collusive regimes we present the following Proposition.

Proposition 5. *(i) Profit under $C1$ is strictly (weakly) greater than under SQ for $\gamma \neq \gamma_1^{C1}$ ($\gamma = \gamma_1^{C1}$). (ii) Profit under $C2$ is strictly greater than under SQ for $\gamma_3^{C2} > \gamma > \gamma_1^{C2}$. (iii) The key substitutability thresholds are ordered as follows: $\gamma_3^{C2} < \gamma_1^{C1} < \gamma_1^{C2}$.*

Hence, $C1$ weakly dominates SQ for profit, whilst profit under $C2$ falls below SQ levels between two contours (the interval between the solid and dashed cyan contours in Figure 3 representing γ_3^{C2} and γ_1^{C2}, respectively), but otherwise $C2$ is more profitable than SQ, too.

The following Proposition sets out the corresponding analysis for consumer surplus and welfare across the regimes.

Proposition 6. *Under the collusive regimes $R \in \{C1, C2\}$: consumer surplus and welfare are strictly greater than under SQ below some substitutability threshold ($K \in \{S, W\}$):*

$$K^{C1} > K^{SQ} \text{ for } \gamma < \gamma_1^{C1}$$
$$K^{C2} > K^{SQ} \text{ for } \gamma < \gamma_1^{C2}$$

Combining Propositions 5 and 6 yields the following result.

Corollary 3. *(i) The collusive regimes offer a win-win opportunity relative to SQ, with higher profit, consumer surplus and welfare, below some substitutability threshold ($K \in \{S, W\}$):*

$$K^{C1} > K^{SQ}, \text{ for } \gamma < \gamma_1^{C1}$$
$$K^{C2} > K^{SQ}, \text{ for } \gamma < \gamma_3^{C2}$$

(ii) The win-win substitutability threshold is more restrictive for $C2$ then $C1$: $\gamma_3^{C2} < \gamma_1^{C1}$

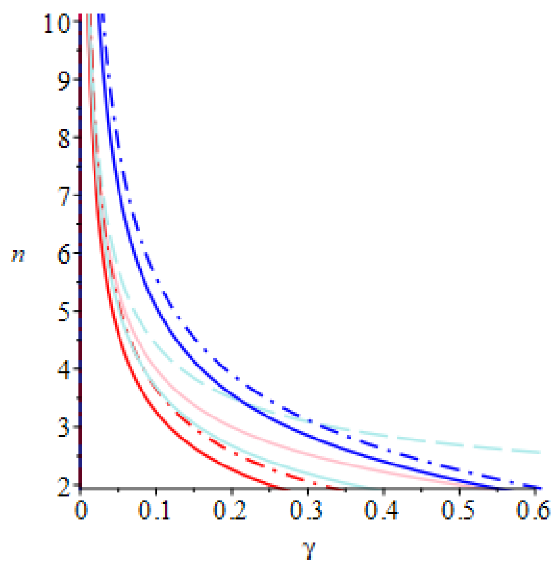
Hence, the lower prices under collusion relative to SQ, for sufficiently low levels of substitutability, translate into, not only higher consumer surplus, but also profit and therefore welfare. Regime $C1$ is more robust to higher levels of substitutability in terms of this win-win outcome, relative to SQ, whilst $C2$ offers consumer surplus and welfare gains over SQ for a wider range of substitutability than $C1$. Increasing the number of operators reduces the scope of the win-win outcome. Hence, if it is not possible to accommodate higher n under SQ and γ is below γ_3^{C2}, then either of the collusive models will improve profit and consumer surplus and the regulating body might choose between the two based on whether emphasis is particularly on enhancing consumer surplus ($C2$) or enhancing profit ($C1$), with the latter perhaps to incentivise an increase in the number of operators and services. If it is thought that γ is below γ_1^{C1}, but might not be below γ_3^{C2}, then $C1$ would be the safe bet to ensure win-win gains. However, in such circumstances, if consumer surplus gains are sought over profit incentives, then $C2$ might be selected as it ensures higher consumer surplus and welfare but profit may be lower than under SQ.

We now reproduce the calibration exercise in $M22$ which takes long-run own-price elasticity of demand estimates for transport modes and fits this data to the equilibrium under SQ with constant marginal cost, c (see $M22$ Appendix B for a derivation).[22] This results in contours in (γ, n)-space which the market would be on approximately, if it behaved according to the SQ model at a given elasticity, η. Let $n_{\eta,c}^{CAL}(\gamma)$ (with inverse $\gamma_{\eta,c}^{CAL}(n)$) represent the number of operators identified under the calibration assuming the market is operating in line with SQ and for a given level of elasticity and constant marginal cost. In Figure

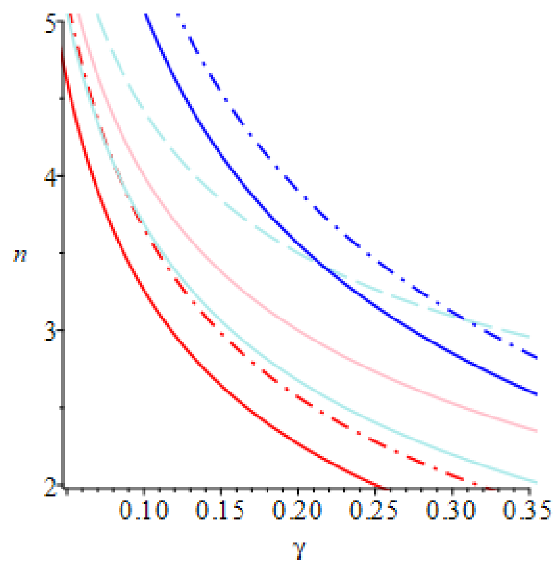
[22]Marginal cost relates to the cost to service providers of providing one x and one y component passenger journey. Hence, it is the marginal cost of a single $O-D$ trip.

