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Multidisciplinary Design Optimization

Decomposition Based Evolutionary Algorithm for Many Objective Optimization with Systematic Sampling and Adaptive Epsilon Control

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- Introduction and Background
- Proposed Method
- Illustrative Example
- Experimental Results
- Conclusion and Future Work

Introduction

- Many objective optimization typically refers to problems with the number of objectives greater than four.
- The commonly used dominance based methods for multi-objective optimization, such as NSGA-II, SPEA2 etc. are known to be inefficient for many-objective optimization as non-dominance does not provide adequate selection pressure to drive the population towards convergence.
- There are also radically different approaches to deal with many objective optimization, such as attempts to identify the reduced set of objectives or corners of the Pareto front. Interactive use of decision makers preferences .
- Use of reference points from systematic sampling or solution of the problem as a hypervolume maximization problem.

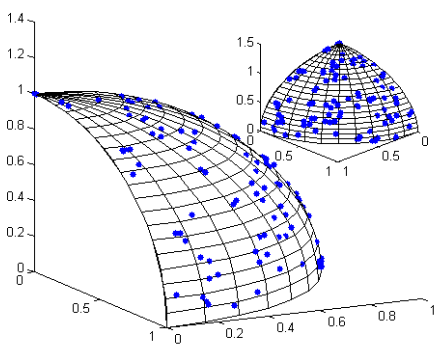

Fig: 1. Using Traditional Approach (NSGA-II)

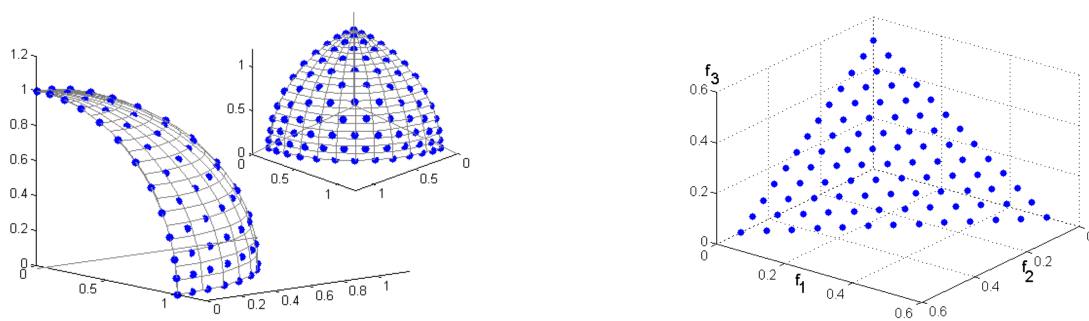

Fig: 2. Using Systematic Sampling

Introduction

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- Decomposition based evolutionary algorithms are yet another class of algorithms originally introduced as MOEA/D

- The multi/many-objective optimization problem is decomposed into a series of scalar optimization problems using different scalarization approaches (i.e. Weighted Sum Approach, Tchebycheff Approach or Normal Boundary Intersection Method)

- In the context of many objective optimization, the first issue relates to the design of a computationally efficient scheme to generate *W* uniform reference directions for a *M objective optimization problem.*

- The second issue related to scalarization has been addressed via two fundamental means i.e. through a systematic association and niche preservation mechanism.

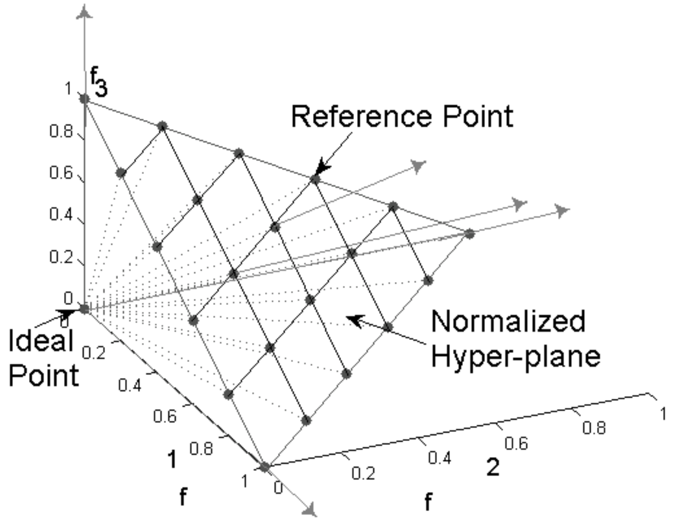


Fig. 3. A set of reference points in a normalized hyper-plane for number of objectives, *M = 3.*

Asafuddoula, A. , **Ray, T.** , Sarker, R. and Alam, K. “An adaptive constraint handling approach embedded MOEA/D,” in Proceedings of the IEEE Congress on Evolutionary Computation, (Brisbane, Australia), pp. 2516-2513, 2012.

