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Revisiting Real Wage Rigidity

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Abstract

In this paper, we provide empirical evidence that real wage rigidity is not a major cause of unemployment volatility. We argue that there is a disconnect between the theoretical and empirical literatures on this topic. While theoretical studies define real wage rigidity as the response of wages to changes in unemployment following productivity shocks, the empirical literature measures real wage rigidity as the estimated semi-elasticity of wages with respect to unemployment, averaged over all shocks. We show that averaging over shocks gives a biased measure of real wage rigidity, as the impact of other shocks confounds the response to productivity shocks. Our results indicate that the estimated semi-elasticity with respect to productivity shocks is twice as large as the estimated semi-elasticity averaged over all shocks. This implies that one cannot attribute unemployment volatility to real wage rigidity.

Keywords: real wage rigidity, time-varying parameter model, real wages, search frictions,

JEL Classification: *E23, E32, J23, J30, J64*

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1 Introduction

The link between unemployment and real wages is central to debates on business cycles. The real wage rigidity hypothesis is a leading candidate to explain the lack of movement in real wages relative to unemployment which prior studies show is evident within the data. New Keynesian DSGE models widely use real wage rigidity, including models using labour market frictions (e.g. Gertler and Trigari, 2009; Blanchard and Gali, 2010; Gertler et al., 2020). Real wage rigidity also appears in the new generation of heterogeneous agent New Keynesian (HANK) models (e.g. Broer et al., 2020), as well as the Diamond-Mortensen-Pissarides models of equilibrium unemployment to account for the volatility of unemployment and vacancies (see e.g. Shimer, 2005; Hall, 2005; Christiano et al., 2015).

In this paper, we argue that there is a disconnect between the theoretical and empirical literatures on real wage rigidity. The empirical literature uses a regression approach to estimate the semi-elasticities of real wages with respect to unemployment (see e.g. Pissarides, 2009; Gertler et al., 2020), with a stronger response of wages to unemployment implying lower values of real wage rigidity[1]. There are alternative approaches to real wage rigidity in the theoretical literature[2]. In this paper, we use the inverse of the semi-elasticity of real wages with respect to unemployment, since this allows us to compare measures of rigidity across the empirical and theoretical literatures. The theoretical literature assigns a prominent role to productivity shocks in driving business cycle fluctuations, focusing on the role of real wage rigidity in generating a large volatility of unemployment in response to productivity shocks. The disconnect arises because the semi-elasticities estimated in the empirical literature reflects the impact of all shocks, not just productivity shocks. This implies that evidence from the empirical literature cannot currently be used to inform the debate in the theoretical literature. In order to address this, one requires an empirical estimate of the semi-elasticity of real wages with respect to unemployment in response to different shocks, especially productivity shocks.

In this paper, we provide this. We present estimates of the semi-elasticities of real wages with respect to unemployment in response to productivity and other shocks. We find that the estimated semi-elasticity in response to productivity shocks is large. This implies a lower value of real wage rigidity thereby suggesting a lack of support for the real wage rigidity hypothesis. We show that the measure used in the current literature overstates the degree of real wage rigidity because it confounds the impact of productivity shocks by averaging over all identified shocks, including

[1] The value of this semi-elasticity is controversial. Much of the debate concerns which measure of wages one should use. In the data, the response of average wages to unemployment is small, suggesting a high degree of real wage rigidity. Skeptics argue that it is more appropriate to use the wages of newly hired workers, since these wages are more relevant for job creation. Many studies, including Pissarides (2009), find that the wages of new hires are more flexible than the wages of incumbent workers. This suggests a low degree of wage rigidity. Gertler et al. (2020) challenges this view and argues that the relevant margin of adjustment is the wages of workers newly hired from unemployment, rather than the wages of all new hires. The latter includes the wages of workers upgrading to a better job match. After controlling for these composition effects, they find that the wages of new hires are no more cyclical than those of existing workers.

[2] As discussed by Hall (2005) and Christoffel and Linzert (2010), among others.

shocks that move real wages and unemployment in the same direction. By correcting for this confounding effect, we show that the underlying response of real wages relative to unemployment following productivity shocks is much larger than the values used in the current empirical literature.

In order to do this, we depart from the regression approach used by the existing literature to measure real wage rigidity. We estimate a structural time-varying parameter VAR model with stochastic volatility (TVP VAR) in order to track temporal evolutions in the relationships among US productivity, real wages, vacancies, the unemployment rate, and inflation. We identify four transitory structural shocks using robust sign restrictions that stem from a DSGE model with search frictions (DSGE-SF) (similar to Mumtaz and Zanetti, 2012) following the procedure in Canova and Paustian (2011). We calculate the semi-elasticities of real wages with respect to unemployment for: a productivity shock; an aggregate demand shock; a job destruction shock; and a wage bargaining power shock. The latter is crucial to our analysis since it moves unemployment and wages in the same direction. The impact of the wage bargaining power shock in the data reduces the average semi-elasticity, thereby making this a biased estimate of the response of real wages to unemployment following productivity shocks. Evidence on the importance of this shock is in, among others, Fujita and Ramey (2007), Pizzinelli et al. (2020), Drautzburg et al. (2021) and Ellington et al. (2021). Our results quantify the size of the bias. We calculate the average semi-elasticity of real wages with respect to unemployment, averaging over all shocks. This is consistent with the existing literature; and coherent with substantial real wage rigidity. But the semi-elasticity with respect to productivity shocks is over twice as large, implying that the degree of real wage rigidity in response to productivity shocks, the focus of the theoretical literature, is far smaller than the values this literature widely uses.

Our paper proceeds as follows. Section 2) describes our data and outlines our econometric model. Section 3) contains our structural analysis, including our strategy for identifying structural shocks and our estimates of the semi-elasticities of real wages to unemployment following different shocks. Finally, in Section 4) we conclude and consider options for future work.

2 Data and Econometric Model

We use quarterly US data from 1954Q3 to 2019Q4 on productivity, real wages, the vacancy rate, the unemployment rate, and inflation[3]. Our measures of US productivity and real wages are Nonfarm Business Sector: Real Output Per Hour of all Persons, and Nonfarm Business Sector: Real Compensation Per Hour[4]. The vacancy rate is the Help Wanted Index in Barnichon (2010) and the unemployment rate is from the Bureau of Labor Statistics (BLS). For inflation, we take the Nonfarm Business Sector: Implicit Price Deflator[5]. We take the natural logarithm of productivity, real wages and the implicit price deflator before applying the Hamilton (2018) filter to every variable.

[3]We end our sample in 2019, to avoid the turbulence of the Covid-19 pandemic.

[4]Both series are available from the Federal Reserve Bank of St. Louis (FRED) database with codes OPHNFB and COMPRNFB for productivity and wages respectively.

[5]Also from the FRED database with code: IPDNBS.

These data are plotted in the Online Appendix.

We work with the following TVP VAR model, with $p = 2$ lags and $N = 5$ variables:

$$Y_t = \beta_{0,t} + \beta_{1,t}Y_{t-1} + \cdots + \beta_{p,t}Y_{t-2} + \epsilon_t \equiv X_t^{'}\theta_t + \epsilon_t \tag{1}$$

where $Y_t \equiv [y_t,\, w_t,\, v_t,\, u_t,\, \pi_t]^{'}$ is a vector of endogenous variables. Here y_t is the filtered value of labour productivity, w_t is the filtered value of real wages, v_t, u_t, and π_t are filtered values of the unemployment rate, the vacancy rate and the implicit price deflator respectively. $X_t^{'}$ contains lagged values of Y_t and a constant.

Stacking the VAR's time-varying parameters in the vector θ_t, they evolve as a driftless random walk

$$\theta_t = \theta_{t-1} + \gamma_t \tag{2}$$

with $\gamma_t \equiv [\gamma_{1,t}, : \gamma_{2,t}, ..., : \gamma_{N\cdot(Np+1),t}]'$. We consider two specifications for the variance of γ_t. The first case is where $\gamma_t \backsim N(0, Q)$, with Q is a full matrix containing parameter innovation variances and covariances (Primiceri (2005)). The second is where $\gamma_t \backsim N(0, Q_t)$ with Q_t being a diagonal matrix where such diagonal elements of Q_t follow independent log-stochastic volatility processes as in Baumeister and Benati (2013). Bayesian DIC statistics suggest that the Primiceri (2005) model fits our data best and we proceed in this case. Results using the specification in Baumeister and Benati (2013) have the same conclusions as we report here and are available upon request.

The innovations in (1) follow $\epsilon_t \backsim N(0, \Omega_t)$. Ω_t is the time–varying covariance matrix which is factored as

$$\Omega_t = A_t^{-1} H_t (A_t^{-1})^{'} \tag{3}$$

with A_t being a lower triangular matrix with ones along the main diagonal, and the elements below the diagonal contain the contemporaneous relations. H_t is a diagonal matrix containing the stochastic volatility innovations. Collecting the diagonal elements of H_t and the non-unit non-zero elements of A_t in the vectors $h_t \equiv [h_{1,t}, : h_{2,t}, ..., h_{N,t}]'$, $\alpha_t \equiv [\alpha_{21,t}, : \alpha_{31,t}, \ldots, \alpha_{NN-1,t}]'$ respectively, they evolve as

$$\begin{aligned} \ln h_{i,t} &= \ln h_{i,t-1} + \eta_t & (4) \\ \alpha_t &= \alpha_{t-1} + \zeta_t & (5) \end{aligned}$$

where $\eta_t \backsim N(0, Z_h)$, and $\zeta_t \backsim N(0, S)$. The innovations in the model are jointly Normal, and the structural shocks, ψ_t are such that $\epsilon_t \equiv A_t^{-1} H_t^{\frac{1}{2}} \psi_t$. Similar to Primiceri (2005), S is a block diagonal matrix; this implies the non-zero and non-unit elements of A_t evolve independently. The specification of the priors of our model are similar to Baumeister and Benati (2013). To calibrate the initial conditions of the model, we use the point estimates of the coefficients and covariance matrix from a time-invariant VAR model using the first 10 years of data. Therefore the estimation sample of our results span 1964Q2–2019Q4. We estimate the model using Bayesian methods allowing for 20,000 runs of the Gibbs sampler. Upon discarding the initial 10,000 iterations as burn-in, we sample every 10^{th} draw to reduce autocorrelation which leaves 1000 draws from the posterior distribution. The Online Appendix contains details of our prior specification, and an outline of the posterior simulation algorithm as well as estimates of the total prediction variation of our model, the stochastic volatilities of each variable and the reduced form correlations between our variables.