3, we reproduce the contours for the following parameter selections: (i) constant marginal cost, $c \in \{0, \frac{\alpha}{20}\}$, and, (ii) elasticity, $\eta \in \{-0.6, -1\}$.[23] Contours with $\eta = -0.6$ are blue and those with $\eta = -1.0$ are red, and solid lines represent $c = 0$ whilst dash-dot lines represent $c = \frac{\alpha}{20}$. These are plotted alongside the contour thresholds, $\gamma_1^{C1}$and γ_3^{C2} (solid pink and cyan, respectively), below which collusive outcomes yield superior profit, consumer surplus and welfare outcomes to SQ in (γ, n)-space. As noted earlier parameter combinations between the contours γ_3^{C2} and γ_1^{C2} (dashed cyan line) under $C2$ yield higher consumer surplus and welfare than SQ but lower profit.

Figure 3: Calibration contours $n_{\eta,c}^{CAL}(\gamma)$ for $\eta \in \{-0.6, -1\}$ and $c \in \{0, \frac{\alpha}{20}\}$, Collusion Regime Threshold Contours $n(\gamma_1^{C1})$, $n(\gamma_1^{C2})$ and $n(\gamma_3^{C2})$



(a) Wide view



(b) Close-up

$n_{-0.6,0}^{CAL}(\gamma)$, $n_{-0.6,\frac{\alpha}{20}}^{CAL}(\gamma)$, $n_{-1.0,0}^{CAL}(\gamma)$, $n_{-1.0,\frac{\alpha}{20}}^{CAL}(\gamma)$, $n(\gamma_1^{C1})$, $n(\gamma_3^{C2})$ $n(\gamma_1^{C2})$

From Figure 3, under $\eta = -1.0$ and zero marginal cost, the market would be operating at a point on the solid red line, which appears to sit, at least over some range of substitutability, below both threshold contours giving win-win outcomes under both collusive regimes. It is also clear that win-win outcomes are also available under non-zero costs, although the introduction of non-zero marginal costs and less elastic demand appear to make this less likely. However, earlier we noted that $C1$ may not produce interior solutions for sufficiently high levels of substitutability. Therefore, in formally unpicking the implications of the win-win observation in the Figure, we begin with the following Lemma.

[23]The selection of elasticities is based on evidence of long-run bus elasticities in the UK of around -1, (e.g., see Paulley et al., 2006; Dunkerley et al., 2018), with the lower elasticity value corresponding to lower-end estimates in the U.S.: e.g., Goodwin (1992), who finds long-run price elasticities for bus and rail to be between -0.6 and -1.1.

Lemma 2. *Regime $C1$ (i) improves profit, consumer surplus and welfare relative to SQ for levels of substitutability where the former produces internal solutions: $\gamma_1^{C1} < \min \tilde{\gamma}(n)$, and, (ii) produces interior solutions over the entire set of γ for which all four calibrations suggest the market might be operating: $\min \tilde{\gamma}(n) > \gamma_{\eta,c}^{CAL}(n)|_{n=2}$ for $\eta \in \{-0.6, -1.0\}$ and $c \in \{0, \frac{\alpha}{20}\}$.*

Hence, regime $C1$ produces the win-win outcome over SQ within the parameter selection consistent with interior solutions, and, none of the calibrations suggests the market is operating at a point where the regime might not produce interior solutions.

We can now formalise the earlier observation around the win-win potential for the collusive regimes.

Proposition 7. *If the market is accurately charaterised by the SQ regime, marginal cost is constant and sufficiently close to zero, and the own-price elasticity of demand is -1.0, then implementing either collusive regime will strictly increase profit, consumer surplus and welfare, relative to SQ.*

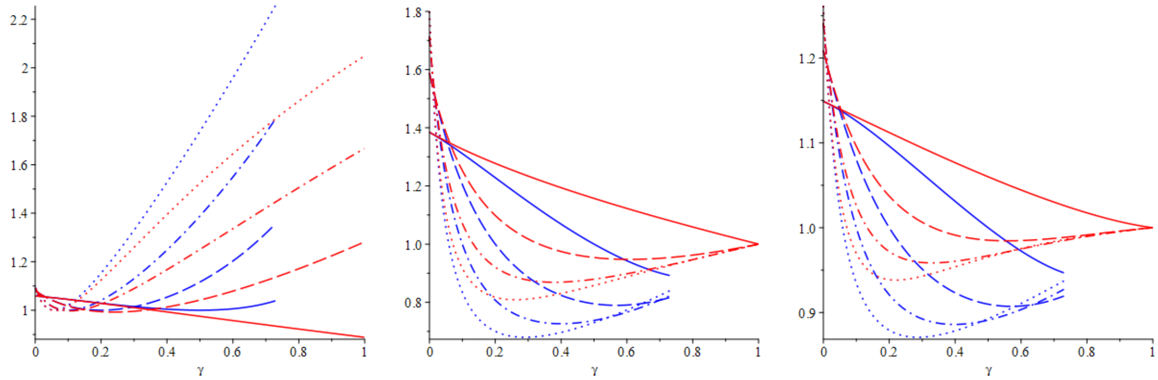
It is straightforward to show that, even with constant marginal cost of $\frac{\alpha}{20}$, $C1$ everywhere still offers a win-win relative to SQ with $\eta = -1.0$. The win-win is still available too, under $C2$, with consumer surplus and welfare gains everywhere, although, for sufficiently low substitutability and high n there are some parameter combinations with profit below SQ levels. The dashed cyan line representing $n(\gamma_1^{C2})$, is more substantially above the $\eta = -1.0$ calibration contour than the pink $n(\gamma_1^{C1})$ contour, guaranteeing higher welfare and consumer surplus under $C2$, but possibly not higher profit, than SQ. Indeed, even under $\eta = -0.6$ and marginal costs of $\gamma = \frac{\alpha}{20}$, there are welfare and consumer surplus gains under $C2$ relative to SQ as, for sufficiently high γ, $n(\gamma_1^{C2})$ lies above $n^{CAL}_{-0.6,\frac{\alpha}{20}}$. Figure 4 reports profit, consumer surplus and welfare for each collusive regime relative to SQ. Note that the blue 'u'-shaped curves in Figure 4(a) are tangent to the horizontal line at 1, and in (b) and (c) the red 'u'-shaped curves return to meet unity only at $\gamma = 1$, which lies outside the relevant parameter set under Assumption 1. In Figure 4(a) the red 'u'-shaped lines sink very slightly below unity consistent with the lower profit under $C2$ relative to SQ between the two critical levels of substitutability: γ_1^{C2} and γ_3^{C2}. It's apparent from the Figure that for low levels of n the parameter set of substitutability over which collusive regimes dominate SQ on consumer surplus and welfare is quite extensive. Indeed, as we have seen, the collusive regimes dominate on consumer surplus and welfare grounds under the parameterisations associated with the calibration with $\eta = -1.0$ with constant marginal cost of zero.

