A Dimensionally-Aware Genetic Programming Architecture for Automated Innovization

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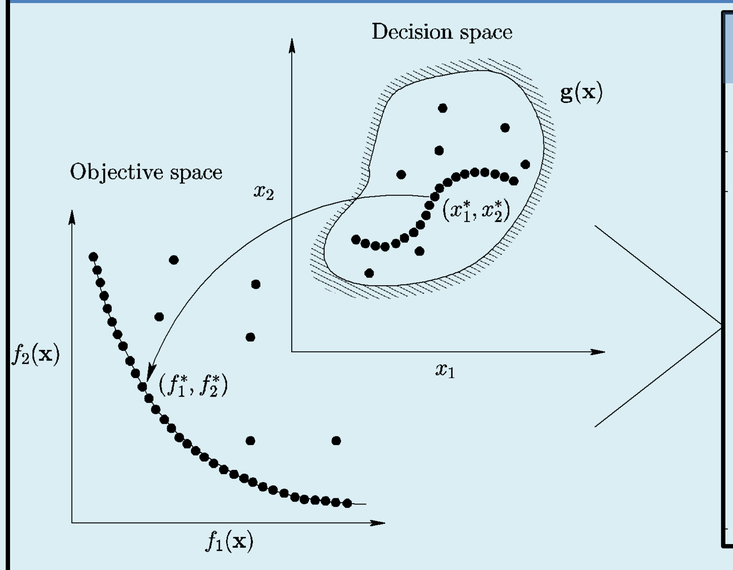
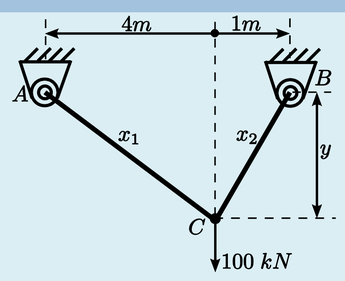
Innovization: *Innov*ation through Multi-objective Optim*ization*



Illustrative Example



$$\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = V = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2},\\
\text{Minimize} \quad & f_2(\mathbf{x}) = S = \max(\sigma_{AC}, \sigma_{BC}),\\
\text{Subject to} \quad & \max(\sigma_{AC}, \sigma_{BC}) \le 10^5 \text{ kPa},\\
& 0 \le x_1, x_2 \le 0.01 \text{ m}^2,\\
& 1 \le y \le 3 \text{ m}.
\end{aligned}$$

$$\sigma_{AC} = \sigma_{BC} \Rightarrow S = \frac{20\sqrt{16+y^2}}{yx_1} = \frac{80\sqrt{1+y^2}}{yx_2}$$

