An Alternative Preference Relation to Deal with Many-Objective Optimization Problems

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Outline of the Presentation

Motivation

The Chebyshev Preference Relation

Approximating the Entire Pareto Front

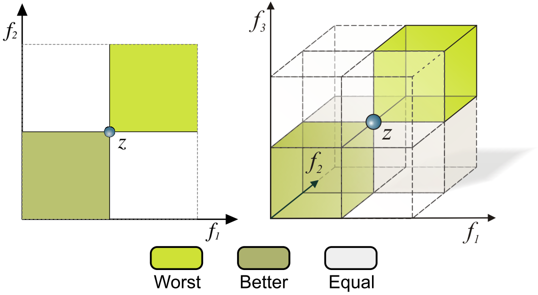
Experimental Results

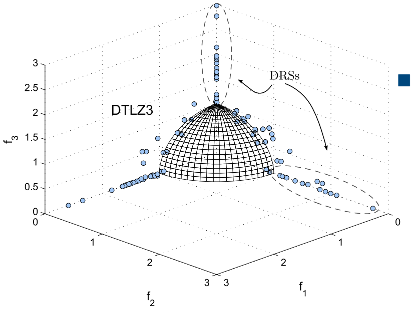
Conclusions

Outline of the Presentation

Difficulties of Many-objective Problems (1/2)

- **Number of nondominated vectors** quickly increases with the number of objectives: $(2^k - 2)/(2^k)$.





- Number of **dominance resistant solutions (DRSs)** increases with the objectives:
  - Solutions with good values in most objectives, but a poor value in at least one objective.

Motivation and Proposal (2/2)

1. We propose an alternative preference relation to deal with Many-objective problems.
   - Good convergence without sacrificing distribution.
   - Suitable for parallel implementation.
   - Easy to use it just by replacing the Pareto dominance relation.

2. We also study the effect of DRSs in the widely used DTLZ test problem suite and also in some WFG problems.

Outline of the Presentation

Augmented Chebyshev Achievement Function

Definition (Wierzbicki 1980, Ehrgott 2005)

The augmented Chebyshev achievement function is defined by

$$s(\mathbf{z}\,|\,\mathbf{z}^{\text{ref}}) = \max_{i=1,\ldots,k}\{\lambda_i(z_i - z_i^{\text{ref}})\}$$

- The reference point: aspiration levels for each objective.
- The weight vector, λ, normalize the objective values.

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- The second term helps to **avoid weakly Pareto optimal solutions**.

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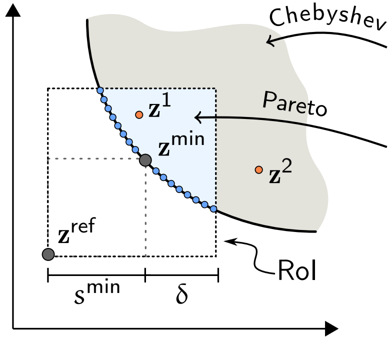
- The reference point: aspiration levels for each objective.
- The weight vector, λ, normalize the objective values.
- The second term helps to **avoid weakly Pareto optimal solutions**.
- **Any solution of the Pareto front can be generated by $s(\mathbf{z} \,|\, \mathbf{z}^{\text{ref}})$.**

The Chebyshev Preference Relation

- Combines **Pareto dominance** + **Chebyshev function**.
- Defines a **Region of Interest (RoI)** around a reference point.

solutions with $s(\mathbf{z} \,|\, \mathbf{z}^{\text{ref}}) \leqslant s^{\text{min}} + \delta$,

best achievement   size of the RoI



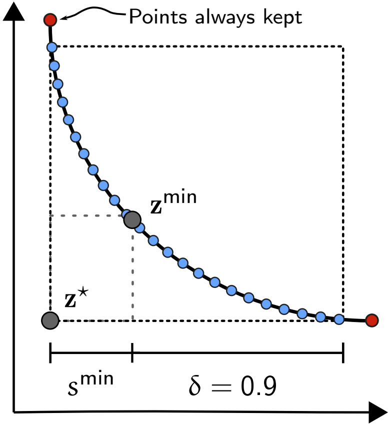
- **Outside RoI:** solutions compared with Chebyshev value.
- **In RoI:** solutions compared using Pareto dominance.
- Solutions **in RoI** dominate solutions **outside RoI**.

