Nonlinear wave propagation in solar flux tubes

Fedun Viktor

The University of Sheffield,
Department of Applied Mathematics

v.fedun@sheffield.ac.uk

The aim of the present work is to investigate the excitation, time dependent dynamic evolution and interaction of weakly nonlinear propagating (i.e. solitary) waves on vertical cylindrical magnetic flux tubes in a compressible atmosphere. Solitons are excited by a footpoint driver. The propagation of the nonlinear signal is investigated by solving numerically a set of fully nonlinear 2D MHD equations in cylindrical coordinates. For the initial conditions we use the solutions of the linear dispersion relation for the wave modes (in our case sausage mode) in a magnetic flux tube. This dispersion relation is solved numerically for arange of plasma parameters. A natural application of our studies is spicule formation in the chromosphere, as suggested by *Roberts & Mangeney, MNRAS 1982*, where it was demonstrated theoretically, that a solar photospheric magnetic flux tube can support the propagation of solitons governed by the Benjamin-Ono (slow mode) equations. Future possible improvements in modeling and the relevance of the photospheric chromospheric transition region coupling by spicules is suggested.

Outline of the talk

- The basic equations and assumptions

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part
  - Numerical calculations

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part
  - Numerical calculations
- Few words about solitons.

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part
  - Numerical calculations
- Few words about solitons.
- The full MHD simulations

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part
  - Numerical calculations
- Few words about solitons.
- The full MHD simulations
  - Linear stage of the wave propagation along the tube

Outline of the talk

- The basic equations and assumptions
- The general dispersion relation
  - The linear part
  - Numerical calculations
- Few words about solitons.
- The full MHD simulations
  - Linear stage of the wave propagation along the tube
  - Nonlinear stage

The basic equations and assumptions

The set of the full magnetohydrodynamics equations read as follows:

$$\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V} = -\frac{\vec{\nabla}p}{\rho} + \frac{1}{\mu\rho}(\vec{\nabla}\times\vec{B})\times\vec{B}$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$$

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{V}\times\vec{B})$$

$$p = p_0\left(\frac{\rho}{\rho_0}\right)^\gamma$$

where ρ is the density;

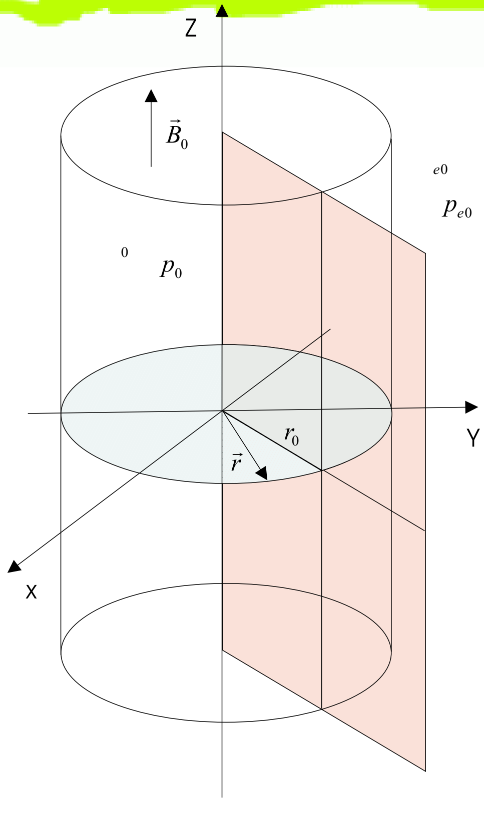
p, the pressure;

the $\vec{V}$ the velocity;

$\vec{B}$,magnetic induction;

γ is the adiabatic index.

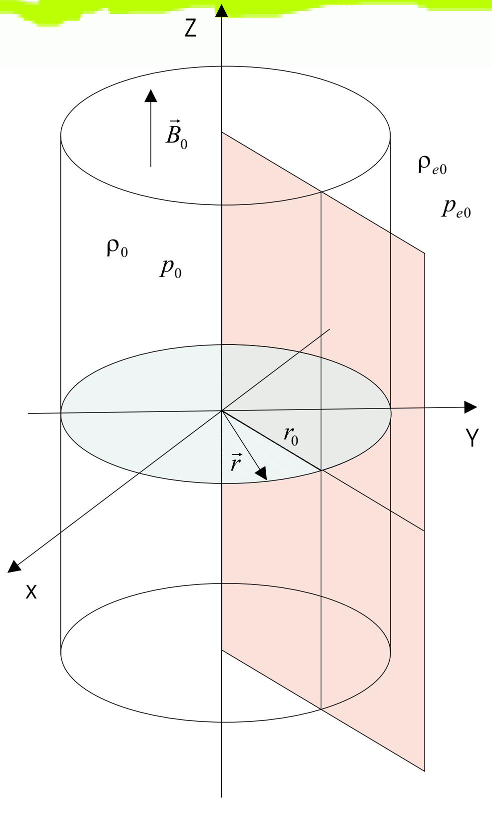
The geometry of the problem



The coordinate system

We denote the tube radius r_0,
all dependent variables inside the tube fi t without the index,
outside the tube - e.

The geometry of the problem



The coordinate system

We denote the tube radius r_0,
all dependent variables inside the tube fi t
without the index,
outside the tube - e.

undisturbed variables

ρ_0 - density,

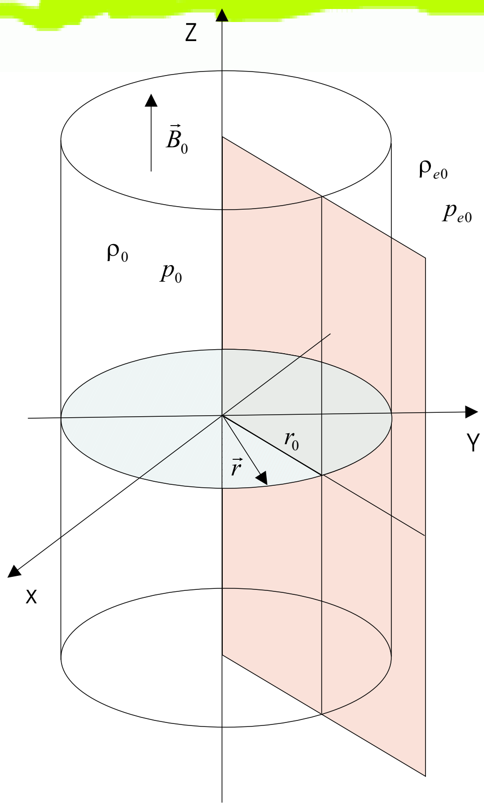
p_0 - pressure,

$V_0 = 0$ - velocity.

the static state of equilibrium

$$p_0 + B_0^2/2\mu = p_e$$

The geometry of the problem



The coordinate system

We denote the tube radius r_0,
all dependent variables inside the tube fi t without the index,
outside the tube - e.

undisturbed variables

ρ_0 - density,

p_0 - pressure,

$V_0 = 0$ - velocity.

the static state of equilibrium

$$p_0 + B_0^2/2\mu = p_e$$

$$\overrightarrow{V} = (u(r,0,z), 0, w(r,0,z)),$$

$$p(r,0,z),$$

$$\rho(r,0,z),$$

$$\overrightarrow{b} = (b_r(r,0,z), 0, b_z(r,0,z))$$

Lighthill equation

$$\frac{\partial^2}{\partial t^2}\left\{\frac{\partial^2}{\partial t^2} - (V_A^2 + C_0^2)\nabla^2\right\} + V_A^2 C_0^2 \frac{\partial^2}{\partial z^2}\nabla^2 \Delta = 0$$

where $\Delta = div\,\overrightarrow{u}$

Lighthill equation

$$\frac{\partial^2}{\partial t^2}\left\{\frac{\partial^2}{\partial t^2}-(V_A^2+C_0^2)\nabla^2\right\}+V_A^2C_0^2\frac{\partial^2}{\partial z^2}\nabla^2\triangle=0$$

where $\triangle = div\overrightarrow{u}$

Assuming that $\triangle = R(r)e^{i(\omega t-n\phi-kz)}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right)+\left(k_0^2-\frac{n^2}{r^2}\right)R=0$$

where $k_0^2=\frac{(\omega^2-k^2V_A^2)(\omega^2-k^2C_0^2)}{(V_A^2+C_0^2)(\omega^2-k^2C_t^2)}$

$V_A=B_0/\sqrt{\mu\rho}$ - Alfvén speed,

$C_0=(\gamma p_0/\rho_0)^{1/2}$ - sound speed inside the tube,
$C_t=C_0V_A/\sqrt{(C_0^2+V_A^2)}$ - tube speed

