Selection Operators based on Maximin Fitness Function for Multi-Objective Evolutionary Algorithms

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Motivation

When designing multi-objective evolutionary algorithms (MOEAs), there are two main types of approaches that are normally used as selection mechanisms:

1. those that incorporate the concept of Pareto optimality, and
2. those that do not use Pareto dominance to select individuals.

However, **the use of Pareto-based selection has several limitations**. From them, its poor scalability is, perhaps, the most remarkable. In this work, **we are interested in the maximin fitness function** (belonging to the type (2)).

Maximin Fitness Function (MFF)

The maximin fitness function of individual i is defined as:

$$fitness^i = max_{j \neq i}(min_k(f_k^i - f_k^j)) \quad (1)$$

where the min is taken over all the objectives, and the max is taken over all the individuals in the population, except for the same individual i. From eq. (1), we can say the following:

1. If $fitness^i > 0$ then i is a dominated individual,
2. If $fitness^i < 0$ then i is a non-dominated individual.
3. Finally, if $fitness^i = 0$ then i is a weakly-dominated individual.

This scheme is computationally efficient (its complexity is linear with respect to the number of objectives).

Modified MFF

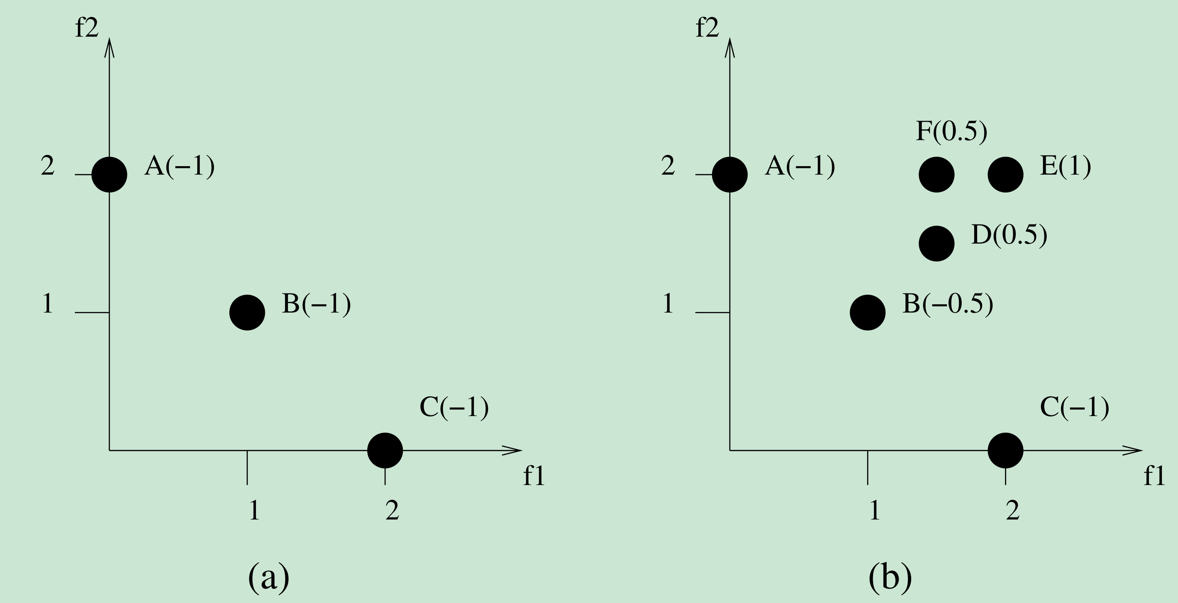
The author of the MFF proposed the following modified MFF:

$$fitness^i = max_{j \neq i, j \in P}(min_k(f_k^i - f_k^j)) \quad (2)$$

where *P is the set of non-dominated individuals*. Using eq. (2), we only penalize clustering between non-dominated individuals.

Some properties of MFF

1. MFF **penalizes clustering** of non-dominated individuals.
2. The maximin fitness of dominated individuals **is a metric of the distance to the non-dominated front**.



