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Evenly Spaced Pareto Front Approximations for Tricriteria Problems Based on Triangulation

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Outline

- Introduction
  - Averaged Hausdorff Distance
  - Main Idea for d = 2
- Extension to d = 3
  - Construction of Benchmark & Algorithm
  - Experiments & Results
- Conclusions

Hausdorff Metric for Populations

Let a, b $\in \mathbb{R}^n$ and A, B $\subset \mathbb{R}^n$ and d: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ be a metric.

Distance point to set $d(a, B) = \inf \{ d(a, b) : b \in B \}$

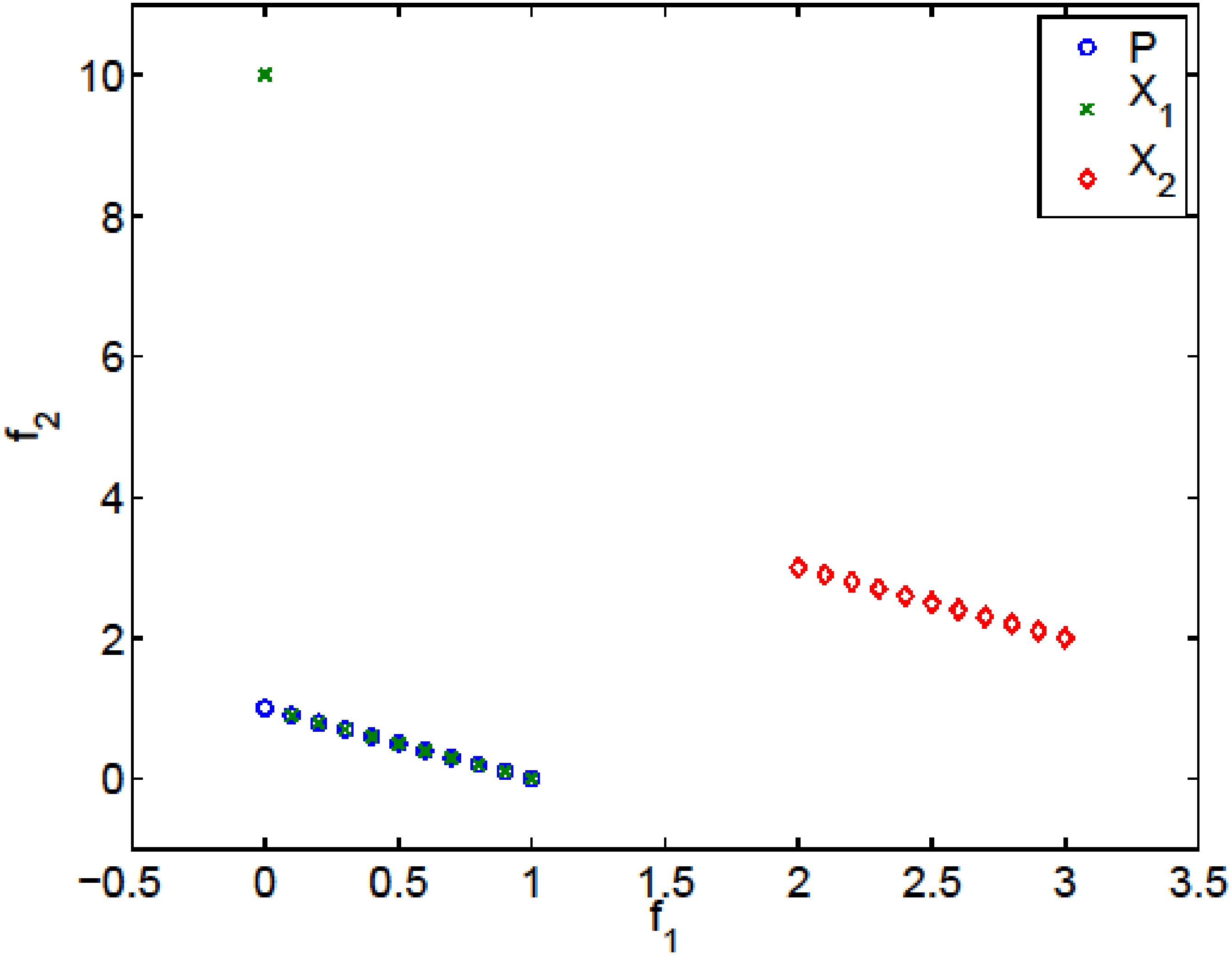
Distance set to set $d(A, B) = \sup \{ d(a, B) : a \in A \}$

Hausdorff metric $d_H(A,B) = \max \{ d(A, B), d(B, A) \}$ if A, B compact

Scenario:
evenly spaced approximation of Pareto front desired for some application

Conjecture:
Good approximations in Hausdorff sense should be evenly spaced

Hausdorff Metric: Problems



$d_H(X_1, P) > d_H(X_2, P)$

Distance depends on outliers only!

⇓

Hausdorff metric penalizes outliers too strongly!

⇓

“More gentle“ measure needed!

Let a, b $\in \mathbb{R}^n$ and A, B $\subset \mathbb{R}^n$ with |A|, |B| < ∞

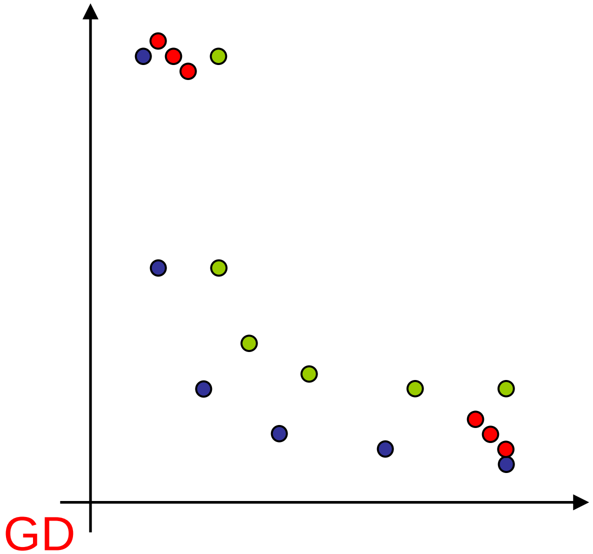
Generational Distance

$$d_{GD}(A,B) = \frac{1}{|A|}\left(\sum_{a\in A} d(a,B)^p\right)^{\frac{1}{p}}$$

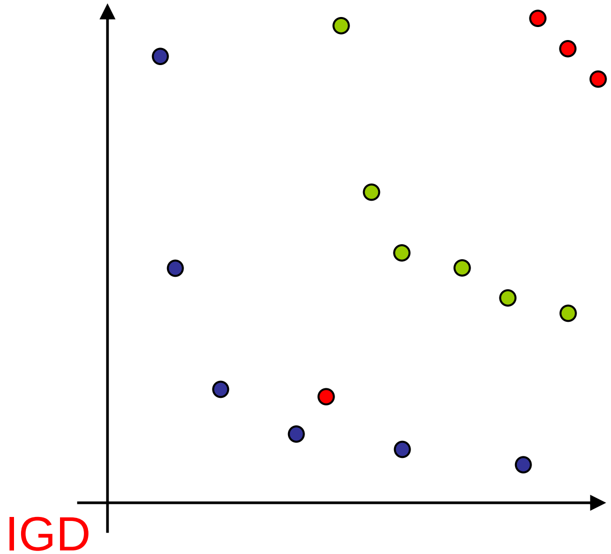
Inverted Generational Distance

$$d_{IGD}(A,B) = \frac{1}{|B|}\left(\sum_{b\in B} d(b,A)^p\right)^{\frac{1}{p}}$$