Turning to the comparisons of the collusive regimes with the MTC, we have the following result.

Proposition 8. *Under the MTC, consumer surplus and welfare are (profit is) strictly greater (lower) than under the collusive regimes in the relevant range: $W^{MTC} > W^R$, $S^{MTC} > S^R$, $\pi^{MTC} < \pi^R$, $R \in \{C1, C2\}$.*

For the MTC, engagement with which is non compulsory under the Block Exemption, consumer surplus and welfare are strictly higher than under either collusion regime but profit is strictly lower. Contrast this with the comparison of SQ with MTC in $M22$, where welfare and consumer surplus are superior under the MTC, however, profit is not everywhere

Figure 4: Profit, Consumer Surplus and Welfare under $C1$ and $C2$ relative to SQ



(a) Profit (b) Consumer Surplus (c) Welfare

—— $n = 2$, - - - $n = 3$, -·-·- $n = 4$, ······ $n = 5$, Blue=C1, Red=C2

superior under SQ. The point remains that, despite its potential to raise consumer surplus and revenue, the MTC, in the form recommended by the Competition Commission, does not appear to be widely evident in practice, perhaps victim of incentive incompatibility issues associated with operator profit under Eq. (10). However, whilst making the MTC compulsory on the recommended model clearly offers welfare gains through lower prices, it may harm provision in the face of fixed costs potentially having unintended consequences, lowering the number of services and overall lowering consumer surplus and welfare.

The story does not end here though. We have seen the potential for collusive pricing to improve welfare relative to SQ, and we have seen the capacity for it to do so through a profit channel with consumer surplus gains as well when all regimes offer the same services. And though the collusive regimes offer profit but not welfare gains over the MTC under common service provision, where SQ or MTC have left a network inadequately served in terms of network coverage or density, due to inadequate profit, the above analysis suggests possible additional gains reachable via the collusive pricing structure in terms of its higher profitability incentivising/sustaining a larger/denser network. We turn to this issue in the next section. It also follows from the analysis that there are clear consumer benefits from facilitating better connectivity on urban transport networks between existing services/modes which is more likely to be incentive compatible under $C1$ and $C2$ than SQ or MTC where associated profitability is higher.

Finally, it is well known that there are other benefits to urban public transport provision beyond direct consumer surplus and profit e.g., reduced congestion, pollution and accidents from attracting passengers to switch from private car to public transport. These factors are not captured in our welfare analysis but are generally thought to be increasing in use of the public transport mode. For instance, the sizeable externality benefits of urban public transport identified in Adler and van Ommeren (2016) suggest that the potentially substantial increases in total quantities under regimes $C1$ and $C2$, see Figure 2(c), relative to SQ under low levels of substitutability, could well add heavily to the welfare benefits of the proposed pricing regimes.[24] Indeed, by improving transport efficiency the proposed pricing structure may help improve city density, especially in Britain's second-tier cities which do not tend to benefit from extensive public transit rail and underground networks and so are more reliant on bus provision, with associated agglomeration effects improving productivity (e.g., see Glaeser and Gottlieb, 2009) contributing to the current leveling-up priority in Britain.

5 Network Expansion with Fixed Costs

Up until now we have compared the regimes in terms of key performance indicators assuming each regime supports the same level of service provision. However, we have also found that profits vary across regimes, which in the presence of non-zero fixed costs may have implications for the number of services that are sustainable under each regime, with the potential that a higher profit regime may result in a higher number of operators and services with possible additional welfare benefits. A priori, we cannot say that affording additional services will result in higher welfare, as the utility gains associated with additional services, assumed through our choice of utility function and potentially enhanced by increased competition, must be offset against the additional fixed cost of the new x and y component. Of course, in the case of regimes that are better disposed to generating profit, it is certainly not clear the extent to which any additional surplus accruing through additional services will be appropriated by the operators as profit, and so we will be interested in potential welfare gains associated with having more services but also whether these result in gains to passengers in terms of consumer surplus relative to the position under an alternative regime that cannot sustain the larger network.

In order to investigate the potential of the collusive regimes to support larger networks than the, broadly lower (strictly lower) profit, SQ (MTC) regime, we set a fixed cost such that n operators would not be viable at the equilibrium under SQ and the MTC regimes. Following $M22$ we define these prohibitive fixed costs for SQ and MTC, as, respectively:[25]

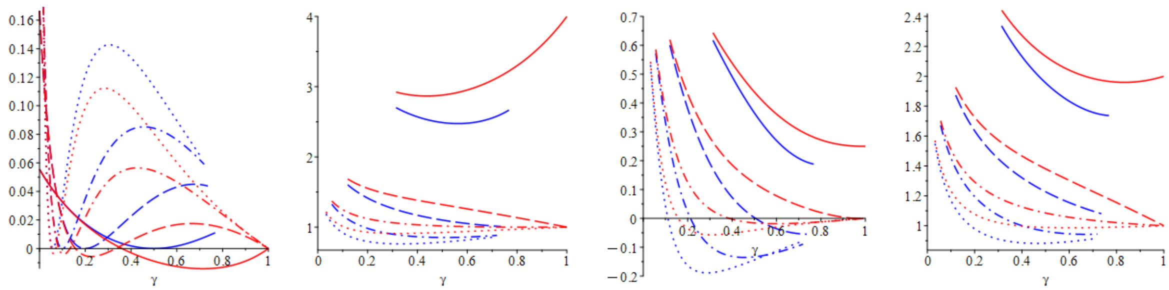
$$\begin{aligned}F_n^{SQ} &\equiv \frac{(8\gamma n^3 - 15\gamma n^2 - 2\gamma + 8n + 1)(1-\gamma)\alpha^2 n}{(2\gamma n^2 - 3\gamma n - 5\gamma + 6)^2(\gamma n^2 - \gamma + 1)} + \epsilon \\ F_n^{MTC} &\equiv \frac{\alpha^2 n(2n-1)(\gamma - 1)(\gamma n^3 - \gamma n^2 - 2\gamma n + \gamma + 2n - 1)}{(\gamma n^3 - \gamma n^2 - 4\gamma n + 2\gamma + 4n - 2)^2(\gamma n^2 - \gamma + 1)} + \epsilon\end{aligned} \tag{25}$$

where ϵ is an arbitrarily small, positive number. Hence, for example, under the fixed cost per operator of F_n^{MTC} the network cannot support n operators with pricing according to the stylised MTC. The questions of interest are then as follows. Under these prohibitive fixed cost scenarios, can either or both of the collusive regimes sustain n operators, and if so does this result in welfare and consumer surplus gains relative to the, then smaller, network under the SQ and MTC regimes?

