Force-based Cooperative Search Directions in Evolutionary Multi-objective Optimization

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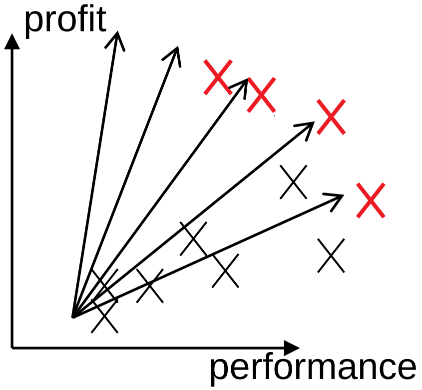
INVENTORS FOR THE DIGITAL WORLD

Multiobjective Optimization Scenario

Three main approaches in EMO:

- classical dominance-based algorithms: NSGA-II, SPEA2, ...
- indicator-based algorithms: IBEA, AGE, HypE, ...
- scalarization-based algorithms: MSOPS, MOEA/D, ...

Scalarization approaches:



- solve several scalarized problems simultaneously
- #scalarizations = #solutions desired

Problems:

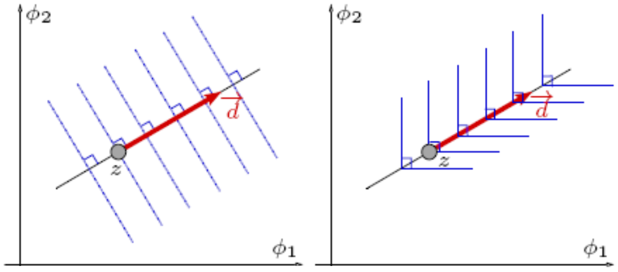
- defining search directions a priori is ~~difficult~~ problem-dependent
- given a direction in objective space, finding good scalarizations in terms of a direction in decision space is non-trivial

at least for comb. problems

Goal: adapting search directions cooperatively during search

Main Idea of Force-Based Scalarization

μ scalarization functions = μ x (1+λ)-EA



$$\mathcal{W}(x) = \sum_{m=1}^{M} d_m^i \cdot \phi_m(x)$$

$$\mathcal{A}(x) = \max_{m\in\{1,\dots,M\}} \{w_m^i \cdot (z_m^i - \phi_m(x))\}$$

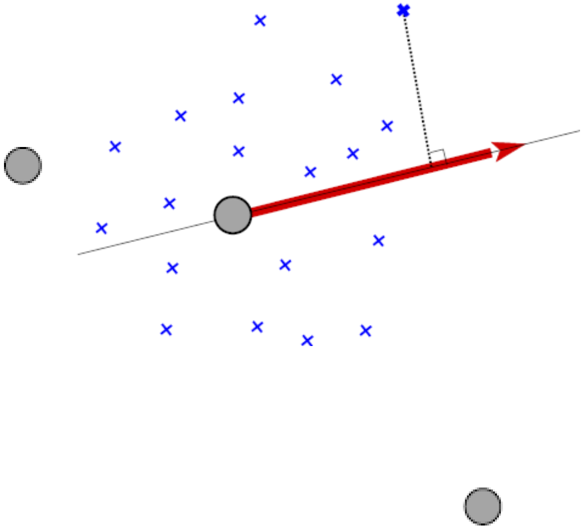
adaptation of search directions inspired by Newton's laws of motion, especially F = -ma

in each iteration:

❶ compute force of each particle based on positions of others

e.g. $\vec{f}_j^{\,i} = \dfrac{z^i - z^j}{\|z^i - z^j\|^\alpha}$

$$\vec{d}^{\,i} = \sum_{j\in\{1,\dots,n\}\setminus\{i\}} \vec{f}_j^{\,i}$$



❷ generate λ offspring from each particle

❸ update particle to best in current direction

Related Work

Nothing is totally new:

- adapting weights in MOEA/D, e.g. [Jiang et al. 2011]
  - assumption on estimated Pareto front: $\sum_{i=1}^{M} f_i^p = 1$
- force-based approach in PSO and other algorithms [see paper]
  - but typically in decision space

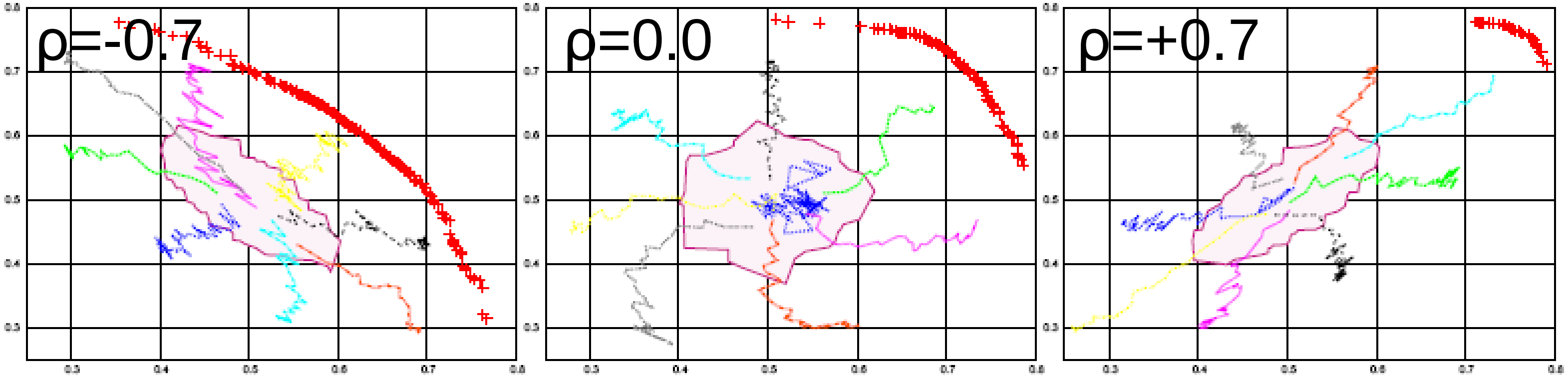
Here: a force-based algorithm
adapting search directions in objective space during search