$d_{GD}(A, R) > d_{GD}(B, R)$



$d_{IGD}(A, R) > d_{IGD}(B, R)$



Definition

$$d_{GD_p}(A,B) = \left(\frac{1}{|A|} \sum_{a \in A} d(a,B)^p \right)^{\frac{1}{p}}$$

$$d_{IGD_p}(A,B) = \left(\frac{1}{|B|} \sum_{b \in B} d(b,A)^p \right)^{\frac{1}{p}}$$

„averaging“ Hölder norm

[Schütze et al. 2008]

Averaging Hausdorff Measure (Δ_p)

Definition

$$\Delta_p(X,Y) = \max\{ d_{GD_p}(X,Y), d_{IGD_p}(X,Y) \} \text{ für } p \in [1,\infty)$$

$\Rightarrow$ The smaller p, the more gentle the penalties on outliers!

let P be the current population of EMOA

then $f(P) = \{ y_1, ..., y_\mu \}$ is current approximation of Pareto front

compute piecewise linear curve K with support points $y_1, ..., y_\mu$

place μ points uniformly on polygonal line K → yields reference set R

generate offspring x from population P

update the archive A:

$A = \text{nds}(A \cup \{x\})$

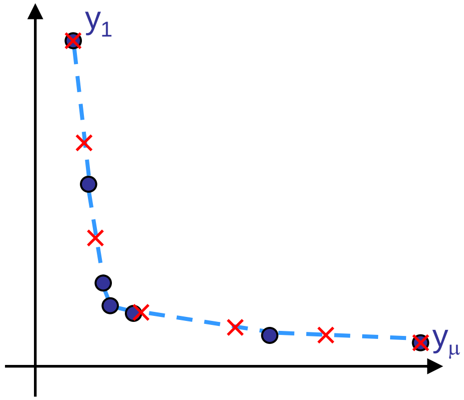
if $|A| > N_R$ then

$\forall a \in A: h(a) = \Delta_1(A \setminus \{a\}, R)$

$a^* = \text{argmin}\{ h(a) : a \in A \}$

$A = A \setminus \{ a^* \}$

endif



decide, if x is accepted and integrated in population

[GRST11]

Offline version: run EMOA and save all points, build R, put all points in archive

Δ_1 - Update

Algorithm 1 Δ_1-update

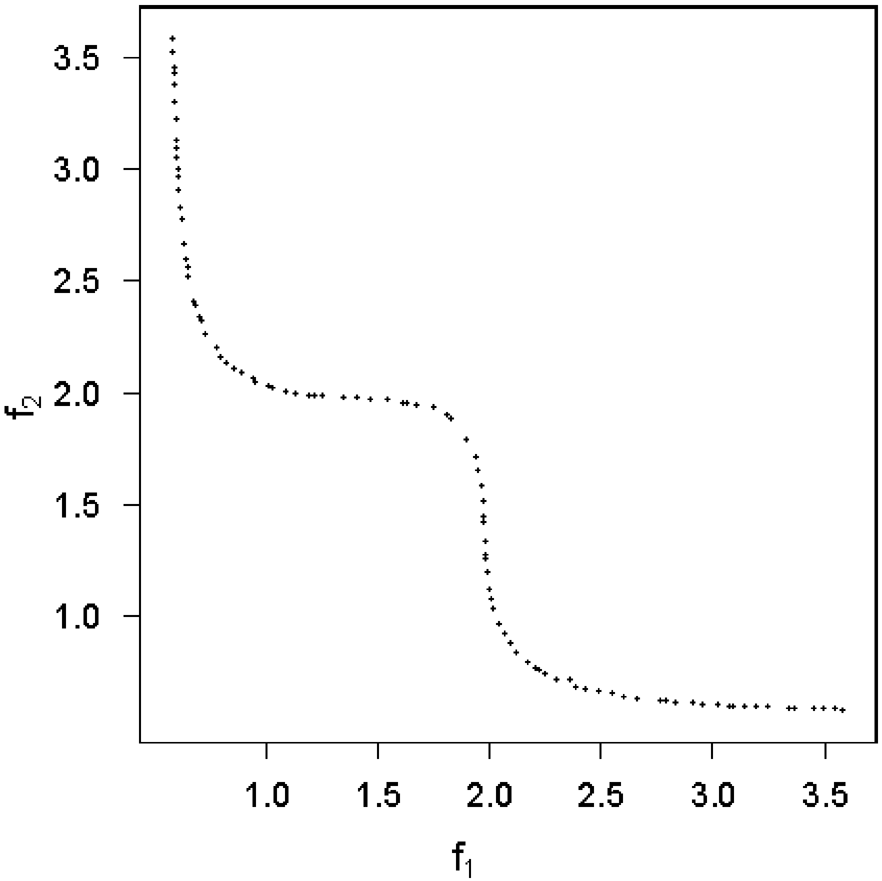
Input: archive set A, reference set R, new element x

$A = \mathsf{ND}_f(A \cup \{x\}, \preceq)$
if $|A| > N_R := |R|$ **then**
  for all $a \in A$ **do**
    $h(a) = \Delta_1(A \setminus \{a\}, R)$
  end for
  $A^* = \{a^* \in A : a^* = \mathsf{argmin}\{h(a) : a \in A\}\}$
  if $|A^*| > 1$ **then**
    $a^* = \mathsf{argmin}\{GD_P(A \setminus \{a\}, R) : a \in A^*\}$ {ties broken at random}
  end if
  $A = A \setminus \{a^*\}$
end if

Naive approach : $\Theta(|A| \times (|A| \times |R| \times d))$

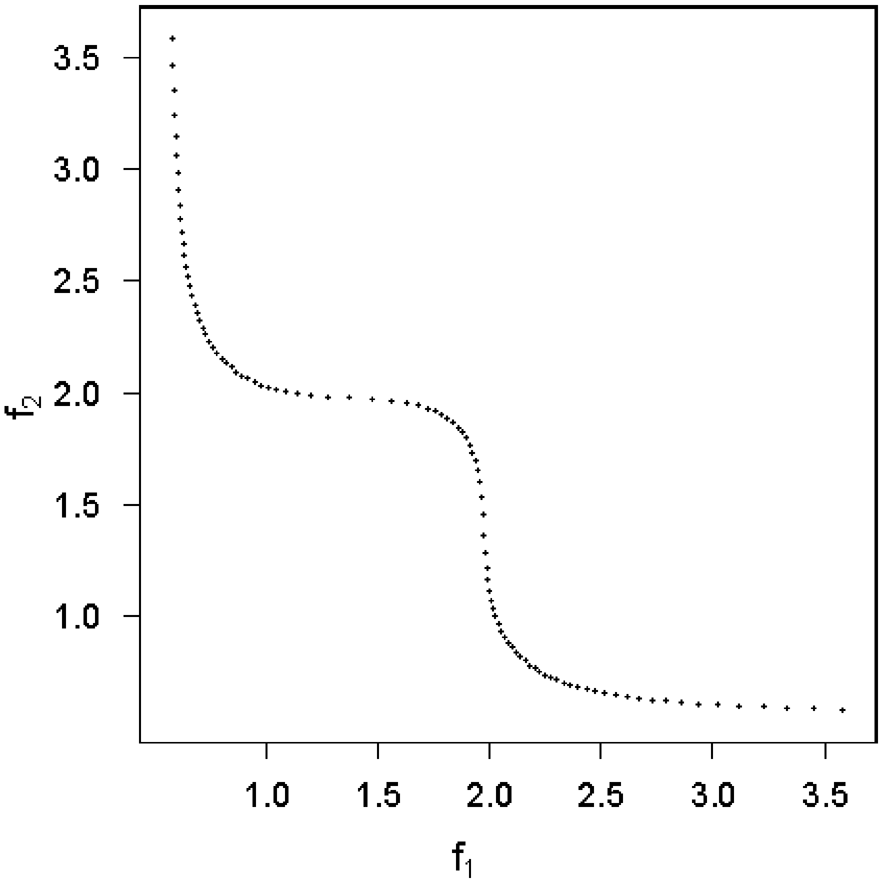
Smart approach : $\Theta(|A| \times |R| \times d)$