[24]The idea of quantity being a focus for public policy makers is not new. Maximising passenger-miles was adopted as a target by London Transport (see Glaister and Collings, 1978, and the references therein). It was also put forward by Sir Peter Parker, when Chairman of British Rail, in his 1978 Haldane Lecture (Parker, 1978).

[25]Recall, the stylised characterisation of the MTC here is one that includes the limiting assumption of a zero discount for the MO price relative to the weighted average of SO prices across the network.

Figure 5: Profit, Consumer Surplus, Welfare and Aggregate Quantities under $C1$ and $C2$ with an n Operator Extended Network and SQ (MTC) in a $n-1$ Operator Network, with all operators facing Fixed Cost, F_n^{SQ} (F_n^{MTC})

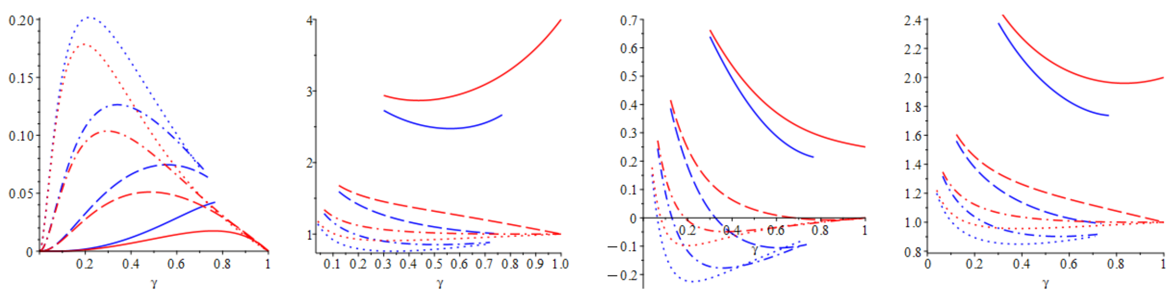


(a) Profit: π^R

(b) Consumer Surplus: $\frac{S^R}{S^{SQ}}$

(c) Welfare: $\frac{W^R - W^{SQ}}{W^R}$

(d) Aggregate Quantity: $\frac{\sum_t Q_t^R}{\sum_t Q_t^{SQ}}$



(e) Profit: π^R

(f) Consumer Surplus: $\frac{S^R}{S^{MTC}}$

(g) Welfare: $\frac{W^R - W^{MTC}}{W^R}$

(h) Aggregate Quantity: $\frac{\sum_t Q_t^R}{\sum_t Q_t^{MTC}}$

——$n = 2$, - - -$n = 3$, -·-·- $n = 4$, ······$n = 5$, Blue $R = C1$, Red $R = C2$

Figures 5(a) and (e) report the profit levels under the collusive regimes (divided by α^2) with n operators facing fixed costs F_n^{SQ} and F_n^{MTC}, respectively, for $n \in \{2, 3, 4, 5\}$. Whilst both collusive regimes can everywhere bear the prohibitive MTC fixed cost with n operators, this is not the case for the prohibitive SQ fixed cost. In particular, the lower profit collusive regime, $C2$, does become loss-making when n is small for some levels of substitutability under F_n^{SQ}, but broadly the prohibitive fixed costs are sustainable under the collusive regimes with n operators. We now our turn attention to Figures 5(c) and (g) to see the extent to which the extra operator under the collusive regimes has the potential to generate welfare gains over the smaller SQ and MTC networks, respectively.[26] The Figures appear to show an extensive ability for the larger collusive networks to convert the additional operator into welfare gains, especially at lower levels of substitutability and n. However, there

[26]Note, since profit under SQ and the MTC become zero with $n-1$ operators at sufficiently low levels of substitutability, and negative for levels of substitutability below this, the lines become discontinuous at this point. It is straightforward to show that at all the points where these regimes have zero or negative profit with $n-1$ operators, they also have zero or negative profit with any smaller number of operators. Hence, the market entirely fails under these regimes.

are also areas where negative ratios (welfare is lower under the collusive regimes) result and to understand whether these are within the range of the calibration we now undertake the following analysis. Let $\gamma^W_{R,T}(n)$ be the level of substitutability which equates the welfare under regime $R \in \{C1, C2\}$ with regime $T \in \{SQ, MTC\}$ with n operators in the former and $n-1$ operators in the latter and fixed cost F^T_n, hence:

$$W^R(n, F^T_n, \gamma^W_{R,T}(n)) = W^T(n-1, F^T_n, \gamma^W_{R,T}(n)), \quad R \in \{C1, C2\}, T \in \{SQ, MTC\}$$

Table 1 reports the left-most critical values of γ for which welfare under each collusive regime is the same as under the smaller SQ and MTC regimes, $\gamma^W_{R,T}(n)$ for $R \in \{C1, C2\}$ and $T \in \{SQ, MTC\}$. Levels of γ below this point guarantee welfare being higher under the relevant collusive regime, R, than the alternative, T. In addition, the Table also reports the levels of substitutability required under SQ for a given level of n to be consistent with the calibration exercise, $\gamma^{CAL}_{\eta,c}(n)$, with $\eta \in \{-0.6, -1.0\}$ and zero marginal cost.

