Fractions: adding and subtracting

In this unit we shall see how to add and subtract fractions. We shall also see how to add and subtract mixed fractions by turning them into improper fractions.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- add and subtract fractions;
- add and subtract mixed fractions.

Contents

1. Introduction

Here is a simple example of adding fractions: calculate $\frac{1}{5} + \frac{2}{5}$.

To understand this, suppose we have a cake and divide it into five equal pieces. Each piece is a fifth, or $\frac{1}{5}$, of the cake. If we take one fifth, and then a further two fifths, we have taken a total of three fifths:

$$\frac{1}{5} + \frac{2}{5} \quad = \quad \frac{3}{5} \quad .$$

Example

Calculate $\frac{1}{8} + \frac{1}{8} + \frac{5}{8}$.

Solution

$$\frac{1}{8} + \frac{1}{8} + \frac{5}{8} \quad = \quad \frac{7}{8}\,.$$

In both of these examples we were adding 'like' things. In the first example we were adding fifths, and in the second we were adding eighths. So in both cases the denominators were the same. So to add 'like' fractions we just add the numerators.

The process is similar for subtraction, but we take away instead of adding.

Example

Calculate $\frac{5}{8} - \frac{3}{8}$.

Solution

$$\frac{5}{8} - \frac{3}{8} \quad = \quad \frac{2}{8} \quad = \quad \frac{1}{4}\,.$$

Example

Calculate $\frac{3}{5} + \frac{4}{5}$.

Solution

$$\frac{3}{5} + \frac{4}{5} \quad = \quad \frac{7}{5} \quad = \quad 1\tfrac{2}{5}\,.$$

Key Point

When adding or subtracting 'like' fractions, when the denominators are the same, just add or subtract the numerators.

Exercises

1. Add the following fractions:

(a) $\frac{1}{6}+\frac{3}{6}$ (b) $\frac{1}{8}+\frac{1}{8}+\frac{3}{8}+\frac{5}{8}$ (c) $\frac{3}{10}+\frac{4}{10}+\frac{7}{10}$

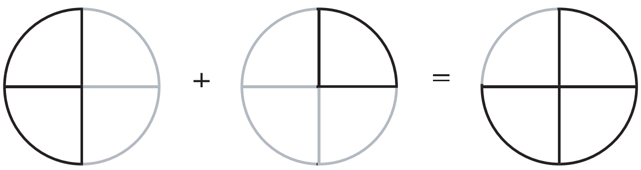
2. Subtract the following fractions:

(a) $\frac{7}{8}-\frac{3}{8}$ (b) $\frac{9}{10}-\frac{2}{10}$ (c) $\frac{11}{12}-\frac{4}{12}$

2. Fractions with different denominators

What happens when we want to add or subtract fractions where the denominators are not the same? Let us look at a simple case. What is $\frac{1}{2}+\frac{1}{4}$?

If we think of a pizza cut in half and then into quarters, we can see that if we take a half and then a quarter we will have taken a total of three quarters.



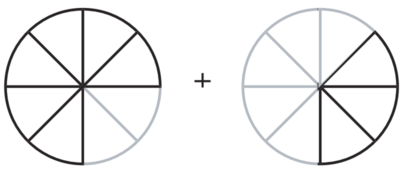
So $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, and then adding $\frac{1}{4}$ gives us $\frac{3}{4}$ in total:

$$\frac{1}{2}+\frac{1}{4} \quad = \quad \frac{2}{4}+\frac{1}{4} \quad = \quad \frac{3}{4}\,.$$

Example

Calculate $\frac{3}{4}+\frac{3}{8}$.