Outline of the Presentation

Approximating the Entire Pareto Front

To approximate the entire Pareto front we used as reference point the **approximation of the ideal point** maintained by the Chebyshev relation.

- Use a stringent criterion for solutions far from the Pareto front for guiding the solutions towards the ideal point,
- and Pareto dominance for solutions near the Pareto front for covering the entire Pareto front.



Replacing Pareto Dominance by Another Relation

In order to improve convergence...

- Instead of the Pareto dominance, another secondary preference relation can be used instead.
- Here we used a preference relation based on the **binary ϵ-indicator.**

Replacing Pareto Dominance by ϵ-indicator Relation

Epsilon Indicator, I_ϵ

$$I_\epsilon(\mathbf{z}^2, \mathbf{z}^1) = \min_{\epsilon \in \mathbb{R}} \{z_i^2 \leqslant \epsilon + z_i^1 \ \text{ for } i = 1, \ldots, k\}$$

"Extent" by which a solution dominates another one:

$\epsilon < 0$ $\qquad$ $\epsilon > 0$

$\mathbf{z}^2 \prec \mathbf{z}^1$ dominates by $\qquad$ $\epsilon = 0$ $\qquad$ needs to dominate $\mathbf{z}^1$

- Using a fitness function, $F_\epsilon(\mathbf{z})$, based on the ϵ-indicator we can define a preference relation:

Epsilon relation, R_ϵ

A solution $\mathbf{z}^1$ "dominates" $\mathbf{z}^2$ wrt the relation R_ϵ if and only if:
$F_\epsilon(\mathbf{z}^1) > F_\epsilon(\mathbf{z}^2)$.

Replacing Pareto Dominance by ϵ-indicator Relation

Two examples of fitness functions (P is the current population):

Maximim fitness function (Balling 2003)

$$F_{\epsilon}^{\min}(\mathbf{z}^1) = \min_{\mathbf{z}^2 \in P \setminus \{\mathbf{z}^1\}} I_{\epsilon}(\mathbf{z}^2, \mathbf{z}^1)$$

Sum of I_ϵ values (Zitzler and Künzli 2004)

$$F_{\epsilon}^{\mathrm{sum}}(\mathbf{z}^1) = \sum_{\mathbf{z}^2 \in P \setminus \{\mathbf{z}^1\}} - \exp\left(-I_{\epsilon}(\{\mathbf{z}^2\}, \{\mathbf{z}^1\})\right)$$

$\epsilon < 0$ | $\epsilon = 0$ | $\epsilon > 0$

$\mathbf{z}^2 \prec \mathbf{z}^1$ dominates by | $\epsilon = 0$ | needs to dominate $\mathbf{z}^1$

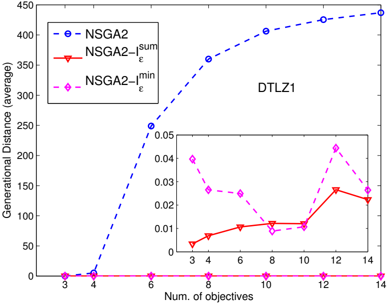
Outline of the Presentation

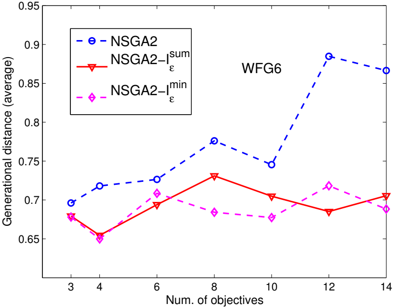
Algorithms and Parameter Settings

- NSGA2 used as baseline optimizer:
  - Pareto dominance.
  - Chebyshev relation with $I_{\epsilon}^{\text{sum}}$.
  - Chebyshev relation with $I_{\epsilon}^{\text{min}}$.
- Test Problems: DTLZ1, DTLZ2, DTLZ3, DTLZ4, DTLZ7, WGF2, WGF6
  - Objectives were varied from 3 to 14 objectives.
  - Number of distance-related variables: 20 (5 for DTLZ1).
  - Number of position-related variables: $k - 1$.
- Performance Indicators:
  - Generational Distance (GD), Inverted Generational Distance (IGD), Epsilon Indicator.