- quite simple
- easy to implement
- in principle independent of search space
- (quite) efficient on ρMNK landscapes (compared with a (μ+λ)-SMS-EMOA)

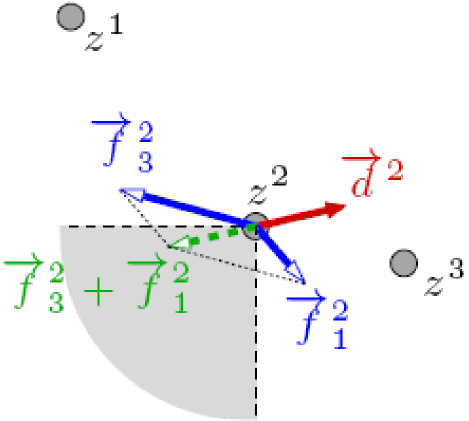
The Naive Idea

Simple repelling forces do not allow to optimize all particles:

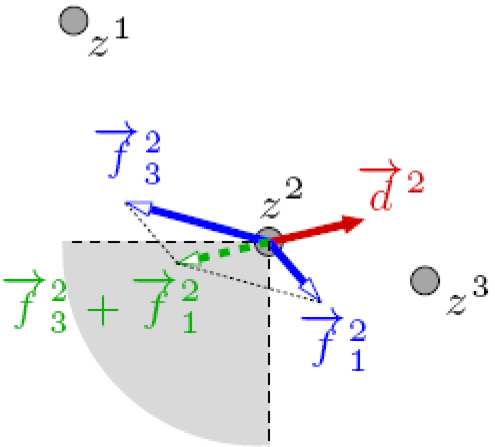
$$\vec{f}^{\,i}_{j} = \frac{z^i - z^j}{\|z^i - z^j\|^{\alpha}} \qquad \vec{d}^{\,i} = \sum_{j\in\{1,\ldots,n\}\setminus\{i\}} \vec{f}^{\,i}_{j}$$



...because it only maximizes the distances among the particles

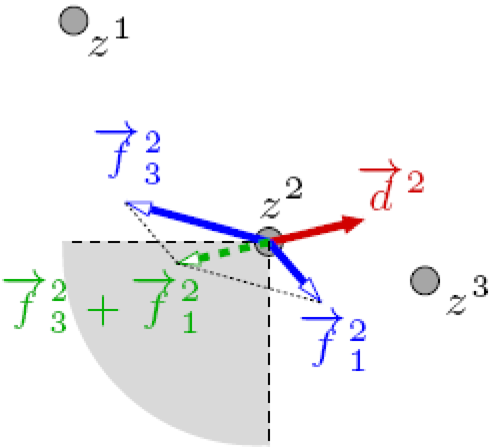


Different Strategies to Incorporate Dominance



no backwards directions

Different Strategies to Incorporate Dominance



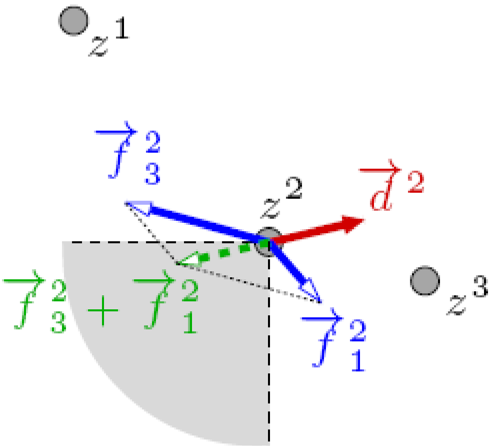
no backwards directions

$$\xcancel{\vec{f}\,^i_j = \frac{z^i - z^j}{\|z^i - z^j\|^\alpha}}$$

$$\vec{f}\,^i_j = \begin{cases} \dfrac{z^j - z^i}{\|z^i - z^j\|^\alpha} & \text{if } x^i \prec x^j \\ \dfrac{z^i - z^j}{\|z^i - z^j\|^\alpha} & \text{otherwise} \end{cases}$$

dominating particles attract

Different Strategies to Incorporate Dominance



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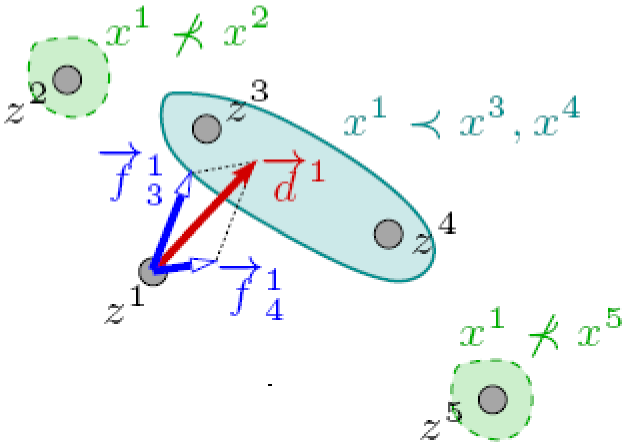
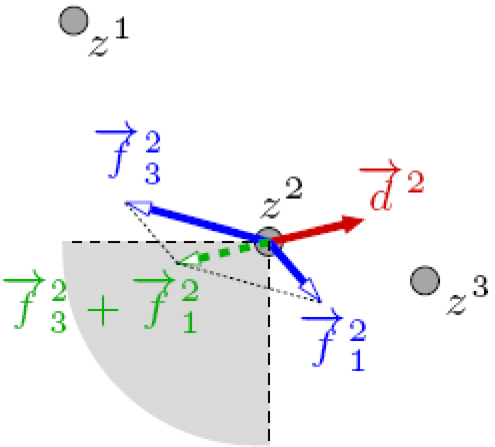


if dominated, non-dominated particles play no role

Different Strategies to Incorporate Dominance



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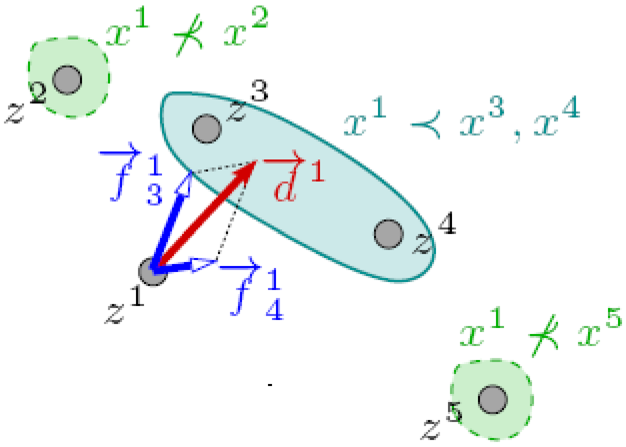
dominating particles attract