Lighthill equation

$$\frac{\partial^2}{\partial t^2}\left\{\frac{\partial^2}{\partial t^2} - (V_A^2 + C_0^2)\nabla^2\right\} + V_A^2 C_0^2 \frac{\partial^2}{\partial z^2}\nabla^2 \triangle = 0$$

where $\triangle = div\overrightarrow{u}$

Assuming that $\triangle = R(r)e^{i(\omega t - n\phi - kz)}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \left(k_0^2 - \frac{n^2}{r^2}\right)R = 0$$

where $k_0^2 = \frac{(\omega^2 - k^2 V_A^2)(\omega^2 - k^2 C_0^2)}{(V_A^2 + C_0^2)(\omega^2 - k^2 C_t^2)}$

$k_e^2 = \frac{\omega^2 - k^2 C_e^2}{C_e^2}$

$V_A = B_0/\sqrt{\mu\rho}$ - Alfvén speed,

where $C_e = (\gamma p_{0e}/\rho_{0e})^{1/2}$

$C_0 = (\gamma p_0/\rho_0)^{1/2}$ - sound speed inside the tube,

$C_t = C_0 V_A/\sqrt{(C_0^2 + V_A^2)}$ - tube speed

The general solution

$$R(r) = \begin{cases} A_{n0} J_n(k_0 r) + B_{n0} Y_n(k_0 r), r < r_0; \\ A_{ne} J_n(k_e r) + B_{ne} Y_n(k_e r), r > r_0 \end{cases}$$

for $k_0^2, k_e^2 > 0$

The general solution

$$R(r) = \begin{cases} A_{n0} J_n(k_0 r) + B_{n0} Y_n(k_0 r), r < r_0; \\ A_{ne} J_n(k_e r) + B_{ne} Y_n(k_e r), r > r_0 \end{cases}$$

for $k_0^2, k_e^2 > 0$

$$R(r) = \begin{cases} C_{n0} I_n(m_0 r) + D_{n0} K_n(m_0 r), r < r_0; \\ C_{ne} I_n(m_e r) + D_{ne} K_n(m_e r), r > r_0 \end{cases}$$

for $\begin{array}{l} k_0^2 = -m_0^2 < 0 \\ k_e^2 = -m_e^2 < 0 \end{array}$

The general solution

$$R(r) = \begin{cases} A_{n0} J_n(k_0 r) + B_{n0} Y_n(k_0 r), r < r_0; \\ A_{ne} J_n(k_e r) + B_{ne} Y_n(k_e r), r > r_0 \end{cases}$$

for $k_0^2, k_e^2 > 0$

$$R(r) = \begin{cases} C_{n0} I_n(m_0 r) + D_{n0} K_n(m_0 r), r < r_0; \\ C_{ne} I_n(m_e r) + D_{ne} K_n(m_e r), r > r_0 \end{cases}$$

for $\begin{array}{l} k_0^2 = -m_0^2 < 0 \\ k_e^2 = -m_e^2 < 0 \end{array}$

D_{n0}, B_{n0}, C_{n0} must be set to zero

The general solution

$$w = -C_{n0}\frac{C_0^2}{\omega^2} ik I_n(m_0 r)$$

$$u = C_{n0}\frac{\omega^2 - k^2 C_0^2}{m_0^2 \omega^2}\frac{d}{dr} I_n(m_0 r)$$

$$p = iC_{n0}\frac{\rho_0 C_0^2}{\omega} I_n(m_0 r)$$

$$b_r = \frac{k}{\omega} u B_0$$

$$b_z = iC_{n0}\frac{\omega^2 - k^2 C_0^2}{\omega^3} B_0 I_n(m_0 r)$$

$$\rho = iC_{n0}\rho_0 \frac{1}{\omega} I_n(m_0 r)$$

The general solution

$$w = -C_{n0}\frac{C_0^2}{\omega^2} ikI_n(m_0 r)$$
$$u = C_{n0}\frac{\omega^2-k^2C_0^2}{m_0^2\omega^2}\frac{d}{dr}I_n(m_0 r)$$
$$p = iC_{n0}\frac{\rho_0 C_0^2}{\omega}I_n(m_0 r)$$
$$b_r = \frac{k}{\omega}uB_0$$
$$b_z = iC_{n0}\frac{\omega^2-k^2C_0^2}{\omega^3}B_0 I_n(m_0 r)$$
$$\rho = iC_{n0}\rho_0\frac{1}{\omega}I_n(m_0 r)$$

$$w_e = -D_{ne}\frac{C_e^2}{\omega^2} ikK_n(m_e r)$$
$$u_e = D_{ne}\frac{\omega^2-k^2C_e^2}{m_0^2\omega^2}\frac{d}{dr}K_n(m_e r)$$
$$p_e = iD_{ne}\frac{\rho_e C_e^2}{\omega}K_n(m_e r)$$
$$b_{re} = 0$$
$$b_{ze} = 0$$
$$\rho_e = iD_{ne}\rho_{0e}\frac{1}{\omega}K_n(m_e r)$$

The general solution

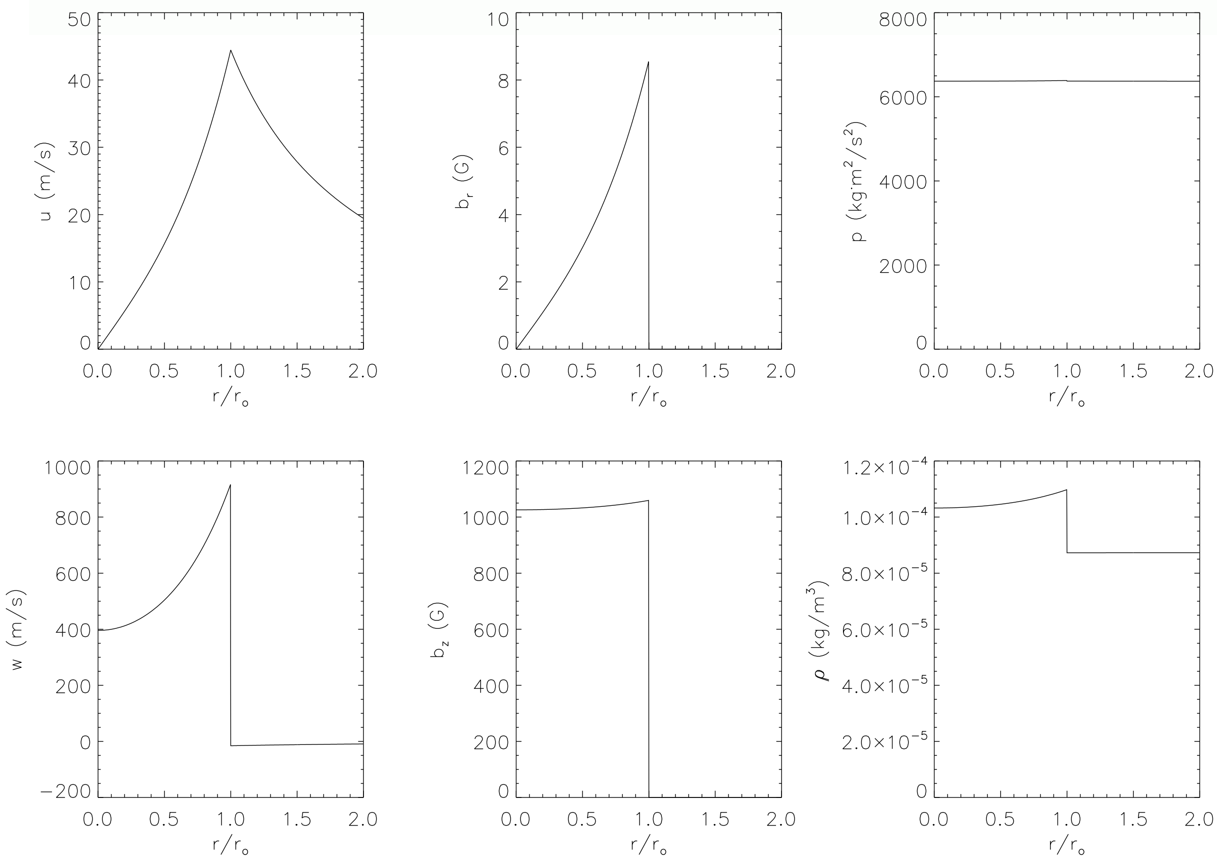
$$w = -C_{n0}\frac{C_0^2}{\omega^2}ikI_n(m_0 r)$$
$$u = C_{n0}\frac{\omega^2-k^2C_0^2}{m_0^2\omega^2}\frac{d}{dr}I_n(m_0 r)$$
$$p = iC_{n0}\frac{\rho_0 C_0^2}{\omega}I_n(m_0 r)$$
$$b_r = \frac{k}{\omega}uB_0$$
$$b_z = iC_{n0}\frac{\omega^2-k^2C_0^2}{\omega^3}B_0 I_n(m_0 r)$$
$$\rho = iC_{n0}\rho_0\frac{1}{\omega}I_n(m_0 r)$$

$$w_e = -D_{ne}\frac{C_e^2}{\omega^2}ikK_n(m_e r)$$
$$u_e = D_{ne}\frac{\omega^2-k^2C_e^2}{m_0^2\omega^2}\frac{d}{dr}K_n(m_e r)$$
$$p_e = iD_{ne}\frac{\rho_e C_e^2}{\omega}K_n(m_e r)$$
$$b_{re} = 0$$
$$b_{ze} = 0$$
$$\rho_e = iD_{ne}\rho_{0e}\frac{1}{\omega}K_n(m_e r)$$