NSGA-2

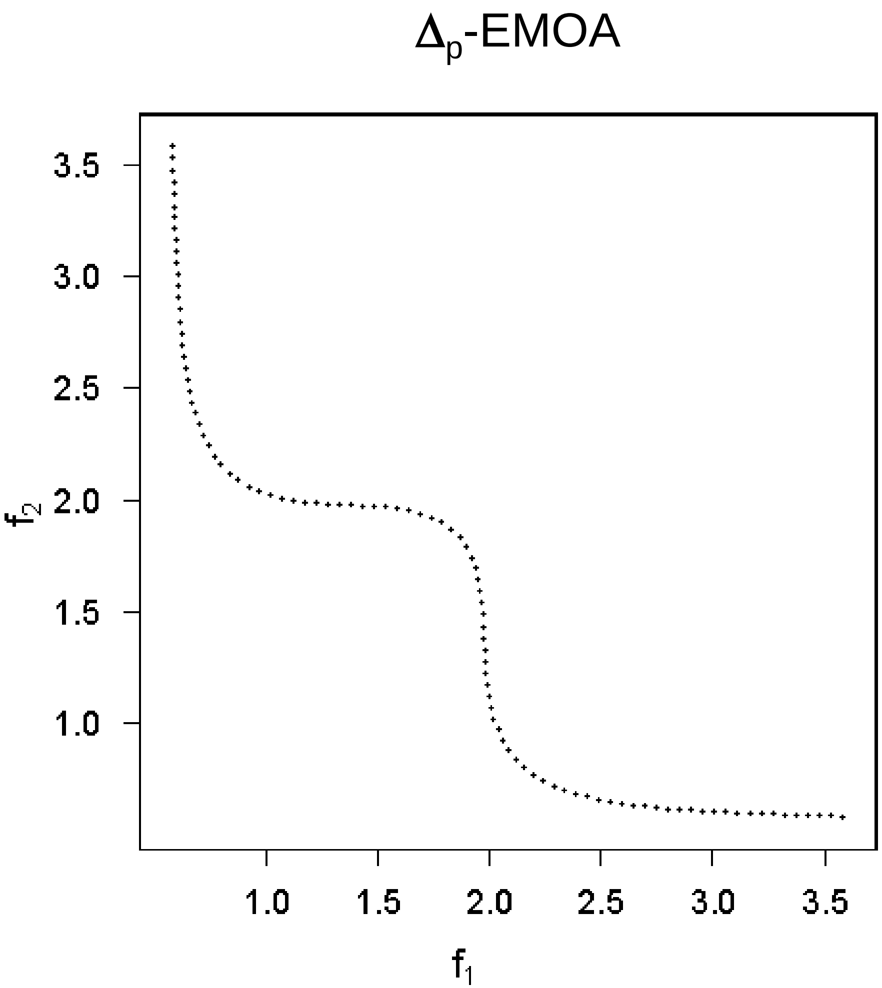


population: 100 individuals – archive: 100 points – evaluations: 50.000

SMS-EMOA



population: 100 individuals – archive: 100 points – evaluations: 50.000



population: 100 individuals – archive: 100 points – evaluations: 50.000

Two problems:

1. **Benchmark problem**:

   How to place M points “uniformly“ on Pareto front?