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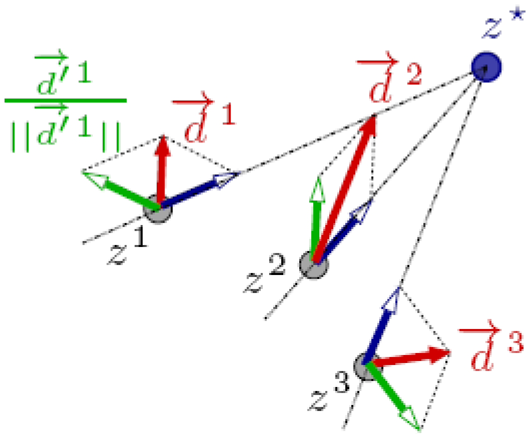
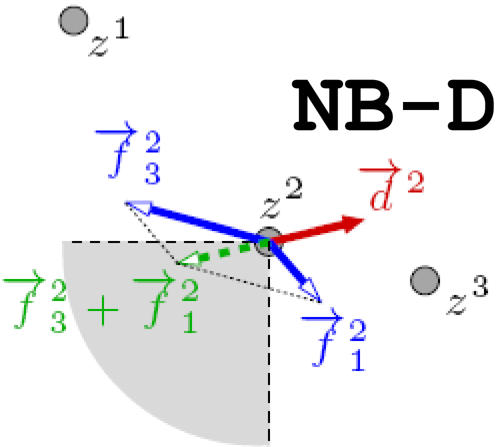


blackhole attracts as well

Different Strategies to Incorporate Dominance

NB-D



no backwards directions

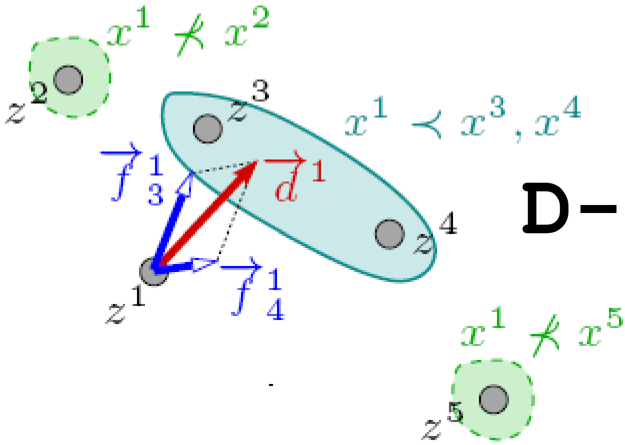
RA-D

$$\xcancel{\vec{f}^{\,i}_{j} = \frac{z^i - z^j}{\|z^i - z^j\|^\alpha}}$$

$$\vec{f}^{\,i}_{j} = \begin{cases} \dfrac{z^j - z^i}{\|z^i - z^j\|^\alpha} & \text{if } x^i \prec x^j \\ \dfrac{z^i - z^j}{\|z^i - z^j\|^\alpha} & \text{otherwise} \end{cases}$$