The conditions for the matching of the inside and outside solutions at the $r = r_0$ are:

$$u_e(r_0) = u_0(r_0) \qquad p_e = p_0 + \frac{1}{\mu}B_0 b_z$$

The general solution



see also
Spruit, Sol.Phys. 1982
Zhugzhda & Goossens, A&A. 2002

The eigenfunctions of radial and longitudinal components of velocities and magnetic fi eld, pressure, density for $(kr_0 = 0.3)$ and $V_f = 5183m/s$.

Dispersion relations

The dispersion relations for the cylindrically symmetric mode (sausage mode) given by $n = 0$

$$\omega^2 \rho_e m_0 \frac{I_1(r_0 m_0)}{I_0(r_0 m_0)} = \rho_0 (V_A^2 k^2 - \omega^2) m_e \frac{K_1(r_0 m_e)}{K_0(r_0 m_e)}$$

The asymmetric mode (kink) given by $n = 1$.
In the case of body waves:

$$\omega^2 \rho_e n_0 \frac{J_1(r_0 m_0)}{J_0(r_0 m_0)} = \rho_0 (V_A^2 k^2 - \omega^2) m_e \frac{K_1(r_0 m_e)}{K_0(r_0 m_e)}$$

see
Wilson, A & A. 1980
Spruit, Sol.Phys. 1982
Edwin & Roberts, Sol.Phys. 1983)

Dispersion relations

The typical photospheric conditions:

	$B_0(G)$	$V_A(m/s)$	$C_0(m/s)$	$C_e(m/s)$
tube which is cooler than its surroundings	1000	$9 \cdot 10^3$	$\approx 6.4 \cdot 10^3$	$\approx 11 \cdot 10^3$
intense cool tube	1000	$9 \cdot 10^3$	$4.5 \cdot 10^3$	$\approx 7.8 \cdot 10^3$

$R = V_A/C_0$, $Q = C_e/C_0$

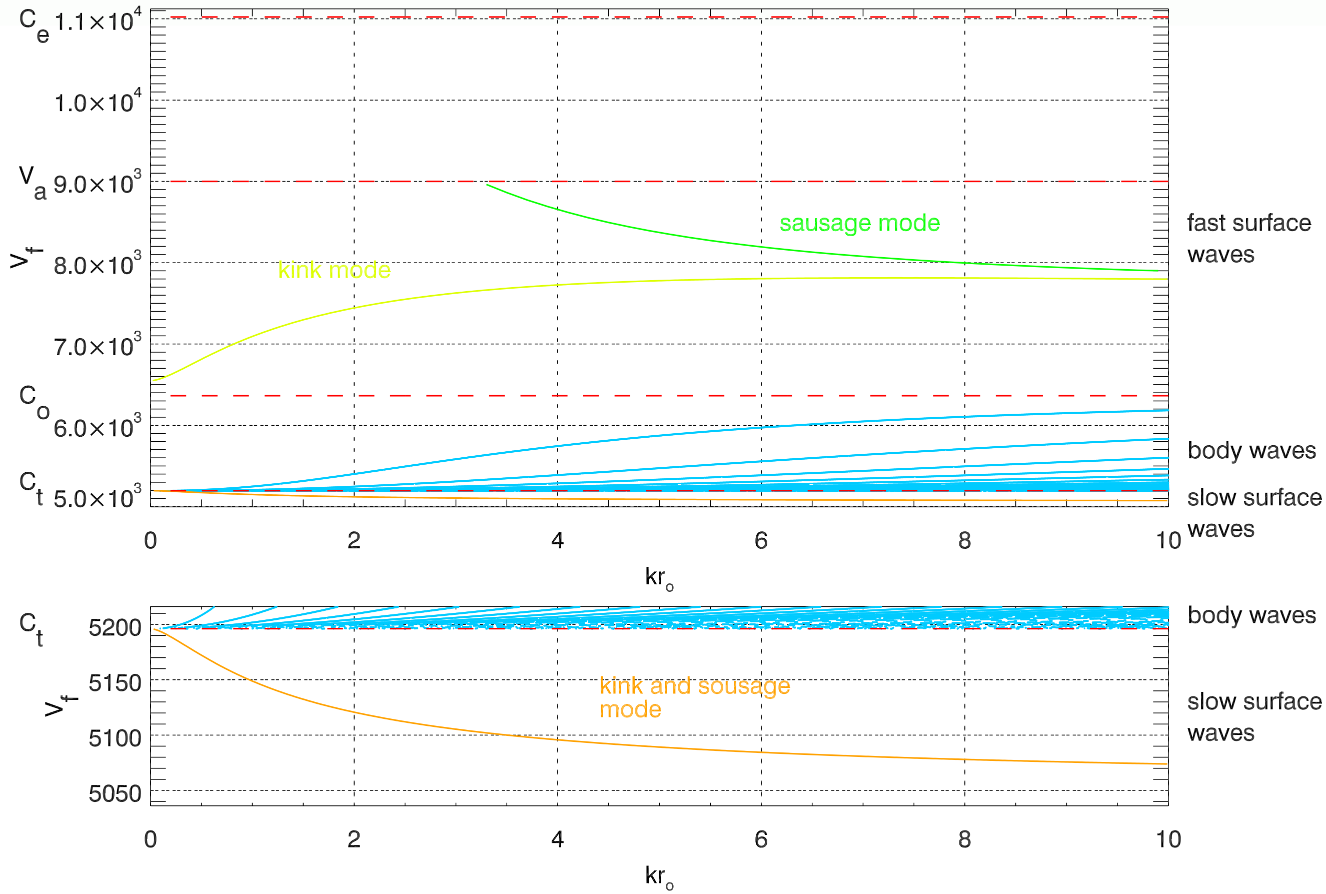
The tube of moderate intensity in an isotermal atmosphere : $R^2 = 2$, $Q^2 = 1$

