A Comparative Study on Evolutionary Algorithms for Many-Objective Optimization

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Abstract

This paper provides a systematic comparison of eight representative evolutionary multiobjective algorithms from the six angles to solve many-objective optimization problems. The compared algorithms are tested on four groups of well-defined test functions, by three performance metrics as well as a visual observation in the decision space. We conclude that none of the algorithms has a clear advantage over the others, although some of them are competitive on most of the problems. In addition, different search abilities of these algorithms on the problems with different characteristics suggest a careful choice of algorithms for solving a many-objective problem in hand.

Tested Algorithms and Problems

Table 1: List of some existing comparison studies on many-objective optimization

	Algorithm Class	Test Problem
Khare et al. (2003)	C1	DTLZ
Hughes (2005)	C1, C2	Custom
Purshouse and Fleming (2007)	C1	DTLZ
Corne and Knowles (2007)	C4, C6	TSP
Wagner et al. (2007)	C1, C2, C3, C4	DTLZ
Ishibuchi et al. (2008)	C1, C2, C4, C5, C6	Knapsack
Jaimes and Coello (2009)	C1, C4, C6	DTLZ
Hadka and Reed (2012)	C1, C2, C4, C6	DTLZ, WFG, UF

C1: Pareto-based algorithms;
C2: Aggregation-based algorithms;
C3: Indicator-based algorithms;
C4: Improved Pareto dominance-based algorithms;
C5: Improved diversity maintenance-based algorithms;
C6: Non-Pareto-based algorithms.

In this work, we consider eight EMO algorithms selected from the above six classes.

- Nondominated Sorting Genetic Algorithm II (NSGA-II)
- Multiobjective Evolutionary Algorithm based on Decomposition with PBI (MOEA/D+PBI)
- Multiple Single Objective Pareto Sampling (MSOPS)
- Hypervolume Estimation Algorithm (HypE)
- ε-dominance Multiobjective Evolutionary Algorithm (ε-MOEA)
- Diversity Management Operator (DMO)
- Average Ranking (AR)
- Average Ranking combined with Grid (AR+Grid)

We consider four groups of test functions and two performance metrics

- DTLZ, WFG, TSP, and Pareto-Box problems
- HV and IGD

Results

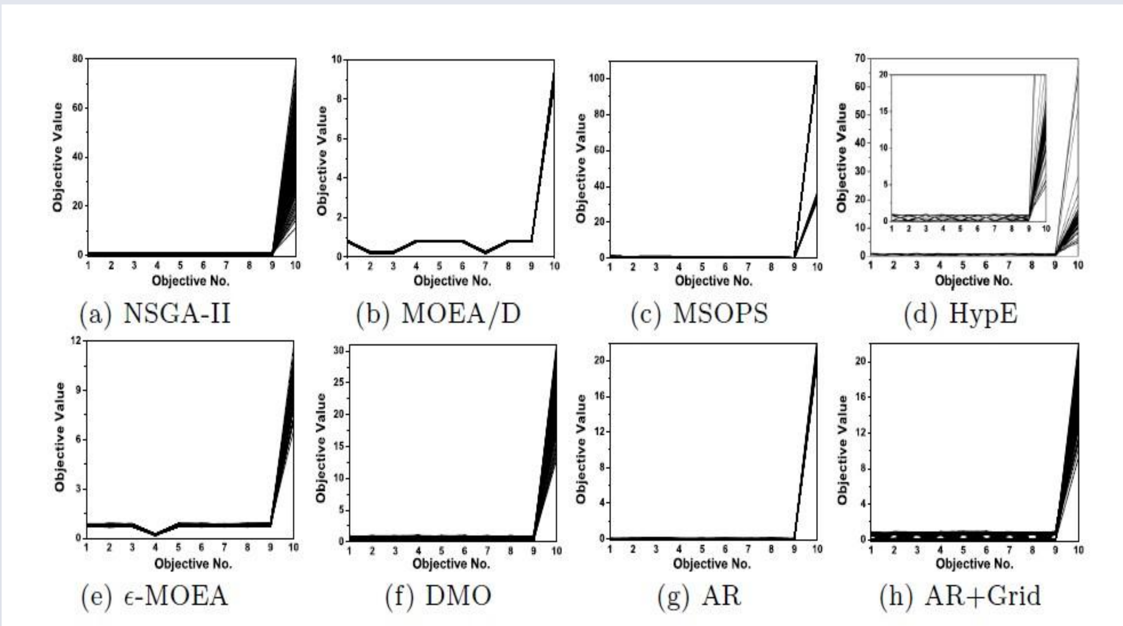
Table 2: Comparative Results of IGD or HV on the 5-objective instance