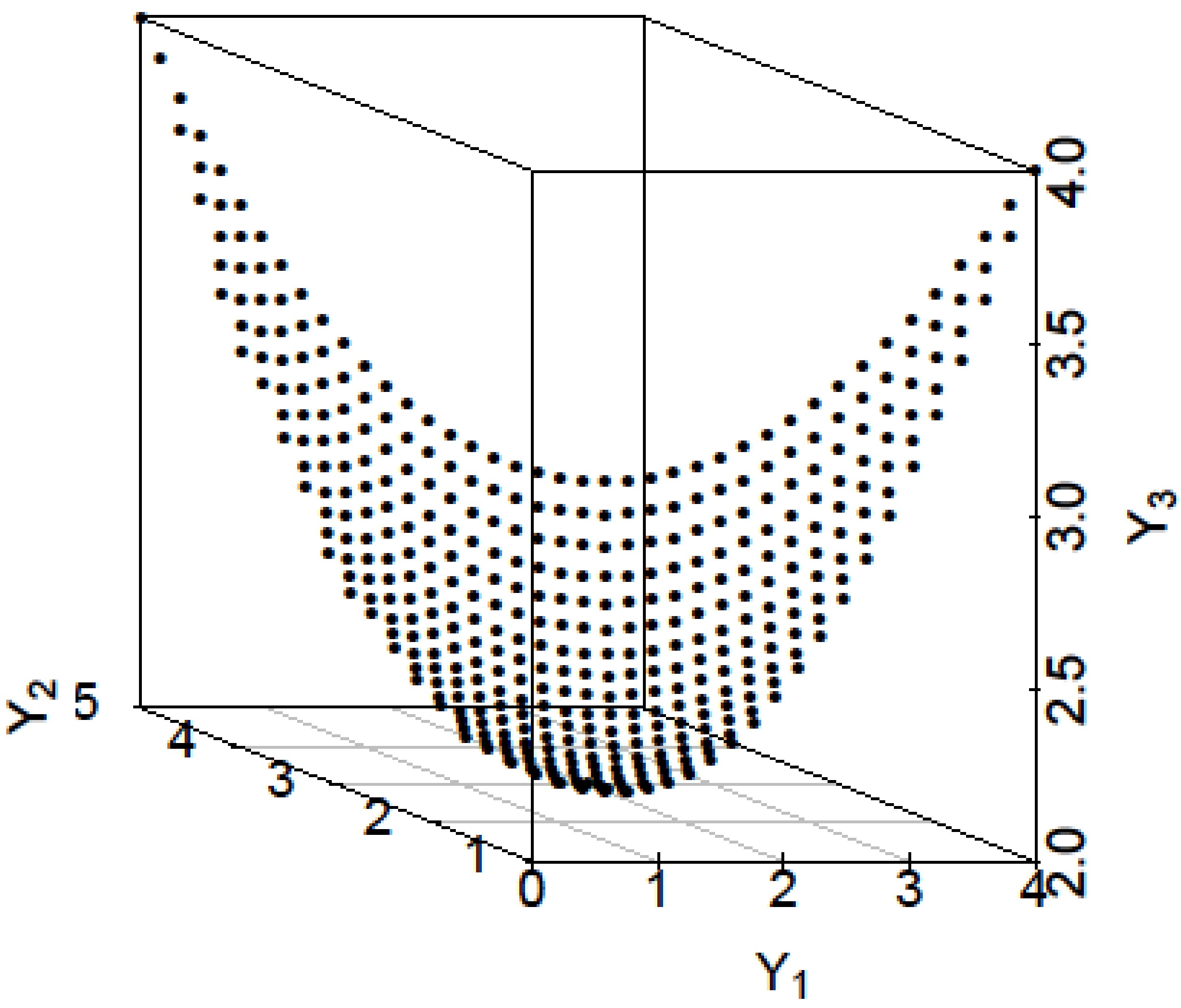
2. **Construction of reference front:**

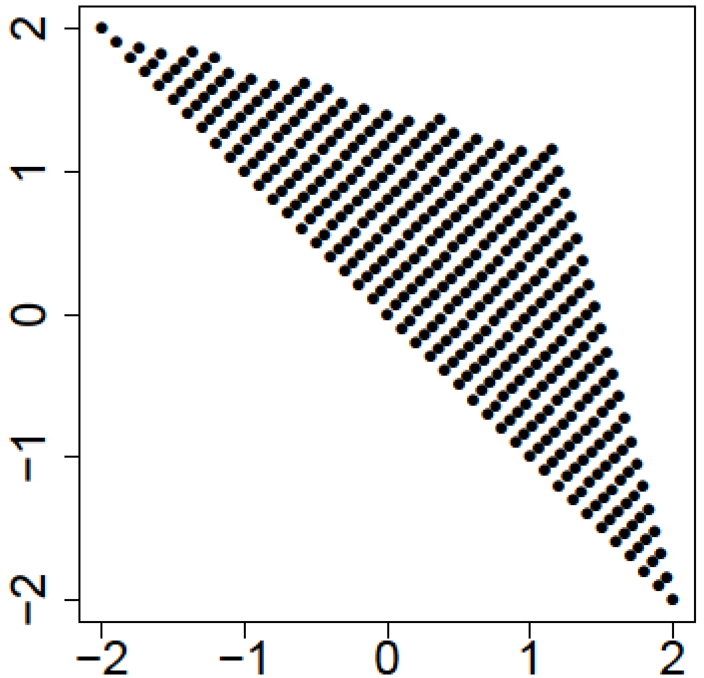
   How to create reference front and place M points uniformly?

   → EVOLVE 2012 : multidimensional scaling 3D to 2D

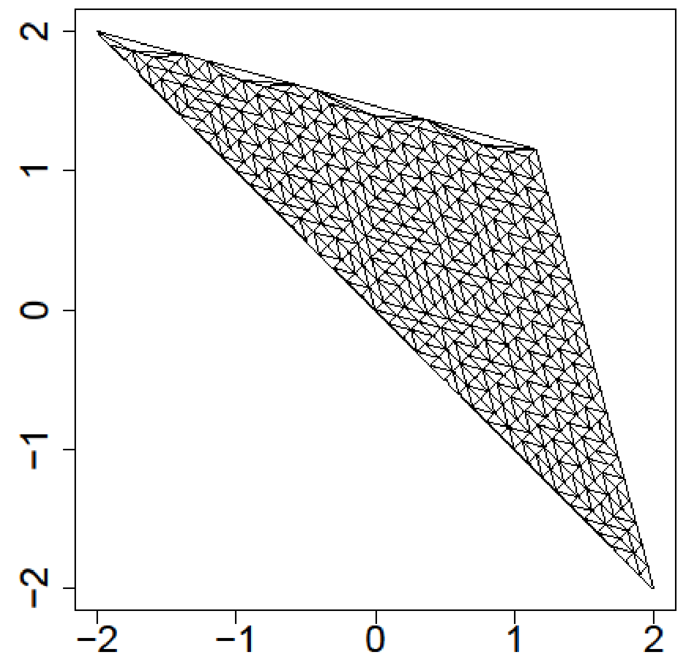
   → EMO 2013 : work directly in 3D with triangulations

Test Problem: Viennet

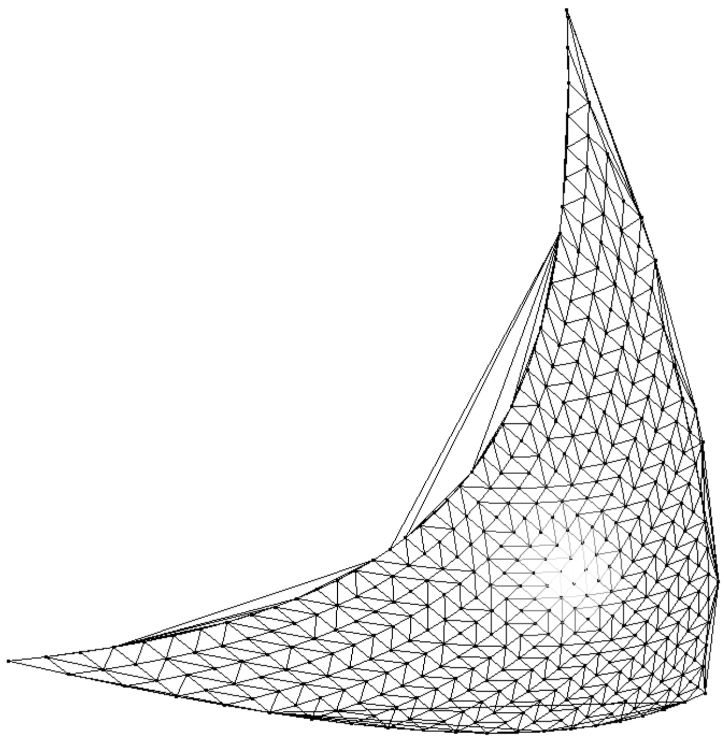




After parallel projection on plane with normal (1,1,1)