3 Structural Analysis

In this section we outline structural identification and analysis of our model. Our identification strategy follows Canova and Paustian (2011) and Mumtaz and Zanetti (2015). We simulate a theoretical model using a range of alternative calibrations, based on randomly sampling parameter values within a specified range, constructing a distribution of impulse responses of our endogenous variables to a variety of shocks. We identify structural shocks for which the sign of the impulse responses on impact is unambiguous across this distribution. In this way, we ensure that our identifying sign restrictions are credible, robust to alternative calibrations of the structural parameters. Our identifying restrictions are based on a standard New Keynesian DSGE model without capital but with search frictions in the labour market, similar to Faia (2008), Krause and Lubik (2007), Blanchard and Gali (2010), Mumtaz and Zanetti (2012) and others. Details of our procedure and the model used are contained in the Online Appendix[6].

Table 1: **Contemporaneous Impact of Short-run Shocks on Labour Market Variables**
Notes: This table shows the contemporaneous sign restrictions imposed on variable $x = \{y_t,\ v_t,\ u_t,\ w_t\}$ to a productivity shock, ψ_t^{Prod}; a job separation shock, ψ_t^{JS}; a shock to workers bargaining power, ψ_t^{W}; and a demand shock, ψ_t^{D}, respectively. y_t is the log-level of productivity; w_t is the log-level of real wages; v_t is the vacancy rate; u_t is the unemployment rate; and π_t is inflation. x denotes no restriction.

	y_t	w_t	v_t	u_t	π_t
ψ_t^{Prod}	+	+	+	−	−
ψ_t^{JS}	x	−	+	+	x
ψ_t^{W}	x	+	−	+	x
ψ_t^{D}	x	+	+	−	+

We identify four temporary structural shocks within our empirical model as in Table 1). We identify: a productivity shock, ψ_t^{Prod}; a job separation shock, ψ_t^{JS}; a shock to workers' bargaining power, ψ_t^{W}; and a demand shock ψ_t^{D}. The productivity shock increases productivity, wages and vacancies, while reducing unemployment and inflation. The demand shock increases wages, inflation and vacancies but reduces unemployment; we are agnostic as to its impact on productivity. The job separation shock increases unemployment and vacancies, thus shifting out the Beveridge Curve.

[6]This approach is similar to Ellington et al. (2021). That paper works with permanent productivity shocks and focuses on structural change in the labour market. By contrast, this paper addresses issues around real wage rigidity using a model that, in line with the literature, examines responses to temporary productivity shocks.

It also reduces wages; we are agnostic about its impact on productivity and inflation. The shock to wage bargaining increases wages and unemployment but reduces vacancies; we are again agnostic about its impact on productivity and inflation. As noted above, the positive relationship between wages and unemployment implied by this shock is important for our results.

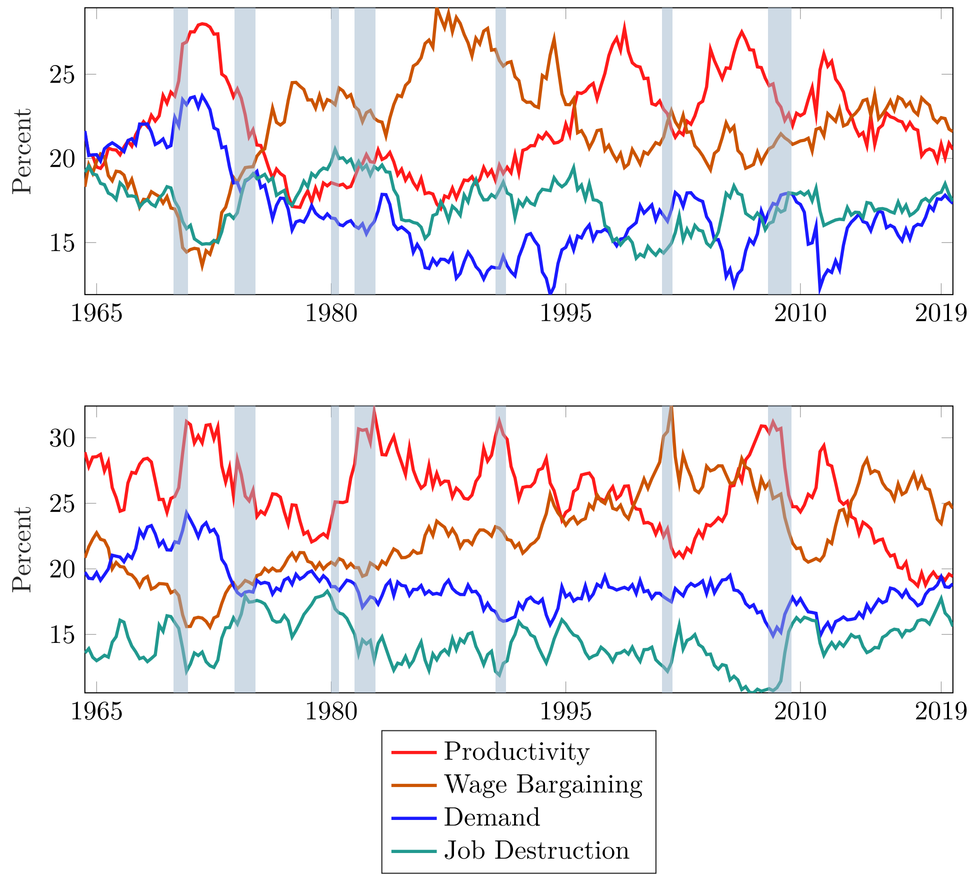

Figure 1: **Variance Decomposition of the One-Period Ahead Forecast Error Variances of Wages and Unemployment**
Notes: This figure plots the contribution of (i) productivity shocks (red); (ii) wage bargaining power shocks (brown); (iii) demand shocks (blue) and (iv) job destruction shocks (green) in explaining the volatility of the one-period ahead forecast error variances of wages (top panel) and unemployment (lower panel) across our sample.

Figure 1) shows the forecast error variance decompositions of wages and unemployment that emerge from our structural estimates. Movements in wages and unemployment across our sample reflect the impact of all the shocks, with no single shock accounting for more than 35% of the variance of unemployment and more than 30% of the variance of wages. Productivity and wage bargaining shocks make the largest contribution to explaining the volatility of both variables across our sample. Productivity shocks have the strongest impact on unemployment until around 2000. Thereafter, wage bargaining shocks become more prominent. Productivity shocks have the strongest impact on wages until 1975 and in 1995-2010. Wage bargaining shocks make a larger contribution in 1975-1995; the two shocks have roughly equal importance in recent years. The relative importance of the shock to worker wage bargaining power in Figure 1) is consistent with evidence in Fujita and Ramey (2007), Pizzinelli et al. (2020), Drautzburg et al. (2021) and Ellington et al. (2021)[7].

Using our structural estimates, we estimate impulse response functions for wages and unemployment in response to each of the structural shocks, for every data point in our sample and for each of K periods after the incidence of the shock as

$$\zeta_{t+k,t}^{\mathrm{w,s}} = \frac{\partial log w_{t+k}}{\partial \psi_t^{\mathrm{s}}} \tag{6}$$

and

$$\zeta_{t+k,t}^{\mathrm{u,s}} = \frac{\partial u_{t+k}}{\partial \psi_t^{\mathrm{s}}} \tag{7}$$

for $\mathrm{s} \in \{\mathrm{Prod}, \mathrm{JS}, \mathrm{W}, \mathrm{D}\}$ and for $k = 1, ..., K$. From these, we construct estimates of semi-elasticities of wages with respect to unemployment, for all four structural shocks as[8]

$$se_{t+k,t}^{\mathrm{s}} = \frac{\zeta_{t+k,t}^{\mathrm{w,s}}}{\zeta_{t+k,t}^{\mathrm{u,s}}} \tag{8}$$

for $\mathrm{s} \in \{\mathrm{Prod}, \mathrm{JS}, \mathrm{W}, \mathrm{D}\}$

To compare our estimates to the existing literature, we calculate a weighted average of the four semi-elasticities as

$$\bar{se}_{t+k,t} = \sum_{\mathrm{s} \in \{\mathrm{Prod,JS,W,D}\}} \phi_{t+k,t}^{s} se_{t+k,t}^{\mathrm{s}} \tag{9}$$

where $\phi_{t+k,t}^{s}$ is the share of shock s in the Forecast Error Variance Decomposition, as shown in Figure 1), so that a shock that explains a larger share of the FEVD has a larger weight. The average value of this statistic across our sample corresponds to the point estimate of the semi-elasticity of wages with respect to unemployment in the existing literature, and so allow us to compare our estimates with previous results.

Table 2 contains sample averages of the estimated average semi-elasticity and the estimated semi-elasticities with respect to our four structural shocks, for different values of k. Several features are worth noting. First, our estimates of the average value semi-elasticity, $\bar{se}$, are within the range of estimates in the existing literature. Our average semi-elasticity lies between -1.259 and -0.613, depending on the value of k. By comparison, Gertler et al. (2020) estimate a continuing worker semi-elasticity of -0.46; the same semi-elasticities are estimated as -0.6 in Bils (1985) and as -2.6 in Barlevy (2001). Second, underlying the average semi-elasticity are very different responses to different shocks. In particular, our estimates of semi-elasticities in response to productivity shocks are substantially larger than the average semi-elasticity. For example, our estimates of

[7]For example, Drautzburg et al. (2021) find that bargaining power shocks account for 28% of aggregate fluctuations. This is consistent with the evidence we present in Figure 1).

[8]Our approach is similar to Barnichon and Mesters (2019), who estimate a "Phillips Multiplier" showing the cumulated response of inflation to a demand shock relative to the cumulated response of unemployment, ie $PM = \frac{\sum_{k=0}^{K} \zeta_{t+k,t}^{\pi,\mathrm{D}}}{\sum_{k=0}^{K} \zeta_{t+k,t}^{u,\mathrm{D}}}$

semi-elasticities with respect to productivity shocks are -2.416 for $k = 0$ and -2.165 for $k = 1$. These are approximately twice as large as the corresponding average semi-elasticities. Third, the weak average response of wages to unemployment stems from a strong positive semi-elasticity of wages with respect to unemployment following shocks to wage bargaining power. Finally, the estimated semi-elasticity in response to job destruction shocks is more volatile than the responses to other structural shocks. The impact of this on our results is limited, since shocks to job destruction explain much less of the variation in unemployment and wages than do productivity and wage bargaining power shocks.

