Emergence and inhibition of synchronization in robot swarms - Appendix

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Appendix: Synchronization threshold

In Section 4.3 of the main text we claim that a standard deviation of 2 frames (approx. 130 ms) in the robotic experiments was approximately equivalent to $r_{sync} = 0.95$ as set for the simulations.

We start by considering the distribution of instantaneous firings ($\phi_{LED} \to 0$) that yields to the worst order parameter, r, for a system with N oscillators. This is displayed in Fig. 1(a). Half of the firings occur at $t = -\sigma_A$ and the other half occurs at $t = +\sigma_A$. The standard deviation of this distribution is thus σ_A.

Remember that the complex order parameter is calculated as follows,

$$r(T_k)e^{\mathrm{i}\frac{2\pi\phi(T_k)}{\tau}} = \frac{1}{N}\sum_{j=1}^{N} e^{\mathrm{i}\frac{2\pi\phi_j(T_k)}{\tau}}. \tag{1}$$

Solving it for the standard deviation, σ_A, that would yield $r = 0.95$ in this case,

$$0.95 = \frac{1}{N}\frac{N}{2}\left(e^{-\mathrm{i}\frac{2\pi\sigma_A}{\tau}} + e^{\mathrm{i}\frac{2\pi\sigma_A}{\tau}}\right) = \cos\left(\frac{\tau}{2\pi\sigma_A}\right), \tag{2}$$

yields $\sigma_A = 0.25\,\mathrm{s}$ or, approximately, 3.8 frames (at the frame rate used for tracking of approximately 15-16 FPS).

In reality, however, the firings are not instantaneous, but the LEDs are turned on for a period of time. In our case $\phi_{LED} = 0.075\tau = 0.375\,\mathrm{s}$ or, approximately, 5.7 frames. If we measure the number of agents that are flashing at any particular frame, and the extreme points occur at $t = -\sigma_A$ and $t = +\sigma_A$, the distribution we obtain will have a standard distribution, $\sigma_B \leq \sigma_A$.

Therefore, in the worse case scenario, where σ_B is greatest, and approximating $\sigma_A \approx 4$ frames and $\phi_{LED} \approx 5$ frames to minimize skewness (see Fig. 1(b)), we have that:

$$\sigma_B = \sqrt{\frac{2}{5N}\left(\frac{N}{2}(4^2+3^2+2^2)+1^1\right)} \approx 2.49\,\text{frames}, \tag{3}$$

which is consistent with the choice of synchronization threshold in our experiments.

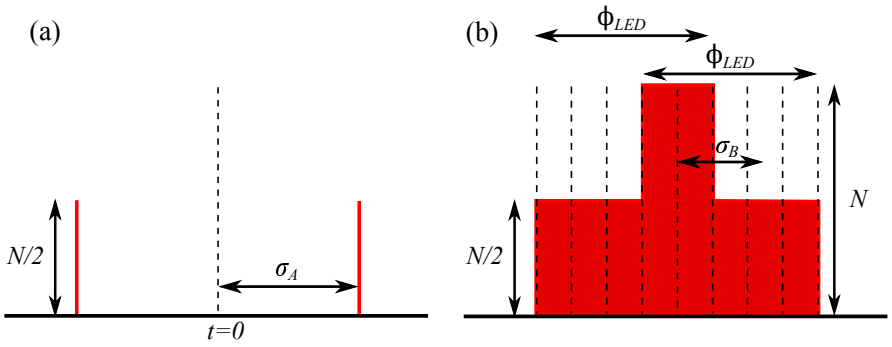

Fig. 1