	DTLZ2	DTLZ3	DTLZ5	DTLZ7	WFG1	WFG8	WFG9	TSP
NSGA-II	Fair	Poor	Fair	Good	Poor	Fair	Fair	Poor
MOEA/D	Good	Good	Good	Poor	Good	Fair	Good	Good
MSOPS	Fair	Poor	Good	Poor	Poor	Fair	Fair	Fair
HypE	Fair	Fair	Fair	Good	Poor	Fair	Good	Poor
ε-MOEA	Good	Good	Fair	Good	Poor	Fair	Fair	Good
DMO	Fair	Poor	Poor	Good	Poor	Fair	Fair	Poor
AR	Poor	Fair	Poor	Fair	Poor	Poor	Poor	Poor
AR+Grid	Good	Fair	Fair	Fair	Fair	Good	Good	Good

Table 3: Comparative Results of IGD or HV on the 10-objective instance

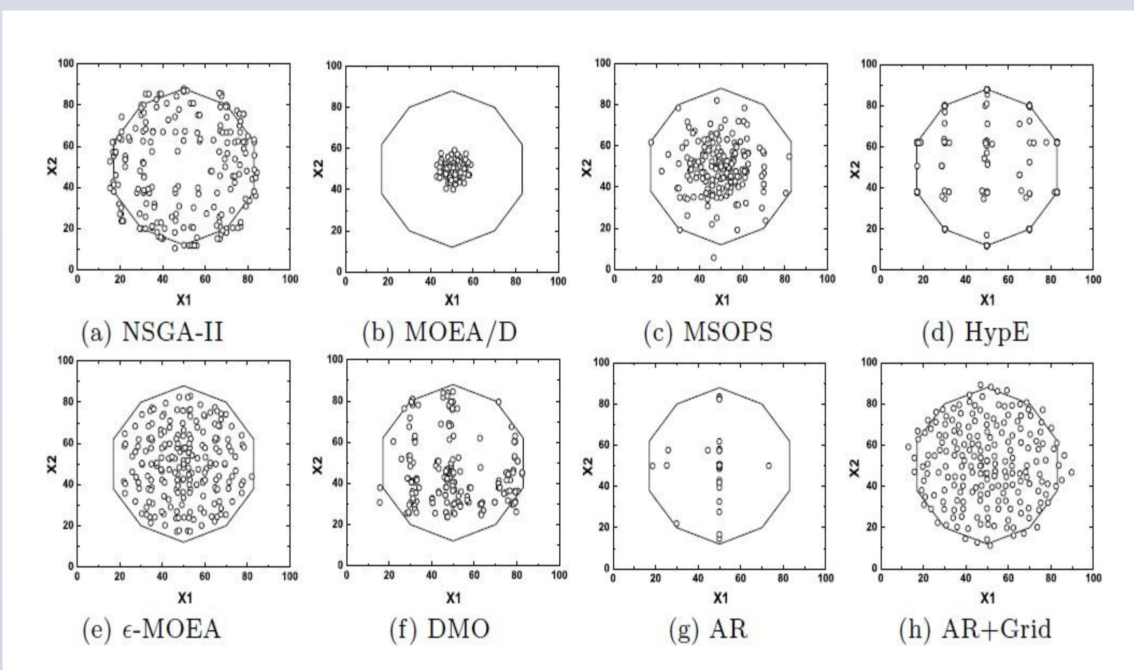
	DTLZ2	DTLZ3	DTLZ5	DTLZ7	WFG1	WFG8	WFG9	TSP	Pareto-Box
NSGA-II	Poor	Poor	Fair	Fair	Fair	Fair	Fair	Poor	Fair
MOEA/D	Good	Good	Good	Poor	Good	Poor	Poor	Good	Poor
MSOPS	Fair	Poor	Good	Poor	Poor	Fair	Fair	Good	Fair
HypE	Fair	Fair	Fair	Good	Good	Good	Good	Poor	Poor
ε-MOEA	Good	Poor	Fair	Fair	Poor	Poor	Poor	Fair	Good
DMO	Fair	Poor	Fair	Poor	Fair	Fair	Fair	Poor	Fair
AR	Poor	Fair	Poor	Poor	Poor	Good	Fair	Fair	Poor
AR+Grid	Good	Fair	Poor	Good	Good	Good	Fair	Good	Good

