

# Knowledge Transfer Strategies for Vector Evaluated Particle Swarm Optimization

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## Introduction

- Particle swarm optimization (PSO) is a population-based, stochastic optimization technique based on the social dynamics of a flock of birds
- Vector evaluated particle swarm optimization (VEPSO) is a multi-swarm variant of PSO used to optimize multi-objective optimization problems (MOPs)
  - Utilizes a knowledge transfer strategy (KTS) to determine global guides and propagate information between sub-swarms

**Objective:** Investigate new random, probabilistic, and hybrid strategies to select the global guide and transfer information between sub-swarms

## Vector Evaluated Particle Swarm Optimization

- Inspired by the vector evaluated genetic algorithm (Schaffer [1985]), Parsopoulos and Vrahatis [2002] proposed a multi-objective PSO variant, VEPSO
  - Fitness evaluated as a vector with each sub-objective being a vector component
- Each sub-objective allocated a single sub-swarm dedicated solely to optimizing this sub-objective
- Information passed between sub-swarms to optimize MOP as a whole using a KTS

## Existing and Proposed Knowledge Transfer Strategies

### Existing Knowledge Transfer Strategies

- Ring KTS** - Parsopoulos and Vrahatis [2002] selected the global guide for a sub-swarm as the global best particle from the neighboring sub-swarm, according to a directed ring topology
- Random Global Best KTS** - Grobler [2008], under the supervision of Engelbrecht, selected the global guide for a sub-swarm as the global best position from a randomly-selected sub-swarm

### Proposed Random and Probabilistic Knowledge Transfer Strategies

The following proposed strategies used random and probabilistic approaches to select a global guide:

- Random Personal Best KTS** - The global guide for a sub-swarm is a randomly selected personal best position from a randomly selected sub-swarm
- Roulette Wheel Personal Best KTS** - The global guide for a sub-swarm is a personal best position selected using roulette wheel selection from a random sub-swarm
- Tournament Personal Best KTS** - The global guide for a sub-swarm is a personal best solution chosen from a random sub-swarm using tournament selection with 10% of the sub-swarm
- Rank-Based Personal Best KTS** - The global guide for a sub-swarm is a personal best solution selected from a random sub-swarm using rank-based selection

### Proposed Hybrid Knowledge Transfer Strategies

The following proposed strategies applied the parent-centric crossover operator (PCX) (Deb *et al.* [2002]) to compute a global guide:

- PCX GBest KTS** - The global guide for a sub-swarm is computed as the offspring of PCX applied to the global best position of three randomly selected sub-swarms
- PCX Archive KTS** - The global guide for a sub-swarm is computed as the offspring of PCX applied to three randomly selected non-dominated solutions from the archive

## Performance Measures

### Hypervolume Measure

- Zitzler and Thiele [1999] defined a unary performance measure which measures hypervolume of space weakly dominated by an approximation set
- Fleischer [2003] proved hypervolume is maximized if and only if an approximation set contains maximally-distributed, Pareto optimal solutions

### Solution Distribution Measure

- Goh and Tan [2007] developed a measure of the distribution of solutions along the approximation front based on average nearest-neighbor distance

## Statistical Analysis of Results

- Each experiment consisted of 30 independent runs
- Pairwise Mann-Whitney U tests performed to determine if a significant difference in performance existed
  - If a difference exists, a win is recorded for the better optimizer and a loss for the other
  - Optimizers are assigned a rank based on the subtractive difference between wins and losses

## Parameterization

- All experiments performed using the Computational Intelligence Library (Clib)
- Three dimensional formulations of the nine minimization problems from the Walking Fish Group (WFG) toolkit (Huband *et al.* [2006]) were used as benchmark functions

### Vanilla PSO Parameters

- $\omega = 0.729844$
- $c_1 = c_2 = 1.496180$
- 100 particles for 250 generations
- Clamping boundary constraint

### VEPSO Parameters

- 1 vanilla PSO sub-swarm (as above) per sub-objective
- Archive size: 500
- Distance based archive removal
  - Remove solution with smallest nearest-neighbor distance

## Results: Pairwise Mann-Whitney U Tests for Hypervolume Measure

Table : Proposed Random-Based Knowledge Transfer Strategies vs. Existing Strategies

Knowledge Transfer Strategy	Result	WFG Function								
		1	2	3	4	5	6	7	8	9
Ring KTS	Wins	1	0	0	1	4	5	0	0	4
	Losses	1	5	5	2	0	0	5	5	0
	Difference	0	-5	-5	-1	+4	+5	-5	-5	+4
	Rank	3	6	6	3	1	1	6	6	1
Random Global Best KTS	Wins	5	5	5	5	3	3	5	5	0
	Losses	0	0	0	0	0	2	0	0	5
	Difference	+5	+5	+5	+5	+3	+1	+5	+5	-5
	Rank	1	1	1	1	2	3	1	1	6
Random Personal Best KTS	Wins	0	1	2	0	0	0	1	1	1
	Losses	3	2	2	5	3	4	3	2	2
	Difference	-3	-1	0	-5	-3	-4	-2	-1	-1
	Rank	6	4	3	6	4	5	5	3	3
Tournament Personal Best KTS	Wins	2	3	3	4	3	4	4	4	4
	Losses	1	1	1	1	1	1	1	1	0
	Difference	+1	+2	+2	+3	+2	+3	+3	+3	+4
	Rank	2	2	2	2	3	2	2	2	1
Roulette Wheel Personal Best KTS	Wins	0	1	1	1	0	0	1	1	1
	Losses	2	1	3	2	3	4	2	2	2
	Difference	-2	0	-2	-1	-3	-4	-1	-1	-1
	Rank	5	3	5	3	4	5	4	3	3
Rank-Based Personal Best KTS	Wins	0	1	1	1	0	2	2	1	1
	Losses	1	2	1	2	3	3	2	2	2
	Difference	-1	-1	0	-1	-3	-1	0	-1	-1
	Rank	4	4	3	3	4	4	3	3	3