Table 1: Critical Values of γ

n	$\gamma^{CAL}_{-0.6,0}(n)$	$\gamma^{CAL}_{-1.0,0}(n)$	$\gamma^W_{C1,SQ}$	$\gamma^W_{C2,SQ}$	$\gamma^W_{C1,MCT}$	$\gamma^W_{C2,MCT}$
2	0.526	0.250	-	-	-	-
3	0.274	0.118	0.506	0.957	0.325	0.680
4	0.160	0.067	0.213	0.385	0.130	0.190
5	0.103	0.043	0.111	0.157	0.063	0.079

Recall, if the market is currently behaving in line with SQ under zero or very low marginal cost, with elasticities consistent with one our selection, η, then for a given level of n, substitutability between the services will be characterised by $\gamma^{CAL}_{\eta,c}(n)$. Therefore, if $\gamma^{CAL}_{\eta,c}(n)$ is below the left-most critical value, $\gamma^W_{R,T}$, for $R \in \{C1, C2\}$ and $T \in \{SQ, MTC\}$, then we have a situation where the market is operating where welfare is strictly greater under regime R with n-operators, relative to the smaller network under regime T. We can see from the Table that under both elasticity selections, welfare under both collusive regimes is superior to SQ under the calibration. Whilst it is clear that for low levels of n, both collusive regimes also dominate MTC on welfare, the shaded cells highlight cases where the larger network harms welfare, with $C1$ more prone than $C2$ for falling short against MTC on welfare. Similar inspection of Figures 5(b) and (f) indicate that the welfare gains available under the collusive regimes are not driven purely by profit gains, with obvious consumer surplus gains in the relevant range.

There are several conclusions to draw. First, not only do the collusive regimes offer potential profit, consumer surplus and welfare gains over SQ, but they also offer the possibility of a more extensive network which can yield additional welfare gains. Second, whilst the MTC dominates the collusive regimes in consumer surplus and welfare terms, the Competition Commission's recommended pricing framework (which our stylised regime seeks to represent) appears to be unattractive to operators with evidence of their use in practice being limited. Third, if the market is struggling to support more than a small number of operators, even if the operators can be convinced to adopt the MTC, higher consumer surplus and welfare might be available if the operators were allowed to pursue one or other of our collusive pricing regimes, to the extent it might encourage a wider/denser network.The calibration suggests exactly this if n is low.

Finally, Figures 5(d) and (h) illustrate aggregate quantity ratios of collusive regimes relative to the smaller networks under SQ and MTC. The quantity gains available under the larger collusive regime networks suggest additional potential welfare benefits through the replacement effect of private car journeys with public transport lowering pollution, congestion and accident externalities (e.g., see Adler and van Ommeren, 2016) and offering productivity boosting agglomeration economies over and above those discussed in the previous section.

6 Conclusions

The importance of a well-functioning urban public transport system has long been understood in relation to facilitating growth and equality of access to urban facilities whilst helping alleviate pollution, accidents and congestion and stimulating agglomeration economies. However, urban transport networks often exhibit rival and complementary service aspects. Here, theory suggests that private operation, with service providers setting prices independently, a scenario we characterise as the 'Status Quo' (SQ), will result in inefficiencies, in particular through inflated Multi-Operator (MO) prices, which can damage profit as well as consumer surplus. Evidence in the deregulated urban public transport system in Britain (outside London) appears to lend support to these theoretical priors. This is despite the introduction of a Block Exemption (Competition Commission, 2001) permitting collusive pricing on MO tickets aimed at addressing the externalities which drive up MO prices. In particular, the UK Competition Commission recommends a pricing framework under a Multi-operator Ticketing Card system (MTC) where MO prices are set at an agreed discount of the weighted average of the prevailing Single-Operator (SO) ticket prices on the network. Analysis of this pricing framework in McHardy (2022) suggests that, whilst it may help address excessive MO prices, raising consumer surplus and welfare, it may be less attractive to operators than SQ, and therefore joining it may not be incentive compatible. Indeed, usage of tickets conforming to such a pricing regime (MO being discounted relative to SO prices) do not appear to be widespread in Britain.

Department for Transport (2021) explicitly recognises the urgent work that is needed to fix urban public transport in Britain. It focuses on correcting long-standing shortcomings in the deregulated bus sector, in particular, excessive MO ticket prices, declining patronage, geographical and temporal deficiencies in service provision and poor integration with other public transport modes. Whilst much emphasis is placed on MO tickets, evidence suggests the MTC, as it is currently imagined, is not providing the answer. Further, the Block Exemption, which underpins it, is due to expire in 2026 and is currently under statutory review.

In this paper we propose an alternative pricing system that can help address inflated MO prices under SQ in a way that can also benefit from incentive compatibility. This system involves operators setting their SO prices independently in one stage and colluding with other operators to set MO prices in another stage. We consider two collusive regimes, distinguished by the stage in which collusion on MO prices takes place and analyse both in an n-operator differentiated transport framework, which captures rival and complementary service aspects. Comparing the two collusive regimes against SQ, we show both have the potential to create a win-win situation where profit, consumer surplus and welfare are improved relative to

SQ. Using a calibration exercise, we demonstrate that these gains occur close to where the market may be functioning. Comparing the collusion regimes against a stylised MTC, whilst the latter dominates in consumer surplus and welfare terms, it is inferior in profit terms. Hence, whilst the collusive regimes offer a potential win-win relative to SQ, the MTC is better for consumers and welfare. The MTC is optional, however, and suffers from incentive incompatibility, meaning the associated benefits may not arise under the current policy environment with the evidence pointing to this being the case to some extent.

In terms of the perennial problems hindering urban public transport in Britain that we identified at the out-set, the new regimes offer the potential for lower MO prices, incentivising greater MO use, and driving increased patronage, relative to SQ. However, inadequate service provision and an inability for markets to sustain sufficient numbers of rivals to engender a competitive environment were also cited as ongoing problems. Our analysis indicates that the collusive two-stage regimes may be able to sustain a higher number of operators in the network than would be profitable under the $SQ\ and\ MTC$ regimes, resulting in further potential consumer surplus and welfare gains. Hence, even though the collusive regimes offer lower consumer surplus and welfare than the MTC when all regimes have the same number of operators, the higher profit available under the former can result in more operators providing more services than under the MTC to the extent that these regimes now dominate the MTC in consumer surplus and welfare terms. Indeed, we find that the collusive regimes can support larger networks with associated consumer surplus and welfare gains relative to SQ and MTC where a calibration exercise indicates the market might be operating.