The maximin fitness of a dominated individual is always controlled by a non-dominated individual and is indifferent to clustering. The maximin fitness of a non-dominated individual may be controlled by a dominated or a non-dominated individual.

Proposed selection mechanism

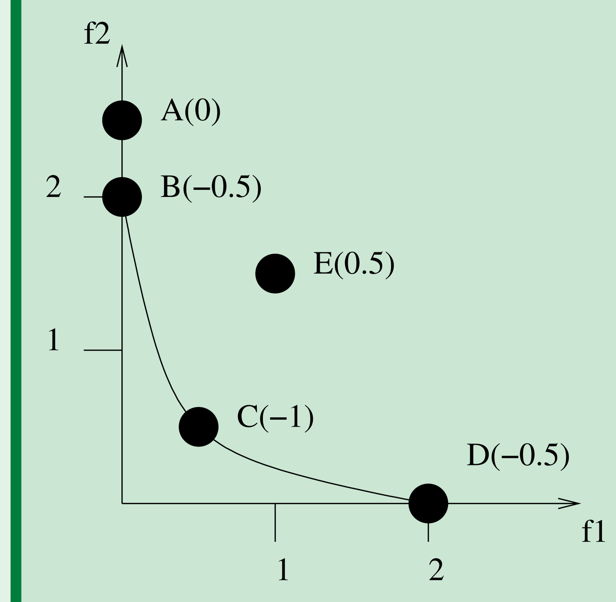
Input : X (Current population) and S (number of individuals to choose).
Output: Y (individuals selected).

MaximinFitnessFunction(X);
if *The number of nondominated individuals is greater to* S **then**
  $Y \leftarrow$ Maximin-Clustering(X, S);
else
  $Y \leftarrow$ Maximin-Constraint(X, S);
Returns Y;

MFF vs Modified MFF

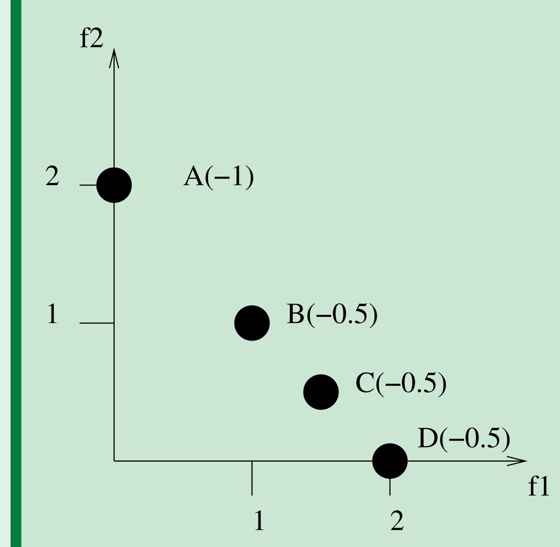
We propose three operators based on MFF. The first uses MFF. The second uses MFF when applies Maximin-Constraint and uses modified MFF when applies Maximin-Clustering. The third uses modified MFF. According to the results, the three operators are competitive to solve multi-objective optimization problems having both low dimensionality (two or three) and high dimensionality (more than three) in objective function space.

Disadvantage 1



Is it better to prefer weakly dominated individuals than dominated individuals? In the Figure, solution A is a weakly dominated individual and solution E is a dominated individual. To guarantee convergence to the Pareto optimal set, we must choose individual E. Otherwise, it is possible that the MOEA converges to a weak Pareto optimal solution. Problem ZDT2 is an example.

