MATHEMATICS

SUPPORT CENTRE

Title: Trigonometric graphs.

Target: On completion of this worksheet you should be able to sketch graphs of the sine, cosine and tangent functions and be able to use them to solve simple trigonometry equations.

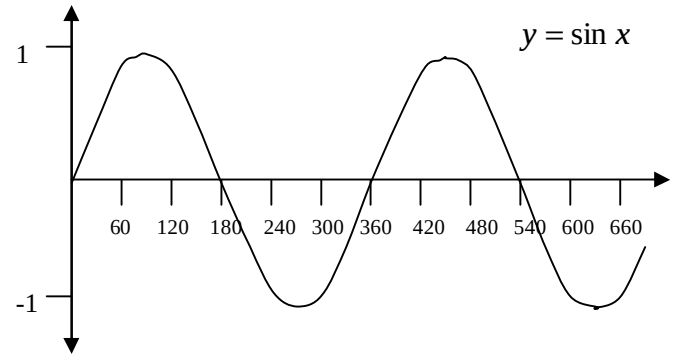
To sketch the graphs of

$y = \sin x,\ y = \cos x$ and $y = \tan x$

we can plot several points. We will choose x-coordinates at 30° intervals.

x	0	30	60	90	120	150	180	210
y=sin$x$	0	0.5	0.87	1	0.87	0.5	0	-0.5

x	240	270	300	330	360	390	420	450
y=sin$x$	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Exercise.

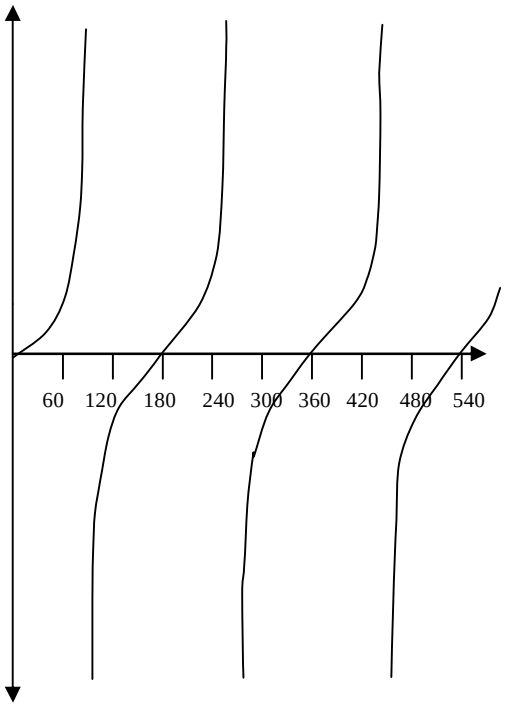
Sketch the graph of $y = \cos x$ by plotting points at 30° intervals.

(No Answers.)

In a similar way we will sketch the graph of $y = \tan x$.

x	0	30	60	90	120	150	180	210
y=tan$x$	0	0.58	1.73	∞	-1.73	-.058	0	0.58

x	240	270	300	330	360	390	420
y=tan$x$	1.73	∞	-1.73	-0.58	0	0.58	1.73



We can see that the graphs of sin tan and cos "repeat themselves". That is they are periodic.

We can solve some equations involving sine, tangent and cosine by using the inverse sine, cosine and tangent functions.

Example:
Find an angle x such that:

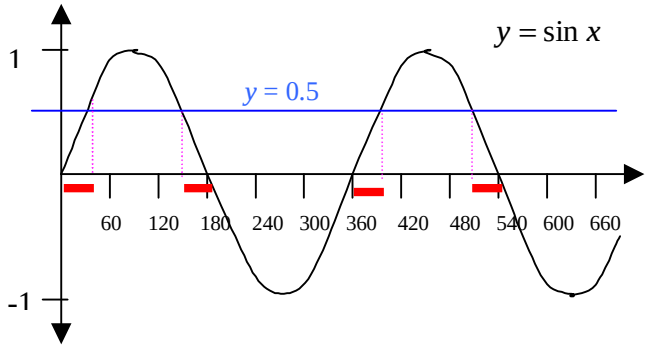
1. $\sin x = 0.5$.
2. $\tan x = 1$.

1. $\sin x = 0.5$.
   Doing $\sin^{-1}$ to both sides gives:
   $x = 30°$.

2. $\tan x = 1$.
   Doing $\tan^{-1}$ to both sides gives:
   $x = 45°$.

If you have difficulty with this refer to the trigonometry sheet on finding angles of right angled triangles.

From the graph we have sketched of $y = \sin x$, we can see that there are many angles x such that $\sin x = 0.5$.



If you have difficulty with this refer to the graph sheet on solving equations.

We can find all the angles satisfying $\sin x = 0.5$ by looking at the symmetry of the graph.
All the red lines are the same length. The length of the line is 30°. (We know this from the previous example.)
Therefore x = 30°, or 180°-30° = 150°, or 360° + 30° = 390° or 540° - 30° = 510° etc.

Therefore to find all the solutions of an equation involving sine, cosine or tangent we should:

- Find one answer using our calculators.
- On the graph of the function mark all the other solutions.
- Using the symmetry of the graph evaluate the other solutions.

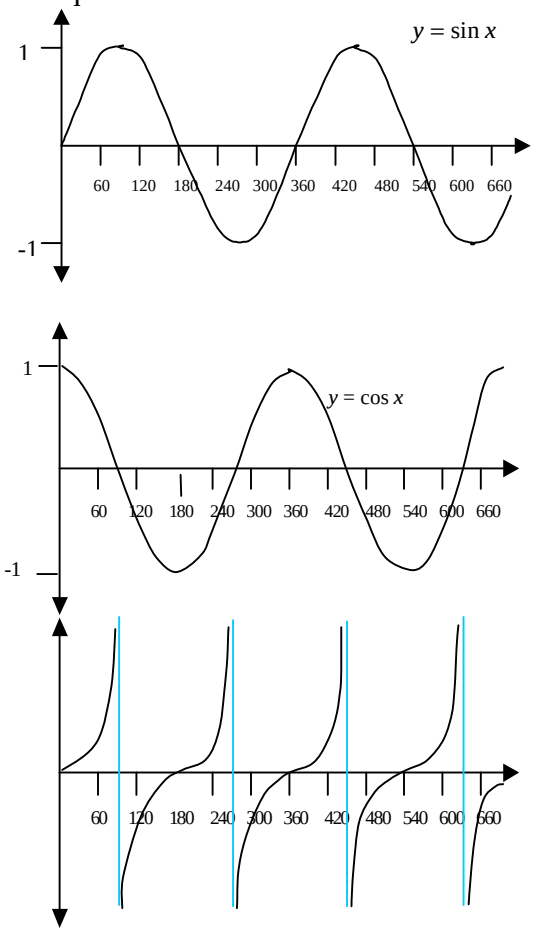
Exercise.

1. Find an angle x satisfying the following equations:

   a) $\cos x = 0.5$ b) $\sin x = \frac{\sqrt{3}}{2}$ c) $\tan x = -1$

   d) $\cos x = -\frac{\sqrt{3}}{2}$ e) $\sin x = -\frac{1}{\sqrt{2}}$.

2. For each of the above equations use the graphs below to find all the angles x between 0° and 600° that satisfy the equations.



(Answers: {60°, 300°, 420°};{60°, 120°, 420°, 480°};{135°, 225°, 405°, 485°};{150°, 210°, 510°, 570°};{ 225°, 315°, 585°}.)