Table 2: **Sample Averages of Semi-Elasticities of Wages With Respect to Unemployment**
Notes: This table presents estimated semi-elasticities of real wages with respect to unemployment, calculated as the ratios of the estimated impulse response functions as in (8), for different values of k and averaged across 1964Q2–2019Q4. The first row shows the value of $\bar{se}_{t+k,t}$, calculated using (9). The second row shows the value of $se^{\mathrm{PROD}}_{t+k,t}$, calculated using (8). The other rows show the values of $se^{\mathrm{W}}_{t+k,t}$, $se^{\mathrm{D}}_{t+k,t}$ and $se^{\mathrm{JS}}_{t+k,t}$, also calculated using (8).

	$k=0$	$k=1$	$k=2$	$k=4$
$\bar{se}$	-1.259	-1.025	-0.818	-0.613
se^{PROD}	-2.416	-2.165	-1.598	-1.031
se^{W}	2.273	1.808	1.848	1.020
se^{D}	-2.2207	-1.531	-1.406	-1.289
se^{JS}	-3.387	0.020	0.632	-0.182

Summarising these results, we note that our average semi-elasiticities are in line with the existing literature. As such, they show a weak response of wages to unemployment which indicates substantial real wage rigidity. However, as we show above, these results are misleading, because of the influence of the strong positive response of wages to unemployment following shocks to worker wage bargaining power. The object of interest to the theoretical literature is the response of wages to unemployment following productivity shocks. As shown in Figure 2), we find this to be far larger than the average semi-elasticity. Overall, this implies that the degree of real wage rigidity in response to productivity shocks, the focus of the theoretical literature, is much smaller than the values the empirical literature widely use[9].

Our approach enables us to go beyond the literature by examining movements in semi-elasticities over time. Figure 2) shows estimates of the semi-elasticity of wages with respect to unemployment following productivity shocks and the semi-elasticity of wages with respect to unemployment wage averaged over shocks. Table 3) shows the average values of these semi-elasticies for the periods 1964Q2-1979Q4; 1980Q1-2008Q4 and post-2008. We note that the average semi-elasticity has remained stable over time. This suggests that the existing literature would find no evidence of changes to wage rigidity over time. By contrast, the absolute value of the semi-elasticity in response to productivity shocks has risen across our sample. This implies that the degree of real wage rigidity

[9] As further evidence against wage rigidity, we note that the estimated semi-elasticities in Table 2) decline as k increases; this reflects the fact that real wages respond more quickly than unemployment to shocks.

has fallen throughout our sample.

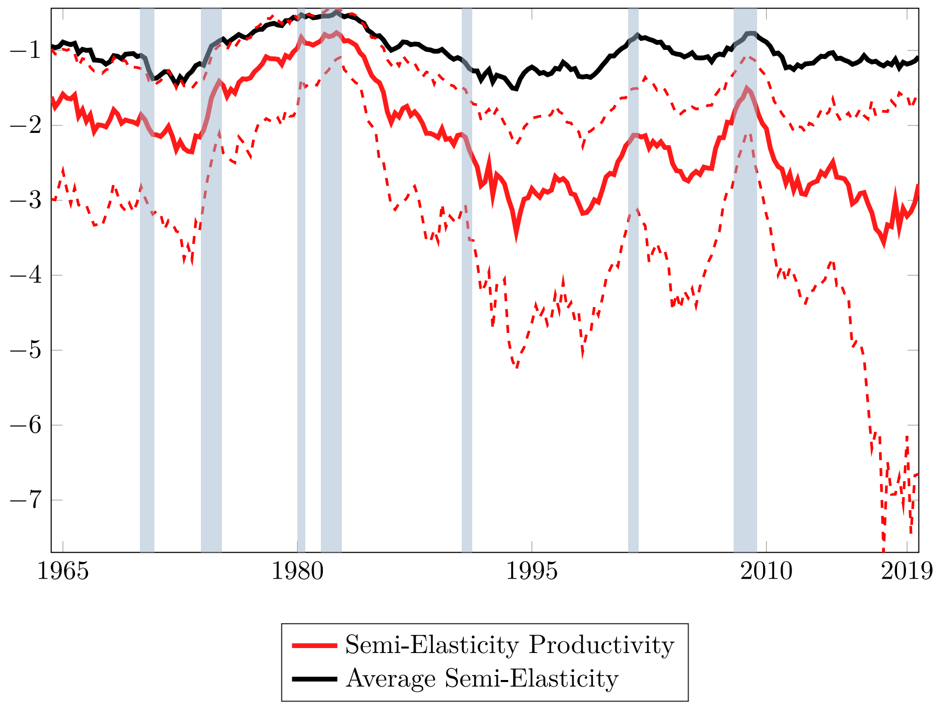

Figure 2: **Variation in Estimated Semi-Elasticities Over Time**
Notes: This figure plots estimated semi-elasticities of real wages with respect to unemployment, calculated as the ratios of the estimated impulse response functions, using $k = 1$. The figure plots (i) the estimated semi-elasticity of wages with respect to unemployment following productivity shocks (red), calculated using (8), with associated credibility bands; (ii) the estimated semi-elasticity of wages with respect to unemployment wage averaged across all shocks (9) (**black**), calculated using (9).

Table 3: **Estimated Sample Averages of Semi-Elasticities of Wages With Respect to Unemployment At Different Dates**
Notes: This table presents estimated semi-elasticities of real wages with respect to unemployment, calculated as the ratios of the estimated impulse response functions as in (8), for $k = 1$ and averaged across 1964Q2-1979Q4; 1980Q1-2008Q4 and 2009Q1-2019Q4. The first row shows the values of $\bar{se}_{t+k,t}$, calculated using (9). The second row shows the values of $se^{\text{PROD}}_{t+k,t}$, calculated using (8).

	1964Q2 − 1979Q4	1980Q1 − 2008Q4	2009Q1 − 2019Q4
$\bar{se}$	−1.006	−1.009	−1.105
se^{PROD}	−1.764	−2.171	−2.781

We explore the robustness of these findings in two ways. First, we use the alternative measure of productivity constructed by Fernald (2014), which adjusts for variations in factor utilisation. Second, we use an alternative empirical identification strategy, which combines the maximum forecast error variance procedure of Uhlig (2004) with our sign restrictions that stem from the theoretical model. This is in a similar vein to Pizzinelli et al. (2020). As the Online Appendix documents, both experiments yield similar conclusions to those we report in the main text. Productivity and wage bargaining shocks account for the majority of wage and unemployment variation with an increasing relative importance of wage bargaining shocks as we move through our sample. The absolute value of the semi-elasticity in response to productivity shocks has risen throughout the sample while the semi-elasticity averaged over all shocks remains relatively stable.

3.1 Implications

Our results imply that many arguments in the existing theoretical literature rely on implausibly large values for real wage rigidity, as measured by the responsiveness of real wages to unemployment in the context of productivity shocks. To assess the implications of this, we calibrate a workhorse New Keynesian model with matching frictions in two scenarios. In the first, we calibrate the model in order to match the semi-elasticity of wages with respect to unemployment that is used in the current empirical literature. In the other, we calibrate in order to match the larger value of the semi-elasticity of wages with respect to unemployment that we estimate in this paper. We then calculate key business cycle statistics under these alternative scenarios.

To do this, we adapt the model used to derive credible identifying restrictions in section 3)[10]. In scenario 1), we calibrate the opportunity cost of employment (b) and the average value of worker bargaining power (z) in order to match a semi-elasticity of wages with respect to productivity shocks of $se^{\text{PROD}} = -0.46$ (the value obtained by Gertler et al. (2020)); in scenario 2), we target a semi-elasticity of $se^{\text{PROD}} = -2.17$ (the value estimated in this paper). The other parameters are the same as those used in our identification exercise. For scenario 1), this implies a high value for the opportunity cost and a small value for bargaining power ($b = 0.71$ and $z = 0.085$). For scenario 2), this implies a lower opportunity cost and much higher bargaining power ($b = 0.4$ and $z = 0.88$).

Table 4: **Simulation Results**

Parameter	Interpretation	Scenario 1	Scenario 2
σ_u	Volatility of Unemployment	0.031	0.01
σ_w	Volatility of the Wage	0.014	0.02
$\rho_{w,u}$	Correlation Between Wage and Unemployment	−0.987	-0.983
ψ_w	First-Order Autocorrelation of the Wage	0.878	0.878
ψ_u	First-Order Autocorrelation of unemployment	0.935	0.935

Our results are summarised in Table 4). We find similar values for the correlation between wages and unemployment, and for the first-order auto-correlations of wages and unemployment. But the volatility of unemployment, relative to the volatility of wages, is three times larger with scenario 1). Although our simple DSGE model is not designed to replicate the high value of unemployment volatility that is observed in the data, it is clear from this that our finding of a low value for wage rigidity challenges existing models that are able to generate a high value for unemployment volatility.

[10]The Online Appendix contains details.

To explore this further, we used a calibration similar to that of Hagedorn and Manovskii (2008), a well known paper that is able to generate a large volatility of unemployment. In particular, we set $b = 0.955$ and $z = 0.052$. The resultant semi-elasticity of wages with respect to unemployment is only $se^{\text{PROD}} = -0.05$, much lower than any estimate in the literature. We also used a calibration similar to that of Shimer (2005), whose calibration does not generate a large unemployment volatility. In this case, we set $b = 0.4$ and $z = 0.72$; the resultant semi-elasticity is $se^{\text{PROD}} = -1.56$, which is consistent with existing evidence, although somewhat lower than our estimate. These experiments highlight how our results create a challenge to the theoretical literature, since it is not clear whether any existing model can match the high value of unemployment volatility in the data while also matching the small value for real wage rigidity that we estimate in this paper.

4 Conclusions

This paper argues there is a disconnect between the theoretical and empirical literatures on real wage rigidity. The theoretical literature assigns a prominent role to productivity shocks in driving business cycle fluctuations, focusing on the role of real wage rigidity in generating a large volatility of unemployment in response to productivity shocks. The empirical literature uses estimates of the semi-elasticity of wages with respect to unemployment to measure real wage rigidity. We point out that this measure is not specific to productivity shocks because it reflects the impact of the different shocks that drive the economy. The impact of other shocks therefore induce bias into estimates of the object of interest; namely the semi-elasticity of wages with respect to unemployment following a productivity shock. This issue is important since the data reflect the impact of shocks to the wage bargaining power of workers as a main driver of unemployment and wage variation. This shock drives wages and unemployment in the same direction and therefore leads to a semi-elasticity one averages over all shocks that indicates substantial wage rigidity.