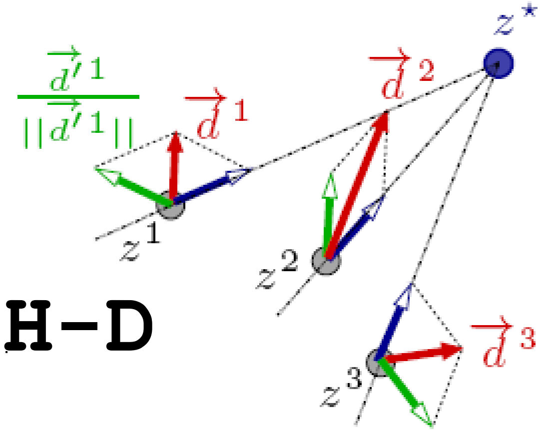
dominating particles attract

D-D



if dominated, non-dominated particles play no role

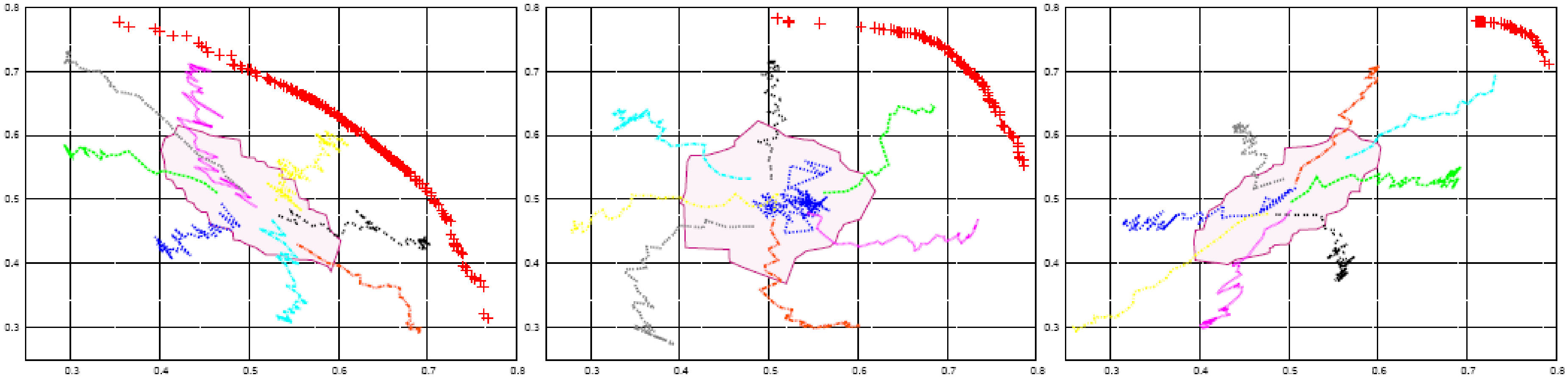
BH-D



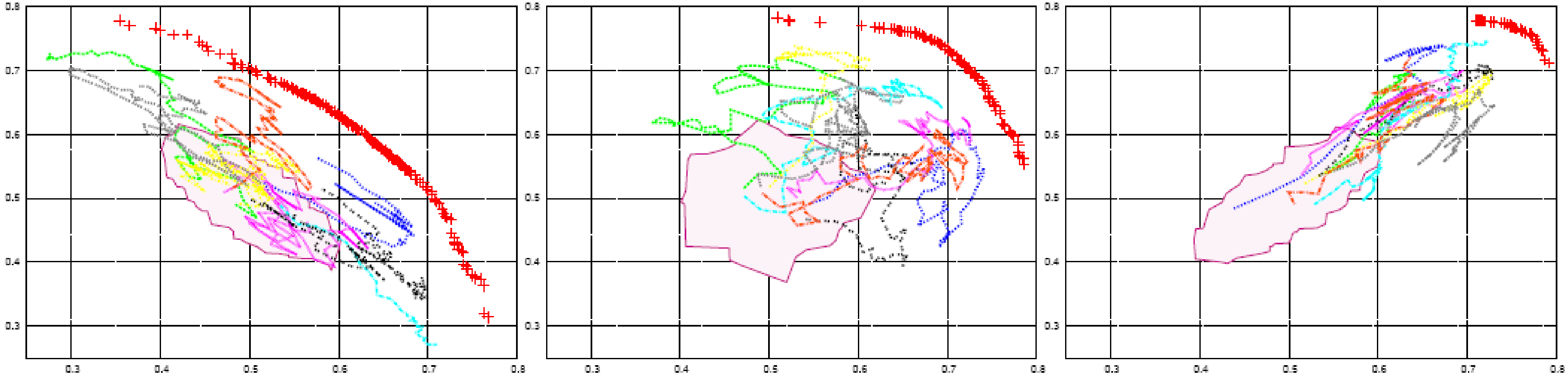
blackhole attracts as well

Repelling and Attracting Forces

only repelling forces

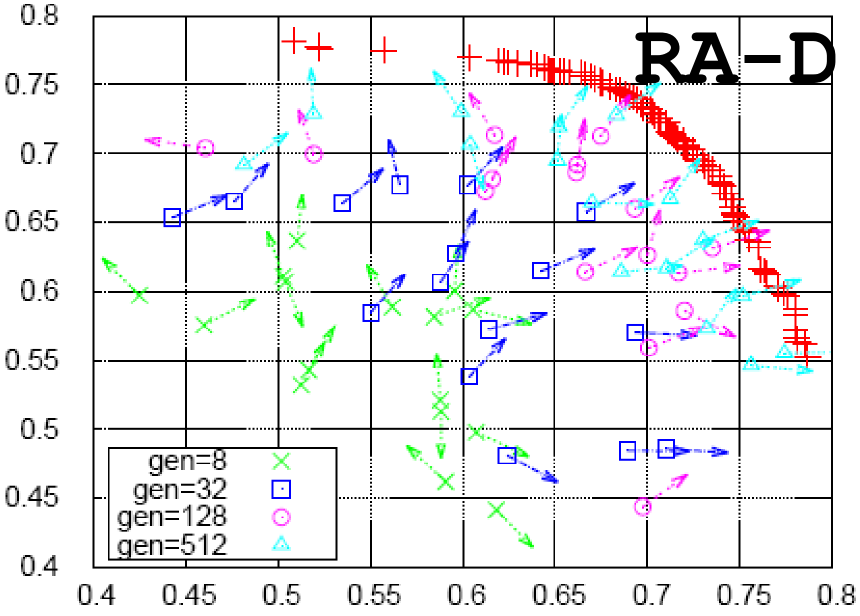


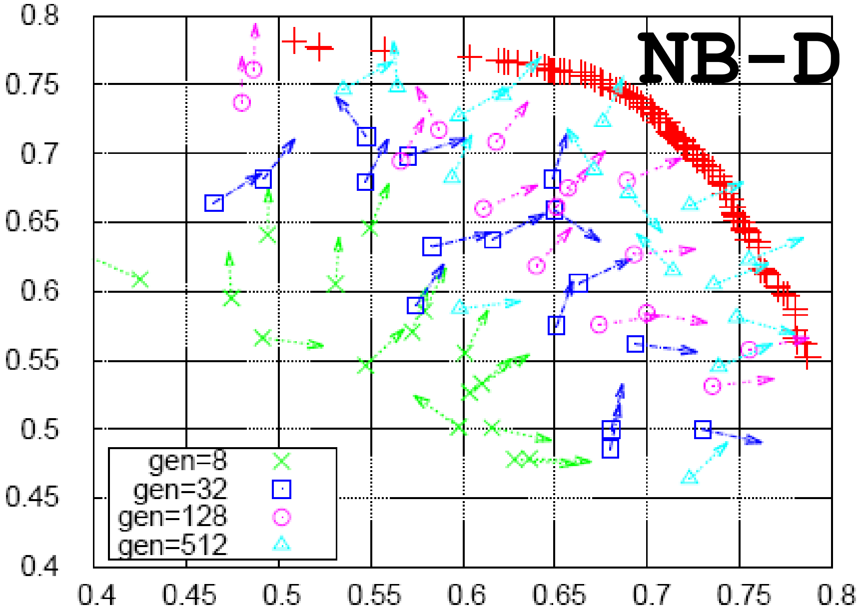