The tube which is cooler then surroundings : $R^2 = 2$, $Q^2 = 3$

The intense cool tube : $R^2 = 4$, $Q^2 = 3$

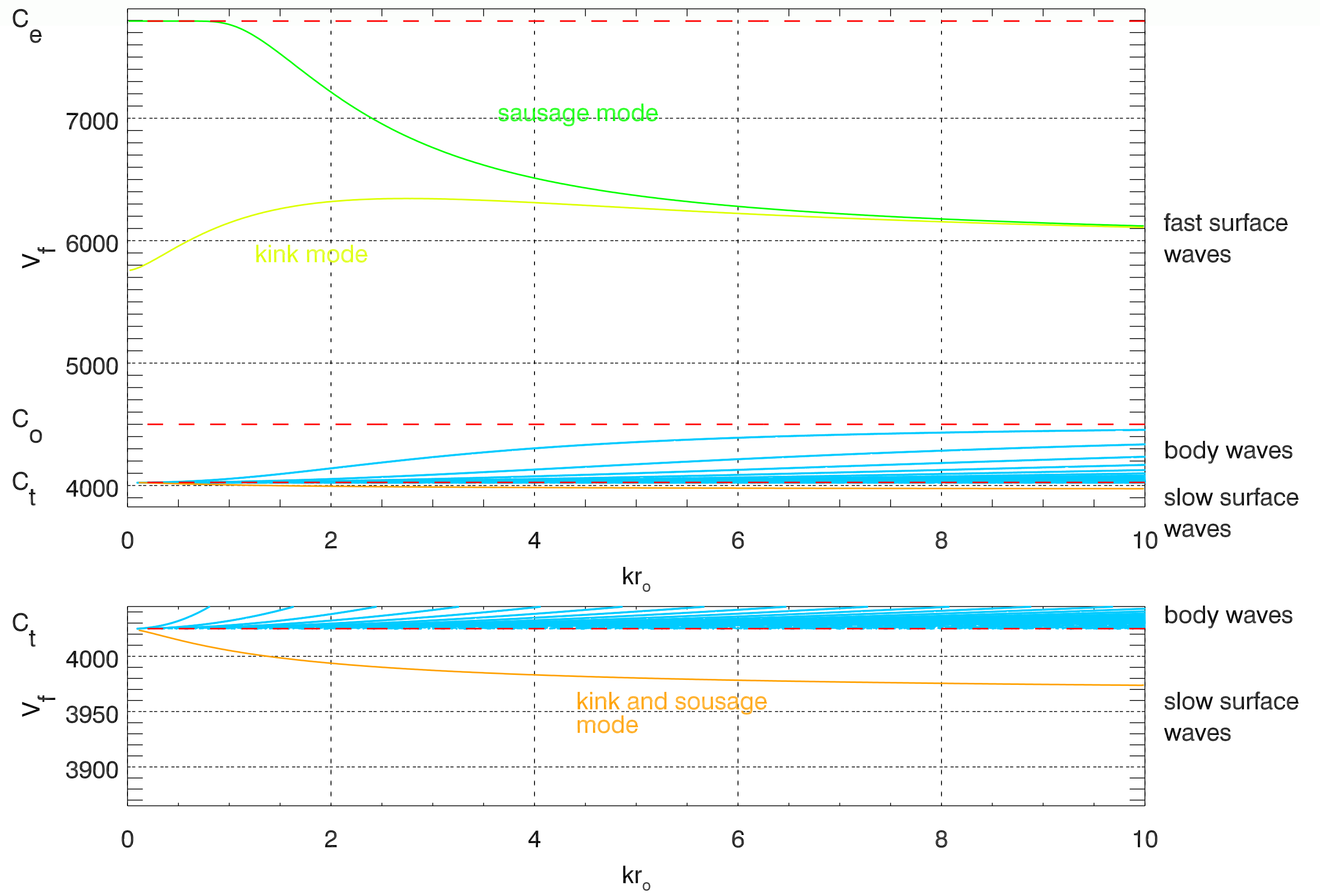
The intense tube which is hotter then its surroundings : $R^2 = 4$, $Q^2 = 0.8$

Dispersion relations



The solution of the dispersion relations for the case $C_e > V_A > C_0 > C_t$, $(R^2 = 2, Q^2 = 3)$

Dispersion relations



The phase speed of modes under photospheric conditions $V_A > C_e > C_0 > C_t$, ($R^2 = 4$, $Q^2 = 3$). Many slow body waves are shown.

The thin-flux tube

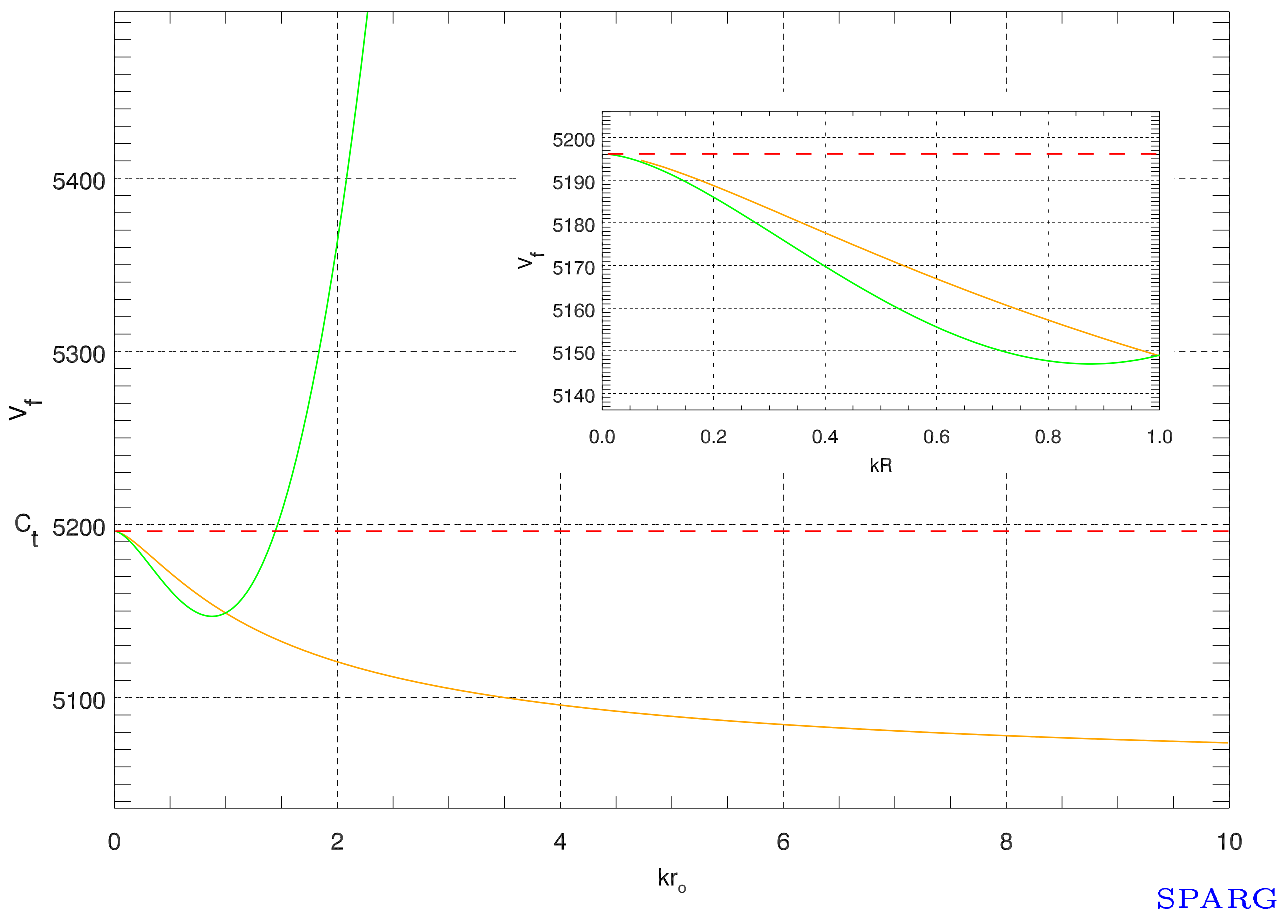
In the long-wavelength limit $(kr_0 << 1)$ equation for the surface waves has the solution *Molotovshchikov & Ruderman, Sol. Phys. 1987*:

$$\omega = c_t k + 2\beta k^3 \left(ln \frac{\alpha |k|}{2} + 0.577 \right) + O(k^5 ln|k|), \quad \beta = \frac{\rho_{e0} C_t^5}{8\rho_0 V_A^4} r_0^2, \quad \alpha^2 = 1 - \frac{C_t^2}{C_e^2} r_0^2$$

The thin-flux tube

In the long-wavelength limit $(kr_0 << 1)$ equation for the surface waves has the solution *Molotovshchikov & Ruderman, Sol. Phys. 1987*:

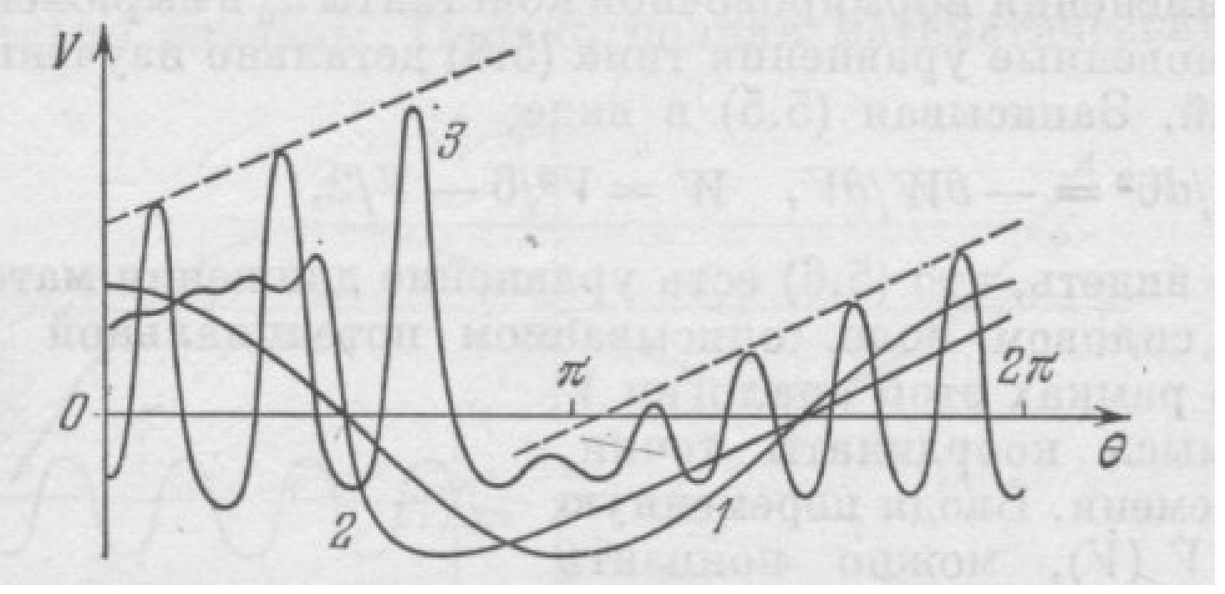
$$\omega = c_t k + 2\beta k^3 \left(ln \frac{\alpha |k|}{2} + 0.577 \right) + O(k^5 ln|k|), \quad \beta = \frac{\rho_{e0} C_t^5}{8\rho_0 V_A^4} r_0^2, \quad \alpha^2 = 1 - \frac{C_t^2}{C_e^2} r_0^2$$