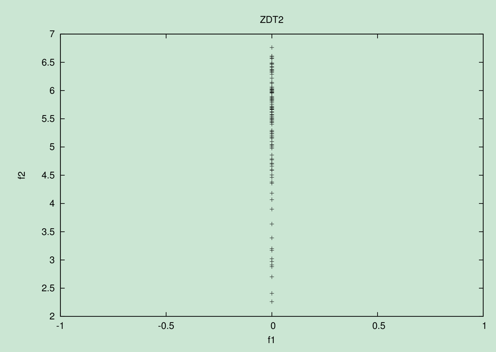
Disadvantage 2



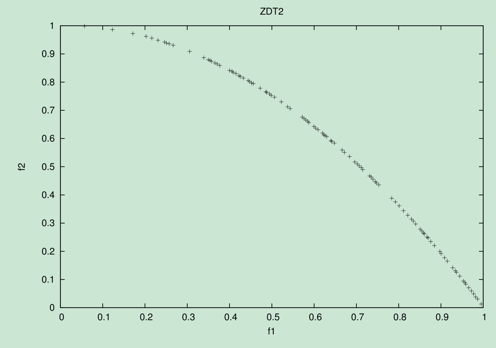
MFF penalizes individuals B, C and D because they are close from each other. However, we can not know which of the three is the best individual to form part of the next generation.

Solution (Checking similarity)

We show that it is not good to prefer weakly dominated individuals or individuals which are close to being weakly dominated. Then, we proposed the following constraint: Any individual that we want to select must not be similar (in objective space) to another (selected) individual.



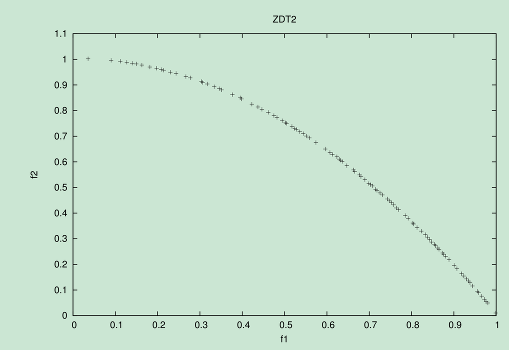
(only MFF)



(MFF and our constraint)

Solution (Maximin - Clustering)

To select S individuals, we choose the best S individuals with respect to their maximin fitness, and use them as centers of their clusters. Then, we proceed to do clustering.



We apply this technique only when more than S individuals are non-dominated.

Results

We designed a MOEA using a simulated binary crossover (SBX) and a polynomial mutation operator (PM) combined with the described selection operators, giving rise to our **MC-MOEA** approach. According to the hypervolume, MC-MOEAs obtained competitive results with respect to both SMS-EMOA and App-SMS-EMOA. Regarding the additive epsilon indicator, we only compared with respect to App-SMS-EMOA and the results indicate that MC-MOEAs outperformed App-SMS-EMOA in most cases. **MC-MOEAs have two advantage: First, they are consistent when we increase the number of objectives. And second, they are computationally efficients. Thus, we argue that the proposed MC-MOEAs can be a good alternative for dealing with many-objective optimization problems**.

Set of problems	Objectives	NSGA-II	SMS-EMOA	App-SMS-EMOA	MC-MOEAs
ZDT	2	$\lessapprox 1s$	$5s - 10s$	$5s - 10s$	$\lessapprox 1s$
DTLZ	3	$2s - 4s$	$4568s - 8468s$	$231s - 307s$	$3s - 9s$
DTLZ	4	$3s - 4s$	$14448s - 14650s$	$378s - 423s$	$5s - 12s$
DTLZ	5	$4s - 5s$	$15423s - 18000s$	$472s - 499s$	$9s - 14s$
DTLZ	6	$5s - 6s$	-	$531s - 584s$	$8s - 16s$
DTLZ	7	$5 - 6s$	-	$536s - 583s$	$9 - 18s$
DTLZ	8	$5s - 7s$	-	$525s - 583$	$9s - 16s$

Running time required per run, s = seconds.

References

[1] R. Balling and S. Wilson. The Maximin Fitness Function for Multi-objective Evolutionary Computation: Application to City Planning. In Lee Spector and Erik D. Goodman and Annie Wu and W.B. Langdon and Hans-Michael Voigt and Mitsuo Gen and Sandip Sen and Marco Dorigo and Shahram Pezeshk and Max H. Garzon and Edmund Burke, editors, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2001)*, pages 1079-1084, San Francisco, California, 2001. Morgan Kaufmann Publishers.