ρ=-0.7 ρ=0.0 ρ=+0.7

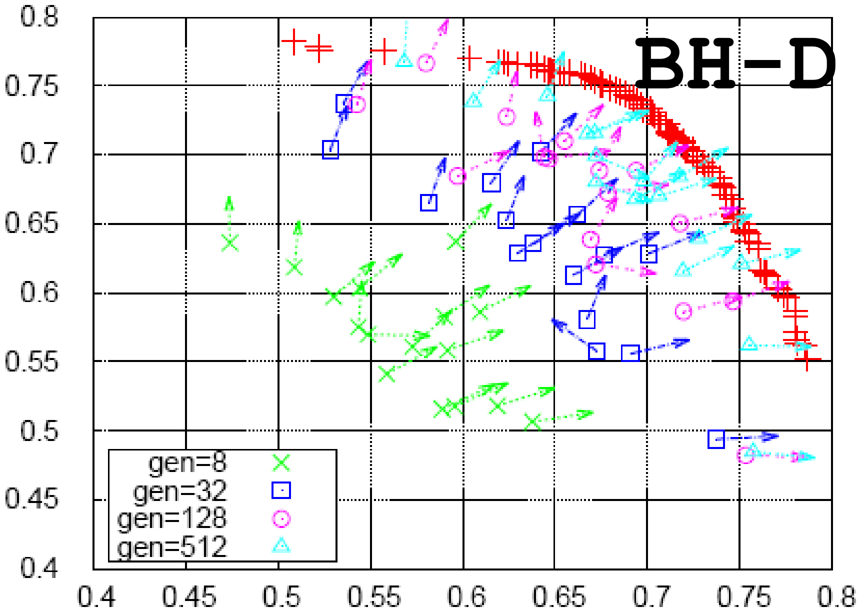


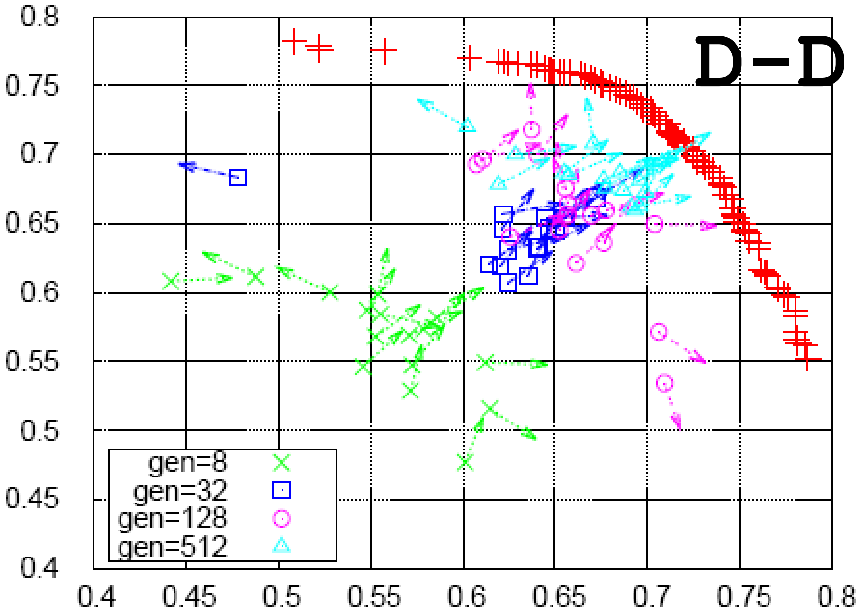
RA-D

Qualitative Differences Between the Strategies









Quantitative Comparison

5 strategies: **`RA-D`**, **`BH-D`**, **`D-D`**, **`NB-D`** and **`I-D`**

weighted sum vs. Chebyshev scalarization

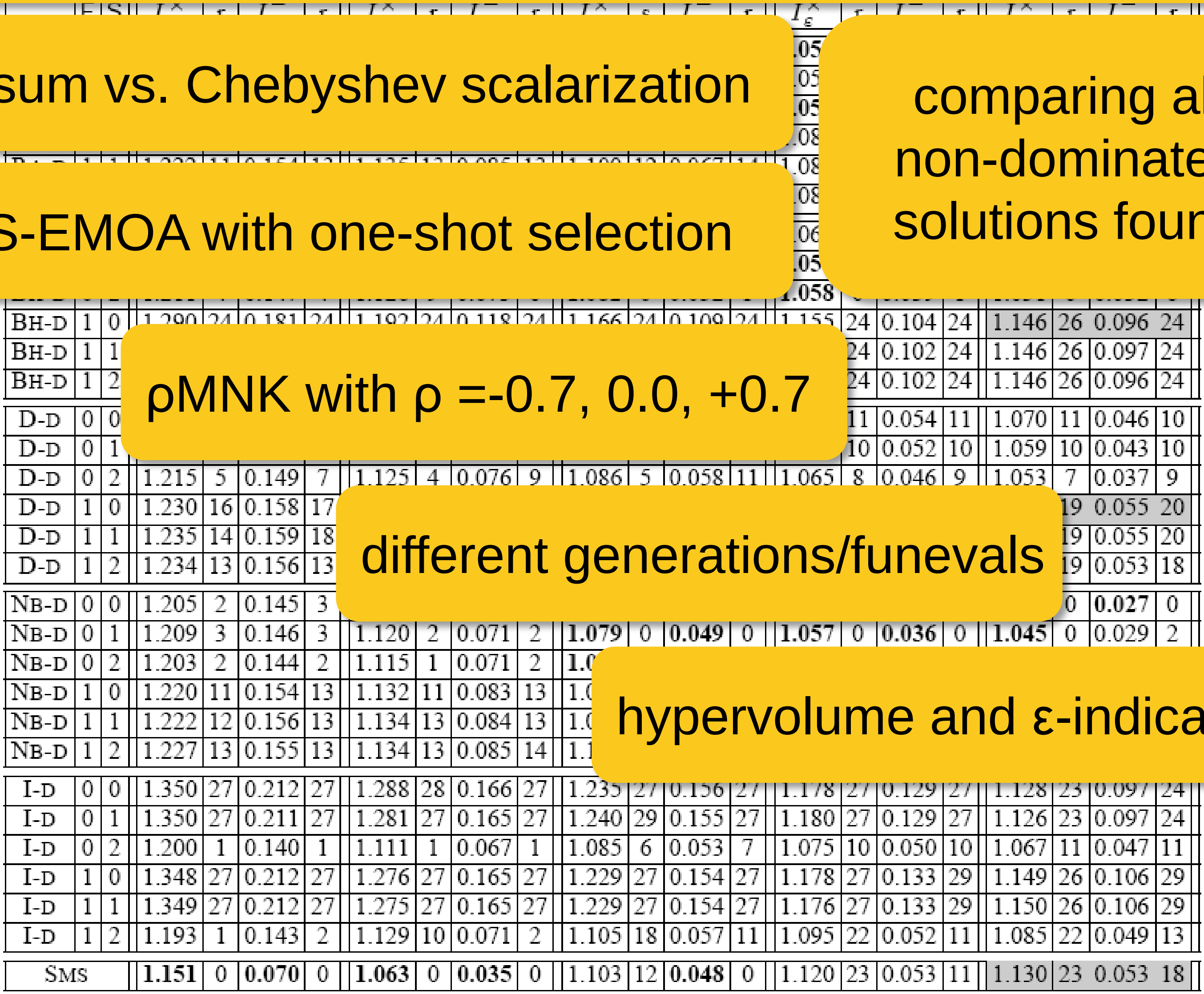
comparing all non-dominated solutions found