Figure 1: The final solution set of the eight algorithms on the ten-objective DTLZ7, shown by parallel coordinates



With many disconnected Pareto optimal regions, DTLZ7 tests an algorithm's ability to maintain subpopulations in disconnected portions of the objective space. The upper bound of the last objective in the Pareto front of DTLZ7 is equal to 20 for the 10-objective instance

Figure 2: The final solution set of the eight algorithms in the decision space on the ten-objective Pareto-Box problem



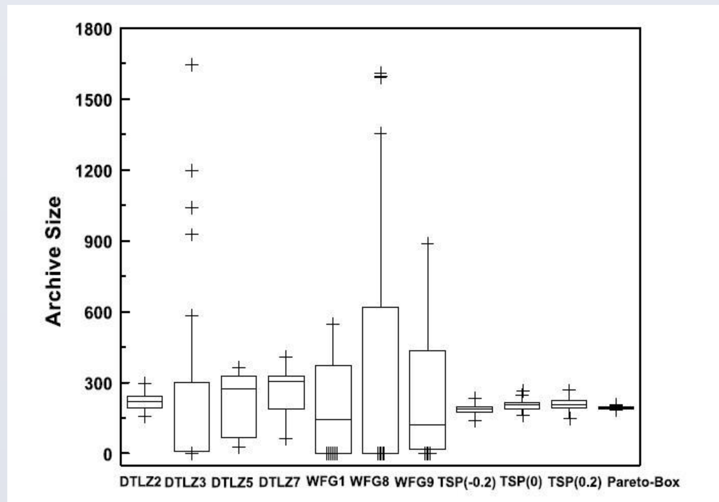
With a high-dimensional objective space and a two-dimensional decision space, the Pareto-Box problem is used to visually investigate the distribution of solutions in the decision space. The Pareto optimal region is the inside of the regular decagon.

The solutions of MOEA/D shown here are obtained by MOEA/D+PBI with penalty parameter 5.0; A much better result can be obtained by MOEA/D+TCH and MOEA/D+PBI with penalty parameter 0.1.

Based on the examination on these continuous and combinatorial problems, a summary observation of the eight algorithms can be made:

- NSGA-II does not always perform the worst on all many-objective problems. On some problems with relatively low dimensions, such as the 5-objective DTLZ7 and WFG8, NSGA-II outperforms some algorithms designed specially for many-objective optimization.
- The search ability of MOEA/D has sharp contrasts on different problems. It works very well on DTLZ2, DTLZ3, WFG1, and TSP, but encounters great difficulties on the DTLZ7 and WFG8. From the results on the WFG and TSP suites, MOEA/D appears to be more competitive in relatively low-dimensional problems.
- Similar to MOEA/D, MSOPS struggles on the problem with the disconnected Pareto front (DTLZ7). But MSOPS performs the best on the degenerate problem DTLZ5.
- Although favoring the boundary solutions, HypE shows advantages in a higher-dimensional objective space. This can be obtained from the results of the DTLZ7 and three WFG test instances.
- ε-MOEA performs well on most of the 5-objective test instances. However, the instability of the archive size will count against the evolutionary process of the algorithm as the number of objectives further increases.
- By adaptively controlling the diversity maintenance mechanism, DMO has a clear advantage over NSGA-II on the DTLZ suite. However for the WFG and TSP suites, the advantage vanishes, NSGA-II even outperforming DMO on WFG8 and most of the TSP instances.
- Due to the lack of diversity maintenance, AR is the algorithm with poor comprehensive performance on all the test problems, except for the 10-objective TSP, where AR is clearly superior to HypE and DMO.
- Despite being competitive on most of the test instances, AR+Grid has difficulty on the problem with many local optima, such as DTLZ3. This is because the neighbor punishment strategy in AR+Grid may make some "bad" individuals rank higher than their better competitors.

Figure 3: The box plot of the archive size of ε-MOEA on 30 runs for all 10-objective problems



Conclusions

Our study has revealed that there is not a clear performance gap between algorithms for all the test problems. The considered algorithms have their own strengths on different test instances. This means that a careful choice of algorithms must be made when dealing with a many-objective problem in hand.

Another observation of our study is that none of the algorithms can produce a well-converged and well-distributed solution set even for some "easy" problems, such as the Pareto-Box problem. This indicates the infancy of evolutionary many-objective optimization and highlights the need for further development in the area.

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