Proposed Method

The algorithm consists of four major components –

- generation of reference directions and assignment of neighbourhood
- computation of distances along and perpendicular to each reference direction
- method of recombination using information from neighbouring sub-problems and finally
- adaptive epsilon comparison to manage the balance between convergence and diversity.

Algorithm 1. DBEA-Eps

Input: Gen_{max} maximum number of generations, W the number of reference points

Generate the reference points and assign their neighborhood
Initialize the population P; $|P| = \text{W}$
Evaluate the initial population and compute the ideal point $\bar{z}_j = (f_1^{min}, f_2^{min}, \ldots\ldots, f_M^{min})$ and intercepts a_i's for $i = 1$ to M
Scale the individuals of the population
while $(gen \leq Gen_{max})$ **do**
    for *i=1:W* **do**
        Assign the base parent as P_i
        I=Select a mating partner for (P_i)
        Create a child via recombination as C_i
        Evaluate C_i and compute the distances ($d1$ and $d2$) using all reference directions
        Replace the parent P_k with C_i using *single-first encounter*, where k denotes the index of the first parent satisfying the condition of replacement
        Update the ideal point $(\bar{z})$, the intercepts and re-scale the population
    end for
end while

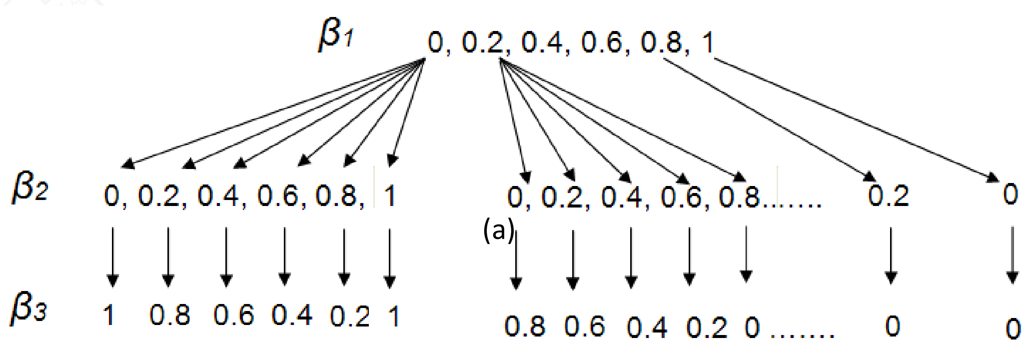
Proposed Method

Generation of reference directions and assignment of neighbourhood

A structured set of reference points (β) is generated spanning a hyper-plane with unit intercepts in each objective axis.

The approach generates *W points on the hyper-plane with a uniform spacing of δ = 1/p for any number* of objectives *M.*

The process of generation of the reference points is illustrated for a 3-objective optimization problem i.e. (*M=3) and with an assumed spacing of δ = 0.2* i.e. (*p = 5) in the Figure below.*

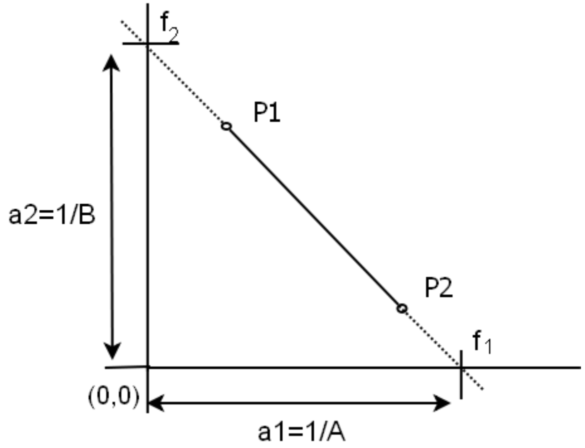


β_1	β_2	$(1-\beta_1-\beta_2)$
0	0	1.0000
0	0.2000	0.8000
0	0.4000	0.6000
0	0.6000	0.4000
0	0.8000	0.2000
0	1.0000	0
0.2000	0	0.8000
0.2000	0.2000	0.6000
0.2000	0.4000	0.4000
0.2000	0.6000	0.2000
0.2000	0.8000	0
0.4000	0	0.6000
0.4000	0.2000	0.4000
0.4000	0.4000	0.2000
0.4000	0.6000	0
0.6000	0	0.4000
0.6000	0.2000	0.2000
0.6000	0.4000	0
0.8000	0	0.2000
0.8000	0.2000	0
1.0000	0	0

Fig. 4. (a) the reference points are generated computing β's recursively (b) the table shows the combination of all β's in each column

Computation of Distances along and Perpendicular to Each Reference Direction

The intercepts of the hyper-plane along the objective axes are denoted by $a_1, a_2, \ldots, a_M$
An example of intercepts computation for a two-objective problem.