Regarding whether the collusive regimes proposed here could be permitted in accordance with the Block Exemption, we return to the four conditions set out earlier. First, we have seen that the collusive regimes can improve on prices, consumer surplus and welfare relative to SQ, with a calibration indicating this may be the reality where the market is operating. With lower prices and higher consumer surplus, this appears to satisfy the second condition, that consumers get a fair share of the gains. Indeed, since the collusive regimes also generate higher profits under these parameterisations, it has been shown that they have the potential to provide more services than SQ supporting the first condition regarding efficiency. At the same time the equilibria under the collusive regimes do not suggest anti-competitive outcomes, satisfying the third condition. Regarding the fourth condition, the two-stage pricing structure is designed to explicitly avoid collusion in the setting of MO prices (which generates the win-win higher profit and consumer surplus outcome) leaking into anti-competitive practices elsewhere. However, whether such a scheme would function this way in practice is likely a function of the way in which the collusion stage is organised and regulated, for instance, in terms of what information operators are allowed to share. But a priori, it is not obvious that it need be more prone to anti-competitive leakage than the existing opportunity to collude under the MTC, although this is clearly an avenue for further enquiry.

Finally, we note that the higher levels of patronage under the collusive two-stage pricing regimes might have further indirect benefits not captured in the modelling. It is well understood that attracting urban travellers from private car to public transport carry additional environmental, congestion and accident benefits, further extending the gains from the increased number of operators and patronage of the proposed pricing system. Indeed, by improving transport efficiency the proposed pricing structure may help improve city density, especially in Britain's second-tier cities which do not tend to benefit from extensive public transit rail and underground networks and so are more reliant on bus provision, with associated agglomeration effects contributing to the current leveling-up priority in Britain.

Appendix

A Second Order Conditions

Under Monopoly, the firm sets N prices, and the Hessian for its profit function is $N \times N$ with $-2b$ on the principal diagonal and $2d$ elsewhere. In the case of 'Status Quo', firm i sets $2n-1$ prices and the $(2n-1) \times (2n-1)$ and the Hessian matrix of the profit function has $-2b < 0$ on the principal diagonal and $2d > 0$ elsewhere. A dominant diagonal requires, $b > (n^2-1)d$ and $b > 2(n-1)d$, respectively, both of which follow under under Assumption 1, guaranteeing the second-order condition is satisfied in the relevant range.[27] Under the MTC, firms set a single price and so the second-order condition is satisfied since $-b < 0$.

In the case of stage-two price setting in regime $C1$, the own-price second derivative of profit for firm i is negative since $-b < 0$. The second derivative of aggregate profit with respect to P_x, after imposing symmetry, can be written:

$$-2nA[bA-(n-1)d(n+A)]-2n(n-1)[b-d(n(A+n-1)-1)] \tag{26}$$

the sign of which can be shown to depend on:

$$-4b(b-d(n^2-1))-nd^2(n-1)^2$$

which is strictly negative under Assumption 1.

In the case of regime $C2$, the stage-one price setting second derivative of operator profit i with respect to P_{ii} can be written:

$$-\frac{2}{b-d(n(n-1)-1)}(b+d)[b-d(n(n-1))] \tag{27}$$

which is strictly negative under Assumption 1. In the case of the stage-two price setting in regime $C2$ the second derivative of joint profit with respect to P_x can be written:

$$-2n(n-1)[b-d(n(n-1)-1)] \tag{28}$$

which is guaranteed under Assumption 1.

B Proof to Propositions

B.A Proof to Lemma 1

From Eqs. (7) and (18), note $H \equiv \frac{P_x^{C1}}{P^M} = \frac{2P_x^{C1}}{\alpha}$, which is continuous, has no solutions for $H = 1$ in the relevant range, and for which feasible combinations of (γ, n) yield values of $H < 1$ completing the proof.

B.B Proof to Proposition 1

(i) From Eqs. (18) and (24), define $H \equiv \frac{P^{C1}}{P^{C2}} = \frac{2(2+(n-3)\gamma)((n^2+n-4)\gamma+2)(1+\gamma(n^2-1))}{(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma)(2+(n^2+2n-3)\gamma)}$ and $H_x \equiv \frac{P_x^{C1}}{P_x^{C2}} = \frac{2(\gamma n^2-2\gamma+1)(\gamma n-3\gamma+2)(\gamma n^2-\gamma+1)}{(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma)(1+(n^2+\frac{n}{2}-\frac{3}{2})\gamma)}$. Note, H and H_x are continuous in (n, γ).

[27](E.g., see Theorem M.D.5, Mas-Colell et al., 1995, p. 939).

It is straightforward to see the maximal solutions for $H = H_x = 1$ are $\gamma = 0$ and $n = 1$, which lie outside the relevant range. However, given continuity and the existence of feasible (n, γ) combinations yielding $P^{C1} > P^{C2}$ and $P_x^{C1} > P_x^{C2}$, completes the proof. (ii) From Eqs. (18) and (24), define $H_1 \equiv \frac{P_x^{C1}}{P^{C1}} = \frac{2\gamma n^2-4\gamma+2}{(n^2+n-4)\gamma+2}$ and $H_2 \equiv \frac{P_x^{C2}}{P^{C2}} = \frac{2+(2n^2+n-3)\gamma}{2+(n^2+2n-3)\gamma}$. Note, H_1 and H_2 are continuous in (n, γ). It is straightforward to see the maximal solutions for $H = H_x = 1$ are $\gamma = 0$ and $n = 1$, which lie outside the relevant range. However, given continuity and the existence of feasible (n, γ) combinations yielding $P_x^{C1} > P^{C1}$ and $P_x^{C2} > P^{C2}$, completes the proof.