Using a structural time-varying parameter VAR with stochastic volatility, we estimate the semi-elasticity of wages with respect to unemployment for four structural shocks, including productivity shocks and wage bargaining power shocks. We find that the semi-elasticity with respect to productivity shocks is twice as large as the semi-elasticity one averages over all shocks. This implies a much lower value for real wage rigidity, providing evidence against the hypothesis that real wage rigidity is a major cause of unemployment volatility.

Although we obtain these results using a specific DSGE model with search frictions, our conclusions about the lack of real wage rigidity in the data are more general and not restricted to this type of model. It is also possible to identify the most important structural shocks in our analysis using a model without search frictions in the labour market. This shows that our results apply in a wider set of models than those considered in this paper.

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Revisiting Real Wage Rigidity
Online Appendix

Michael Ellington Chris Martin Bingsong Wang

September 26, 2022

1 Data

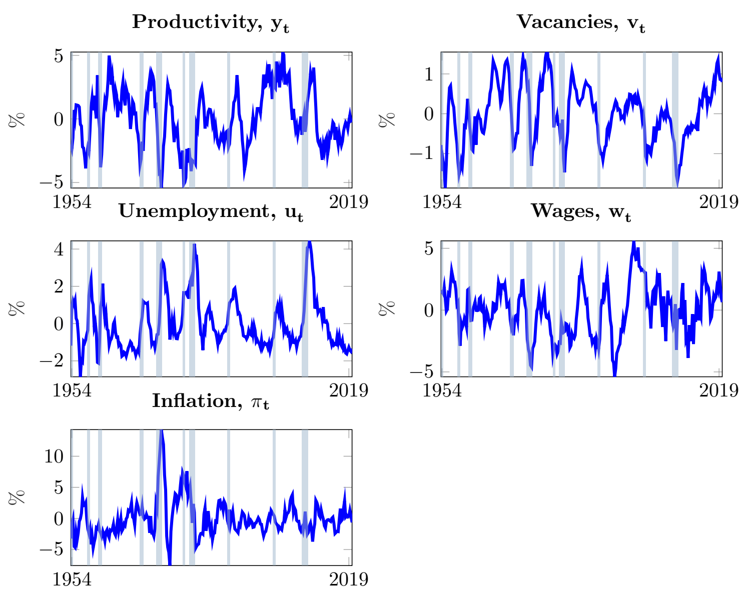

Figure 1: **US Macroeconomic data from 1954Q3 to 2019Q4**
Notes: This figure plots US labour market data from 1954Q3 to 2019Q4. The top left panel plots the log-levels of productivity, y_t; the top right panel plots the vacancy rate, v_t; the middle left panel plots the unemployment rate, u_t; the middle right panel plots the log-levels of the real wage, w_t; the bottom left panel plots the annual inflation rate, π_t. Grey bars indicate NBER recession dates. All variables have been filtered using the Hamilton (2018) filter.

2 Econometric Methodology

Our prior specification involves estimating a Bayesian fixed coefficient VAR (BVAR) model over the training sample. The priors imposed on this BVAR model combine the traditional Minnesota prior of Doan et al. (1984) and Litterman (1986) on the coefficient matrices with an inverse-Wishart prior on the BVAR's covariance matrix. In our specification, the prior mean on the coefficient matrix sets all elements equal zero, except those corresponding to the own first lag of each dependent variable which are set to 0.9. This imposes the prior belief that our variables exhibit persistence whilst simultaneously ensuring shrinkage of the other VAR coefficients to zero. The prior variance of the coefficient matrix is set similar to Litterman (1986). Our prior for the BVAR's covariance matrix follows an inverse-Wishart distribution with the prior scale matrix and degrees of freedom set to an N-dimensional identity matrix and 1+N respectively.

We estimate the BVAR using a standard Gibbs sampler. For the sake of brevity, we do not explicitly outline our algorithm since it is well documented; see e.g. Koop and Korobilis (2010). Our alternative prior specification essentially replaces the conventional Cogley and Sargent (2005) prior with the posterior means from the draws of an estimated BVAR over the training sample

$$\bar{\theta}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\theta_i, \tag{1}$$

$$\overline{\text{V}(\theta)}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\text{V}(\theta_i), \tag{2}$$

$$\bar{\Sigma}_{\text{BVAR}} = \frac{1}{M}\sum_{i=1}^{M}\Sigma_i \tag{3}$$

respectively. Here M denotes the number of saved draws from the estimated BVAR which we set to 20,000. θ_i and $\text{V}(\theta_i)$ denote the ith draw of the coefficient matrix and the variance of the coefficient matrix respectively. Σ_i denotes the ith draw of the BVAR's covariance matrix. From these estimates, the initial conditions of the time-varying coefficient models, θ_0, a_0, h_0 are Normal and independent of one another, and the distributions of the hyperparameters. We set

$$\theta_0 \backsim N\left[\bar{\theta}_{\text{BVAR}},\ 4\cdot\overline{\text{V}(\theta)}_{\text{BVAR}}\right] \tag{4}$$

for α_0, h_0, let $\bar{\Sigma}_{\text{BVAR}}$ be the estimated covariance matrix of the residuals from the time–invariant BVAR. Let C be the lower–triangular Choleski factor such that $CC' = \bar{\Sigma}_{\text{BVAR}}$. The prior for the stochastic volatilities are

$$\ln h_0 \backsim N(\ln\mu_0, 10\times I_5) \tag{5}$$

where μ_0 collects the logarithms of the squared elements along the diagonal of C. Each column of C is divided by the corresponding element on the diagonal; call this matrix $\tilde{C}$. The prior for the contemporaneous relations is

$$\alpha_0 \backsim N\left[\tilde{\alpha}_0,\ \tilde{V}(\tilde{\alpha}_0)\right] \tag{6}$$

with $\tilde{\alpha}_0 \equiv [\tilde{\alpha}_{0,11}, \tilde{\alpha}_{0,21}, \ldots, \tilde{\alpha}_{0,51}]'$ which is a vector collecting all the elements below the diagonal of $\tilde{C}^{-1}$. $\tilde{V}(\tilde{\alpha}_0)$ is diagonal with each element equal to 10 times the absolute value of the corresponding element of $\tilde{\alpha}_0$. This is an arbitrary prior but correctly scales the variance of each element of α_0 to account for their respective magnitudes.

For the time-varying coefficient model assuming $Q_t = Q$, we set Q to follow an inverse Wishart distribution.

$$Q \backsim IW(\underline{Q}^{-1}, \mathrm{T}_0) \tag{7}$$

where $\underline{Q} = (1 + \dim(\theta_t)) \cdot \overline{\mathrm{V}}(\bar{\theta}_{\mathrm{BVAR}}) \cdot 3.4 \times 10^{-4}$. The prior degrees of freedom, $(1 + \dim(\theta_t))$, are the minimum allowed for the prior to be proper. Our choice of scaling parameter of 3.4×10^{-4} is consistent with Cogley and Sargent (2005). We have also estimated our models using different priors, we allowed for a more restrictive scaling parameter of 1.0×10^{-4} and have also set the degrees of freedom to be the length of the training sample; in our case this is 80. The scaling parameter essentially sets the amount of drift within the θ matrices.

With regards to the hyperparameters under the assumption $Q_t = Q_t$, the diagonal elements of Q_t follow a geometric random walk, let $C_{\overline{\mathrm{V}(\bar{\theta}_{\mathrm{BVAR}})}}$ be the lower-triangular Choleski factor such that $C_{\overline{\mathrm{V}(\bar{\theta}_{\mathrm{BVAR}})}} C'_{\overline{\mathrm{V}(\bar{\theta}_{\mathrm{BVAR}})}} = 3.4 \times 10^{-4} \overline{\mathrm{V}(\bar{\theta}_{\mathrm{BVAR}})}$. We then set

$$\ln q_0 \backsim N\left[\ln \mu_{q_0,0}, 10 \times I_{\dim(\theta_t)}\right] \tag{8}$$

with $\ln \mu_{q_0,0}$ collecting the logarithmic squared diagonal elements of $3.4 \times 10^{-4} \overline{\bar{\theta}_{\mathrm{BVAR}})}$. The variances of these stochastic volatility innovations follow an inverse-Gamma distribution for the elements of Z_q,

$$Z_{q,i,i} \backsim IG(\frac{10^{-4}}{2}, \frac{1}{2}) \tag{9}$$

The blocks of S are also assumed to follow inverse–Wishart distributions with prior degrees of freedom equal to the minimum allowed (i.e. $1 + \dim(S_i)$).

$$S_1 \backsim IW(\underline{S}_1^{-1}, 2) \tag{10}$$

$$S_2 \backsim IW(\underline{S}_2^{-1}, 3) \tag{11}$$

$$S_3 \backsim IW(\underline{S}_3^{-1}, 4) \tag{12}$$

$$S_4 \backsim IW(\underline{S}_4^{-1}, 5) \tag{13}$$

we set S_1, S_2, S_3 in accordance with $\tilde{\alpha}_0$ such that $\underline{S}_1 = 10^{-3} \times |\tilde{\alpha}_{0,21}|$, $\underline{S}_2 = 10^{-3} \times \mathrm{diag}([|\tilde{\alpha}_{0,31}|, |\tilde{\alpha}_{0,32}|]')$, $\underline{S}_3 =$

$10^{-3}\times$ diag($[|\tilde{\alpha}_{0,41}|,\ |\tilde{\alpha}_{0,42}|,\ |\tilde{\alpha}_{0,43}|]'$), $\underline{S}_4 = 10^{-3}\times$ diag($[|\tilde{\alpha}_{0,51}|,\ |\tilde{\alpha}_{0,52}|,\ |\tilde{\alpha}_{0,53}|,\ |\tilde{\alpha}_{0,54}|]'$). This calibration is consistent with setting $S_1,\ S_2,\ S_3,\ S_4$ to 10^{-4} times the corresponding diagonal block of $\tilde{V}(\tilde{\alpha}_0)$. The variances for the stochastic volatility innovations, as in Cogley and Sargent (2005), follow an inverse–Gamma distribution for the elements of W,

$$W_{i,i} \backsim IG(\frac{10^{-4}}{2},\ \frac{1}{2}) \tag{14}$$

In order to simulate the posterior distribution of the hyperparameters and states, conditional on the data, we implement the following MCMC that combines elements from Primiceri (2005) and Cogley and Sargent (2005).