Few words about solitons

The Korteweg-de Vries evolution equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} + \delta^2 \frac{\partial^3 v}{\partial z^3} = 0$$

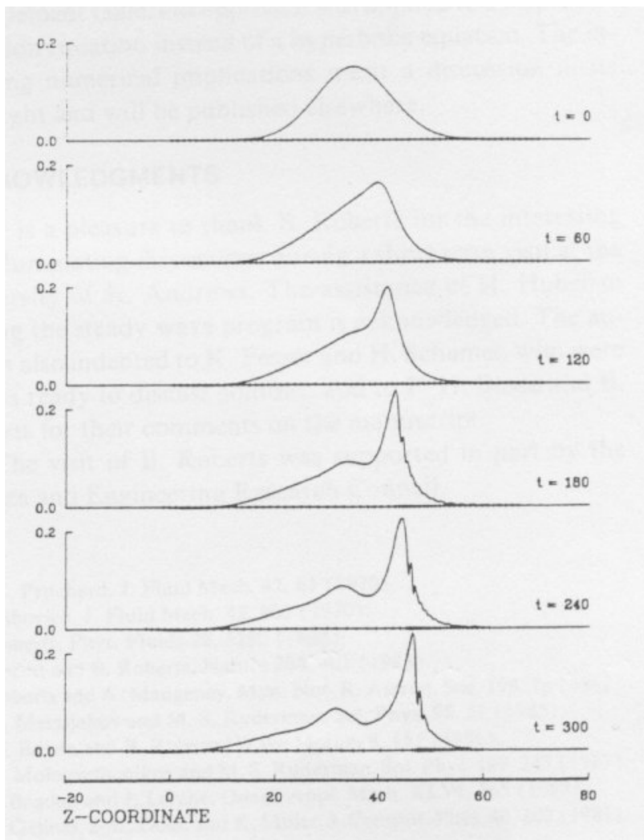


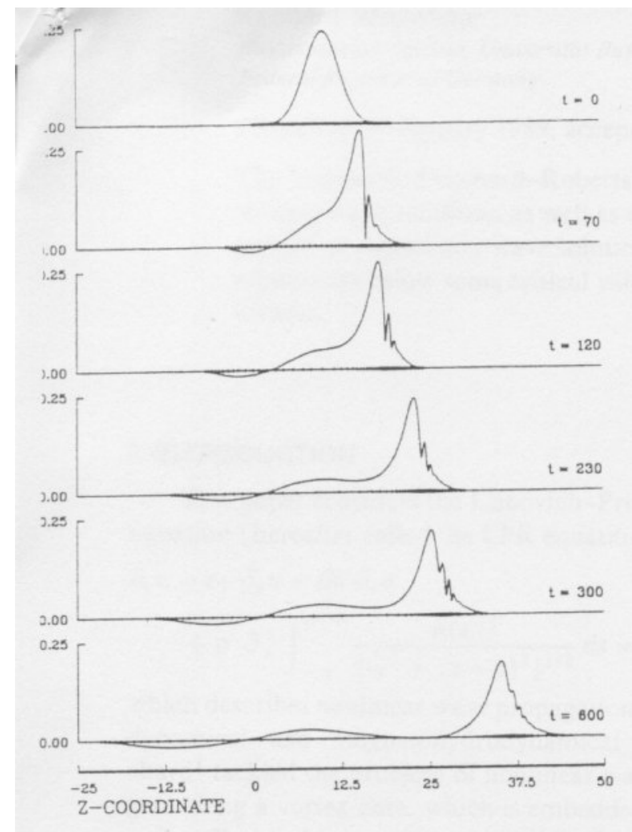
Few words about solitons

The Leibovich-Pritchard-Roberts evolution equation

Weisshaar, Phys. Fluids A. 1989

$$\frac{\partial v}{\partial t} + c_T \frac{\partial v}{\partial z} + \beta v \frac{\partial v}{\partial z} + \alpha \frac{\partial^3 v}{\partial z^3} \int_{-\infty}^{\infty} \frac{v(s,t)ds}{[\lambda^2 + (z-s)^2]^{1/2}} = 0$$





Numerical calculations

$$\frac{\partial \rho \vec{V}}{\partial t} + \vec{\nabla} \cdot (\vec{V} \rho \vec{V} - \vec{B}\vec{B}) + \vec{\nabla} p_{tot} = -(\vec{\nabla} \cdot \vec{B})\vec{B}$$
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0$$
$$\frac{\partial e}{\partial t} + \vec{\nabla} \cdot (\vec{V} e - \vec{B}\vec{B} \cdot \vec{V} + \vec{V} p_{tot}) = -(\vec{\nabla} \cdot \vec{B})\vec{B} \cdot \vec{V}$$
$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot (\vec{V}\vec{B} - \vec{B}\vec{V}) = (\vec{\nabla} \cdot \vec{B})\vec{V}$$
$$p = (\gamma - 1)(e - \rho V^2/2 - B^2/2)$$
$$p_{tot} = p + B^2/2$$

with conservative variables:
density ρ,
momentum of density $\rho \vec{V}$,
total energy density e
magnetic field $\vec{B} = \vec{B}/\sqrt{\mu}$.

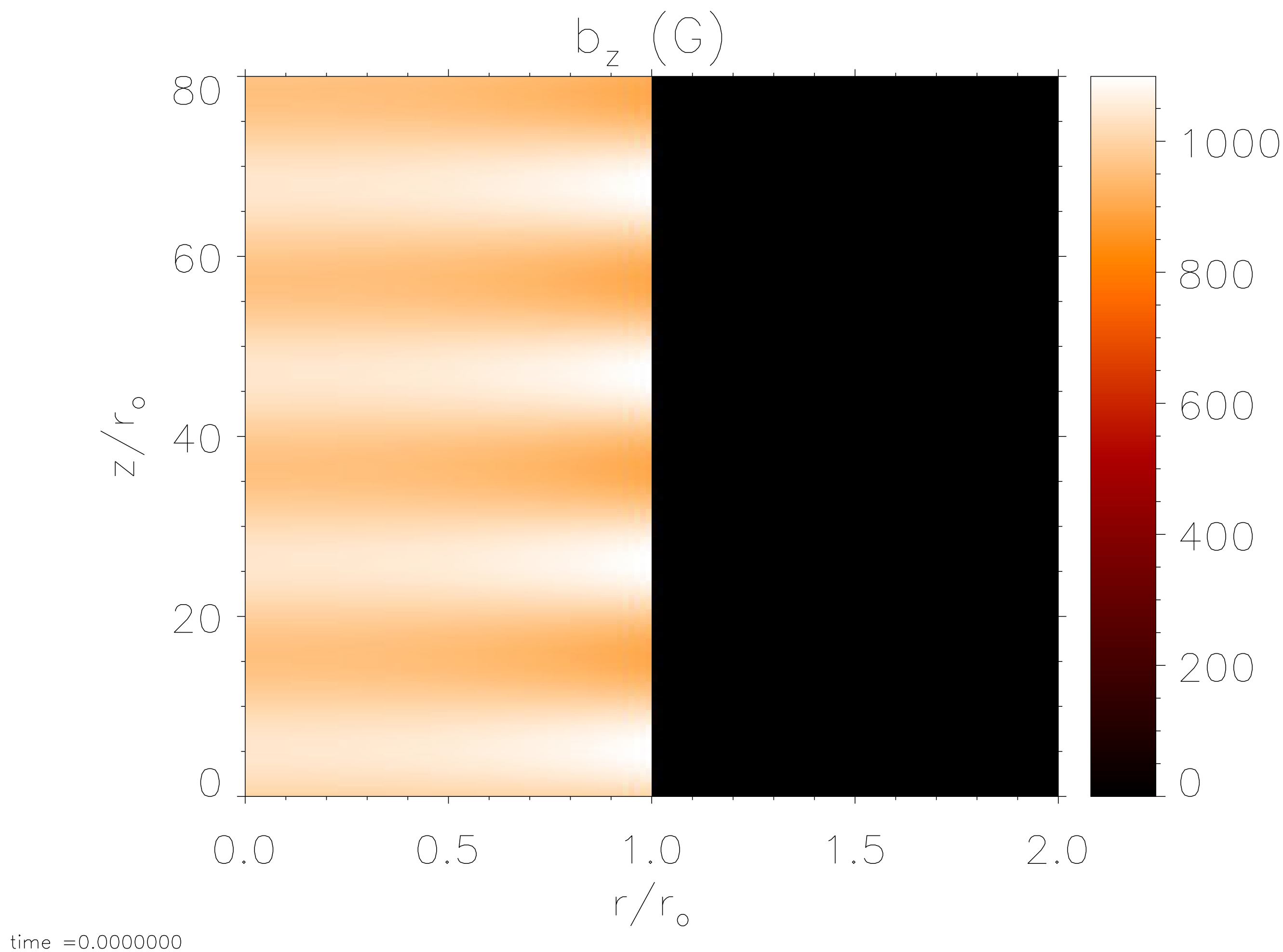
Tóth, Astrophysical Lett. and Comm. 1996