B.C Proof to Proposition 2

(i) From Eqs. (9) and (24), setting $n = 2$ gives: $H \equiv \frac{P^{SQ}|_{n=2}}{P^{C2}|_{n=2}} = \frac{2(1+3\gamma)}{2+5\gamma}$ and $H_x \equiv \frac{P_x^{SQ}|_{n=2}}{P_x^{C2}|_{n=2}} = \frac{8(1+3\gamma)}{6+21\gamma}$, which are both strictly greater than one in the relevant range. (ii) From Eqs. (9), (18) and (24), define $H_1 \equiv \frac{P^{SQ}}{P^{C1}} = \frac{(12+(3n^3-18n^2+3n+24)\gamma^2+(12n^2-36)\gamma)}{(2(3+(n^2-3/2n-5/2)\gamma)((n^2+n-4)\gamma+2))}$, $H_{x1} \equiv \frac{P_x^{SQ}}{P_x^{C1}} = \frac{(8+(2n^3-12n^2+2n+16)\gamma^2+(8n^2-24)\gamma)}{((2\gamma n^2-3\gamma n-5\gamma+6)(\gamma n^2-2\gamma+1))}$, $H_2 \equiv \frac{P^{SQ}}{P^{C2}} = \frac{3(\gamma n-3\gamma+2)(\gamma n^2-\gamma+1)}{((3+(n^2-3/2n-5/2)\gamma)(2+(n^2+2n-3)\gamma))}$, $H_{x2} \equiv \frac{P_x^{SQ}}{P_x^{C2}} = \frac{8(\gamma n-3\gamma+2)(\gamma n^2-\gamma+1)}{((2\gamma n^2-3\gamma n-5\gamma+6)(2\gamma n^2+\gamma n-3\gamma+2))}$. It is straightforward to show that the maximal solution for $H_1 = H_{x1} = 1$ is at the common threshold $\gamma_1^{C1} = \frac{1}{n^2-2n+2}$ and similarly for $H_2 = H_{x2} = 1$ is at the common threshold $\gamma_1^{C2} = \frac{2}{2n^2-5n+3}$, and these are the only solutions in the feasible range. Both thresholds are also clearly strictly decreasing in n in the relevant range. Given H_1, H_{x1}, H_2, and H_{x2} are all continuous functions in their arguments, it follows that since for feasible (n, γ) combinations below those satisfying the relevant critical threshold can be found for which each function is strictly greater than 1, completing the proof. (iii) This follows directly from inspection of γ_1^{C1} and γ_1^{C2} from (ii) above.

B.D Proof to Proposition 3

(i) From Eqs. (11) and (18) define $H \equiv \frac{P_x^{C2}}{P^{MTC}} = \frac{((n^3-n^2-4n+2)\gamma+4n-2)(1+(n^2+1/2n-3/2)\gamma)}{((\gamma n-3\gamma+2)(\gamma n^2-\gamma+1)(2n-1))}$. Note H is continuous on (γ, n) and $H = 1$ has no solutions in the relevant range, whilst solutions exist for $H < 1$. Given $P_x^{C1} > P_x^{C2}$ from Proposition 1(i) completes the proof. (ii) Let $H^R \equiv \frac{P^R}{P^{MTC}}$, $R \in \{C1, C2\}$: $H^{C1} = \frac{((n^3-n^2-4n+2)\gamma+4n-2)((n^2+n-4)\gamma+2)}{(2(n-1/2)(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma))}$, $H^{C2} = \frac{(2+(n^2+2n-3)\gamma)((n^3-n^2-4n+2)\gamma+4n-2)}{(2(\gamma n-3\gamma+2)(\gamma n^2-\gamma+1)(2n-1))}$. Note H^R is continuous on (γ, n) in the relevant range. Setting $H^R = 1$, and solving for γ, yields $\gamma_2^{C1} = \frac{2}{n^2+3}$ and $\gamma_2^{C2} = \frac{2}{n^2-n+2}$, which are both strictly decreasing in n and $\gamma_2^{C1} < \gamma_2^{C2}$ in the relevant range. Feasible (n, γ) combinations below those satisfying the relevant critical threshold can be found for which each function is strictly less than 1, completes the proof.

B.E Proof to Proposition 4

For parts (i), (ii) and (iii), let $H^\pi \equiv \frac{\pi^{C1}}{\pi^{C2}} = \frac{12(\gamma n^2-\gamma+1)(\gamma n-3\gamma+2)^2(\gamma n^2-2\gamma+1)}{(3(4+3(3n^3-5n^2-7n+9)\gamma^2+(4n^2+4n-12)\gamma)(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma))}$, $H^S \equiv \frac{S2}{S1} = \frac{(1+(n^4-4.25n^3+3.75n^2+3.25n-3.75)\gamma^3+(2.25n^3-3.75n^2-5.25n+7.75)\gamma^2+(n^2+2n-5)\gamma)(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma)}{(2+(n-3)\gamma)^2(1+(n^5-3n^4-n^3+6n^2+2.22\times10^{-16}n-3)\gamma^3+(n^4+2n^3-8n^2-n+7)\gamma^2+(2n^2+n-5)\gamma)}$ and $H^W \equiv \frac{W2}{W1} = \frac{(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma)(3+\frac{1}{4}(4n^4-23n^3+25n^2+27n-33)\gamma^3+\frac{1}{4}(15n^3-33n^2-43n+73)\gamma^2+(-13+4n+3n^2)\gamma)}{((\gamma^2n^3-5\gamma^2n^2+7\gamma^2+3\gamma n^2+\gamma n-10\gamma+3)(\gamma(n-3)+2)^2(\gamma(n^2-1)+1))}$ which are continuous on (n, γ) in the relevant range. Note, there are no solutions for $H^X = 1$ $(X = \pi, S, W)$ in the relevant range.[28] However, given continuity and the existence of feasible (n, γ) combinations yielding $H^X > 1$, completes the proof.

[28]In the case of $X^S = 1$ and $X^W = 1$, solve for n and plot over $\gamma \in [0, 1]$ to see solutions only for $n \leq 1$.