1) *Draw elements of* θ_t Conditional on Y^T, α^T and H^T, the observation equation (1) is linear with Gaussian innovations with a known covariance matrix. Factoring the density of θ_t, $p(\theta_t)$ in the following manner

$$p(\theta^T|y^T, A^T, H^T, V) = p(\theta_T|Y^T, A^T, H^T, V) \prod_{t=1}^{T-1} p(\theta_t|\theta_{t+1}, Y^t, A^T, H^T, V) \tag{15}$$

the Kalman filter recursions pin down the first element on the right hand side of the above in the following manner: $p(\theta_T|Y^T, A^T, H^T, V) \backsim N(\theta_T, P_T)$, P_T is the precision matrix of θ_T from the Kalman filter. The remaining elements in the factorisation are obtained via backward recursions as in Cogley and Sargent (2005). Since θ_t is conditionally Normal

$$\theta_{t|t+1} = P_{t|t}P^{-1}_{t+1|t}(\theta_{t+1} - \theta_t) \tag{16}$$

$$P_{t|t+1} = P_{t|t} - P_{t|t}P^{-1}_{t+1|t}P_{t|t} \tag{17}$$

which yields, for every t from $T-1$ to 1, the remaining elements in the observation equation (1). More precisely, the backward recursion begins with a draw, $\tilde{\theta}_T$ from $N(\theta_T,\ P_T)$. Conditional on $\tilde{\theta}_T$, the above produces $\theta_{T-1|T}$ and $P_{T-1|T}$. This permits drawing $\tilde{\theta}_{T-1}$ from $N(\theta_{T-1|T},\ P_{T-1|T})$ until t=1.

2) *Drawing elements of* α_t Conditional on Y^T, θ^T and H^T we follow Primiceri (2005) and note that (1) can be written as

$$A_t\tilde{Y}_t \equiv A_t(Y_t - X_t'\theta_t) = A_t\epsilon_t \equiv \psi_t \tag{18}$$

$$Var(\psi_t) = H_t \tag{19}$$

with $\tilde{Y}_t \equiv [\tilde{Y}_{1,t},\ \tilde{Y}_{2,t},\ \tilde{Y}_{3,t},\ \tilde{Y}_{4,t}]'$ and

$$\begin{aligned}
\tilde{Y}_{1,t} &= \psi_{1,t} && (20)\\
\tilde{Y}_{2,t} &= -\alpha_{21,t}\tilde{Y}_{1,t} + \psi_{2,t} && (21)\\
\tilde{Y}_{3,t} &= -\alpha_{31,t}\tilde{Y}_{1,t} - \alpha_{32,t}\tilde{Y}_{2,t} + \psi_{3,t} && (22)\\
\tilde{Y}_{4,t} &= -\alpha_{41,t}\tilde{Y}_{1,t} - \alpha_{42,t}\tilde{Y}_{2,t} - \alpha_{43,t}\tilde{Y}_{3,t} + \psi_{4,t} && (23)
\end{aligned}$$

These observation equations and the state equation permit drawing the elements of α_t equation by equation using the same algorithm as above; assuming S is block diagonal.

3) *Drawing elements of* H_t Conditional on Y^T, θ^T and α^T, the orthogonal innovations u_t, $Var(\psi_t) = H_t$ are observable. Following Jacquier et al. (2002) the stochastic volatilities, $h_{i,t}$'s, are sampled element by element; Cogley and Sargent (2005) provide details in Appendix B.2.5 of their paper.

4) *Drawing the hyperparameters* Conditional on Y^T, θ^T, H_t and α^T, the innovations in θ_t, α_t and $h_{i,t}$'s are observable, which allows one to draw the elements of $Q_t = Q$, S_1, S_2, S_3 and the $W_{i,i}$.

Note that for the model allowing for stochastic volatility in the innovation variances of the time-varying coefficients, Q_t being a diagonal matrix, we add an extra block into the MCMC algorithm.

3a) *Drawing the elements of* Q_t Conditional on θ_t, the innovations $\kappa_t = \theta_t - \theta_{t-1}$, with $\mathrm{Var}(\kappa_t) = Q_t$ are observable. Therefore we sample the diagonal elements of Q_t applying the Jacquier et al. (2002) algorithm element by element. Following this, we can then sample the $Z_{q,i,i}$ from the inverse-Gamma distribution in step 4 of the above algorithm.

3 Reduced-Form Results

The upper panel of Figure 2 plots the posterior median and 80% highest posterior density intervals for the logarithmic determinant of the time-varying covariance matrices. The lower panel of Figure 2 plots the stochastic volatilities of each variable. Figure 3) contains the reduced-form correlations between our variables

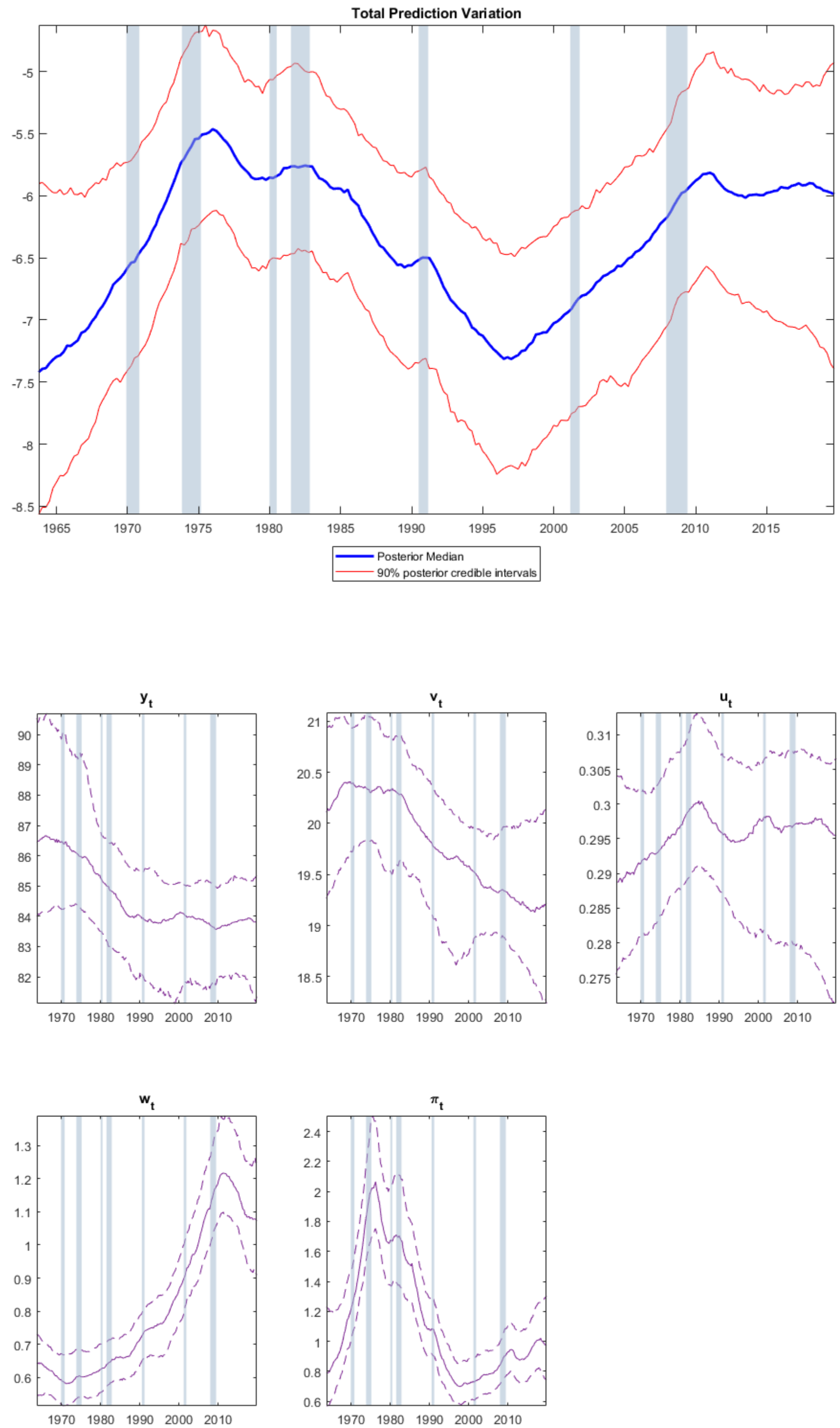

Figure 2: **Total Prediction Variation,** $\ln|\Omega_{t|T}|$**, and Stochastic Volatilities of US Labour Market Variables from 1964Q3 to 2016Q4**

Notes: The upper panel plots the posterior median, and 80% posterior credible intervals of logarithmic determinant of the time-varying reduced-form covariance matrices, $\ln|\Omega_{t|T}|$, from 1964Q3–2016Q4. The lower panel plots the posterior median, and 80% posterior credible intervals of the reduced-from stochastic volatility innovations of productivity, y_t (top left panel); real wages, w_t (top middle panel); the vacancy rate, v_t (top right panel); the unemployment rate, u_t (bottom left panel); and inflation, π_t (bottom middle panel) from 1964Q3–2016Q4. Grey bars indicate NBER recession dates.