Every solution in the population is subsequently scaled as follows:

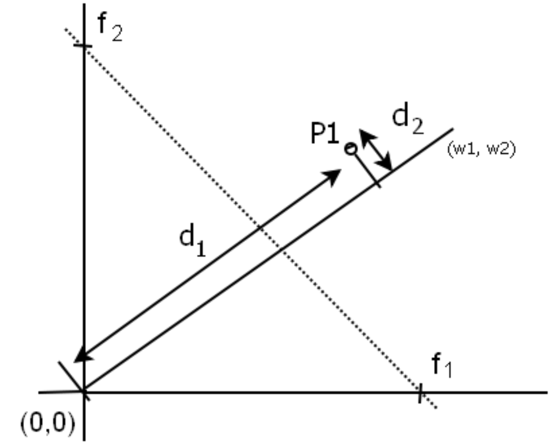
$$f_j^{'}(x) = \frac{f_j(x) - z_j}{a_j - z_j}, \forall j = 1, 2, \ldots M \qquad \textbf{(2)}$$

$z_j = (f_1^{min}, f_2^{min}, \ldots, f_M^{min})$ *represents the ideal point.*

For any given reference direction, the performance of a solution can be judged using two measures d_1 *and* d_2 *as depicted in Figure below.*

The first measure d_1 *is the Euclidean* distance between origin and the foot of the normal drawn from the solution to the reference direction, while the second measure d_2 *is the length of the normal.*

It is clear that a value of $d_2 = 0$ *ensures the solutions are perfectly aligned along the required reference direction* ensuring perfect diversity, while a smaller value of d_1 *indicates superior convergence.*



Distance measures for a point p1 in two objectives

Mating Partner Selection

The mating partner for P_i *(where i is the index of the current* individual in a population) is selected using of the following rules i.e. rule 1: select a parent from the neighbourhood with a probability of τ and rule 2: select a random parent from the population with a probability of (1−τ *).*

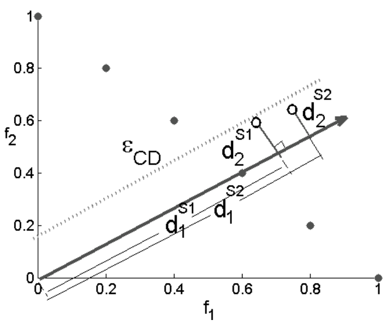
Method of Recombination

In the recombination process, two child solutions are generated using simulated binary crossover (SBX) operator and polynomial mutation. The first child is considered as an individual attempting to replace any parent in the population.

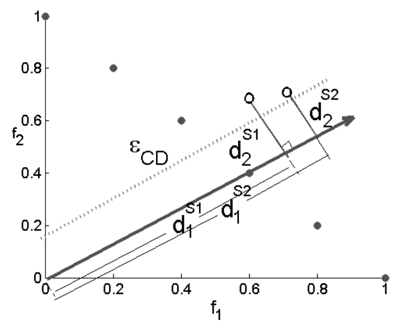
Proposed Method

Adaptive Epsilon Comparison to Manage the Balance between Convergence and Diversity

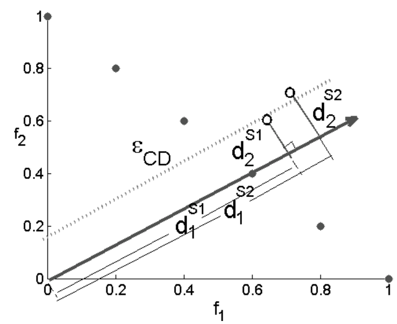
The average deviation ε_{CD} *for* the population of solutions is computed as follows: $\varepsilon_{CD} = \frac{\sum_{i=1}^{W} d_2}{W}$



Case 1: Both the solutions have their d_2 *values less than εCD. One* with the smaller d_1 *is selected i.e.(s1)*



Case 2: Both the d_2 *values are more than εCD.* One with the lower d_2 *value is selected i.e.(s2).*



Case 3: One solution has its d_2 *value* more than *εCD and the other has its* d_2 *value less than εCD(s2 is selected)*

Constraint Handling

The constraint handling approach used in this work is based on epsilon level comparison and has been reported earlier in the following paper.