After standard triangulation algorithm in 2D

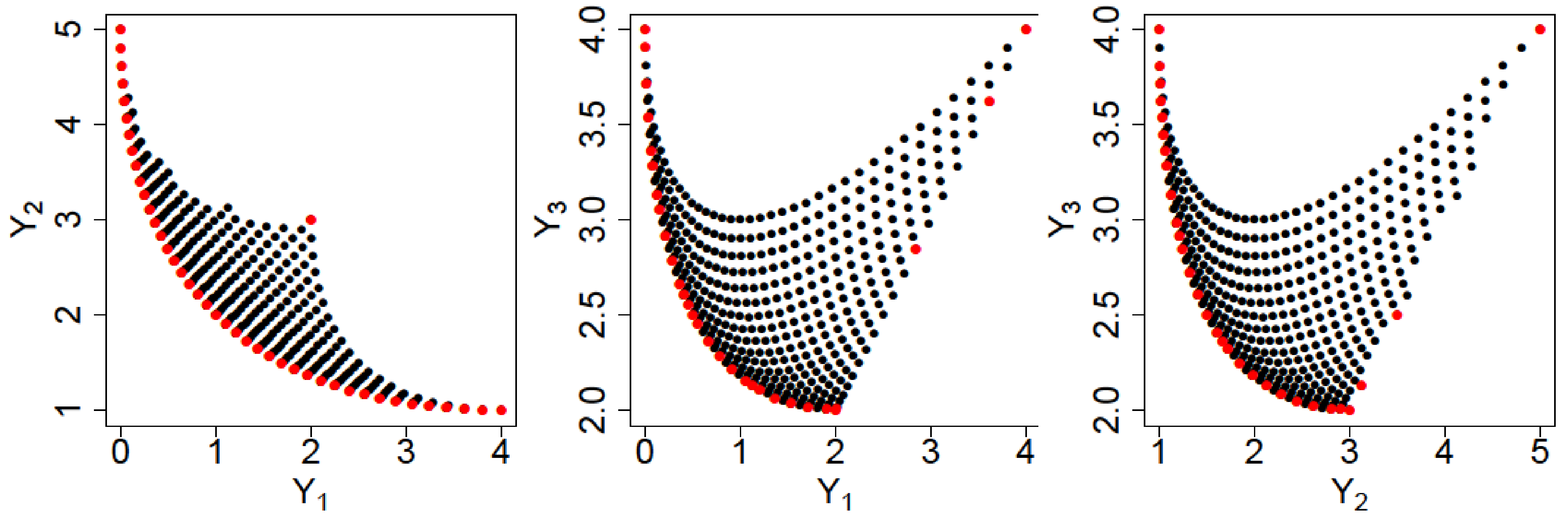


Using 2D triangulation
with 3D coordinates

Evidently: false edges

caused by

- nonconvexity
- numerical inaccuracies



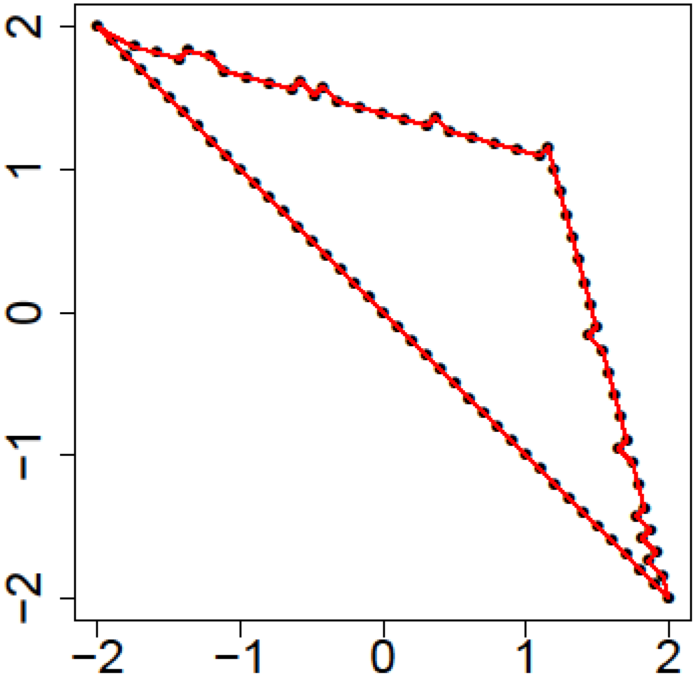
Consider all three 2D projections by omitting one dimension

Determine border points of each projection:

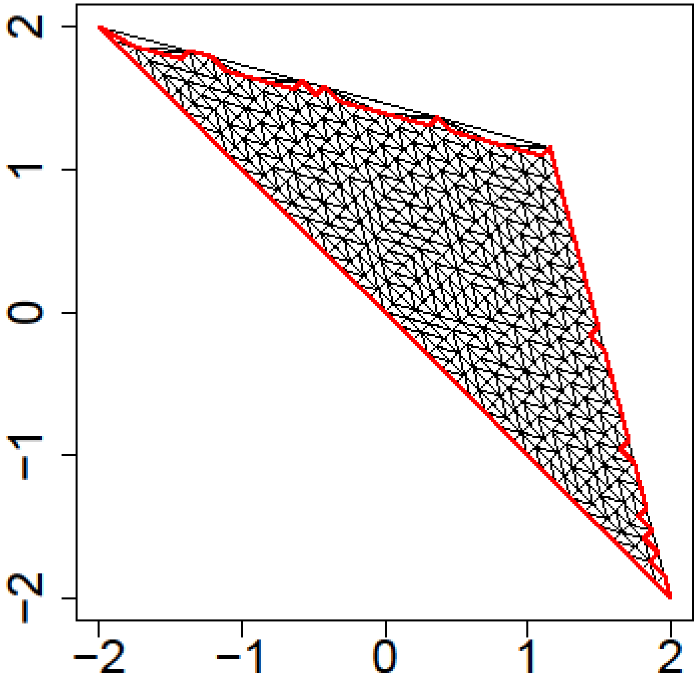
for each edge (u,v) let N(u) and N(v) the neighboring vertices

if | N(u) ∩ N(v) | = 1 then both u and v are border points

Apply TSP Solver to union of border points for identifying border edges

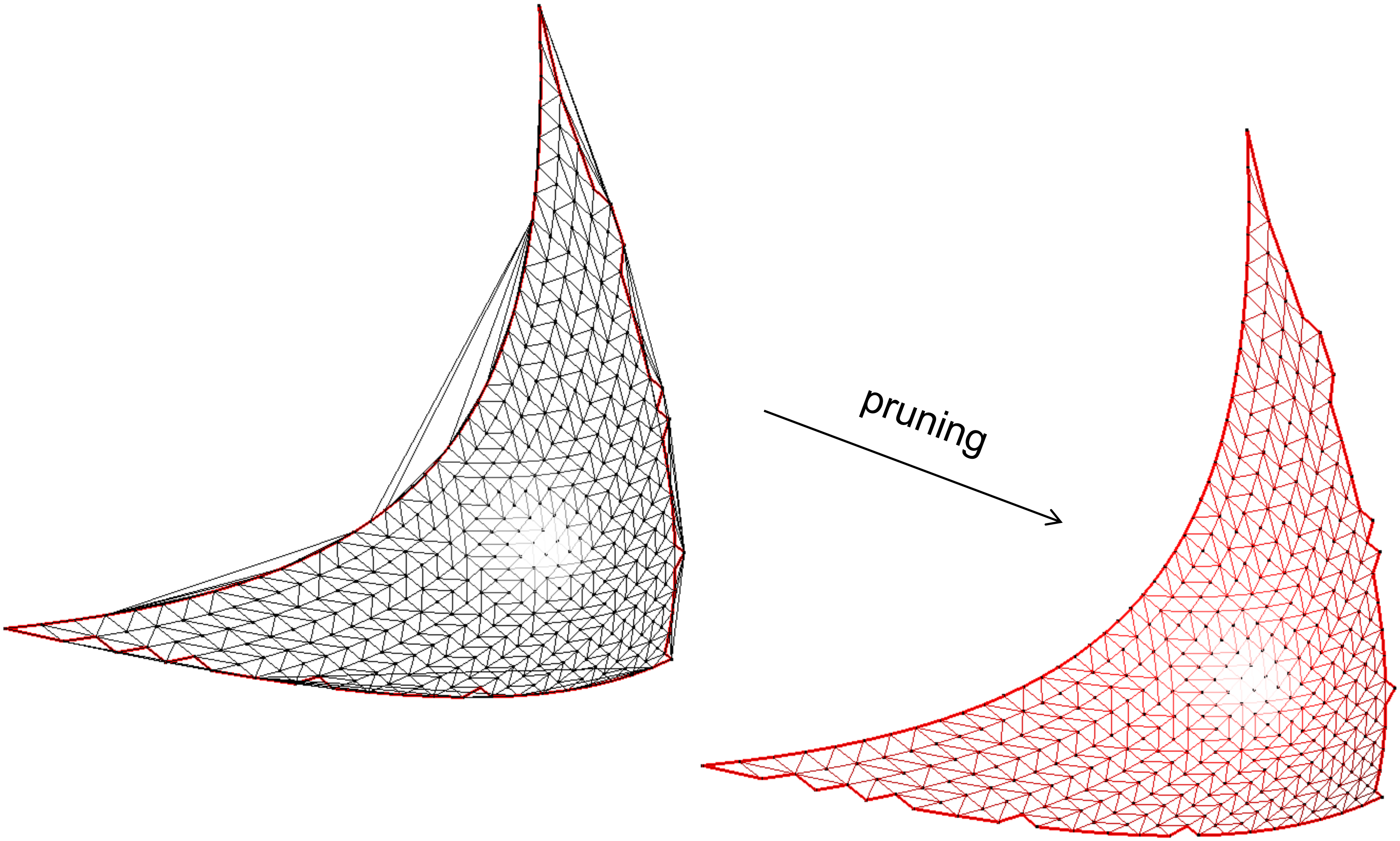


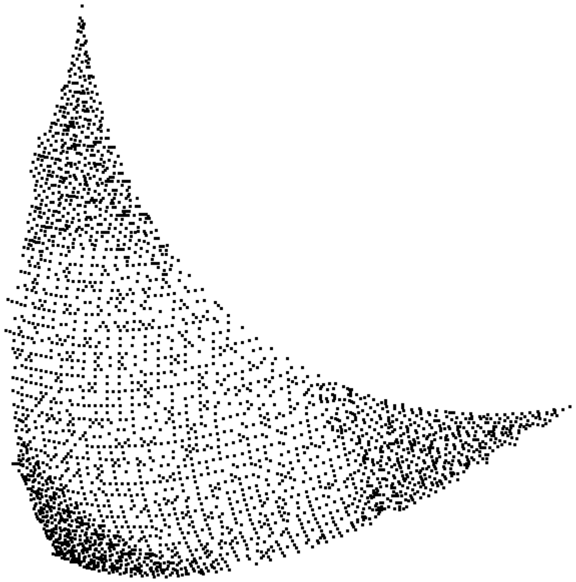
After using TSP solver:
border eges are identified

