

Statistics: 1.1 Paired t-tests

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1 Introduction

A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample. Examples of where this might occur are:

- Before-and-after observations on the same subjects (e.g. students' diagnostic test results before and after a particular module or course).
- A comparison of two different methods of measurement or two different treatments where the measurements/treatments are applied to the same subjects (e.g. blood pressure measurements using a stethoscope and a dynamap).

2 Procedure for carrying out a paired t-test

Suppose a sample of n students were given a diagnostic test before studying a particular module and then again after completing the module. We want to find out if, in general, our teaching leads to improvements in students' knowledge/skills (i.e. test scores). We can use the results from our sample of students to draw conclusions about the impact of this module in general.

Let x = test score before the module, y = test score after the module

To test the null hypothesis that the true mean difference is zero, the procedure is as follows:

1. Calculate the difference ($d_i = y_i - x_i$) between the two observations on each pair, making sure you distinguish between positive and negative differences.
2. Calculate the mean difference, \bar{d} .
3. Calculate the standard deviation of the differences, s_d , and use this to calculate the standard error of the mean difference, $SE(\bar{d}) = \frac{s_d}{\sqrt{n}}$
4. Calculate the t-statistic, which is given by $T = \frac{\bar{d}}{SE(\bar{d})}$. Under the null hypothesis, this statistic follows a t-distribution with $n - 1$ degrees of freedom.
5. Use tables of the t-distribution to compare your value for T to the t_{n-1} distribution. This will give the p-value for the paired t-test.

NOTE:

For this test to be valid the differences only need to be approximately normally distributed. Therefore, it would not be advisable to use a paired t-test where there were any extreme outliers.

Example

Using the above example with $n = 20$ students, the following results were obtained:

Student	Pre-module score	Post-module score	Difference
1	18	22	+4
2	21	25	+4
3	16	17	+1
4	22	24	+2
5	19	16	-3
6	24	29	+5
7	17	20	+3
8	21	23	+2
9	23	19	-4
10	18	20	+2
11	14	15	+1
12	16	15	-1
13	16	18	+2
14	19	26	+7
15	18	18	0
16	20	24	+4
17	12	18	+6
18	22	25	+3
19	15	19	+4
20	17	16	-1

Calculating the mean and standard deviation of the differences gives:

$$\bar{d} = 2.05 \text{ and } s_d = 2.837. \text{ Therefore, } SE(\bar{d}) = \frac{s_d}{\sqrt{n}} = \frac{2.837}{\sqrt{20}} = 0.634$$

So, we have:

$$t = \frac{2.05}{0.634} = 3.231 \quad \text{on 19 df}$$

Looking this up in tables gives $p = 0.004$. Therefore, there is strong evidence that, on average, the module does lead to improvements.

3 Confidence interval for the true mean difference

The in above example the estimated average improvement is just over 2 points. Note that although this is statistically significant, it is actually quite a small increase. It would be useful to calculate a confidence interval for the mean difference to tell us within what limits the true difference is likely to lie. A 95% confidence interval for the true mean difference is:

$$\bar{d} \pm t^* \frac{s_d}{\sqrt{n}} \quad \text{or, equivalently} \quad \bar{d} \pm (t^* \times SE(\bar{d}))$$

where t^* is the 2.5% point of the t-distribution on $n - 1$ degrees of freedom.

Using our example:

We have a mean difference of 2.05. The 2.5% point of the t-distribution with 19 degrees of freedom is 2.093. The 95% confidence interval for the true mean difference is therefore:

$$2.05 \pm (2.093 \times 0.634) = 2.05 \pm 1.33 = (0.72, 3.38)$$

This confirms that, although the difference in scores is statistically significant, it is actually relatively small. We can be 95% sure that the true mean increase lies somewhere between just under one point and just over 3 points.

4 Carrying out a paired t-test in SPSS

The simplest way to carry out a paired t-test in SPSS is to compute the differences (using **Transform, Compute**) and then carrying out a one-sample t-test as follows:

- **Analyze**
- **Compare Means**
- **One-Sample T Test**
- Choose the difference variable as the **Test Variable** and click **OK**

The output will look like this:

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Difference	20	2.0500	2.83725	.63443

One-Sample Test

Test Value = 0						
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval	
					Lower	Upper
Difference	3.231	19	.004	2.0500	.7221	3.3779