Asafuddoula, A. , **Ray, T.** , Sarker, R. and Alam, K. “An adaptive constraint handling approach embedded MOEA/D,” in Proceedings of the IEEE Congress on Evolutionary Computation, (Brisbane, Australia), pp. 2516-2513, 2012.

The feasibility ratio (*FR) of a population refers to* the ratio of the number of feasible solutions in the population to the number of solutions (*W). The allowable violation is calculated as follows:*

$$CV = \sum_{i=1}^{p} max(g_i, 0) + \sum_{i=1}^{q} max(|h_i - \varepsilon|, 0)$$

$$CV_{mean} = \frac{1}{W} \sum_{j=1}^{W} (CV_j)$$

$$\text{Allowable violation}(\varepsilon_{CV}) = CV_{mean} * FR$$

If two solutions have their constraint violation value less than this epsilon level, the solutions are compared based on their objective values i.e. via d_1 *and* d_2 *measures.*

An Illustrative Example

- In order to observe the process of evolution, we computed the average performance of the population i.e. average of the *d_1 and d_2 values for the individuals for DTLZ1 (3* objectives)

- One can observe from Fig. 8, that the average *d_2 converges to near zero* (i.e. near perfect alignment to the reference directions) while the average *d_1 measure* stabilizes at around 0.8 in the normalized plane indicating convergence to the Pareto front

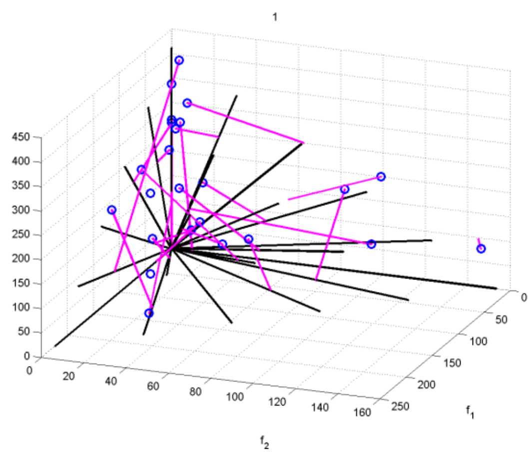


Fig:7. Evolving the best solutions with minimum d_1 and d_2 distances

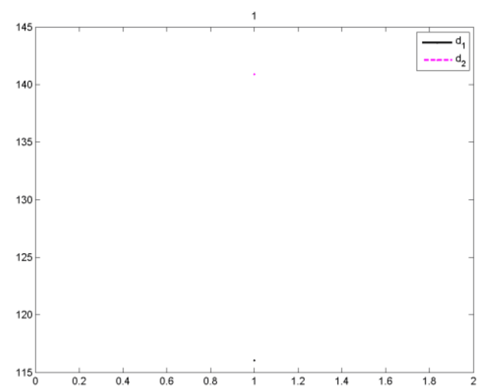


Fig:8. Converging the d_1 and d_2 measures over generations

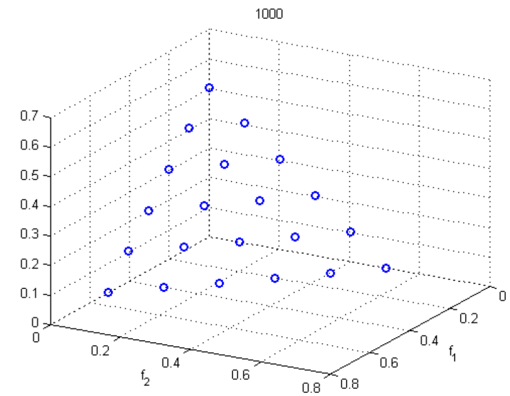
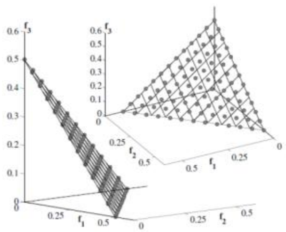


Fig:9. Final non-dominated solutions achieved for DTLZ1 problem

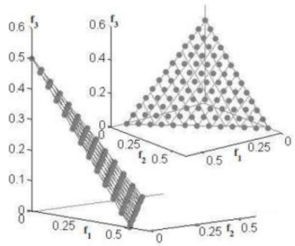
Performance on DTLZ Problems

In this comparison, we have reported the best, median and worst IGD results obtained using 20 independent runs for DTLZ1 and DTLZ2. The results are compared against M-NSGA-II and MOEA/D-PBI.