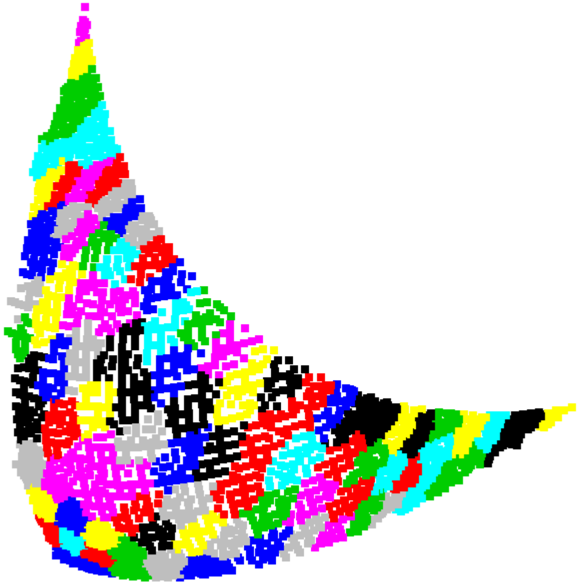


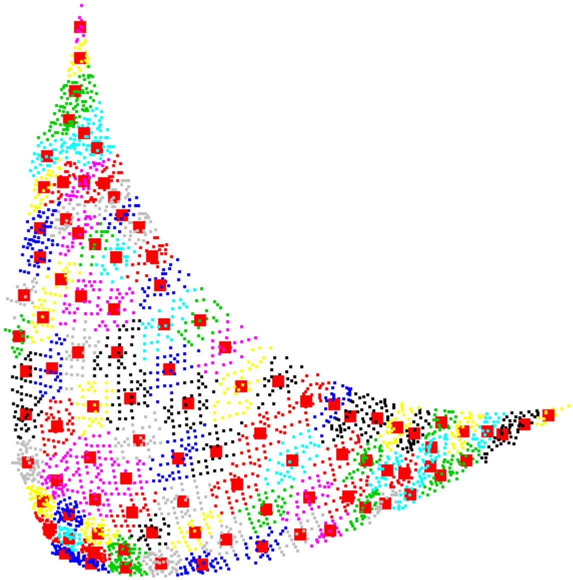
It remains to remove false
edges: pruning!