$$\frac{V}{2} = x_1\sqrt{16+y^2} = x_2\sqrt{1+y^2}$$

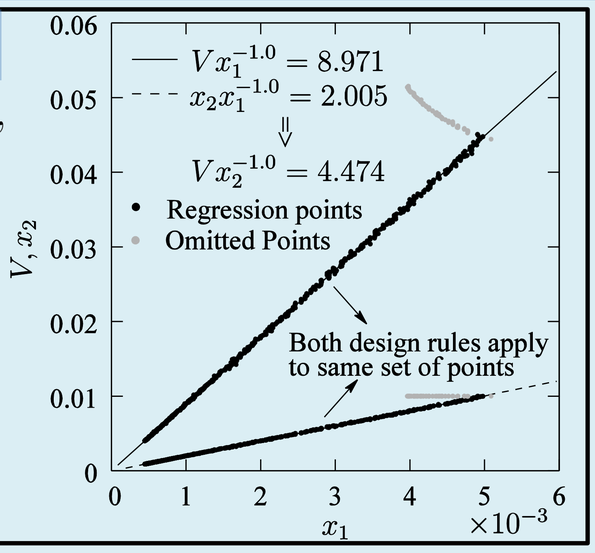
Design principles

$y = 2,$

$V = 4\sqrt{5}x_1 = 8.994x_1$

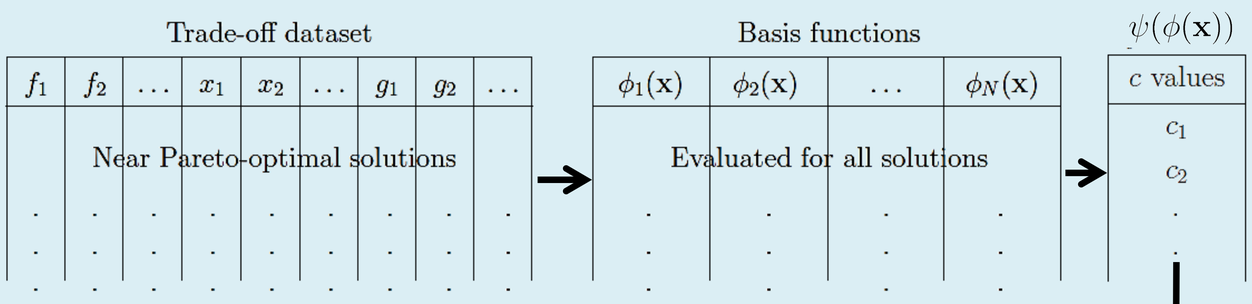
$V = 2\sqrt{5}x_2 = 4.472x_2$

$SV = 400$



Automated Innovization using Dimensionally-Aware Genetic Programming (GP)

Methodology: Grid-based Clustering and Optimization



Objective Function

$$\text{Minimize} \quad \mathcal{C} + \sum_{k=1}^{\mathcal{C}} c_v^{(k)} \times 100\% \quad \text{where } c_v^{(k)} = \frac{\sigma_c}{\mu_c} \quad \forall\, c \in k\text{-th cluster}$$

Number of Clusters

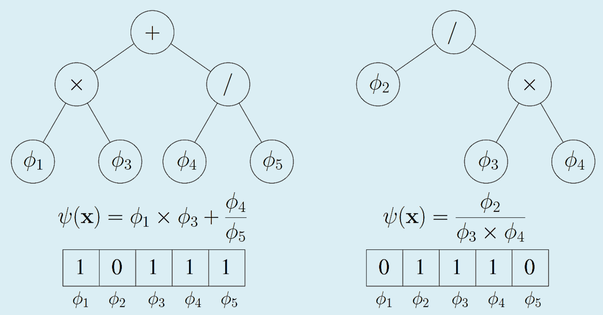
Percentage Coefficient of Variation within Clusters

Extension: Generalization of $\psi(\phi(\mathbf{x}))$ using Parse Tree Representation

Old Representation

$$\psi(\mathbf{x}) = \prod_{j=1}^{N} \phi_j(\mathbf{x})^{a_j b_j}$$

Parse Tree Representation



Niched Tournament Selection

SmallGP Implementation

- **Initialization: Grow Method**
- **Size Safe Subtree Crossover**
- **Point Mutation**
- **Maximum Tree Length (MAXLEN)**
- **Maximum Tree Depth (MAXDEPTH)**

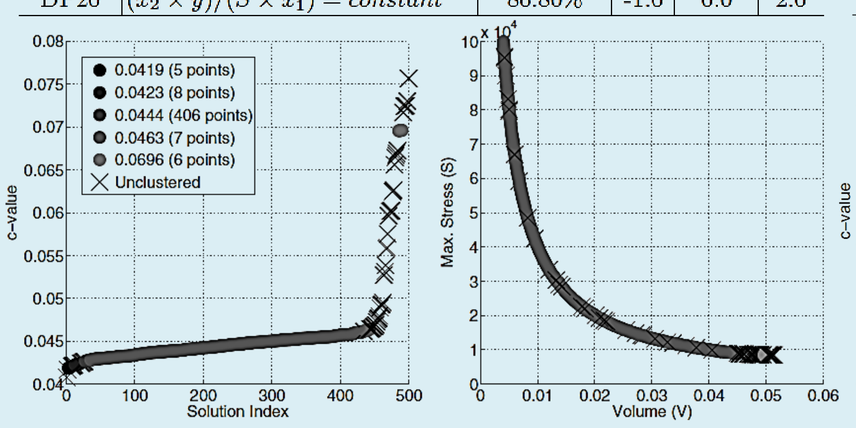
Dimensional Awareness

- **Physically meaningful design principles**
- **Implemented as a constraint on each physical dimension**