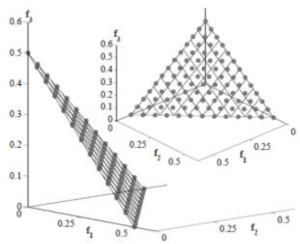
One can observe that our algorithm obtained the best IGD values in 8 instances out of 10



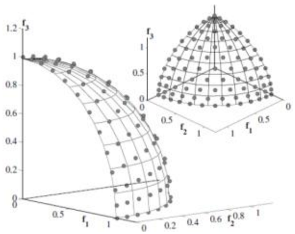
DBEA-Eps



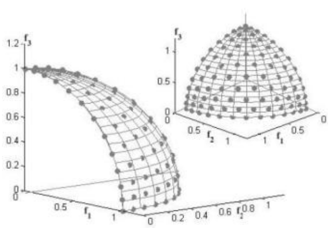
MOEA/D-PBI



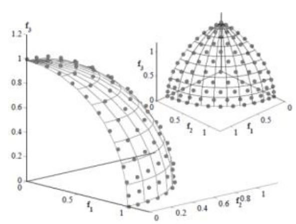
M-NSGA-II



DBEA-Eps



MOEA/D-PBI



M-NSGA-II

Test Problem	Obj.	MaxGen	Strategy	Best	Median	Worst
DTLZ1	3	400	DBEA-Eps	**8.771e-5**	9.521e-3	5.854e-1
			M-NSGA-II	4.880e-4	1.308e-3	4.880e-3
			MOEA/D-PBI	4.095e-4	**1.495e-3**	**4.743e-3**
DTLZ1	5	600	DBEA-Eps	**1.771e-5**	**2.183e-4**	3.782e-1
			M-NSGA-II	5.116e-4	9.799e-4	1.979e-3
			MOEA/D-PBI	3.179e-4	6.372e-4	**1.635e-3**
DTLZ1	8	750	DBEA-Eps	**4.387e-5**	**3.581e-4**	**1.981e-3**
			M-NSGA-II	2.044e-3	3.979e-3	8.721e-3
			MOEA/D-PBI	3.914e-3	6.106e-3	8.537e-3
DTLZ1	10	1000	DBEA-Eps	**7.691e-4**	**1.504e-3**	**2.700e-3**
			M-NSGA-II	2.215e-3	3.462e-3	6.869e-3
			MOEA/D-PBI	3.872e-3	5.073e-3	6.130e-3
DTLZ1	15	1500	DBEA-Eps	**1.696e-3**	**2.606e-3**	**2.686e-3**
			M-NSGA-II	2.649e-3	5.063e-3	1.123e-2
			MOEA/D-PBI	1.236e-2	1.431e-2	1.692e-2
DTLZ2	3	250	DBEA-Eps	2.040e-2	4.138e-2	6.417e-2
			M-NSGA-II	1.262e-3	1.357e-3	2.114e-3
			MOEA/D-PBI	**5.432e-4**	**6.406e-4**	**8.006e-4**
DTLZ2	5	350	DBEA-Eps	**1.199e-3**	3.024e-3	2.272e-2
			M-NSGA-II	4.254e-3	4.982e-3	5.862e-3
			MOEA/D-PBI	1.219e-3	**1.437e-3**	**1.727e-3**
DTLZ2	8	500	DBEA-Eps	**1.172e-3**	**2.899e-3**	6.915e-3
			M-NSGA-II	1.371e-2	1.571e-2	1.811e-2
			MOEA/D-PBI	3.097e-3	3.763e-3	**5.198e-3**
DTLZ2	10	750	DBEA-Eps	3.656e-3	3.657e-3	3.657e-3
			M-NSGA-II	1.350e-2	1.528e-2	1.697e-2
			MOEA/D-PBI	**2.474e-3**	**2.778e-3**	**3.235e-3**
DTLZ2	15	1000	DBEA-Eps	**5.160e-3**	**5.960e-3**	**5.960e-3**
			M-NSGA-II	1.360e-2	1.726e-2	2.114e-2
			MOEA/D-PBI	5.254e-3	6.005e-3	9.409e-3

- Results using systematic sampling for DTLZ1 and DTLZ2 problems for all algorithms

Constrained Engineering Design Problems

- **Car Side Impact Problem**

The problem aims to minimize the weight of a car, the pubic force experienced by a passenger and the average velocity of the V-Pillar responsible for bearing the impact load subject to the constraints involving limiting values of abdomen load, pubic force, velocity of V-Pillar, rib deflection etc