Generational Distance

GD values for DTLZ1 and WFG6 using from 3 to 14 objectives.





Remarkable improvement **Marginal improvement**

Generational Distance

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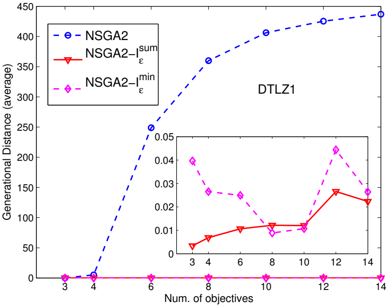
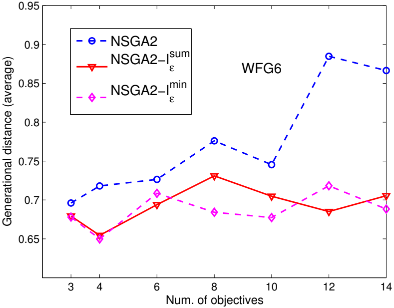




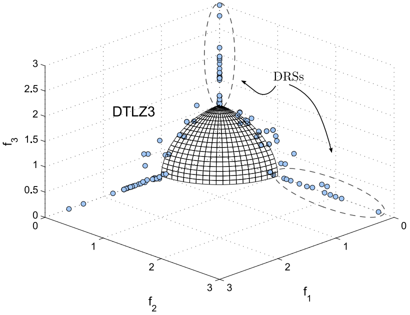
Remarkable improvement **Marginal improvement**

Does Chebyshev approach performs bad or NSGA2 performs well?

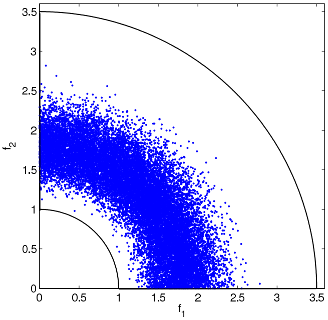
Inverted Generational Distance

MOP	MOEA		3	6	10	14
DTLZ2	NSGA2	mean	0.0151	0.0239	0.0469	0.0443
		std	0.0010	0.0030	0.0113	0.0104
	NSGA2-I_ϵ^{sum}	mean	**0.0079**	0.0132	0.0183	0.0331
		std	0.0025	0.0047	0.0091	0.0185
	NSGA2-I_ϵ^{min}	mean	0.0087	**0.0121**	**0.0165**	**0.0220**
		std	0.0027	0.0028	0.0070	0.0099
DTLZ7	NSGA2	mean	0.0896	0.3300	12.7540	38.0094
		std	0.2740	0.0830	4.8488	9.5044
	NSGA2-I_ϵ^{sum}	mean	**0.0138**	**0.2226**	**4.9511**	**9.1685**
		std	0.0029	0.0083	0.3701	0.3093
	NSGA2-I_ϵ^{min}	mean	0.0150	0.2229	4.9978	9.2333
		std	0.0061	0.0073	0.2918	0.1852
WFG6	NSGA2	mean	0.0075	0.0120	0.0187	0.0271
		std	0.0034	0.0049	0.0052	0.0071
	NSGA2-I_ϵ^{sum}	mean	**0.0073**	0.0091	0.0141	0.0232
		std	0.0024	0.0020	0.0038	0.0240
	NSGA2-I_ϵ^{min}	mean	0.0075	**0.0087**	**0.0132**	**0.0209**
		std	0.0015	0.0025	0.0050	0.0085