(μ+λ)-SMS-EMOA with one-shot selection

ρMNK with ρ =-0.7, 0.0, +0.7

different generations/funevals

hypervolume and ε-indicator



	E	S													$I_\varepsilon^\times$							
BH-D	1	0	1.290	24	0.181	24	1.192	24	0.118	24	1.166	24	0.109	24	1.155	24	0.104	24	1.146	26	0.096	24
BH-D	1	1														24	0.102	24	1.146	26	0.097	24
BH-D	1	2														24	0.102	24	1.146	26	0.096	24
D-D	0	0														11	0.054	11	1.070	11	0.046	10
D-D	0	1														10	0.052	10	1.059	10	0.043	10
D-D	0	2	1.215	5	0.149	7	1.125	4	0.076	9	1.086	5	0.058	11	1.065	8	0.046	9	1.053	7	0.037	9
D-D	1	0	1.230	16	0.158	17														19	0.055	20
D-D	1	1	1.235	14	0.159	18														19	0.055	20
D-D	1	2	1.234	13	0.156	13														19	0.053	18
NB-D	0	0	1.205	2	0.145	3														0	**0.027**	0
NB-D	0	1	1.209	3	0.146	3	1.120	2	0.071	2	**1.079**	0	**0.049**	0	**1.057**	0	**0.036**	0	**1.045**	0	0.029	2
NB-D	0	2	1.203	2	0.144	2	1.115	1	0.071	2												
NB-D	1	0	1.220	11	0.154	13	1.132	11	0.083	13												
NB-D	1	1	1.222	12	0.156	13	1.134	13	0.084	13												
NB-D	1	2	1.227	13	0.155	13	1.134	13	0.085	14												
I-D	0	0	1.350	27	0.212	27	1.288	28	0.166	27	1.235	27	0.156	27	1.178	27	0.129	27	1.128	23	0.097	24
I-D	0	1	1.350	27	0.211	27	1.281	27	0.165	27	1.240	29	0.155	27	1.180	27	0.129	27	1.126	23	0.097	24
I-D	0	2	1.200	1	0.140	1	1.111	1	0.067	1	1.085	6	0.053	7	1.075	10	0.050	10	1.067	11	0.047	11
I-D	1	0	1.348	27	0.212	27	1.276	27	0.165	27	1.229	27	0.154	27	1.178	27	0.133	29	1.149	26	0.106	29
I-D	1	1	1.349	27	0.212	27	1.275	27	0.165	27	1.229	27	0.154	27	1.176	27	0.133	29	1.150	26	0.106	29
I-D	1	2	1.193	1	0.143	2	1.129	10	0.071	2	1.105	18	0.057	11	1.095	22	0.052	11	1.085	22	0.049	13
SMS			**1.151**	0	**0.070**	0	**1.063**	0	**0.035**	0	1.103	12	**0.048**	0	1.120	23	0.053	11	1.130	23	0.053	18

Qualitative Comparison

5 strategies: **`RA-D`**, **`BH-D`**, **`D-D`**, **`NB-D`** and **`I-D`**

weighted sum vs. Chebyshev scalarization

comparing all non-dominated solutions found

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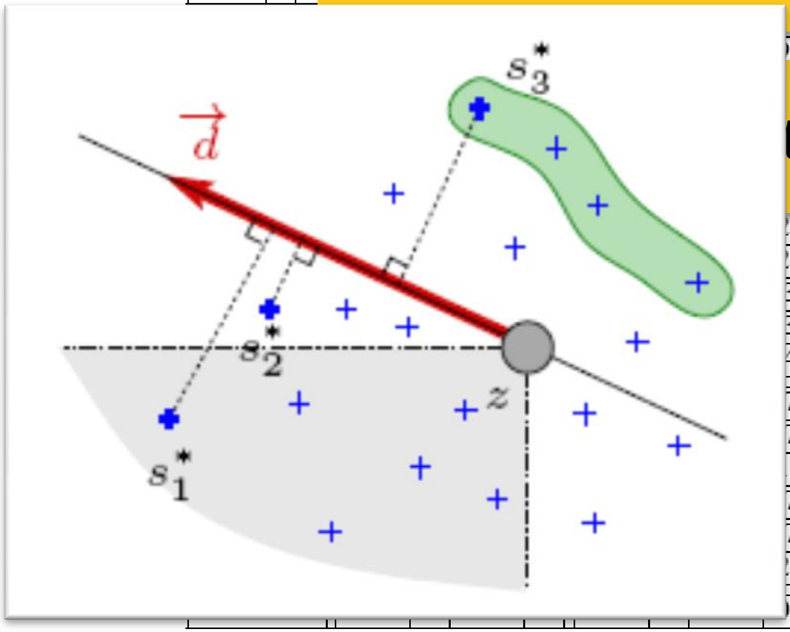
ρMNK with ρ =-0.7, 0.0, +0.7



