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Swiss Federal Institute of Aquatic
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Joint Data Assimilation and Parameter Calibration in online groundwater modelling using Sequential Monte Carlo techniques

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Never heard of filtering?

Start here!

A Introduction

Motivation:

- Groundwater modelling is notoriously limited by avilability of geological data

Classic approach:

- Inverse parameter calibration with batch of available data

Problem:

- Computational effort: with new data, lengthy recalibration is required

B Data assimilation

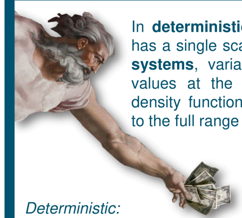
Why

- Models represent reality imperfectly
- For real-time applications, this drift can be corrected
- Assimilating observed data anchors the model in reality

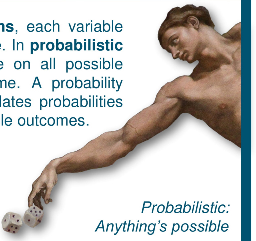
How

- The system is described stochastically
- Introduction of model error: predictions are scattered
- Points close to observations form basis for next prediction
- Common techniques: Kalman filter or particle filter

? Particle Filtering in a nutshell

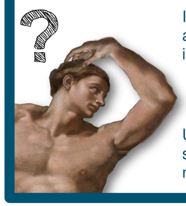


In **deterministic systems**, each variable has a single scalar value. In **probabilistic systems**, variables take on all possible values at the same time. A probability density function (*pdf*) relates probabilities to the full range of possible outcomes.



Deterministic: Absolute knowledge

Probabilistic: Anything's possible

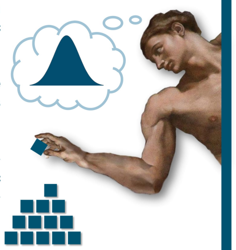


In probabilistic systems, a change in knowledge about a variable requires updating functions. This is done via the **Rule of Bayes**:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Unfortunately, it is impossible to find an analytic solution to this Baysian inference in all but the most simple cases (→ Kalman Filter).

Instead, analytic expressions are often **approximated** by a set of deterministic realizations, the eponymous **particles**:



Given enough of these particles, one could **draw from the set of outcomes** rather than the analytic function itself.

The technique of keeping this set representative of the underlying analytic function through reweighting and re-sampling is known as **particle filtering**.

C Online parameter calibration schemes

Alternatives

Extension of state vector:

- Cell parameters follow data assimilation trend, **requires linear relation**[a]

Untargeted mutation:

- Small, random parameter changes, suffers heavily from **curse of dimensionality**[b]

Chosen approach

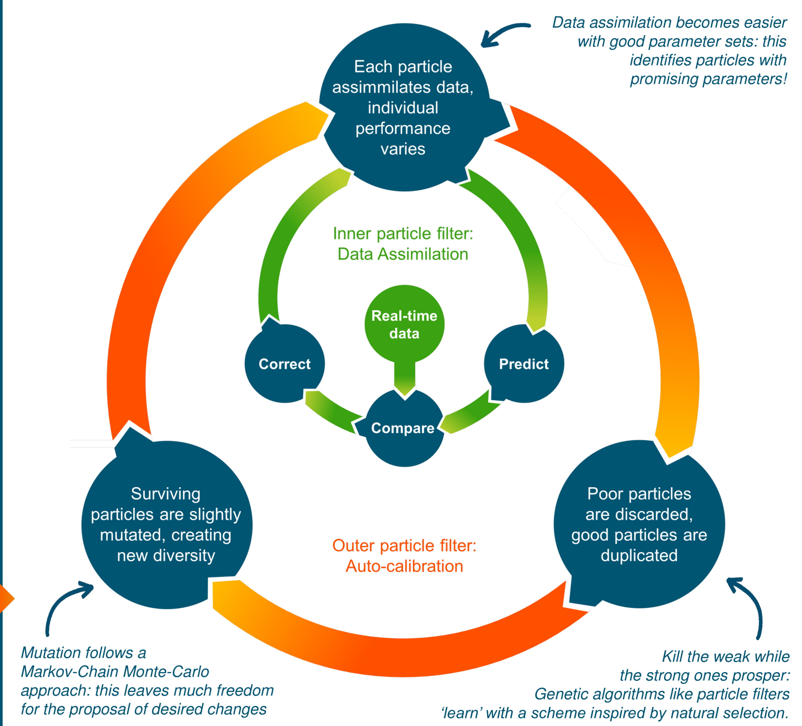
Metropolis-Hastings MCMC:

- User may propose desired change[c]
- **Flexibility at a price**: implementation requires retracing the full model history during calibration step → **not online**

Solution hypothesis:

- The system is **ergodic**: it may suffice to only repeat the last *n* time steps
- Bounded computational effort → **online**

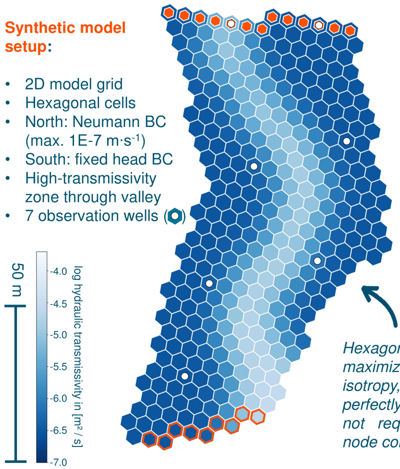
D The concept of nested particle filters



E Preliminary results

Synthetic model setup:

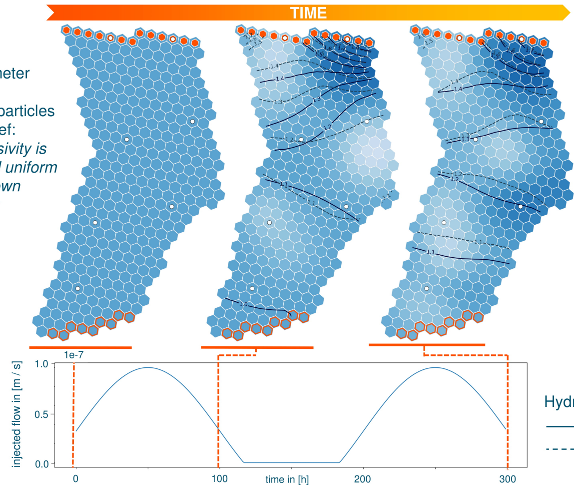
- 2D model grid
- Hexagonal cells
- North: Neumann BC (max. 1E-7 m·s^{-1})
- South: fixed head BC
- High-transmissivity zone through valley
- 7 observation wells



Hexagonal grids maximize structural isotropy, tessellate perfectly and do not require ghost node correction.

Ensemble:

- 40 parameter particles
- 30 state particles
- Prior belief: *transmissivity is assumed uniform*
- BCs: *known precisely*



Hydraulic head isolines:

—— *ensemble mean*

- - - - *synthetic reference*

Results:

- Algorithm quickly identifies parameter sets yielding **equivalent solutions**
- **Best performance** near regions of high head **amplitudes** (e.g. north-eastern edge)

Outlook:

- Draw state observations from more **complex synthetic model** (introducing true structural error)
- Investigate performance for **poorly defined boundary conditions**

Any questions remaining?

Please ask!

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vCard

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References

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[b] Moradkhani, H., Kuo-Lin, H., Gupta, H, and Sorooshian, S. (2005), Uncertainty assessment of hydrologic model states and parameters: Sequential data assimilation using the particle filter. Water Resources Research, 41: W05012.

[c] Chopin, N., Jacob, P. E. and Papaspiliopoulos, O. (2013), SMC^2: an efficient algorithm for sequential analysis of state space models. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 75: 397–426.