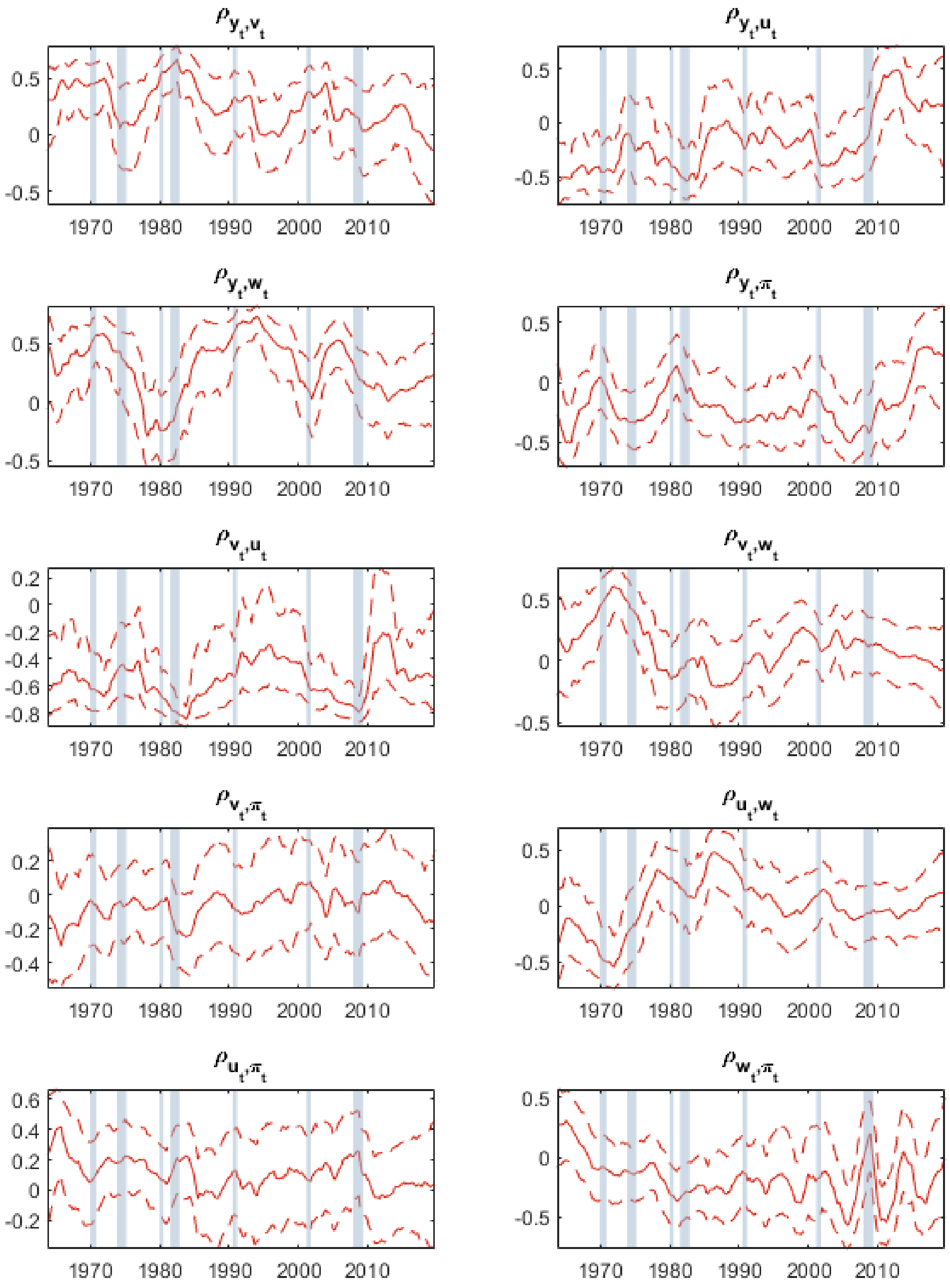

Figure 3: **Reduced-form correlations from 1964Q3 to 2019Q4**

Notes: This figure plots the posterior median, and 80% posterior credible intervals of the reduced-from model implied correlations of variables within the TVP VAR model from 1962Q1–2019Q4. $\hat{\rho}_{i_t,j_t}$ denotes the model implied correlation of variable i and j at time t respectively. y_t, w_t v_t, u_t, π_t denote productivity, real wages, the vacancy rate, the unemployment rate, and inflation, respectively. Grey bars indicate NBER recession dates.

4 Strategy for Identification of Structural Shocks

In this section we outline our identification strategy, which follows Canova and Paustian (2011) and Mumtaz and Zanetti (2015). We simulate a theoretical model using a range of alternative calibrations, based on randomly sampling parameter values within a specified range, constructing a distribution of impulse responses of our endogenous variables to a variety of shocks. We identify structural shocks for which the sign of the impulse responses on impact is unambiguous across this distribution. In this way, we ensure that our identifying sign restrictions are credible, robust to alternative calibrations of the structural parameters. Our identifying restrictions are based on a standard New Keynesian DSGE model without capital but with search frictions in the labour market, similar to Mumtaz and Zanetti (2012) and others.

We summarise the model and structural parameters in the upper panel of Table 1. Equations (T.1)–(T.6) outline the structure of the labour market. Equation T.1 defines the sum of employment (N) and unemployment (u) as the labour force, which is normalised to 1. Equation T.2 outlines employment dynamics and relates employment to hires (h). Equation T.3 defines labour market tightness (θ) as the ratio of vacancies (v) to unemployment. T.4 contains a standard constant returns matching function, while T.5 and T.6 define the vacancy filling rate (q) and the job finding rate (f) respectively. Equation T.7 contains the production function. T.8 defines the marginal cost of hiring labour. Equation T.9 gives the wage, where we have assumed simple Nash bargaining. Equation T.10 defines marginal cost, while T.11 relates price to marginal cost. Equation T.12 is the Euler equation; a summary of these values are in the lower panel of Table 1.

We analyse the impact of four structural shocks. We identify a productivity shock, assuming $A_t = e^{\epsilon_t^P}$. We include a demand shock, ϵ_t^D. We also include a shock to worker relative bargaining power, assuming $z_t = ze^{\epsilon_t^z}$, where ϵ_t^z is a bargaining power shock. And there is a shock to the rate of job destruction, assuming $\tau_t = \tau e^{\epsilon_t^\tau}$, where ϵ_t^τ is a job separations shock. We use impulse response functions to these shocks to impose impact sign restrictions on our structural model.

We specify ranges of values for parameter calibrations and assume that parameters are uniformally distributed within this range. We assume that values of α are uniformally distributed between $0.3 - 0.7$; this is somewhat wider than the range of credible values suggested by Petrongolo and Pissarides (2001). We also consider a wide range of values for matching efficiency, assuming that values of m are uniformally distributed between $0.3 - 1.5$. For the rate of job destruction, Hall and Milgrom (2008) use $\tau = 0.03$, while Pissarides (2009) uses $\tau = 0.036$. These calibrations are designed for monthly data, whereas we use a quarterly frequency, consistent with our data. We therefore consider values between $0.087 - 0.104$. The value of the opportunity cost of employment is also contentious; Shimer (2005) assumes $b = 0.4$, Hall and Milgrom (2008) assume $b = 0.71$. We assume that b is uniformally distributed between 0.4 and 0.8. For the bargaining power of workers, we consider values between $z = 0.1$, so workers have little power to $z = 0.8$, where workers are able to extract most of the surplus from a job match in the form of higher wages. We consider a wide range of values for the probability that prices are fixed, considering values in the range $\theta_\pi = 0$ to $\theta_\pi = 0.9$, encompassing the cases where there is little nominal rigidity and where prices are highly

Table 1: **Contemporaneous Impact of Short-run Shocks on Labour Market Variables**
Notes: Panel a) of this table shows the theoretical model that we simulate. Panel b) shows the range of parameter values from which we sample in our simulations

a) Model Summary

$$N_t + u_t = 1 \tag{T.1}$$

$$N_t = (1-\tau_t)N_{t-1} + h_{t-1} \tag{T.2}$$

$$\theta_t = \frac{v_t}{u_t} \tag{T.3}$$

$$h_t = m u_t^{\alpha} v_t^{(1-\alpha)} \tag{T.4}$$

$$q_t = m\theta_t^{-\alpha} \tag{T.5}$$

$$f_t = q_t\theta_t \tag{T.6}$$

$$Y_t = A_t N_t \tag{T.7}$$

$$\lambda_t = \frac{\kappa}{q_t} - \beta\mathbb{E}_t\frac{\kappa(1-\tau_{t+1})}{q_{t+1}} \tag{T.8}$$

$$w_t = (1-z_t)b + z_t(A_t + \kappa\theta_t) \tag{T.9}$$

$$mc_t = \frac{w_t + \lambda_t}{A_t} \tag{T.10}$$

$$\frac{P_t^*}{P_t} = \frac{\eta}{1-\eta}\left(1-\beta\omega\right)\mathbb{E}_t\sum_{k=0}^{\infty}\left(\beta\omega\right)^k mc_{t+k} \tag{T.11}$$

$$Y_t^{-\eta} = \beta e^{\epsilon_t^D}\mathbb{E}_t Y_{t+1}^{-\eta}\frac{(1+i_t)}{1+\pi_{t+1}} \tag{T.12}$$

$$(1+i_t) = (1+\pi_t)^{\rho_\pi} \tag{T.13}$$

b) Credible Calibration Ranges

Parameter	Interpretation	Range
β	Discount Factor	0.996
α	Elasticity of Matching wrt Unemployment	$0.3 - 0.7$
m	Efficiency of Job Matching	$0.3 - 1.5$
b	Opportunity Cost of Employment	$0.4 - 0.8$
τ	Rate of Job Destruction	$0.087 - 0.104$
z	Worker Relative Bargaining Power	$0.1 - 0.8$
θ_p	Probability Prices Are Fixed	$0. - 0.9$
ρ_π	Monetary Policy Response to Inflation	$1.35 - 2.0$
η	Intertemporal Elasticity of Substitution	1
κ	Cost of Vacancy Posting	0.2

sticky. For the monetary policy response to inflation, we consider values between $\rho_\pi = 1.35$ and $\rho_\pi = 2.0$, encompassing the different estimated values for this parameter in the post-1979 period. We use $\eta = 1$ and set $\kappa = 0.2$.

We simulate our model by randomly selecting a set of calibration values from the distributions we outline above. We calculate the steady-state solution for our model implied by this calibration and construct impulse responses from a log linear expansion of the model around this steady-state. We repeat this process 1000 times, building a distribution of impulse responses. These distributions are shows in Figures 4)-7). We use these to construct the sign restrictions documented in Table 1) of the main paper. In that table, + indicates that all values for the impulse response on impact within the credible range were positive, $-$ indicates that all values for the impulse response on impact within the credible range were negative, and x indicates that the credible range for the impulse response on impact included zero.

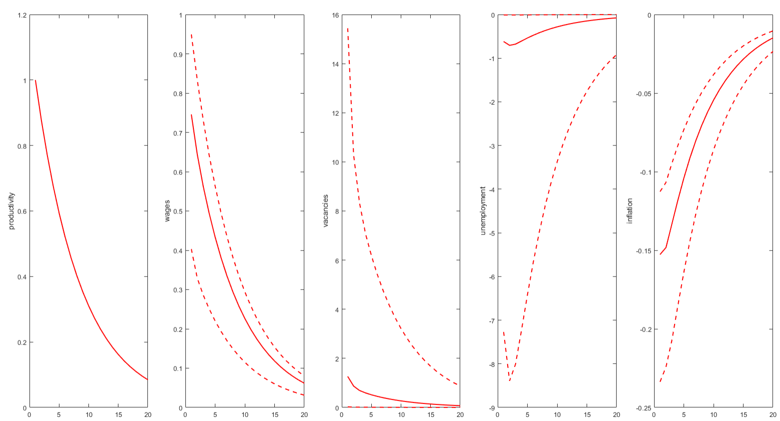

Figure 4: **Median and 10%-90% Bounds of Impulse Responses to Productivity Shocks**
Notes: This figure plots the distribution of impulse response functions following productivity shocks, based on 1000 replications of the model outlined in Table 1 and sampling from the distribution of parameter values outlined in that table.