t generations/funevals

hypervolume and ε-indicator

3 selection strategies

Main Conclusions

Influence of the Neighborhood Selection Strategy

much less than other algorithm design choices

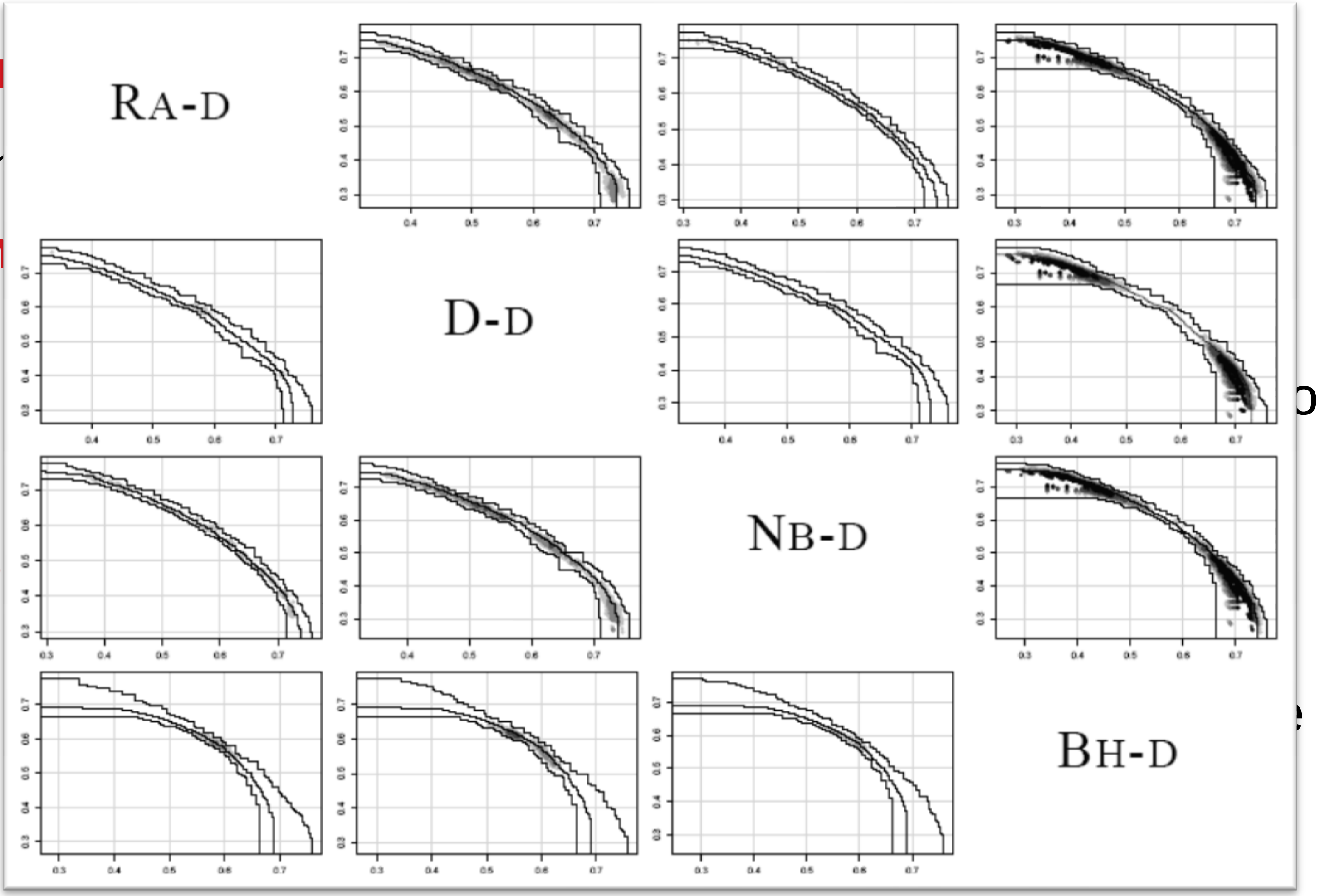
Weighted Sum vs. Achievement Scalarizing Function

- WS consistently better for ρMNK
- Chebyshev/ASF results in more local optima as non-dominated solutions cannot be visited (but with WS can)

Comparison between the Five Scalarizing Strategies

- adapation consistently better than fixed directions
- `D-D` strategy almost always worse than other adaptive ones

Main Conclusions



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- ones
- **BH−D** focuses on middle, **RA−D** more on extremes

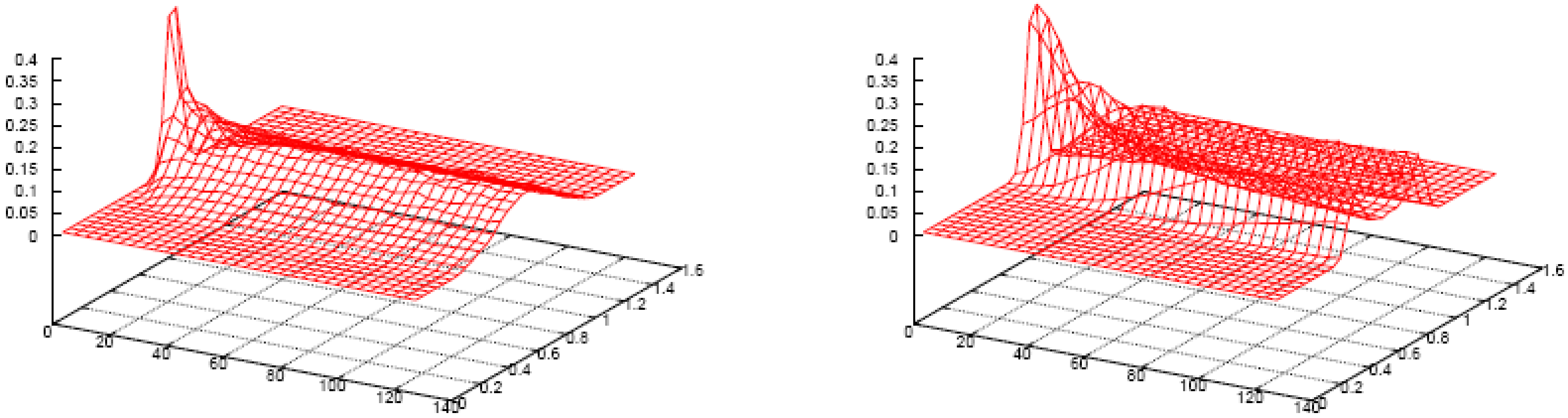
First Conclusion:

use **RA−D** (or **BH−D** if middle is desired and ideal point known)

Main Conclusions II

Distribution of the Population Over the Objective Space

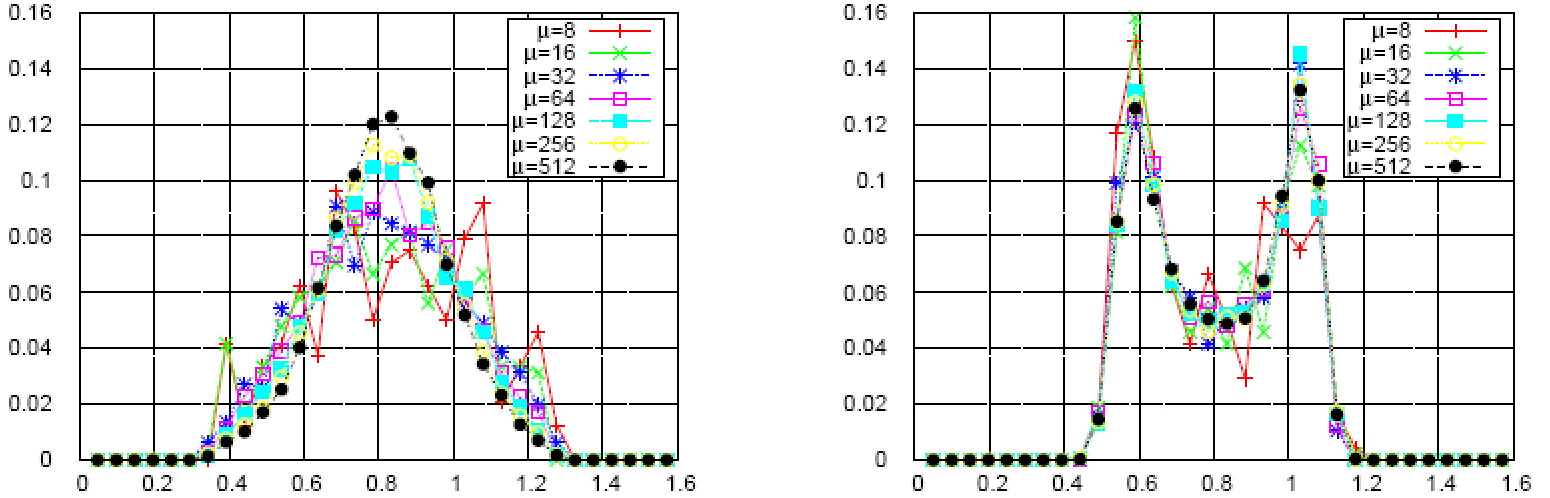
- quickly stable



Main Conclusions II

Distribution of the Population Over the Objective Space

- quickly stable
- smoother and with wider range for weighted sum



Comparison with (μ+λ)-SMS-EMOA with oneshot selection

- SMS-EMOA better on ρ=0.0 and ρ=+0.7 and in early optimization for ρ=-0.7
- force-based approaches only better with larger budgets (> 50μ funevals) on the highly correlated instance

Conclusions

Force-based Cooperative Search Directions in EMO

- first ideas of adapting the search directions in objective space for scalarization approaches
- lots of experimental results on the different strategies on the ρMNK problem

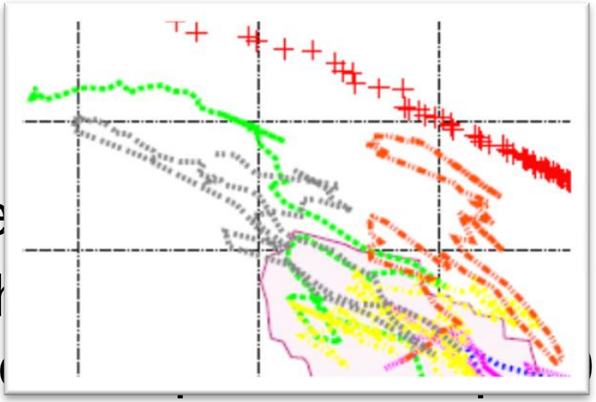
Results

- force-based approach works in principle
- when compared wrt non-dominated archive slightly better than SMS-EMOA only for not too small budgets on ρMNK with ρ=-0.7
- interesting insights into weighted sum vs. Chebyshev

Conclusions

Force-based Cooperative Search Directions in EMO

- first ideas of adapting the search directions in objective space for scalarization approaches
- lots of experimental results on the different strategies on the ρMNK problem



Results

- force-based approach works in principle
- when compared wrt non-dominated arch
SMS-EMOA only for not too small budg .7
- interesting insights into weighted sum vs. Chebyshev
- Final Conclusion: more investigations necessary
  - other problems (started for 0-1-knapsack)
  - comparison with other algorithms
  - influence of scalarizing functions (“landscapes”)

[Jiang et al. 2011] Siwei Jiang, Zhihua Cai, Jie Zhang, Yew-Soon Ong: *Multiobjective Optimization by Decomposition with Pareto-adaptive Weight Vectors*. In 7th International Conference on Natural Computation. 2011.