Results on Engineering Design Problems

Design Principles for Truss Problem (Above)

Notation	Design Principle (DP) $\psi(\mathbf{x}) = constant$	Significance	Basic Dimensions		
			Mass	Length	Time
DP1	$y = constant$	86.60%	0.0	1.0	0.0
DP2	$S \times V = constant$	87.00%	1.0	2.0	-2.0
DP3	$S \times x_1 = constant$	85.00%	1.0	1.0	-2.0
DP4	$S \times V \times y = constant$	87.00%	1.0	3.0	-2.0
DP5	$(V \times y)/x_2 = constant$	86.20%	0.0	2.0	0.0
DP6	$(V \times y)/x_1 = constant$	88.20%	0.0	2.0	0.0
DP7	$V/x_1 = constant$	86.40%	0.0	1.0	0.0
DP8	$V/(S \times x_1 \times x_2) = constant$	87.20%	-1.0	0.0	2.0
DP9	$V^2/(x_1 \times x_2) = constant$	87.40%	0.0	2.0	0.0
DP10	$y/(S \times x_1) = constant$	88.00%	-1.0	0.0	2.0
DP11	$x_2/x_1 = constant$	83.80%	0.0	0.0	0.0
DP12	$(S \times V \times x_2 \times y)/x_1 = constant$	88.00%	1.0	3.0	-2.0
DP13	$V/x_2 = constant$	86.80%	0.0	1.0	0.0
DP14	$(S \times V^2 \times y)/x_1 = constant$	87.20%	1.0	4.0	-2.0
DP15	$(x_2 \times y)/x_1 = constant$	86.40%	0.0	1.0	0.0
DP16	$x_2/(S \times x_1^2) = constant$	86.40%	-1.0	-1.0	2.0
DP17	$V^2/(x_1 \times x_2 \times y) = constant$	91.40%	0.0	1.0	0.0
DP18	$(S \times V^2)/x_2 = constant$	87.20%	1.0	3.0	-2.0
DP19	$S \times x_2 \times y = constant$	87.00%	1.0	2.0	-2.0
DP20	$(x_2 \times y)/(S \times x_1^2) = constant$	86.80%	-1.0	0.0	2.0



Cluster Plot and Mapping of Clusters for DP16

Parse Tree for DP16

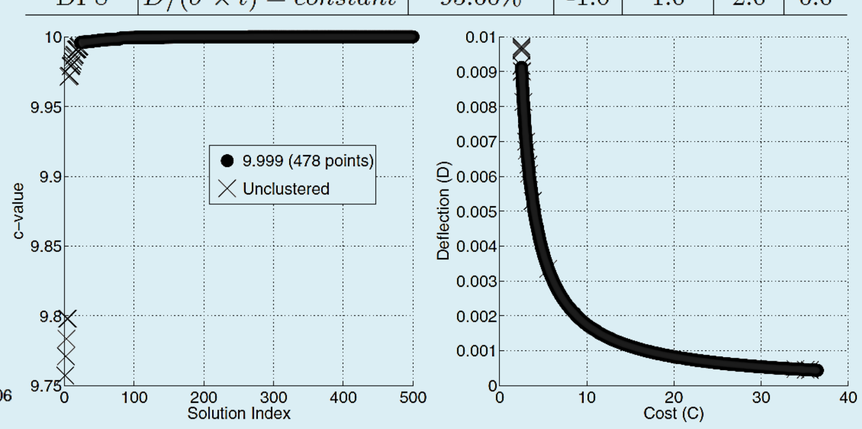
- **Mass, Length and Time are chosen as basic dimensions**
- **All design principles agree with analytical relationships above**