Linear stage of the wave propagation

We use a cylindrically symmetric domain $[0, 80r_0]$ (400 grid points) in the z direction and $[0, 2r_0]$ (140 grid points) in the r direction.

Linear stage of the wave propagation

We use a cylindrically symmetric domain $[0, 80r_0]$ (400 grid points) in the z direction and $[0, 2r_0]$ (140 grid points) in the r direction.



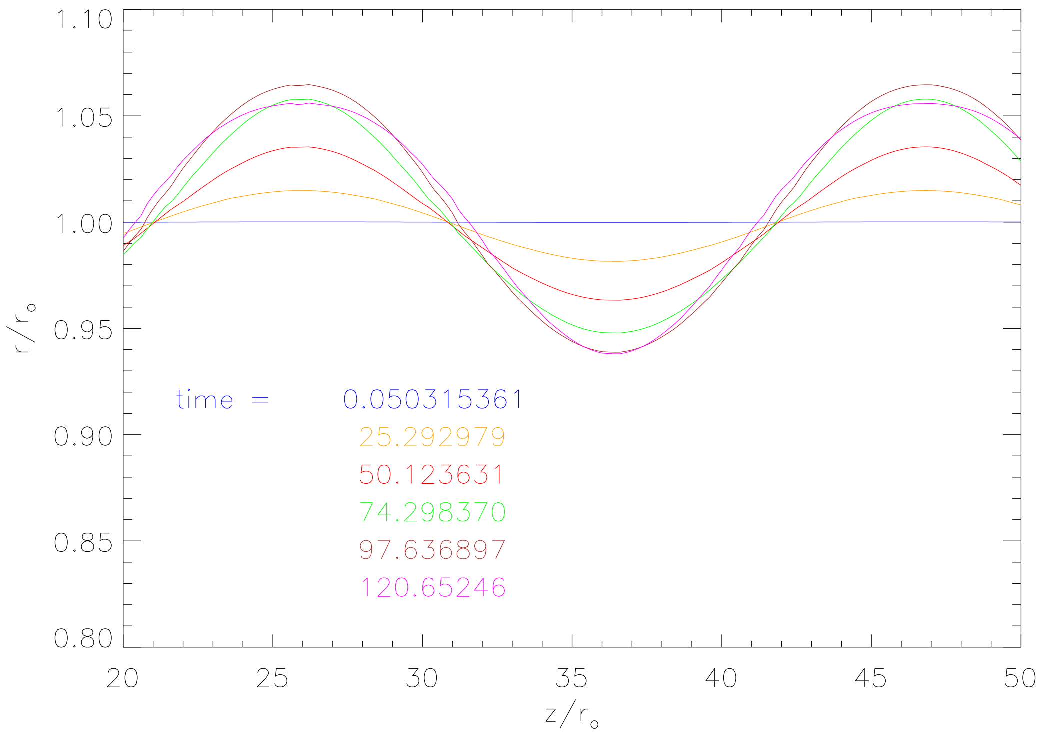
The initial magnetic fields profile $(0.2B_0)$

Linear stage of the wave propagation

The boudary of tube we can fi nd using the equation :

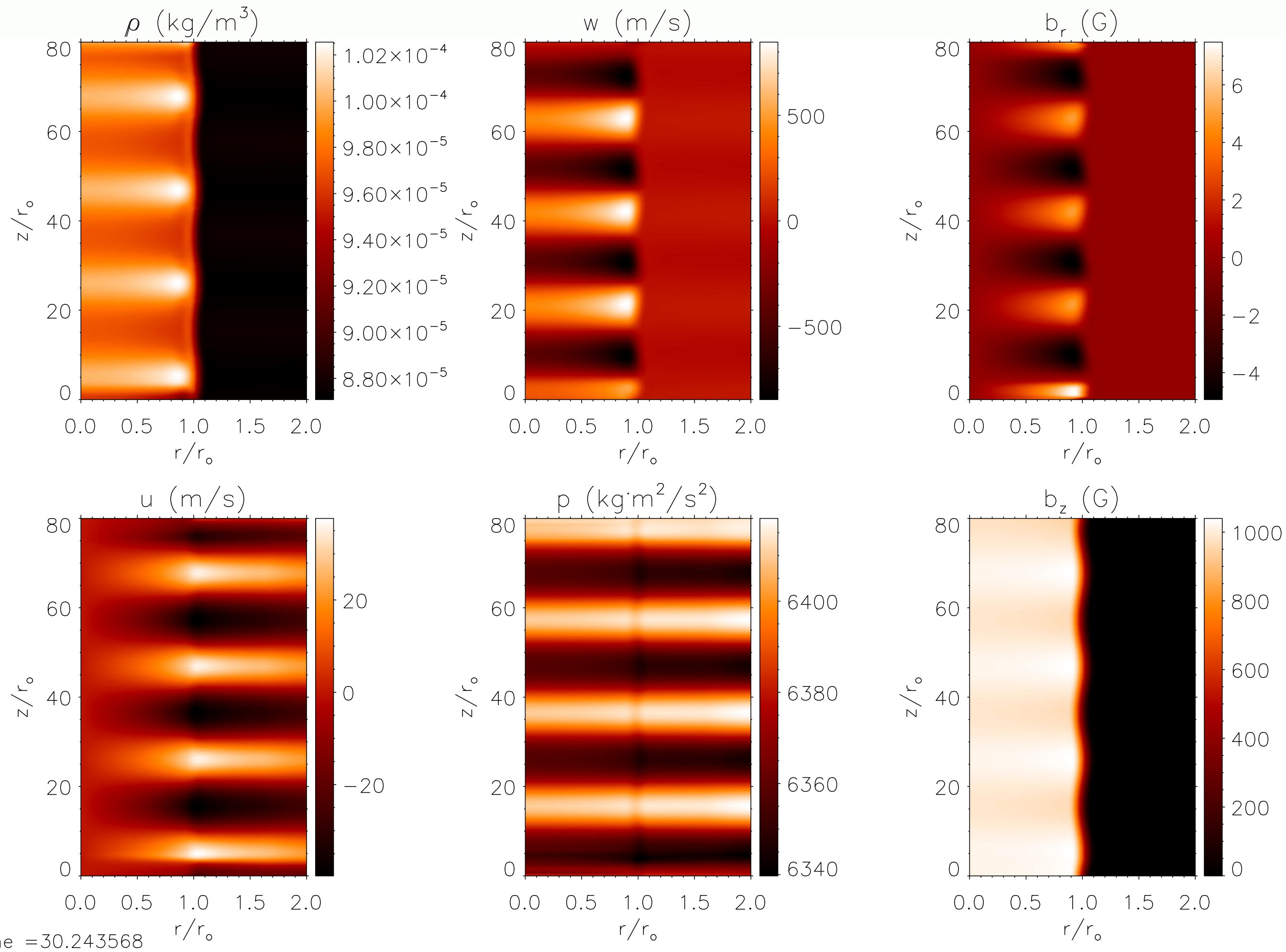
$r = r_0 + \eta(t,z)$, $u = \frac{\partial \eta}{\partial t} + w\frac{\partial \eta}{\partial z}$

(Molotovshchikov & Ruderman, Sol. Phys. 1987)



The time dependent (sec.) evolution of the flux tube boundary for the initial perturbations of magnetic fi elds $0.2B_0$