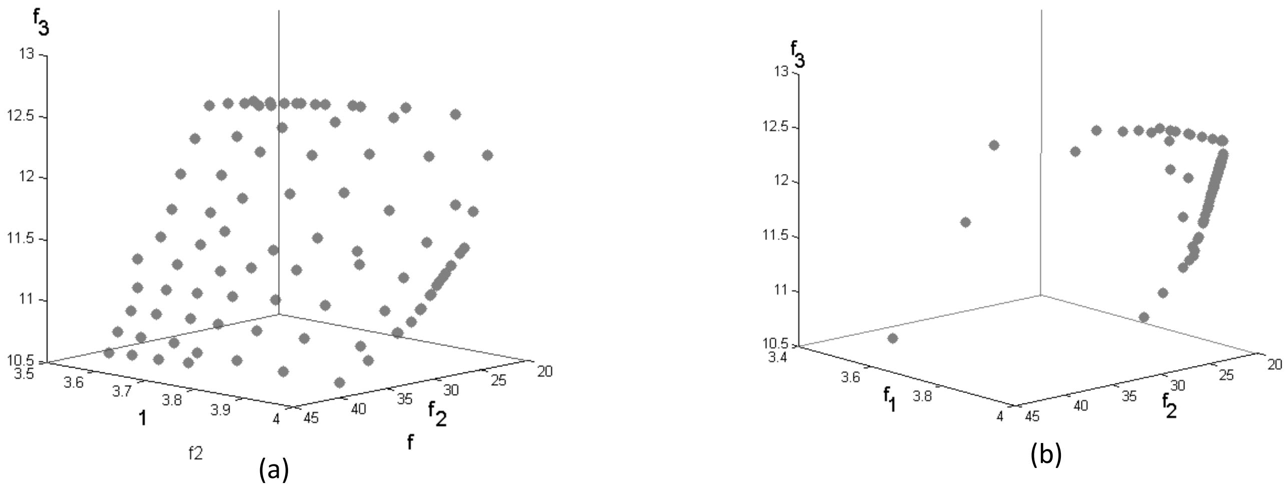


Fig. 10. Solutions obtained using (a) DBEA-Eps (b) MOEA/D-PBI on three-objective car side impact problem

The algorithms are run for 500 generations and the final non-dominated front is shown in Figure. It is important to note that the results of MOEA/D-PBI is derived without scaling which could be a reason among others for poor performance.

Experimental Results

- **Water Resource Management Problem**

This is a five objective problem having seven constraints taken from the literature. The parallel coordinate plot generate using our proposed algorithm (DBEA-Eps) is presented in Fig. 11.

The best IGD value across 20 runs is 3*.29e−2 and the IGD is computed* using the reference set of 2429 solutions

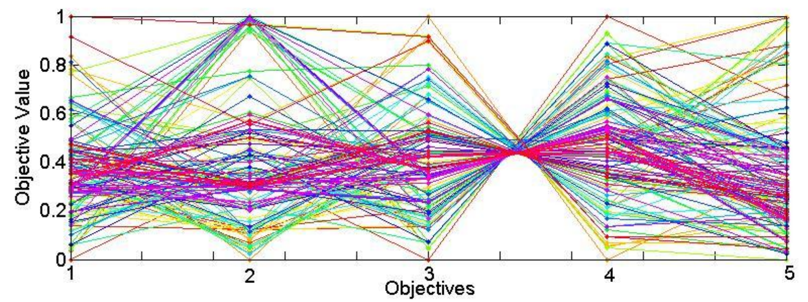


Fig. 11. Solutions obtained using DBEA-Eps on five-objective water problem

- A scatter plot-matrix is presented. The results from the DBEA-Eps are shown in the top-right plots vis-a-vis the known reference set of 2429 solutions (shown in bottom-left plots).

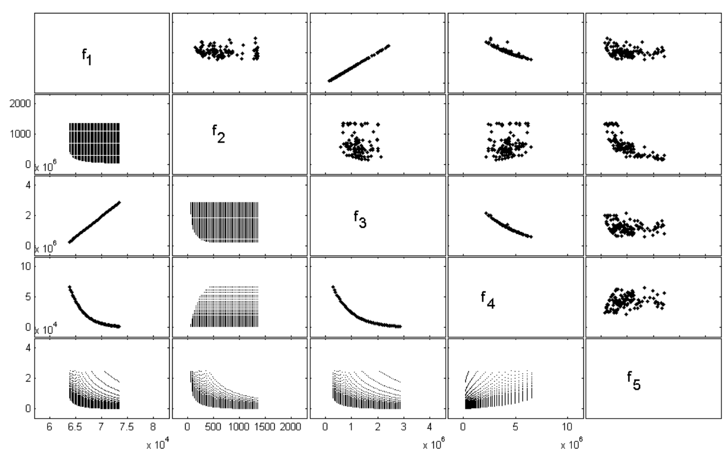


Fig. 12. A scatter plotmatrix showing DBEA-Eps (top-right plots) vis-a-vis the known reference set of 2429 solutions (bottom-left plots)