Analysis of Dominance Resistant Solutions



DRSs in DTLZ3 using NSGA2 (objs. values are divided by 20)



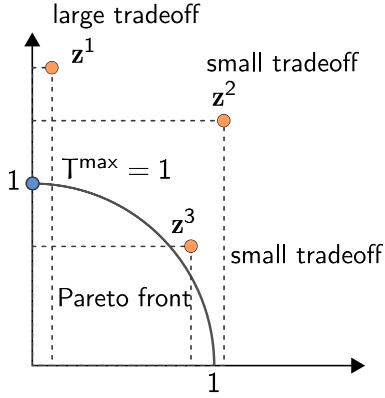
20 000 random solutions for DTLZ2

Maximum Tradeoff to Detect DRSs

Classification of DRSs

Maximum tradeoff: $T^{\max}(\mathbf{z}) = \frac{\max(z_i)}{\min(z_i)+1}$ for all $i = 1, ..., k.$

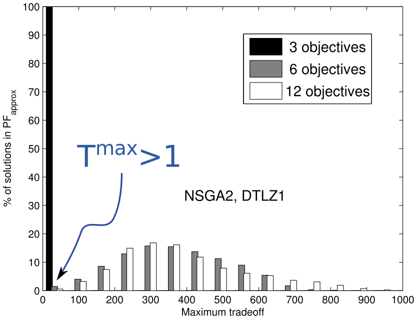
- **DRSs have a very large $T^{\max}$.** Small value in some objective and large value in other objective.
- Not all solutions far from the Pareto front obtain a large $T^{\max}$.



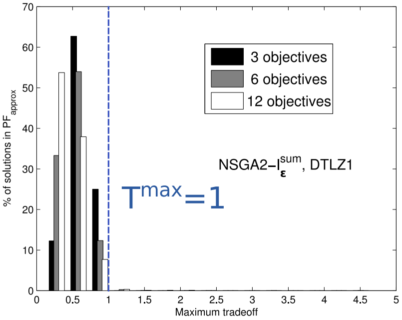
Maximum Tradeoff Results

Maximum tradeoff distribution in DTLZ1

DRSs: $T^{max} >> 1$



NSGA2

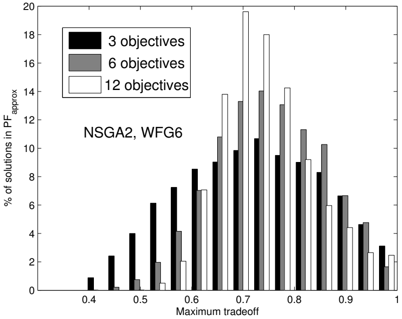


NSGA2-I_{ϵ}^{sum}

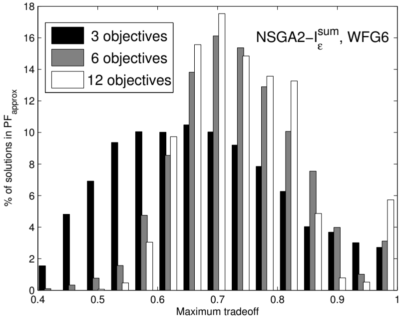
Maximum Tradeoff Results

Maximum tradeoff distribution in WFG6

DRSs: $T^{max} >> 1$



NSGA2



NSGA2-I_ϵ^{sum}

Outline of the Presentation

Conclusions

- The **Chebyshev relation + ϵ-indicator** is also useful to deal with Many-Objective problems.
  - NSGA-II's convergence was improved without sacrificing distribution.
- Comparing solutions using the achievement function and ϵ-indicator helps to eliminate DRSs.
- The main source of difficulty of DTLZ problems is the presence of dominance resistant solutions.
- The difficulty of some WFG problems is not considerably increased with the number of objectives.

Future Work:

- Compare the Chebyshev relation against optimization techniques that have shown good scalability in Many-objective problems. E.g., MOEA/D or ϵ-MOEA.