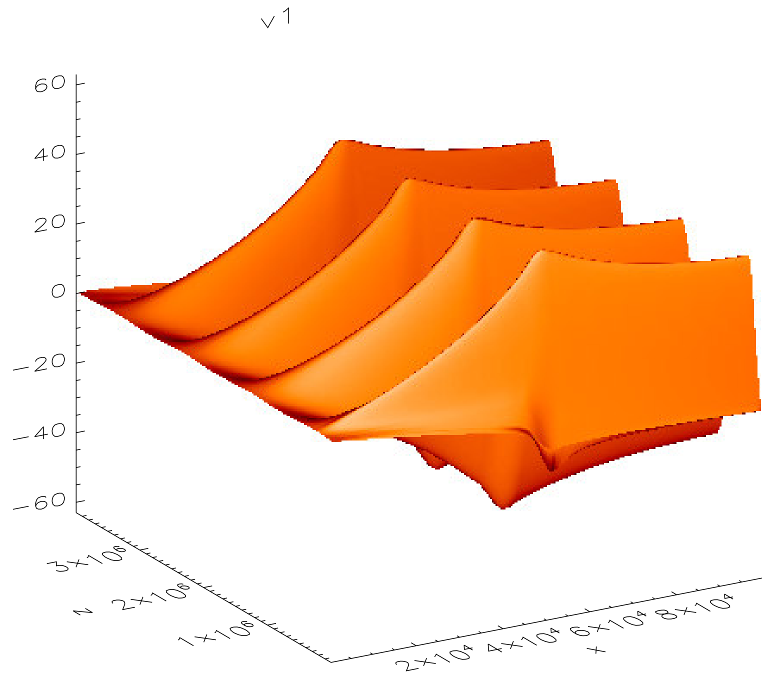
Linear stage of the wave propagation



The vertical variables perturbations for the case (i) 2d snapshot

Linear stage of the wave propagation

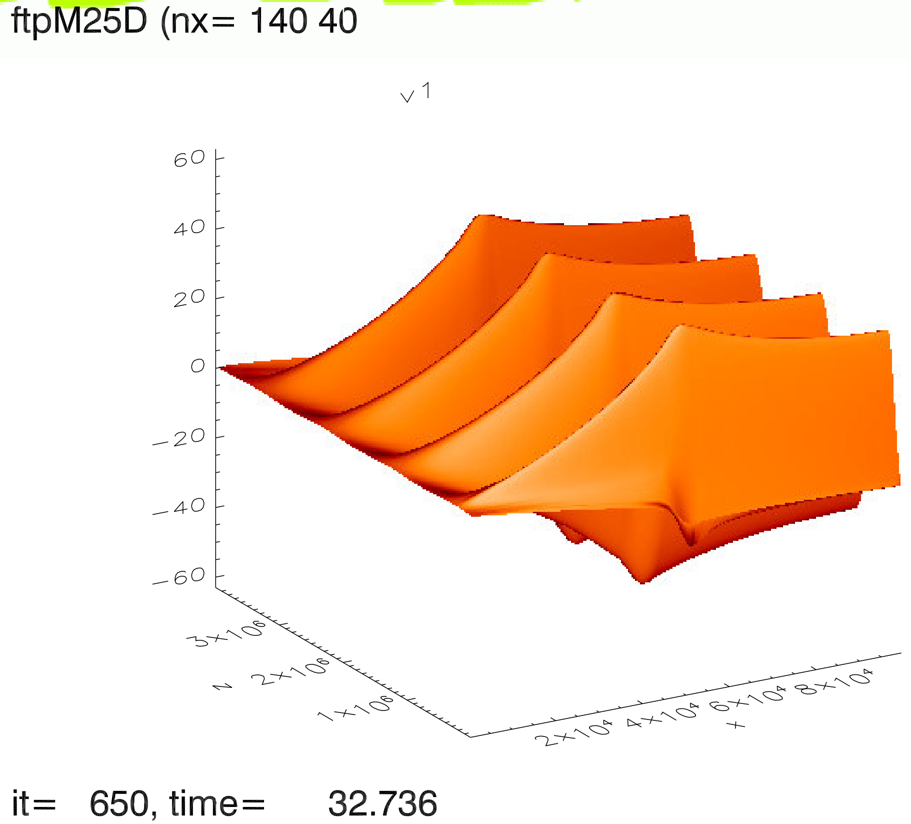
ftpM25D (nx= 140 40

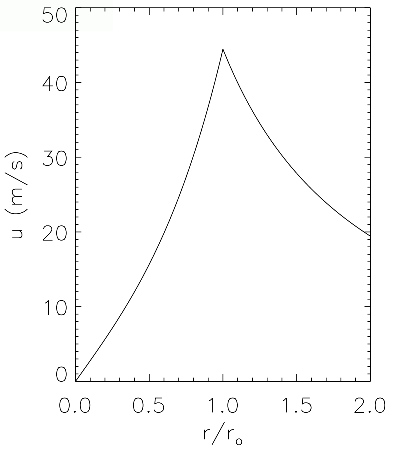


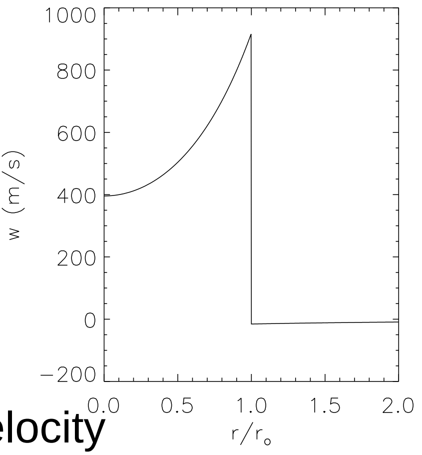
it=   650, time=   32.736

The vertical variables perturbations (2d snapshot). $v1$ - u velocity (m/s), x correspond the r direction (in meters), z the direction along the tube (in meters).

Linear stage of the wave propagation







The vertical variables perturbations (2d snapshot). $v1$ - u velocity (m/s), x correspond the r direction (in meters), z the direction along the tube (in meters).