Test Problem: Viennet







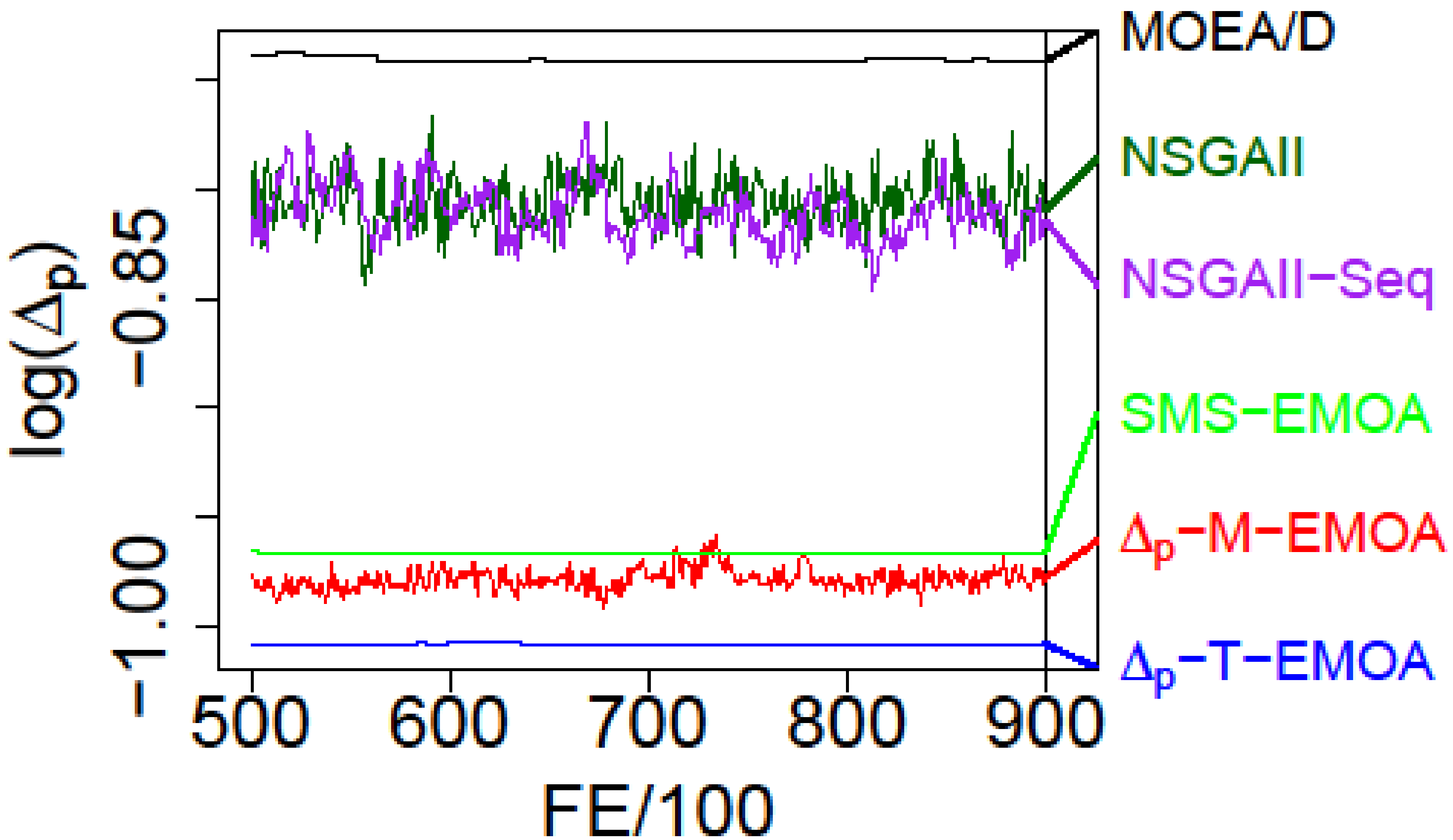


recursively divide triangles until size below some threshold

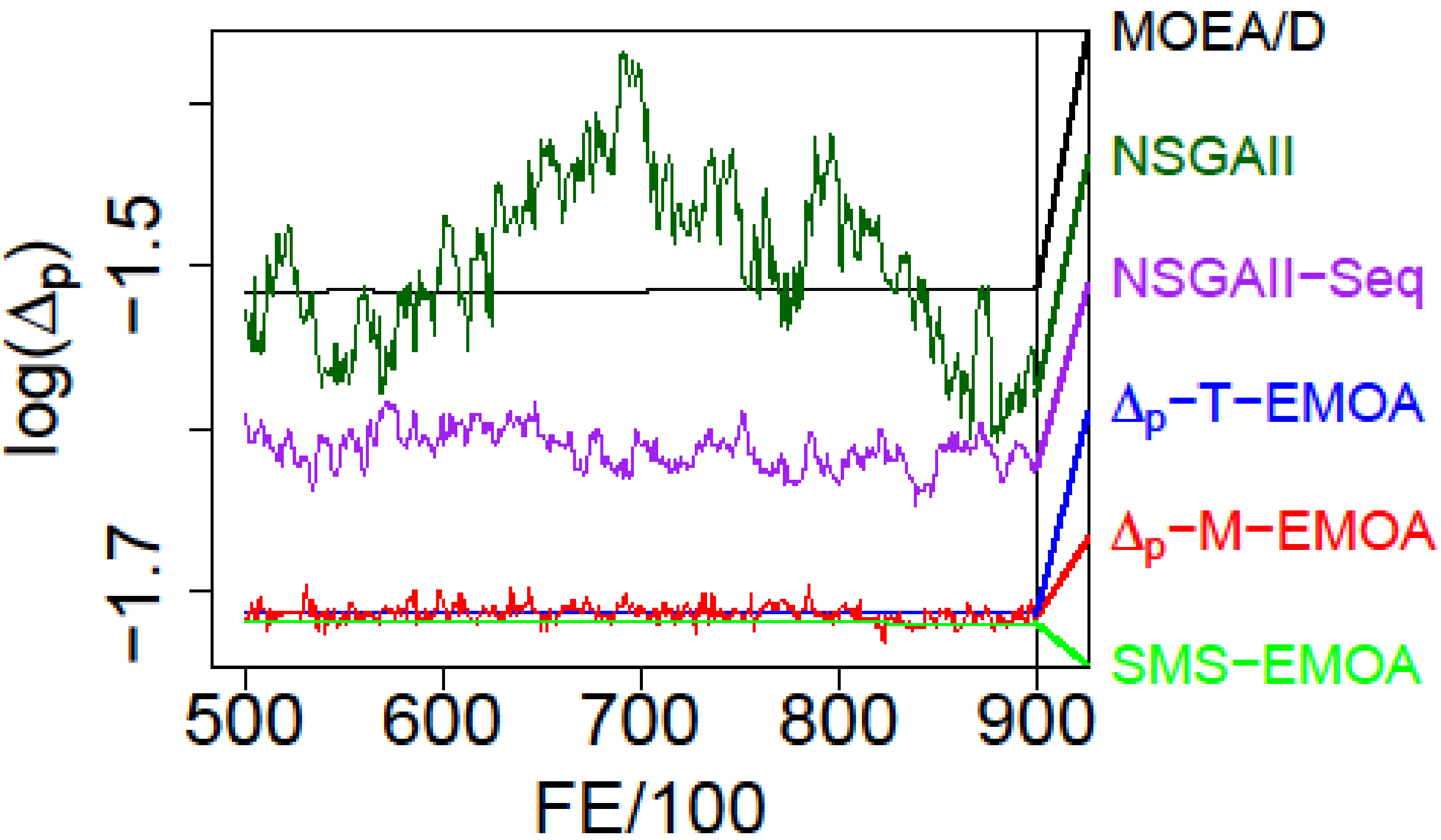
all vertices of new triangles are clustered (here: C-means)