- **General Aviation Aircraft (GAA) Design Problem**

This problem was first introduced by Simpson. The problem involves 9 design variables i.e. cruise speed, aspect ratio, sweep angle, propeller diameter, wing loading, engine activity factor, seat width, tail length/ diameter ratio and taper ratio and the aim is to minimize the takeoff noise, empty weight, direct operating cost, ride roughness, fuel weight, purchase price, product family dissimilarity and maximize the flight range, lift/ drag ratio and cruise speed

In this example, we have used 100 reference points in the population and was allowed to evolve over 5000 generations.

Table 2. Performance metric value of product family design problem using 50 independent runs

Algorithm	Function Evaluation	IGD			
		Best	Mean	Worst	Std
DBEA-Eps	50,000	0.62070	0.80123	0.82430	0.09210
ε-MOEA		0.98312	0.99123	0.99678	0.10312
Borg-MOEA		0.98211	0.99113	0.99337	0.02321
MOEA/D		0.99117	0.99587	0.99723	0.02145
ε-NSGA-II		0.98571	0.98872	0.99131	0.72123

A reference set of 412 non-dominated solutions obtained from ε-MOEA and Borg-MOEA are used to compute the IGD metric.

- **General Aviation Aircraft (GAA) Design Problem**

The performance of the algorithms is compared using the hyper-volume in Table 2 and IGD in Table 3. One can observe that the proposed algorithm performs marginally better than others for this problem.

Table 3. Performance metric value of product family design problem using 50 independent runs

Algorithm	Function Evaluation	Hypervolume			
		Best	Mean	Worst	Std
DBEA-Eps	50,000	0.02899	0.01715	0.00689	0.04561
ε-MOEA		0.02032	0.01032	0.00259	0.04125
Borg-MOEA		0.02245	0.01013	0.00424	0.02327
MOEA/D		0.00092	0.00087	0.00045	0.00145
ε-NSGA-II		0.01636	0.01005	0.00236	0.05232

The figure shows that DBEA-Eps is able to find a widely distributed set of non-dominated points for 10-objective GAA design problem

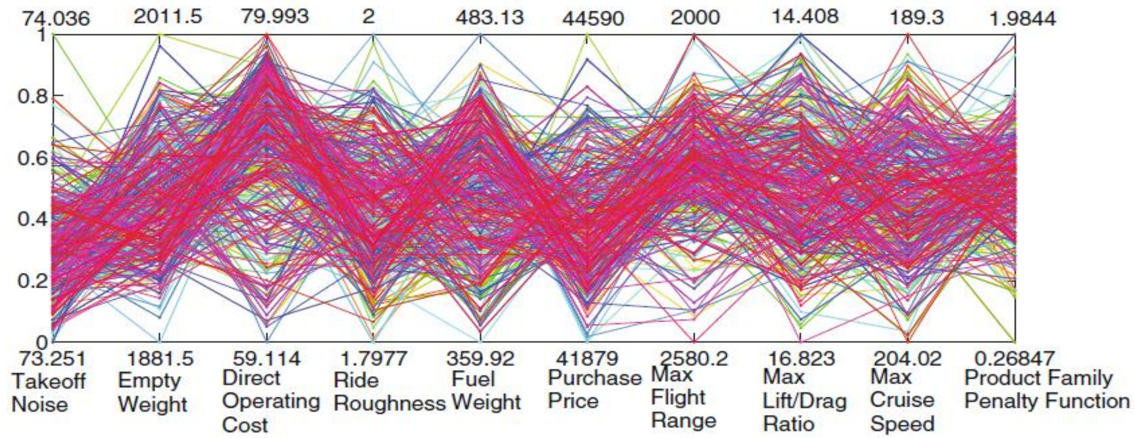


Fig. 11. Parallel coordinate plot of the approximation of Pareto set produced by DBEA-Eps is in a different color trace

Conclusions

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- In this paper, a decomposition based evolutionary algorithm with adaptive epsilon comparison is introduced to solve unconstrained and constrained many objective optimization problems.

- The approach utilizes reference directions to guide the search, wherein the reference directions are generated using a systematic sampling scheme.