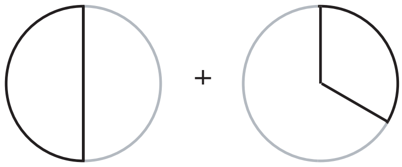
Solution



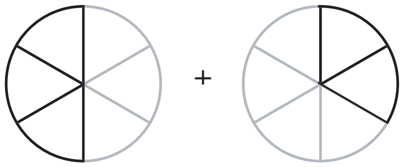
If we change the quarters into eighths, it becomes straightforward. The fraction $\frac{3}{4}$ is equivalent to the fraction $\frac{6}{8}$, and since the fractions now have the same denominator, we can just add the numerators:

$$\frac{3}{4}+\frac{3}{8} \quad = \quad \frac{6}{8}+\frac{3}{8} \quad = \quad \frac{9}{8} \quad = \quad 1\tfrac{1}{8}\,.$$

So far our examples have used fractions within the same family, where it is easy to see a connection between the fractions. For instance, quarters fit exactly into a half, and eighths fit exactly into a quarter. We shall now look at what happens when we add $\frac{1}{2}$ and $\frac{1}{3}$.



This time $\frac{1}{2}$ will not fit exactly into $\frac{1}{3}$, nor will $\frac{1}{3}$ fit exactly into a $\frac{1}{2}$. So we need to find a number that can be divided exactly by both 2 and 3, and then split each whole into that number of pieces. Now 6 can be divided by both 2 and 3, so if we split each whole into 6 pieces then we can see that $\frac{1}{2}$ is $\frac{3}{6}$ and $\frac{1}{3}$ is $\frac{2}{6}$.



So we have

$$\frac{1}{2}+\frac{1}{3} \quad = \quad \frac{3}{6}+\frac{2}{6} \quad = \quad \frac{5}{6}\,.$$

Example

Calculate $\frac{1}{4}+\frac{2}{5}$.

Solution

Again quarters and fifths are different sizes of fraction, and we cannot exactly fit quarters into fifths or fifths into quarters. So we need to find a size of fraction that will fit into both quarters and fifths.

Let us start by listing some numbers that can be divided by 4:

$$4, 8, 12, 16, 20, 24, \ldots\,.$$

And here are some numbers that can be divided by 5:

$$5, 10, 15, 20 \text{ —}$$

and now we see that 20 can be divided by both 4 and 5. It is the smallest number in both the lists, so we shall split both wholes into 20 equal pieces.

If we look at this numerically, what we are doing is finding the smallest number that can be divided by the two denominators. The denominators are 4 and 5, so the number we take is 20. Then we convert the two fractions into equivalent fractions with the same denominator, 20, before adding them. We say that 20 is the *common denominator*. To find the first equivalent fraction we see how many times 4 goes into 20. It goes 5 times, so we multiply both the numerator, 1, and the denominator, 4, by 5. To find the second equivalent fraction, we see how many times 5 goes into 20. It goes 4 times, so we multiply both the numerator, 2, and the denominator, 5, by 4:

$$\frac{1}{4}+\frac{2}{5} \quad = \quad \frac{1\times 5}{4\times 5}+\frac{2\times 4}{5\times 4} \quad = \quad \frac{5}{20}+\frac{8}{20} \quad = \quad \frac{13}{20}\,.$$

Example

Calculate $\frac{3}{4}-\frac{1}{6}$.

Solution

To carry out this calculation, we must find the smallest number that can be divided by both 4 and 6. That number is 12, so we need to convert both our fractions in to twelfths:

$$\frac{3}{4}-\frac{1}{6} \quad = \quad \frac{3\times 3}{4\times 3}-\frac{1\times 2}{6\times 2} \quad = \quad \frac{9}{12}-\frac{2}{12} \quad = \quad \frac{7}{12}\,.$$

In all these cases we have been changing the fractions into equivalent fractions before adding or subtracting. The denominator of the equivalent fraction is chosen so that it is the lowest number that can be divided by the other denominators, and it is called the *lowest common denominator*, or l.c.d. In some cases the l.c.d. can easily be found by multiplying together the denominators of the fractions to be added or subtracted. But, as our last example shows, doing this does not always result in the l.c.d. As you can see, if we had taken 4×6 and used 24 as our common denominator, the result would have been $\frac{14}{24}$ and we would then have needed to find the lowest form of the fraction by dividing both numerator and denominator by any common factors of 14 and 24.