Design Principles for Welded-Beam Problem

$$\begin{aligned}
\text{Minimize} \quad & f_1(\boldsymbol{x}) = C = 1.10471h^2l + 0.04811tb(14.0+l),\\
\text{Minimize} \quad & f_2(\boldsymbol{x}) = D = \tfrac{2.1952}{t^3b},\\
\text{Subject to} \quad & \left\{\begin{array}{l}\tau(\boldsymbol{x}) \le 13,600 \text{ psi}; \sigma(\boldsymbol{x}) \le 30,000 \text{ psi}; b \ge h; P_c(\boldsymbol{x}) \ge 6,000 \text{ lb}\\ 0.125 \le h, b \le 5.0 \text{ in.}; 0.1 \le l, t \le 10.0 \text{ in.}\end{array}\right\}
\end{aligned}$$

$$\begin{aligned}
\text{where} \quad & \tau(\boldsymbol{x}) = \sqrt{(\tau')^2 + (\tau'')^2 + (l\tau'\tau'')/\sqrt{0.25(l^2+(h+t)^2)}},\\
& \tau' = \tfrac{6,000}{\sqrt{2}hl}, \qquad \tau'' = \tfrac{6,000(14+0.5l)\sqrt{0.25(l^2+(h+t)^2)}}{2[0.707hl(l^2/12+0.25(h+t)^2)]},\\
& \sigma(\boldsymbol{x}) = \tfrac{504,000}{t^2b},\\
& P_c(\boldsymbol{x}) = 64,746.022(1-0.0282346t)tb^3.
\end{aligned}$$

Notation	Design Principle (DP) $\psi(\mathbf{x}) = constant$	Significance	Basic Dimensions			
			Mass	Length	Time	Cost
DP1	$(D+t) = constant$	95.20%	0.0	1.0	0.0	0.0
DP2	$t = constant$	95.60%	0.0	1.0	0.0	0.0
DP3	$D \times b = constant$	95.00%	0.0	2.0	0.0	0.0
DP4	$D \times b \times t = constant$	95.60%	0.0	3.0	0.0	0.0
DP5	$\sigma \times b = constant$	94.80%	1.0	0.0	-2.0	0.0
DP6	$\sigma \times b \times t = constant$	95.60%	1.0	1.0	-2.0	0.0
DP7	$D/\sigma = constant$	95.60%	-1.0	2.0	2.0	0.0
DP8	$D/(\sigma \times t) = constant$	95.60%	-1.0	1.0	2.0	0.0



Cluster Plot and Mapping of Clusters for DP2

Observations

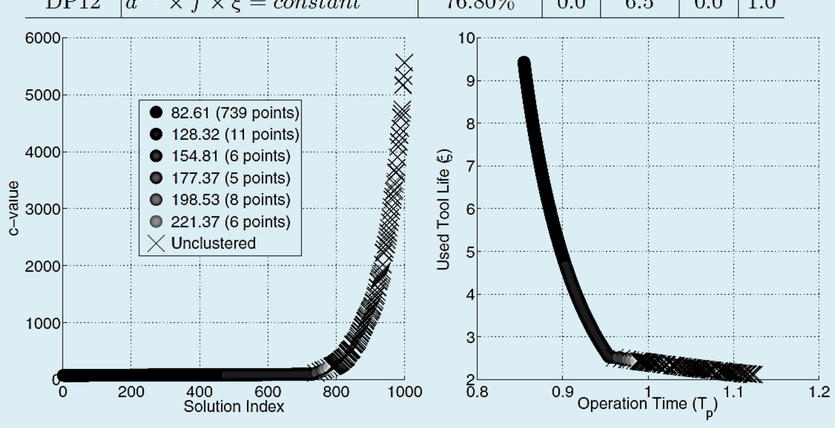
- **A user-defined basic dimension (Cost) has been introduced**
- **Redundant design principles can be seen (For example DP2, DP3 and DP4)**
- **DP1 is a false design principle, arising due to difference in magnitudes of D and t**

