Linear inequalities

Introduction

The ability to sketch regions defined by linear inequalities is helpful when studying linear programming. This leaflet reminds you how to sketch these regions. You will need to be familiar with the manipulation of inequalities and the sketching of linear relationships.

Inequalities

Consider the following problem:

Example
Sketch the region defined by $-4x + 3y \leq -24$.

Solution
The easiest way to proceed is to first sketch the linear relationship defined by the *equality*, rather than the inequality, that is $-4x+3y = -24$. Following the method described in *Linear Relationships* we find two points on the line and join them. Two such points are $(0, -8)$ and $(6, 0)$. The line is shown in Figure 1.

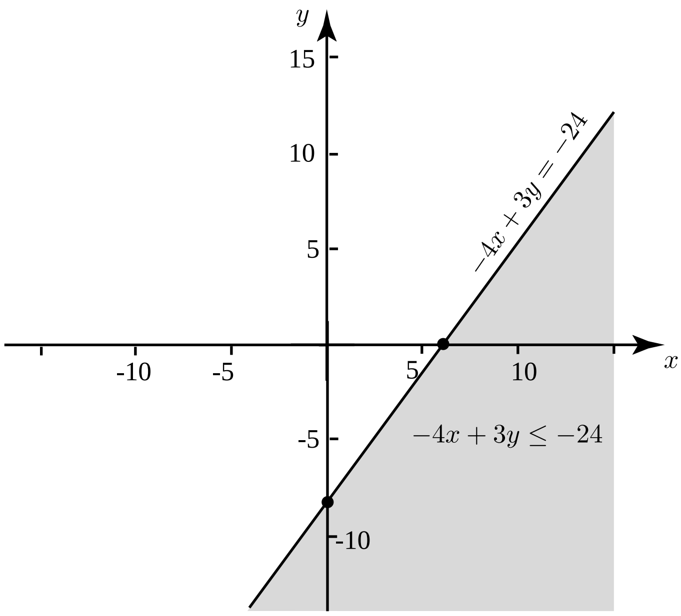

Figure 1

On the line y and x satisfy the equation $-4x + 3y = -24$. This can be rearranged as $y = \frac{4x-24}{3}$. Anywhere <u>below</u> this line, y <u>is less than</u> $\frac{4x-24}{3}$, that is $y < \frac{4x-24}{3}$. Rearranging again we obtain $-4x + 3y < -24$. Hence the inequality $-4x + 3y \leq -24$ is satisfied anywhere on or below this line. This region is shaded in Figure 1.

Example
Sketch the region defined by $x + y < 5$.

Solution
First of all sketch the line defined by $x + y = 5$. This is shown in Figure 2.

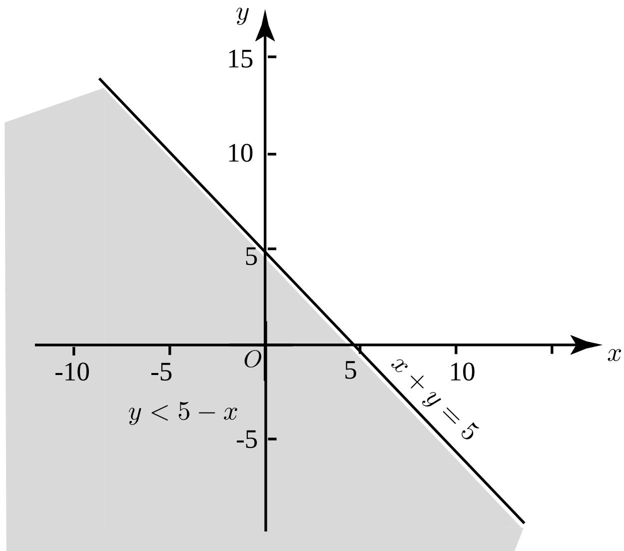

Figure 2

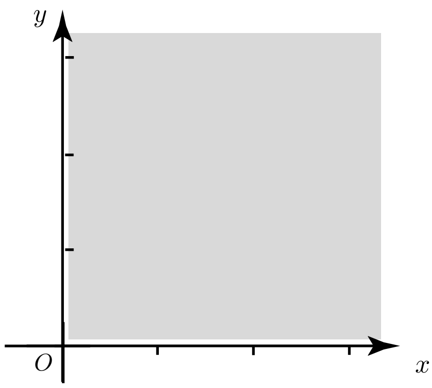

Figure 3

Note that the equation can be rearranged as $y = 5 - x$ and any point on the line satisfies this equation. Anywhere below the line y is less than $5 - x$, that is $y < 5 - x$, or $x + y < 5$. This region is shaded in Figure 2.

Example Sketch the region defined by $x > 0$, $y > 0$.

Solution The inequalities state that both x and y are greater than zero. This corresponds to the region shaded in Figure 3.

Example It will be necessary to shade a region satisfied by two or more inequalities simultaneously. Sketch the region defined by $x \geq 0$, $y \geq 0$, $x + y \leq 10$ and $x + 3y \leq 15$.

Solution The first two inequalities restrict attention to the first quadrant. The shaded region and its boundaries are satisfied by the two remaining inequalities.

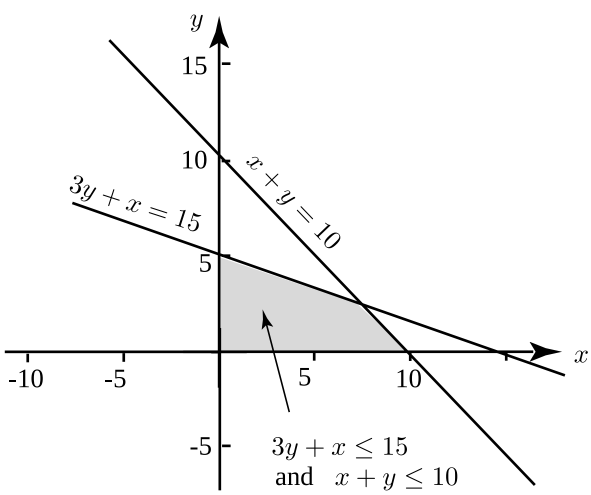

Figure 4