nearest neighbors of cluster centers are target points

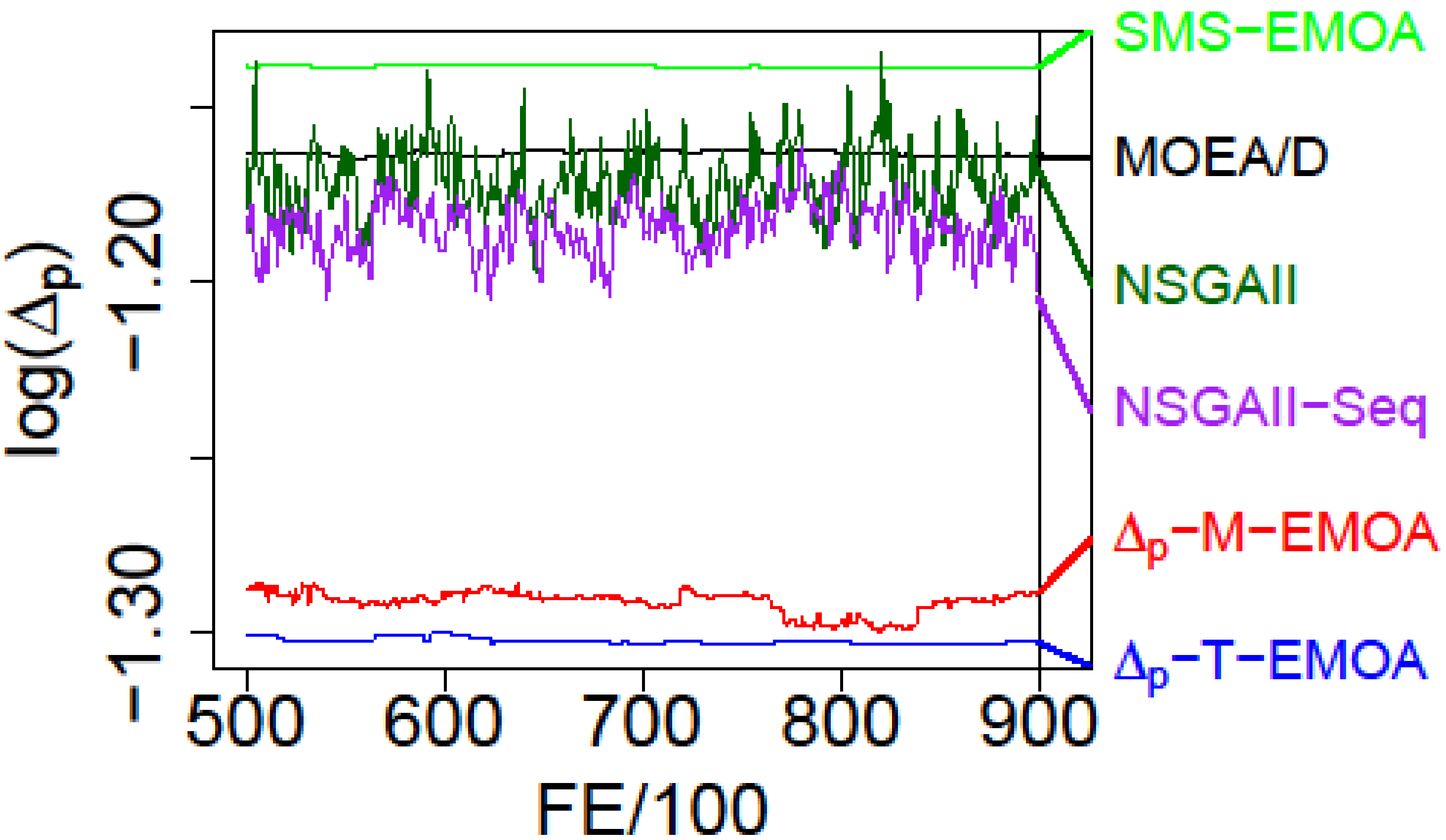
Test Problem: Viennet



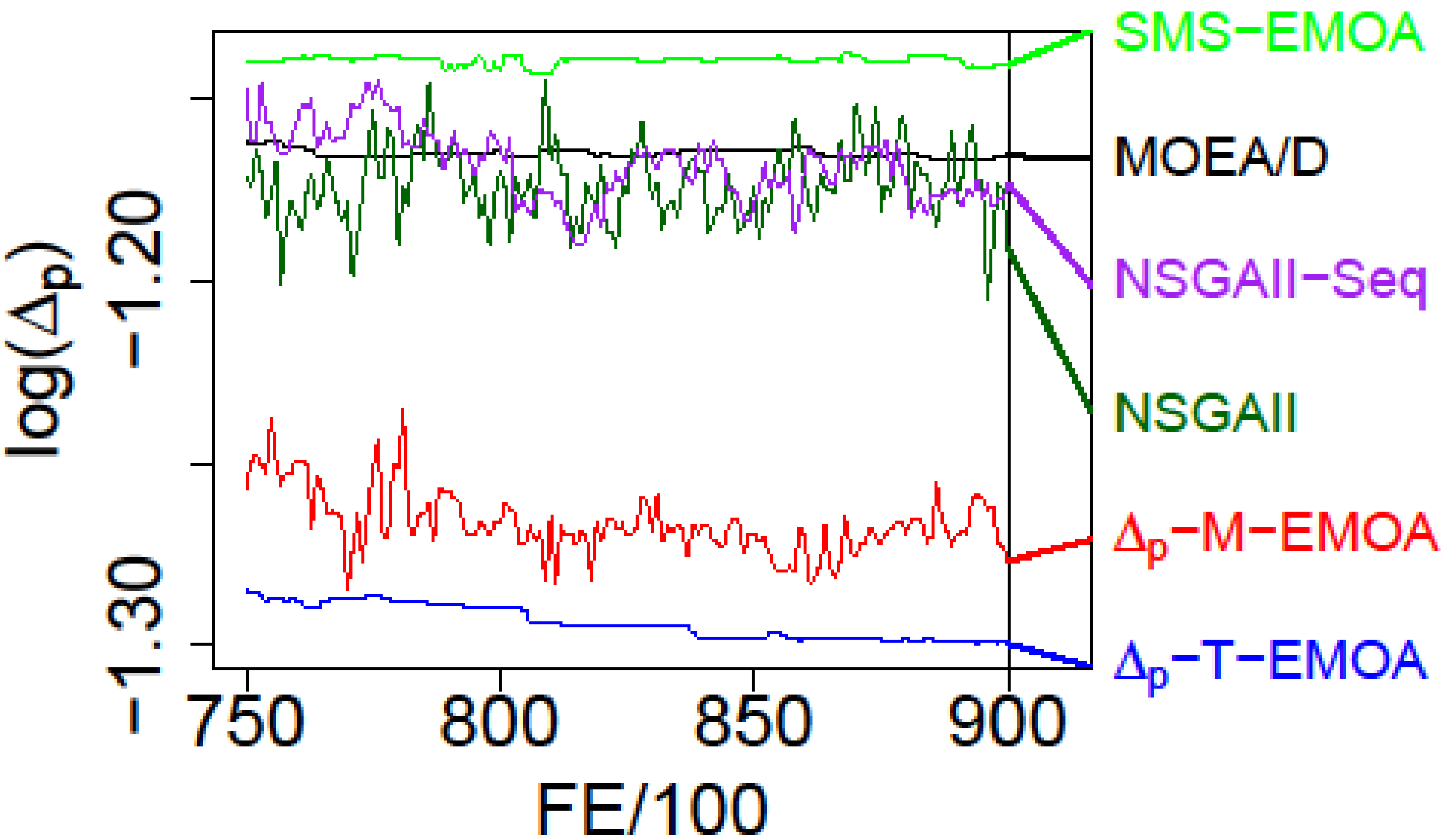
Test Problem: DTLZ1



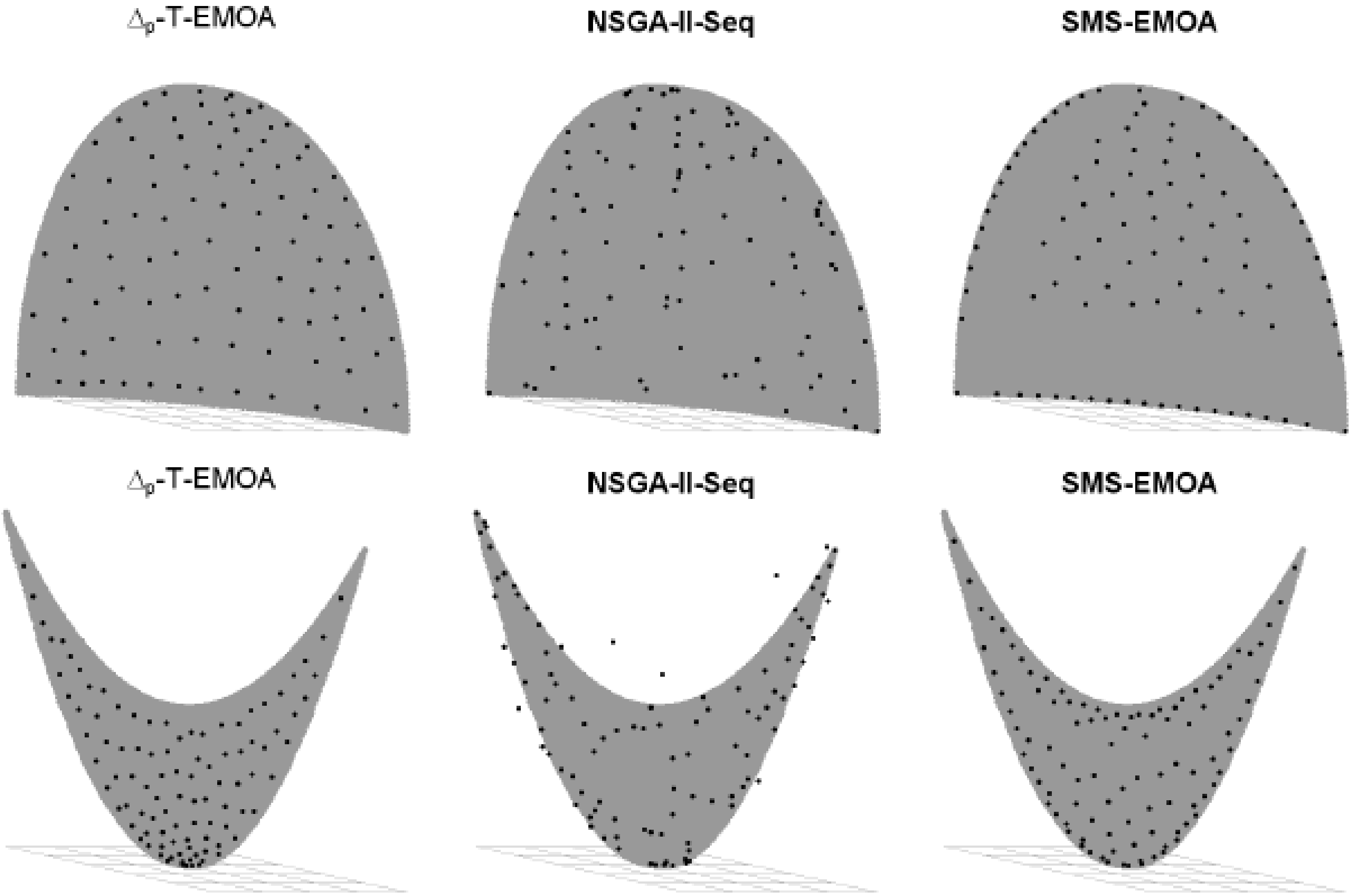
Test Problem: DTLZ2



Test Problem: DTLZ3



Test Problems: DTLZ2 and Viennet



Summary … and open questions

- designed specialized EMOA for evenly spread approximations in d = 3
  - improved efficiency (offline version, fast Δ_p – update)
  - improved quality (better placement of targets by triangulation)

- extension beyond d = 3?
  - what means ‘uniformly spread‘ points?
  - how to place target points ‘uniformly spread‘?
  - how to create hypersurfaces for placing target points?

Work in progress …

Acknowledgements

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