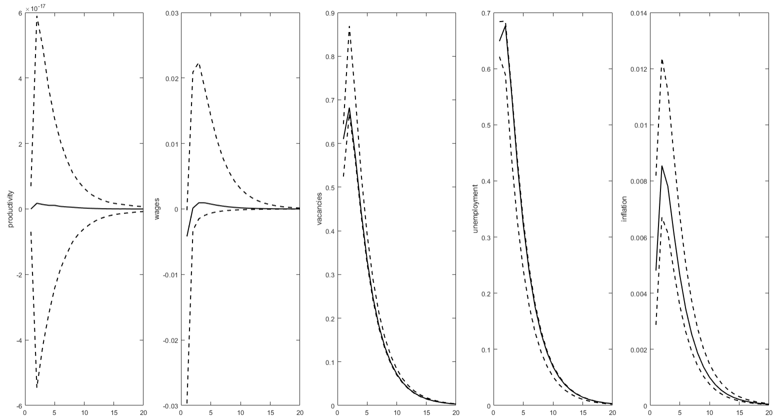

Figure 5: **Median and 10%-90% Bounds of Impulse Responses to Job Separation Shocks**
Notes: This figure plots the distribution of impulse response functions following job separations shocks, based on 1000 replications of the model outlined in Table 1 and sampling from the distribution of parameter values outlined in that table.

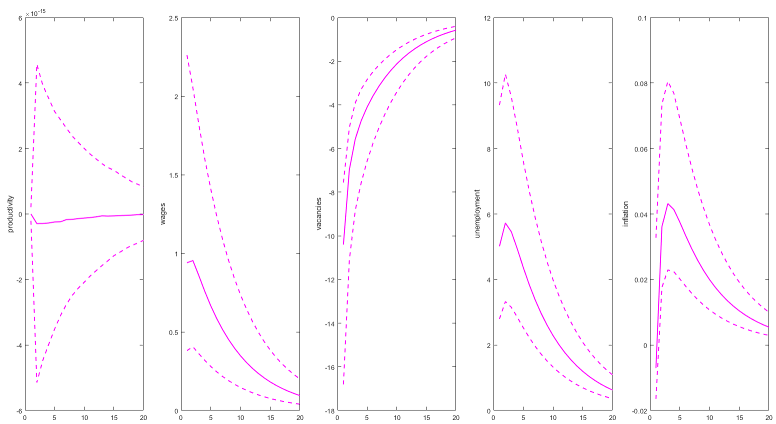

Figure 6: **Median and 10%-90% Bounds of Impulse Responses to Bargaining Power Shocks**
Notes: This figure plots the distribution of impulse response functions following bargaining power shocks, based on 1000 replications of the model outlined in Table 1 and sampling from the distribution of parameter values outlined in that table.

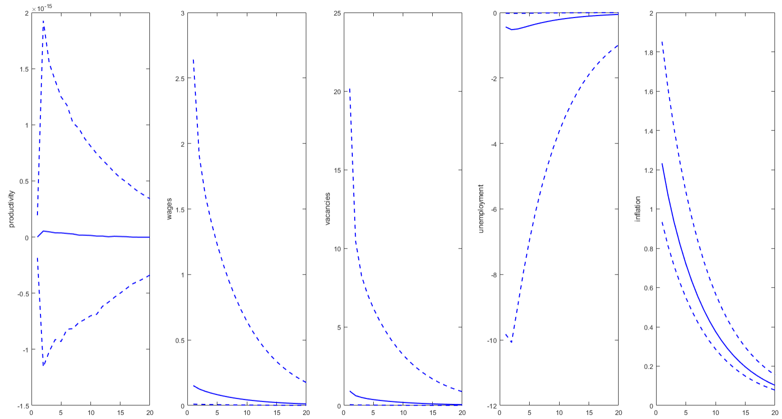

Figure 7: **Median and 10%-90% Bounds of Impulse Responses to Demand Shocks**
Notes: This figure plots the distribution of impulse response functions following demand shocks, based on 1000 replications of the model outlined in Table 1 and sampling from the distribution of parameter values outlined in that table.

5 Robustness Analysis

5.1 An Alternative Productivity Series

To assess the robustness of our main findings, we now replace our original productivity series with that of Fernald (2014). We re-run our baseline model exactly as before and implement the same sign restrictions. Figures 8 and 9 report our results analogous to Figures 1 and 2 in the main text.

Overall, it is clear that replacing our original productivity series with that of Fernald (2014) yields qualitatively similar conclusions to those we report in the main text. Productivity and wage bargaining shocks account for the majority of wage and unemployment variation with an increasing relative importance of wage bargaining shocks as we move through our sample. This provides further evidence supporting Fujita and Ramey (2007), Theodoridis and Zanetti (2020), Drautzburg et al. (2021) and Ellington et al. (2021). We can also see that while the average semi elasticity across all shocks remains stable over the sample, the absolute value of the semi-elasticity in response to productivity shocks has risen.

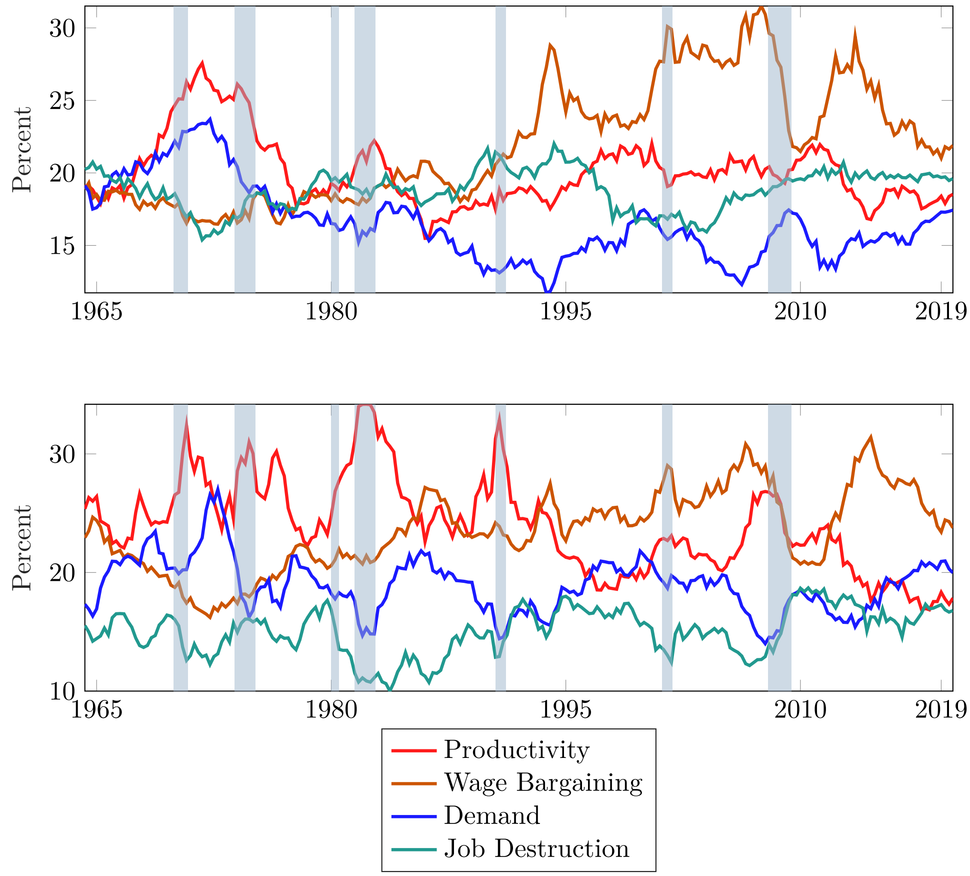

Figure 8: **Variance Decomposition of Wages and Unemployment using Fernald (2014) Productivity**
Notes: This figure plots the contribution of (i) productivity shocks (red); (ii) wage bargaining power shocks (brown); (iii) demand shocks (blue) and (iv) job destruction shocks (green) in explaining the 1-period ahead variation in wages (top panel) and unemployment (lower panel) across our sample.

5.2 An Alternative Identification Strategy

We now focus on an alternative empirical identification strategy. We combine the maximum forecast error variance procedure of Uhlig (2004) with our sign restrictions that stem from the theoretical model. This is in a similar vein to Pizzinelli et al. (2020). In doing so, we impose the restriction that the productivity shock explains the majority of the forecast error variance of labour productivity at business cycle frequencies (i.e. from horizons 0 to 40)[1]. To ease computational burden because we have a TVP VAR model, we compute simple impulse responses and sample every fourth quarter. Figures 10 and 11 report our results analogous to Figures 1 and 2 in the main text.

Again on the whole, using an alternative identification scheme results in similar conclusions to our baseline analysis. Productivity and wage bargaining shocks account for the majority of wage and unemployment variation with an increasing relative importance of wage bargaining shocks as we move through our sample. However, note that the absolute value of the forecast error variance

[1]For technical details on this procedure, see Appendix B.2 in Pizzinelli et al. (2020).

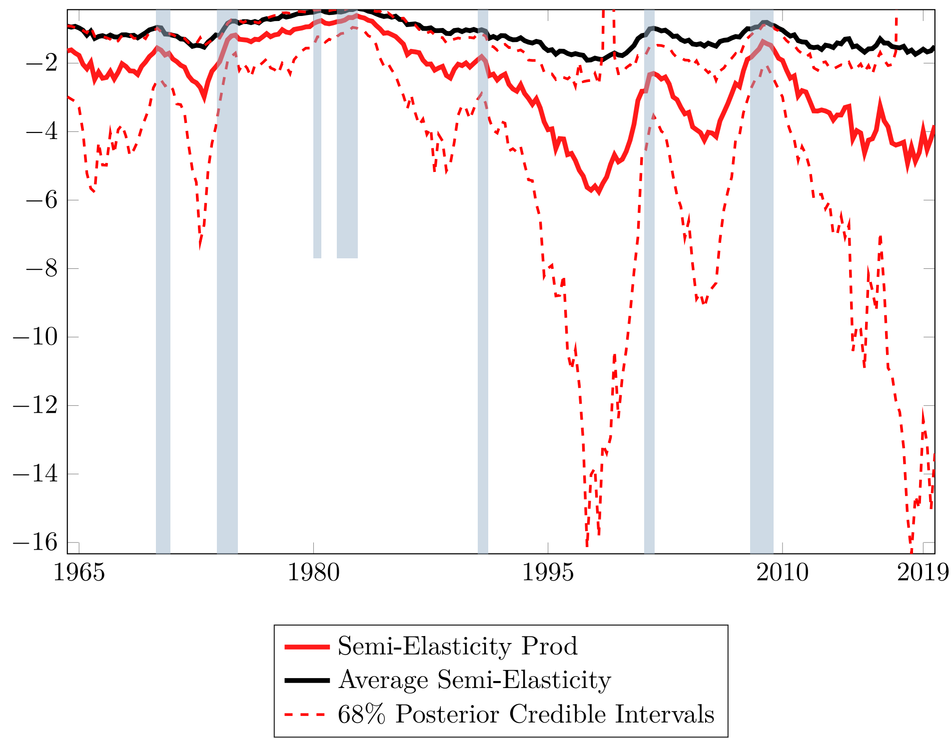

Figure 9: **Variation in Estimated Semi-Elasticities Over Time using Fernald (2014) Productivity**

Notes: This figure plots estimated semi-elasticities of real wages with respect to unemployment, calculated as the ratios of the estimated impulse response functions, using $k = 1$. The figure plots (i) the estimated semi-elasticity of wages with respect to unemployment following productivity shocks (red), with associated credibility bands; (ii) the estimated semi-elasticity of wages with respect to unemployment averaged across all shocks using forecast error variance decompositions to weight shocks.