- In an attempt to alleviate the problems associated with scalarization (commonly encountered in the context of reference direction based methods), the balance between diversity and convergence is maintained using an adaptive epsilon comparison.

- In order to deal with constraints, an epsilon level comparison is used which is known to be more effective than methods employing *feasibility first principles.*

- Three constrained engineering design optimization problems with three to seven constraints (car side impact, water resource management and a general aviation aircraft design problem) have been solved to illustrate the performance of the proposed algorithm.

- The preliminary results indicate that the proposed algorithm is able to deal with unconstrained and constrained many objective optimization problems better or at par with existing state of the art algorithms such as M-NSGA-II and MOEA/D-PBI.

Ongoing Efforts and Challenges

Algorithmic:

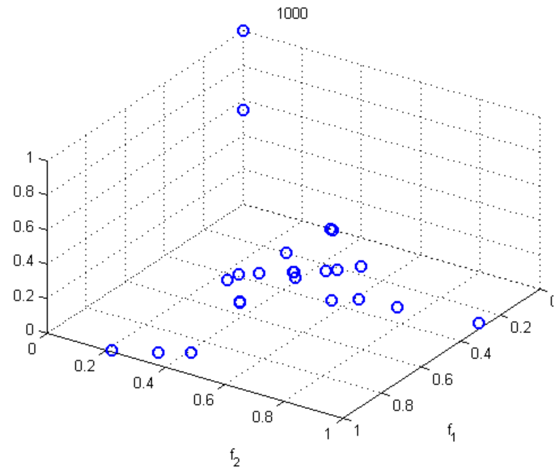
- Moved into DE based scheme, still a steady state.
- Getting rid of neighbourhood parameter.
- Reference directions and population size separated
- Exploring alternative quantum representations.

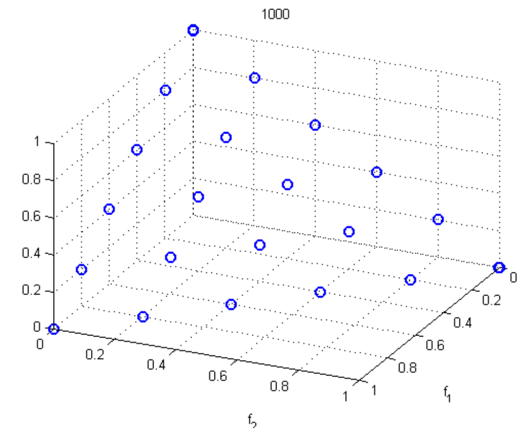
Challenges

- Sensitive to intercept computation.
- Modifying/Addition and deletion of reference directions on the fly.
- Detection of redundancy via d2 measures.
- Performance on degenerate fronts.
- Quantum Gate Operators other than simplistic rotation.
- Control on the distribution of points for degenerate fronts.

Further Improvements

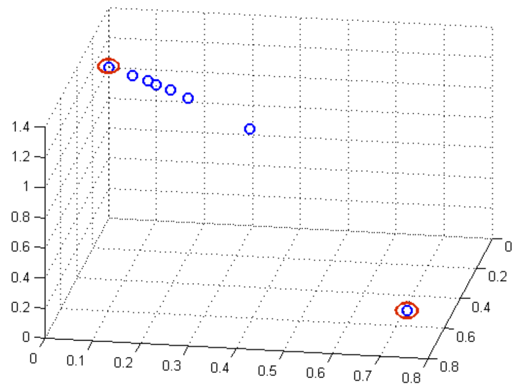
Although epsilon level is adaptive, there are runs where solutions converge to specific areas.

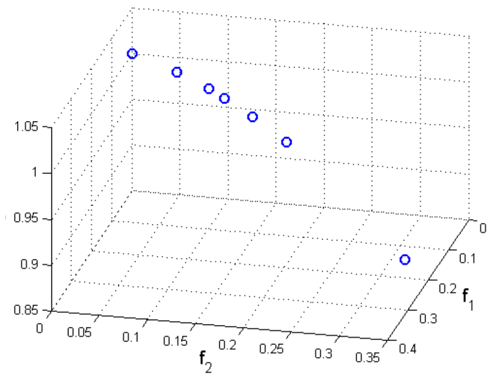




A hard d2 only rule, seems to work better(our preliminary observations). Improved solutions for the same problem DTLZ1 on 3 objectives with same number of function evaluations

Improved Pareto front for DTLZ5(5,2) problem by preserving the corner solutions externally





Singh, H.K. ,Isaacs, A. , and **Ray, T.** , "A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems," IEEE Transactions on Evolutionary Computation, vol. 15, issue 4, pp. 539–556 2011.