Table : Proposed Hybrid Knowledge Transfer Strategies vs. Existing Strategies

Knowledge Transfer Strategy	Result	WFG Function								
		1	2	3	4	5	6	7	8	9
Ring KTS	Wins	0	0	1	0	1	3	0	0	1
	Losses	3	3	2	3	0	0	3	3	1
	Difference	-3	-3	-1	-3	+1	+3	-3	-3	0
	Rank	4	4	4	4	1	1	4	4	2
Random Global Best KTS	Wins	1	1	2	1	1	0	1	1	0
	Losses	2	2	1	1	0	2	1	1	3
	Difference	-1	-1	+1	0	+1	-2	0	0	-3
	Rank	3	3	2	2	1	4	2	2	4
PCX GBest KTS	Wins	3	3	3	3	0	0	3	3	3
	Losses	0	0	0	0	3	1	0	0	0
	Difference	+3	+3	+3	+3	-3	-1	+3	+3	+3
	Rank	1	1	1	1	4	3	1	1	1
PCX Archive KTS	Wins	2	2	1	1	1	1	1	1	1
	Losses	1	1	1	1	0	1	1	1	1
	Difference	+1	+1	0	0	+1	0	0	0	0
	Rank	2	2	3	2	1	2	2	2	2

## Results: Pairwise Mann-Whitney U Tests for Solution Distribution Measure

Table : Proposed Random-Based Knowledge Transfer Strategies vs. Existing Strategies

Knowledge Transfer Strategy	Result	WFG Function								
		1	2	3	4	5	6	7	8	9
Ring KTS	Wins	1	1	2	0	1	5	0	3	5
	Losses	0	0	1	0	0	0	0	0	0
	Difference	+1	+1	+1	0	+1	+5	0	+3	+5
	Rank	1	1	2	1	1	1	1	1	1
Random Global Best KTS	Wins	0	1	5	0	0	3	0	0	0
	Losses	5	0	0	0	5	2	0	1	5
	Difference	-5	+1	+5	0	-5	+1	0	-1	-5
	Rank	6	1	1	1	6	3	1	4	6
Random Personal Best KTS	Wins	1	1	0	0	1	0	0	0	3
	Losses	0	0	2	0	0	3	1	1	1
	Difference	+1	+1	-2	0	+1	-3	0	-1	-2
	Rank	1	1	4	1	1	5	1	4	2
Tournament Personal Best KTS	Wins	1	1	0	0	1	4	0	0	1
	Losses	0	0	1	0	0	1	0	3	2
	Difference	+1	+1	-1	0	+1	+3	0	-3	-1
	Rank	1	1	3	1	1	2	1	6	4
Roulette Wheel Personal Best KTS	Wins	1	0	0	0	1	0	0	1	1
	Losses	0	0	1	0	0	4	0	0	1
	Difference	+1	0	-1	0	+1	-4	0	+1	0
	Rank	1	5	3	1	1	6	1	2	3
Rank-Based Personal Best KTS	Wins	1	0	0	0	1	1	0	1	1
	Losses	0	4	2	0	0	3	0	0	2
	Difference	+1	-4	-2	0	+1	-2	0	+1	-1
	Rank	1	6	4	1	1	4	1	2	4

Table : Proposed Hybrid Knowledge Transfer Strategies vs. Existing Strategies

Knowledge Transfer Strategy	Result	WFG Function								
		1	2	3	4	5	6	7	8	9
Ring KTS	Wins	2	0	1	0	3	2	0	2	1
	Losses	1	1	1	2	0	1	2	0	1
	Difference	+1	-1	0	-2	+3	+1	-2	+2	0
	Rank	2	3	2	3	1	2	4	1	2
Random Global Best KTS	Wins	1	0	3	0	0	1	0	1	0
	Losses	2	2	0	2	3	2	1	2	3
	Difference	-1	-2	+3	-2	-3	-1	-1	-1	-3
	Rank	3	4	1	3	4	3	3	3	4
PCX GBest KTS	Wins	0	1	0	2	1	0	1	0	1
	Losses	3	1	1	0	1	3	0	3	1
	Difference	-3	0	-1	+2	0	-3	+1	-3	0
	Rank	4	2	3	1	2	4	2	4	2
PCX Archive KTS	Wins	3	3	0	1	2	3	2	3	3
	Losses	0	0	2	0	1	0	0	0	0
	Difference	+3	+3	-2	+2	0	+3	+2	+2	+3
	Rank	1	1	4	1	2	1	1	1	1

## Conclusions and Future Work

### Conclusions

- Six proposed knowledge transfer strategies investigated
- Hybrid, PCX-based KTSs shown to outperform existing strategies
  - PCX GBest KTS was the best performing in terms of hypervolume
  - PCX Archive KTS was the best performing in terms of distribution
- Tournament Personal Best KTS observed to be the most consistent random/probabilistic approach
- Contrary to literature, existing Ring KTS outperformed existing Random Global Best KTS when problem is deceptive

### Future Work

- Expanding the number of sub-objectives in a linear fashion
  - Develop strategies to deal with large number of objectives, if necessary
- Compare VEPSO, with both existing and proposed strategies, against other state-of-the-art multi-objective optimization algorithms