Simplifying fractions

Introduction

Fractions involving symbols occur frequently. It is necessary to be able to simplify these and rewrite them in different but equivalent forms. On this leaflet we revise how these processes are carried out. It will be helpful if you have already seen the leaflet: *Fractions*.

Expressing a fraction in its simplest form

An algebraic fraction can always be expressed in different, yet **equivalent** forms. A fraction is expressed in its **simplest form** by cancelling any factors which are common to both the numerator and the denominator. You need to remember that factors are multiplied together.

For example, the two fractions

$$\frac{7a}{ab} \qquad \text{and} \qquad \frac{7}{b}$$

are equivalent. Note that there is a common factor of a in the numerator and the denominator of $\frac{7a}{ab}$ which can be cancelled to give $\frac{7}{b}$.

> To express a fraction in its simplest form, any factors which are common to both the numerator and the denominator are cancelled

Notice that cancelling is equivalent to dividing the top and the bottom by the common factor.

It is also important to note that $\frac{7}{b}$ can be converted back to the equivalent fraction $\frac{7a}{ab}$ by multiplying both the numerator and denominator of $\frac{7}{b}$ by a.

> A fraction is expressed in an equivalent form by multiplying both top and bottom by the same quantity, or dividing top and bottom by the same quantity

Example
The two fractions

$$\frac{10y^2}{15y^5} \qquad \text{and} \qquad \frac{2}{3y^3}$$

are equivalent. Note that

$$\frac{10y^2}{15y^5} = \frac{2 \times 5 \times y \times y}{3 \times 5 \times y \times y \times y \times y \times y}$$

and so there are common factors of 5 and $y \times y$. These can be cancelled to leave $\frac{2}{3y^3}$.

Example
The fractions

$$\frac{(x-1)(x+3)}{(x+3)(x+5)} \qquad \text{and} \qquad \frac{(x-1)}{(x+5)}$$

are equivalent. In the first fraction, the common factor $(x+3)$ can be cancelled.

Example
The fractions

$$\frac{2a(3a-b)}{7a(a+b)} \qquad \text{and} \qquad \frac{2(3a-b)}{7(a+b)}$$

are equivalent. In the first fraction, the common factor a can be cancelled. Nothing else can be cancelled.

Example
In the fraction

$$\frac{a-b}{a+b}$$

there are no common factors which can be cancelled. Neither a nor b is a factor of the numerator. Neither a nor b is a factor of the denominator.

Example
Express $\dfrac{5x}{2x+1}$ as an equivalent fraction with denominator $(2x+1)(x-7)$.

Solution
To achieve the required denominator we must multiply both top and bottom by $(x-7)$. That is

$$\frac{5x}{2x+1} = \frac{(5x)(x-7)}{(2x+1)(x-7)}$$

If we wished, the brackets could now be removed to write the fraction as $\dfrac{5x^2-35x}{2x^2-13x-7}$.

Exercises
1. Express each of the following fractions in its simplest form:

a) $\frac{12xy}{16x}$, b) $\frac{14ab}{21a^2b^2}$, c) $\frac{3x^2y}{6x}$, d) $\frac{3(x+1)}{(x+1)^2}$, e) $\frac{(x+3)(x+1)}{(x+2)(x+3)}$, f) $\frac{100x}{45}$, g) $\frac{a+b}{ab}$.

Answers
1. a) $\frac{3y}{4}$, b) $\frac{2}{3ab}$, c) $\frac{xy}{2}$, d) $\frac{3}{x+1}$, e) $\frac{x+1}{x+2}$, f) $\frac{20x}{9}$, g) cannot be simplified. Whilst both a and b are factors of the denominator, neither a nor b is a factor of the numerator.