3. Mixed fractions

Now let us look at how to add and subtract mixed fractions. Take, for example, $5\frac{3}{4}-1\frac{4}{5}$.

To add or subtract mixed fractions, we turn them into improper fractions first. So

$$\begin{aligned} 5\tfrac{3}{4}-1\tfrac{4}{5} &= \frac{5\times 4+3}{4}-\frac{1\times 5+4}{5} \\ &= \frac{23}{4}-\frac{9}{5}\,. \end{aligned}$$

Now the improper fractions are treated just the same as before. We find the lowest common denominator of 4 and 5. The l.c.d. is 20, so

$$\frac{23}{4}-\frac{9}{5} \quad = \quad \frac{23\times 5}{4\times 5}-\frac{9\times 4}{5\times 4} \quad = \quad \frac{115}{20}-\frac{36}{20} \quad = \quad \frac{79}{20} \quad = \quad 3\tfrac{19}{20}\,.$$

Example

Calculate $1\frac{3}{4}+6\frac{2}{5}+\frac{5}{2}$.

Solution

First of all, write all the mixed fractions as improper fractions:

$$\begin{aligned} 1\tfrac{3}{4} + 6\tfrac{2}{5} + \tfrac{5}{2} &= \frac{1 \times 4 + 3}{4} + \frac{6 \times 5 + 2}{5} + \frac{5}{2} \\ &= \frac{7}{4} + \frac{32}{5} + \frac{5}{2}\,. \end{aligned}$$

We now want the lowest common denominator of 2, 4 and 5. An easy way of finding this is to count up in multiples of the largest denominator, in this case 5, see whether the other denominators, 2 and 4, are factors. So 5 is no good, 10 is no good, 15 is no good, but 20 fits our requirements. So

$$\frac{7}{4} + \frac{32}{5} + \frac{5}{2} \quad = \quad \frac{7 \times 5}{4 \times 5} + \frac{32 \times 4}{5 \times 4} + \frac{5 \times 10}{2 \times 10} \quad = \quad \frac{35}{20} + \frac{128}{20} + \frac{50}{20} \quad = \quad \frac{213}{20}\,.$$

We can then turn the answer back into a mixed fraction by dividing by the denominator and finding the remainder: $213 \div 20$ equals 10 remainder 13, so the answer is $10\frac{13}{20}$.

Key Point

Turn mixed fractions to improper fractions before adding or subtracting them.

Exercises

3. Perform the following calculations:

(a)	$\frac{1}{2} + \frac{1}{5}$	(b)	$\frac{2}{3} + \frac{5}{9}$	(c)	$\frac{2}{7} + \frac{3}{4}$	(d)	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
(e)	$2\frac{3}{5} - \frac{4}{3}$	(f)	$3\frac{2}{3} - 1\frac{1}{4}$	(g)	$1\frac{1}{2} - \frac{7}{10}$	(h)	$4\frac{1}{4} - \frac{2}{5} - \frac{1}{8}$

Answers

1.

(a) $\frac{2}{3}$ (b) $\frac{5}{4}$ or $1\frac{1}{4}$ (c) $\frac{7}{5}$ or $1\frac{2}{5}$

2.

(a) $\frac{1}{2}$ (b) $\frac{7}{10}$ (c) $\frac{7}{12}$

3.

(a)	$\frac{3}{10}$	(b)	$\frac{11}{9}$ or $1\frac{2}{9}$	(c)	$\frac{29}{28}$ or $1\frac{1}{28}$	(d)	$\frac{13}{12}$ or $1\frac{1}{12}$
(e)	$\frac{19}{15}$ or $1\frac{4}{15}$	(f)	$\frac{29}{12}$ or $2\frac{5}{12}$	(g)	$\frac{4}{5}$	(h)	$\frac{149}{40}$ or $3\frac{29}{40}$