B.F Proof to Proposition 5

(i) Let $H \equiv \frac{\pi^{C1}}{\pi^{SQ}} = \frac{(2\gamma n^2-3\gamma n-5\gamma+6)^2 n(\gamma n^2-2\gamma+1)}{(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma)((8n^3-15n^2-2)\gamma+8n+1))}$, which is continuous on (n, γ) in the relevant range. Given there is a unique solution for $H = 1$ in the relevant range, $\gamma_1^{C1}(n)$, defined above, no solutions for $H < 1$ and feasible (n, γ) combinations in the relevant range, above and below γ_1^{C1} for which $H > 1$, with continuity of H completes the proof. (ii) Let $H \equiv \frac{\pi^{C2}}{\pi^{SQ}} \equiv \frac{(4+(3n^3-5n^2-7n+3)\gamma^2+(4n^2+4n-12)\gamma)(6+(2n^2-3n-5)\gamma)^2 n}{2((\gamma n-3\gamma+2)^2(\gamma n^2-\gamma+1)(8\gamma n^3-15\gamma n^2-2\gamma+8n+1))}$, which is continuous on (n, γ) in the relevant range. The are two solutions for $H = 1$: $\bar{\gamma}_1^{C2}$, defined above, and $\gamma_3^{C2} = RootOf(6\gamma^3 Z^5 + (-23\gamma^3 + 8\gamma^2)Z^4 + (21\gamma^3 - 10\gamma^2)Z^3 - \gamma^3 Z^2 + (-27\gamma^3 + 46\gamma^2 - 20\gamma)Z + 24\gamma^3 - 52\gamma^2 + 36\gamma - 8)$. Plotting these reveals γ_1^{C1} lies below γ_1^{C1}, with feasible (γ, n) combinations yielding $H < 1$ ($H > 1$) above the latter and below the former, which given continuity of H completes the proof. (iii) Follows straightforwardly from plotting $n(\gamma_1^{C1}) - n(\gamma_3^{C2})$ over the full interval $\gamma \in [0, 1]$, to see it lies everywhere strictly above zero, completing the proof.

B.G Proof to Proposition 6

(i) Let $H^S \equiv \frac{S^{C1}}{S^{SQ}} = \frac{(2\gamma n^2-3\gamma n-5\gamma+6)^2(\gamma^2 n^3-3\gamma^2 n^2+\gamma n^2+3\gamma^2+\gamma n-4\gamma+1)n}{(((n^5-3n^4+1.25n^3+0.25n^2+2n+0.75)\gamma^2+(2n^3-1.75n^2-2.75n-2)\gamma+n+1.25)(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma))}$ and $H^W \equiv \frac{W^{C1}}{W^{SQ}} = \frac{(2\gamma n^2-3\gamma n-5\gamma+6)^2(\gamma^2 n^3-3\gamma^2 n^2+\gamma n^2+3\gamma^2+\gamma n-4\gamma+1)n}{(((n^5-3n^4+1.25n^3+0.25n^2+2n+0.75)\gamma^2+(2n^3-1.75n^2-2.75n-2)\gamma+n+1.25)(4+(n^3-6n^2+n+8)\gamma^2+(4n^2-12)\gamma))}$, are both continuous on (n, γ) and $H^S = H^W = 1$ for $\gamma = \gamma_1^{C1}$. There exists a feasible (n, γ) combination in the relevant range below γ_1^{C1} for which $H^S > 1$ and $H^W > 1$ is strictly positive, completing the proof. (iv) Let $H^S \equiv \frac{S^{C1}}{S^{SQ}} = \frac{4n(3+(n^2-1.5n-2.5)\gamma)^2(1+(n^4-4.25n^3+3.75n^2+3.25n-3.75)\gamma^3+(2.25n^3-3.75n^2-5.25n+7.75)\gamma^2+(n^2+2n-5)\gamma)}{(2+(n-3)\gamma)^2((4n^7-12n^6+n^5+13n^4+3n^3+2n^2-8n-3)\gamma^3+(12n^5-19n^4-14n^3-33.32\times10^{-16}n^2+19n+11)\gamma^2+(12n^3-2n^2-15n-13)\gamma+4n+5)}$, and, $H^W \equiv \frac{W^{C1}}{W^{SQ}} = \frac{n(3+(n^4-5.75n^3+6.25n^2+6.75n-8.25)\gamma^3+(3.75n^3-8.25n^2-10.75n+18.25)\gamma^2+(-13+3n^2+4n)\gamma)(3+(n^2-1.5n-2.5)\gamma)^2}{(((n^5-3n^4-2.75n^3+7.75n^2+2n+1.75)\gamma^2+(6n^3-9.25n^2-6.75n-3.5)\gamma+5n+1.75)(2+(n-3)\gamma)^2(1+(n^2-1)\gamma))}$, which are continuous on (n, γ) and $H^S = H^W = 1$ for $\gamma = \gamma_3^{C2}$. There exists a feasible (n, γ) combination in the relevant range below γ_3^{C2} for which $H^S > 1$ and $H^W > 1$ are strictly positive and for $n = 2$, γ_3^{C2} is outside the relevant range and there exists no solutions for $H = 1$, completing the proof.

B.H Proof to Lemma 2

First, note that $\tilde{\gamma} = \frac{n^2-3+\sqrt{n^4-2n+1}}{3n^2-n-4}$, which is decreasing in n and $\lim_{n\to\infty} \tilde{\gamma} = \frac{2}{3}$. $\gamma_1^{C1} = \frac{1}{n^2-2n+2}$ is decreasing in n with lower limit in the relevant range $\gamma_1^{C1}|_{n=2} = \frac{1}{2}$, completing the proof.

B.I Proof to Proposition 7

First, note from Propositions 5(ii) and 6(ii), $\gamma_3^{C2} < \gamma_1^{C2} < \gamma_1^{C1}$. Next, the calibration contour in (γ, n)-space with $\eta = -1.0$ and $c = 0$ is given by:

$$n^{CAL}_{-1.0,0}(\gamma) = \frac{\gamma + \sqrt{-15\gamma^2 + 16\gamma}}{4\gamma}$$

and lies everywhere in the relevant range below γ_3^{C2}. To see this, plot $n(\gamma_3^{C2}) - n^{CAL}_{-1.0,0}(\gamma)$ and plot over $\gamma \in [0, 1]$, and see the resulting curve lies strictly above zero in the range where $n \geq 2$ completing the proof.

B.J Proof to Proposition 8

Let $H \equiv \frac{K^R}{K^{MTC}}$, for $K \in \{\pi, S, W\}$ and $R \in \{C1, C2\}$. Note H is a continuous function on its arguments (γ, n). Solving $H = 1$ in each case (e.g., solve $H = 1$ for n and plot over $\gamma \in [0, 1]$ to see there are no solutions for $n \geq 2$. Noting that there are (γ, n) combinations in the relevant range for which $\pi^{MTC} < \pi^R$, $S^{MTC} > S^R$ and $W^{MTC} > W^R$ completes the proof.

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