Design Principles for Metal-Cutting Problem

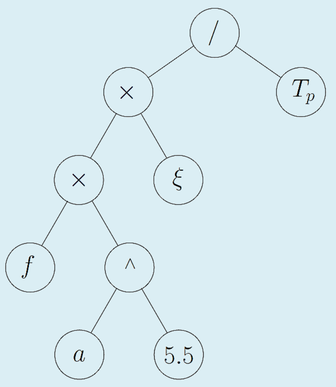
$$\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = T_p(\mathbf{x}) = 0.15 + 219912\left(\tfrac{1+\frac{0.20}{T(\mathbf{x})}}{MRR(\mathbf{x})}\right) + 0.05 \quad \text{min}\\
\text{Minimize} \quad & f_2(\mathbf{x}) = \xi(\mathbf{x}) = \tfrac{219912}{MRR(\mathbf{x})T(\mathbf{x})} \times 100\%\\
\text{Subject to} \quad & \left\{\begin{array}{l}P(\mathbf{x}) \le \eta P^{max}; F_c(\mathbf{x}) \le F_c^{max}; R(\mathbf{x}) \le R^{max}\\ 250 \le v \le 400 \text{ m/min}; 0.15 \le f \le 0.55 \text{ mm/rev}; 0.5 \le a \le 6 \text{ mm}\end{array}\right\}
\end{aligned}$$

$$\begin{aligned}
\text{where} \quad & T(\mathbf{x}) = \tfrac{5.48\times10^9}{v^{3.46}f^{0.696}a^{0.460}}, \qquad MRR(\mathbf{x}) = 1000vfa\\
& P(\mathbf{x}) = \tfrac{vF_c(\mathbf{x})}{60000}, \qquad F_c(\mathbf{x}) = \tfrac{6.56\times10^3 f^{0.917}a^{1.10}}{v^{0.286}}, \qquad R(\mathbf{x}) = \tfrac{125f^2}{r_n}
\end{aligned}$$

Notation	Design Principle (DP) $\psi(\mathbf{x}) = constant$	Significance	Basic Dimensions			
			Mass	Length	Time	Life
DP1	$v/(f^2 \times \xi) = constant$	72.70%	0.0	-1.0	-1.0	-1.0
DP2	$(a \times v)/f = constant$	74.60%	0.0	1.0	-1.0	0.0
DP3	$v/(f^2 \times T_p \times \xi) = constant$	73.40%	0.0	-1.0	-2.0	-1.0
DP4	$f = constant$	72.90%	0.0	1.0	0.0	0.0
DP5	$a/(f \times T_p) = constant$	72.90%	0.0	0.0	-1.0	0.0
DP6	$(a^{5.5} \times f \times \xi)/T_p = constant$	77.50%	0.0	6.5	-1.0	1.0
DP7	$(a \times T_p \times v)/f = constant$	74.20%	0.0	1.0	0.0	0.0
DP8	$a^{5.5} \times T_p \times \xi = constant$	82.60%	0.0	5.5	1.0	1.0
DP9	$a \times T_p \times v = constant$	74.10%	0.0	2.0	0.0	0.0
DP10	$(a^2 \times T_p \times \xi)/v = constant$	74.40%	0.0	1.0	2.0	1.0
DP11	$(a^2 \times \xi)/v = constant$	76.00%	0.0	1.0	1.0	1.0
DP12	$a^{5.5} \times f \times \xi = constant$	76.80%	0.0	6.5	0.0	1.0



Cluster Plot and Mapping of Clusters for DP6



Parse Tree for DP6

- **User-defined dimesion (Life) is introduced**
- **All design principles agree with those obtained in a previous study**