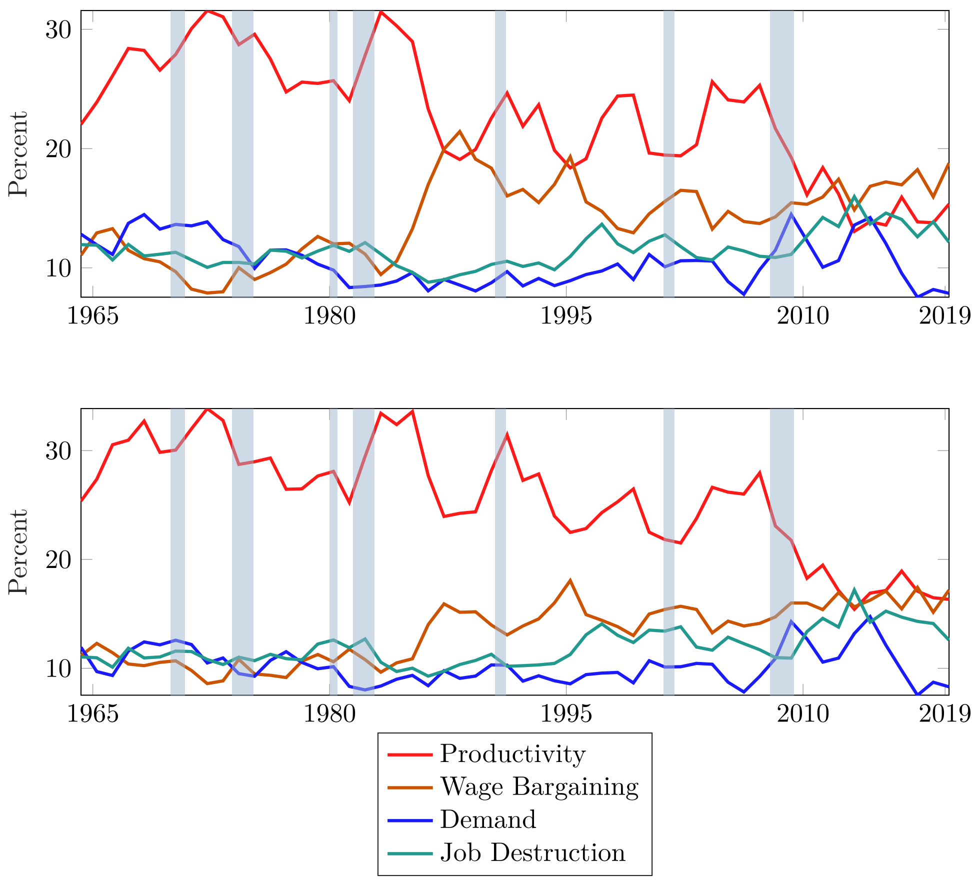

Figure 10: **Variance Decomposition of Wages and Unemployment; An Alternative Identification Scheme**
Notes: This figure plots the contribution of (i) productivity shocks (red); (ii) wage bargaining power shocks (brown); (iii) demand shocks (blue) and (iv) job destruction shocks (green) in explaining the 1-period ahead variation in wages (top panel) and unemployment (lower panel) across our sample.

shares associated to wage bargaining shocks is slightly lower than our baseline analysis. Turning to the semi-elasticity plots in Figure 11 it is clear that the absolute value of the semi-elasticity in response to productivity shocks has risen throughout the sample while the semi-elasticity averaged over all shocks remains relatively stable.

In general these robustness checks further substantiate our main findings and provides additional empirical evidence that one cannot attribute unemployment volatility to real wage rigidity.

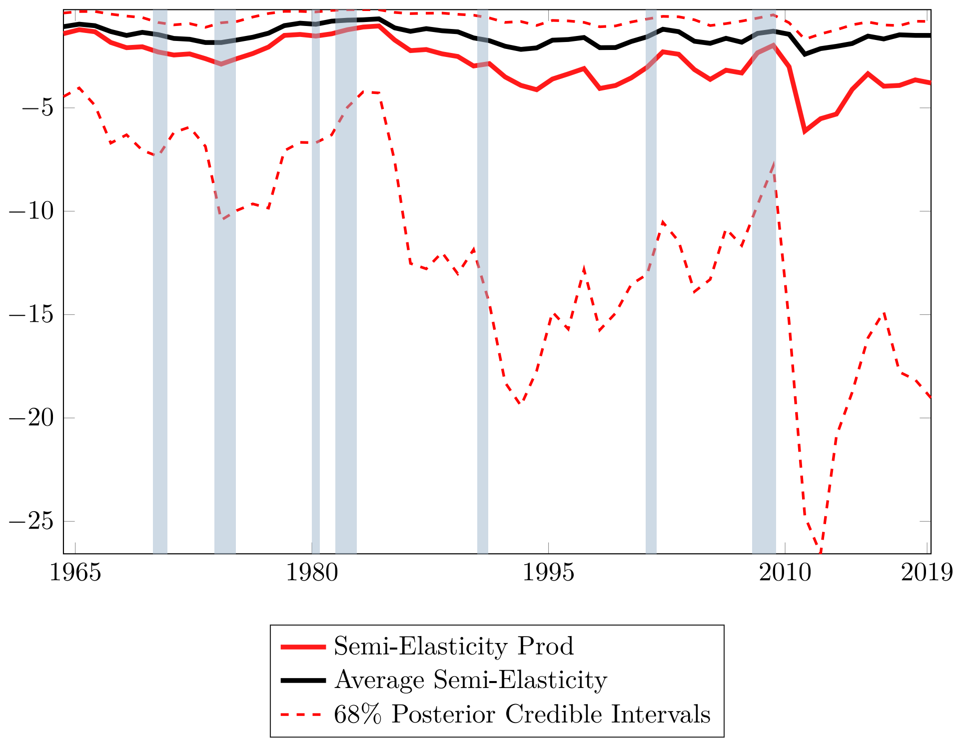

Figure 11: **Variation in Estimated Semi-Elasticities Over Time; An Alternative Identification Scheme**

Notes: This figure plots estimated semi-elasticities of real wages with respect to unemployment, calculated as the ratios of the estimated impulse response functions, using $k = 1$. The figure plots (i) the estimated semi-elasticity of wages with respect to unemployment following productivity shocks (red), with associated credibility bands; (ii) the estimated semi-elasticity of wages with respect to unemployment averaged across all shocks using forecast error variance decompositions to weight shocks.

6 Alternative Calibrations

In this section we explore the consequences of differing values for wage rigidity for macroeconomic modelling. We calibrate a workhorse New Keynesian model with matching frictions in two scenarios. In the first, we calibrate the model in order to match the value for the wage rigidity in response to productivity shocks that we use in the current literature. In the other, we calibrate in order to match the smaller value of wage rigidity that we find in this paper.

To do this, we use the model outlined and used to derive credible sign restrictions, in section 4). We set $\alpha = 0.5$, $m = 1.7$, $\tau = 0.1$, $\theta_p = 0.5$, $\rho_\pi = 1.5$ and $\eta = 1$. We also target an unemployment rate of 5.2%. Given these, we solve for the values of w, κ and λ that satisfy (T.8)-(T.10) in Table 1) above and calibrate z and b to give the desired value of the semi-elasticty of wages with respect to unemployment. In secenario 1), we target a semi-elasticity of wages with respect to productivity shocks of $se^{\text{Prod}}_{t+1,t} = -0.46$, the value obtained by Gertler et al (2020); in scenario 2), we target $se^{\text{Prod}}_{t+1,t} = -2.17$. For scenario 1), we obtain $b = 0.71$, and $z = 0.085$; for scenario 2), we obtain $b = 0.4$ and $z = 0.88$.

Table 2: **Simulation Results**

Parameter	Interpretation	Scenario 1	Scenario 2
σ_u	Volatility of Unemployment	0.031	0.01
σ_w	Volatility of the Wage	0.014	0.02
$\rho_{w,u}$	Correlation Between Wage and Unemployment	-0.987	-0.983
ψ_w	First-Order Autocorrelation of the Wage	0.878	0.878
ψ_u	First-Order Autocorrelation of unemployment	0.935	0.935

Our results are summarised in Table 2). As we might expect, the volatility of unemployment relative to the volatility of wages is higher with the values of real wage rigidity used in the existing literature which are reflected in Scenario 1), compared to our estimated lower value for real wage rigidity, reflected in Scenario 2). Although our simple DSGE model is not designed to replicate the high value of unemployment volatility that is observed in the data, it is clear that our finding of a low value for wage rigidity challenges existing models that are able to generate a high value for unemployment volatility.

To explore this further, we used a calibration similar to that of Hagedorn and Manovskii (2008), a well-known paper that is able to generate a large volatility of unemployment. In particular, we set $b = 0.955$ and $z = 0.052$. The resultant semi-elasticity of wages with respect to unemployment is only -0.05, much lower than any estimate in the literature. We also used a calibration similar to that of Shimer (2005), whose calibration does not generate a large unemployment volatility. In this case, we set $b = 0.4$ and $z = 0.72$; the resultant semi-elasticity is -1.56, which is consistent with existing evidence, although somewhat lower than our estimate. These experiments highlight how our results create a challenge to the theoretical literature, since it is not clear whether any existing model can match the high value of unemployment volatility in the data while also matching the small value for real wage rigidity that we estimate in this paper.

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