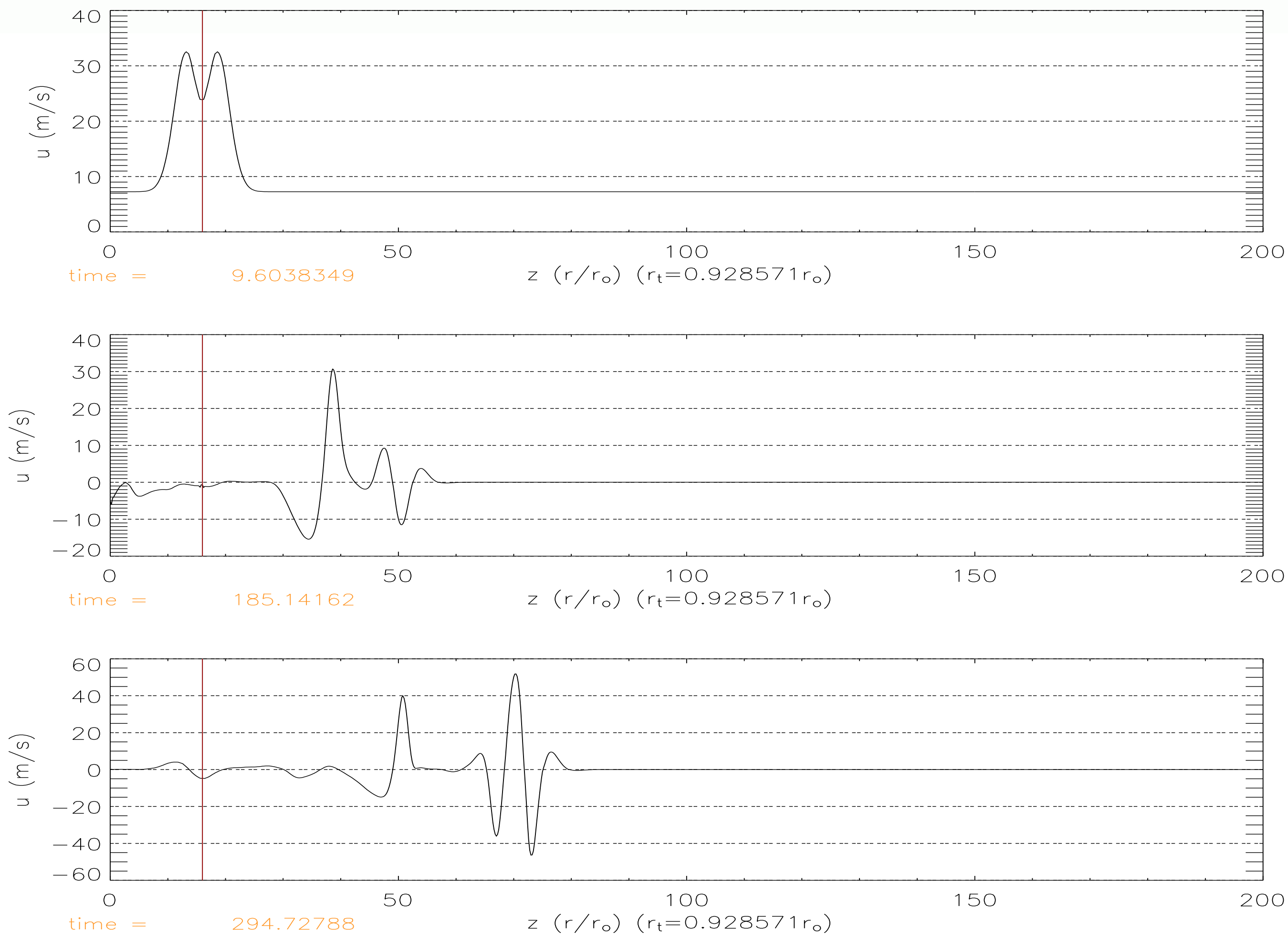
Nonlinear stage of the wave propagation

We use the symmetric domain is $[0, 320r_0]$ (1600 grid points) in the z direction and $[0, 2r_0]$ (140 grid points) in the r direction. Initially the magnetic fi eld is perturbed at the footpoint. This perturbation has a Gaussian spatial distribution $C = 0.12B_0$:

$$b_z = B_0(1 + C\frac{I_0(m_0 r)}{I_0(m_0 r_0)})e^{-(zk-16r_0k)^2}$$

After initial perturbation the evolution of any variables with time can be observed throughout the whole computational domain. For example, here we present the results of the simulations for the u component of velocity.

Nonlinear stage of the wave propagation

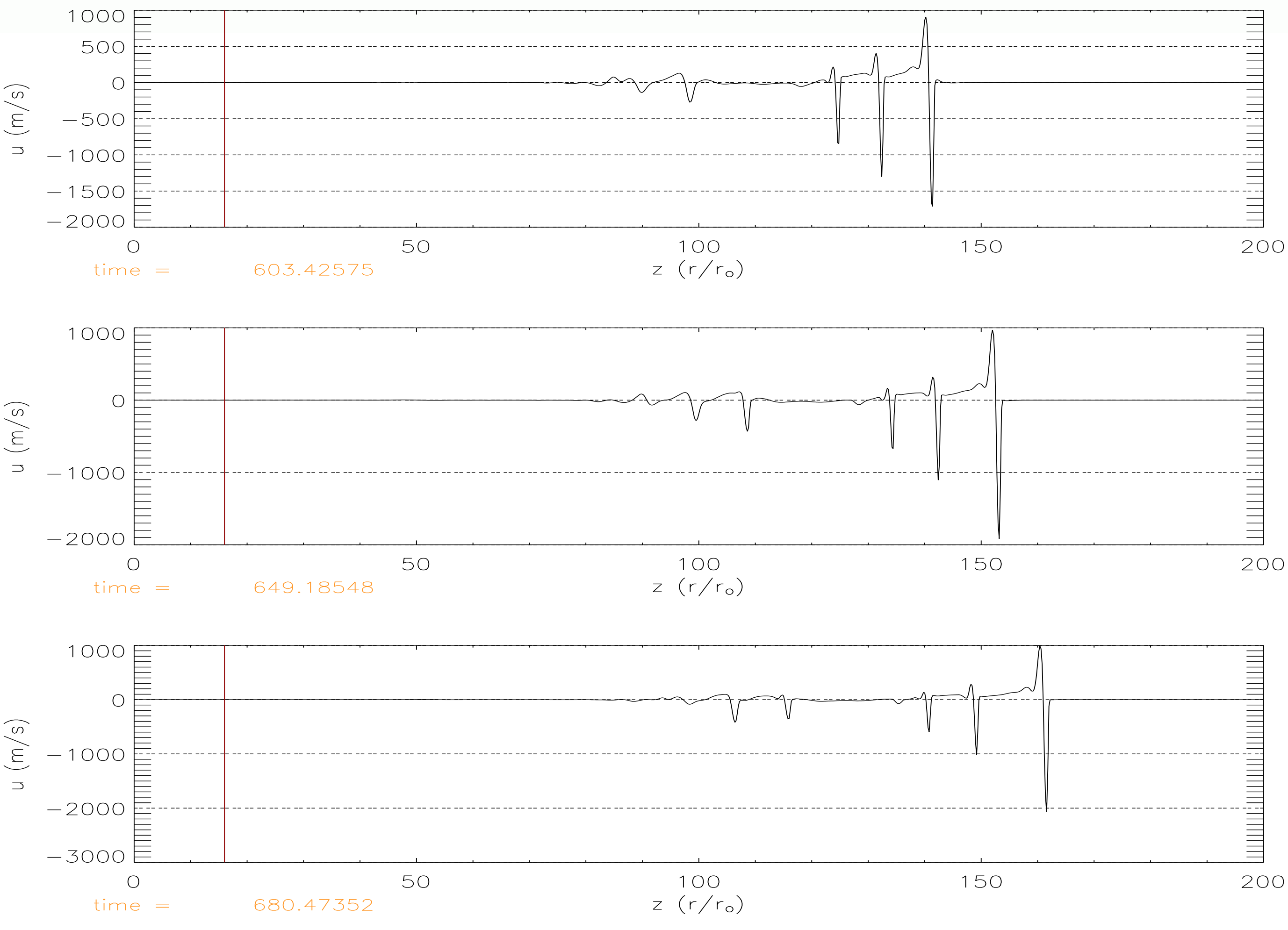


The time dependent evolution of the initial signal. Red line - footpoint.

Nonlinear stage of the wave propagation

The time dependent evolution of the initial signal. The third stage.

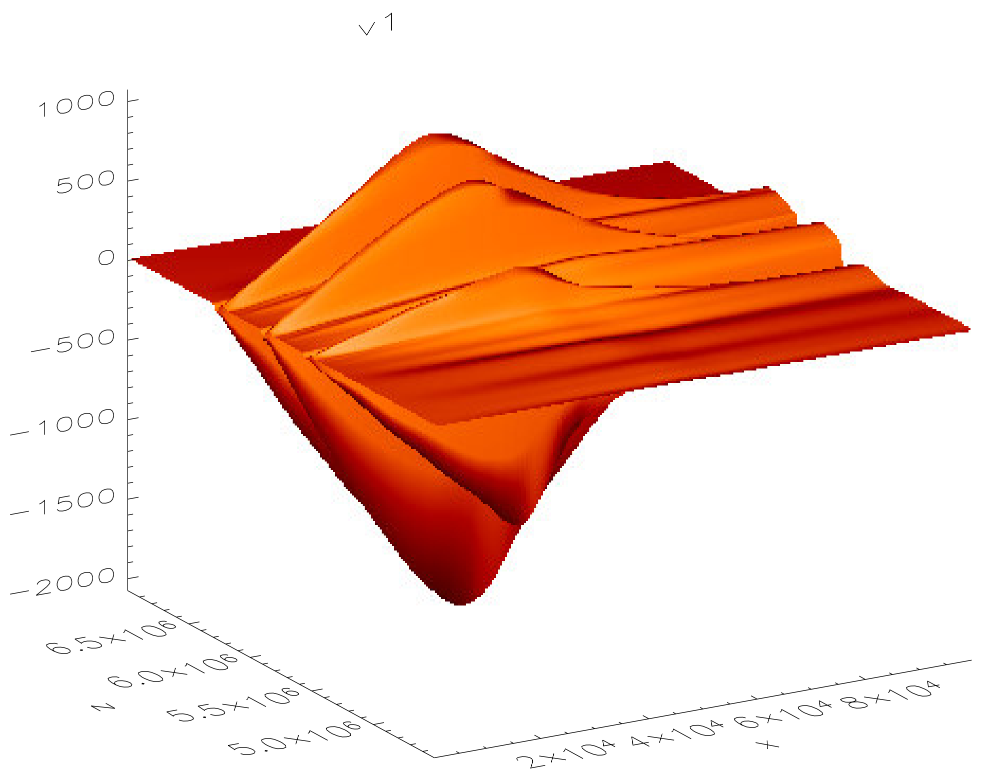
Nonlinear stage of the wave propagation



The time dependent evolution of the initial signal. The fourth stage.

Nonlinear stage of the wave propagation

ftpM25D (nx= 140 250)



it= 15000, time= 540.26

The vertical variables perturbations for the nonlinear case 2d snapshot. $v1$ - u velocity (m/s), x correspond the r direction (in meters), z the direction along the tube (in meters).