Do Hypervolume Regressions hinder EMOA Performance? Surprise and Relief.

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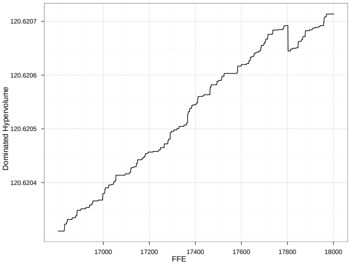
Prior findings

SMS-EMOA

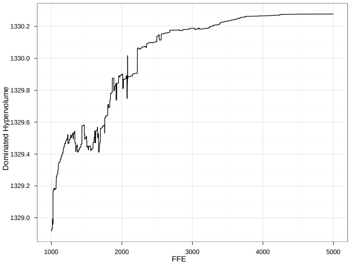
- $\mu + 1$ hypervolume based selection
- least contributer replaced by better one

Hypervolume decreases appear frequently

ZDT1, $\mu = 100$



DTLZ2, $\mu = 10$

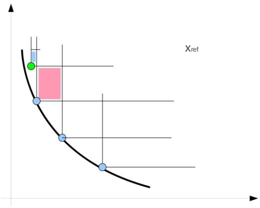


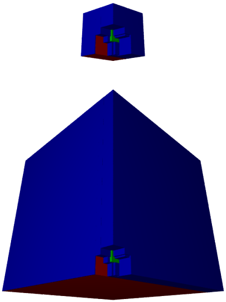
Prior findings

Different reasons identified

(cf. L. Judt et al. @ MCDM 2011)

- 2D case
  special handling of boundary solutions
- 3D case
  adaptive reference point yields different results if
  - reference point changes
  - performance is calculated w.r.t. fixed reference point





Research question

- Does this matter?
  Negative influence on final outcome?

Selection strategies under investigation

4 strategies under investigation, combinations of

- adaptive and fixed reference point
- whether or not decreases are accepted

Adaptive reference point schemes

- `adaptive/with`
  - adaptive reference point
  - decreases in hypervolume are accepted
  - the standard case
- `adaptive/without`
  - adaptive reference point
  - decreases in hypervolume are omitted
  - selection not accomplished
  - a repairing case

Selection strategies under investigation

Fixed reference point schemes

- `fixed/without`
  - reference point fixed like for indicator calculation
  - decreases are omitted
  - the assured implementation

  - decreases may appear in 2D
  - no decreases possible for higher dimensional case
- `fixed/with`
  - reference point fixed
  - decreases in hypervolume are accepted
  - the impossible case (decreases are not expected)

  - just to have all combinations?

Experimental setup

- Different dimensions under investigation
  - 2D: ZDT1 - ZDT4
  - 3D: DTLZ1 - DTLZ3
  - 4D: DTLZ2
- fixed parameterization for variation
- $\mu = 100$
- 50 runs per combination
- 100 000 fitness function evaluations each

Reference points considered

ZDT1 – ZDT3:	$[11, 11]$	DTLZ1:	$[1000, 1000, 1000]$
ZDT4:	$[1000, 1000]$	DTLZ2:	$[11, 11, 11]$
4-dim. DTLZ2:	$[11, 11, 11, 11]$	DTLZ3:	$[2000, 2000, 2000]$

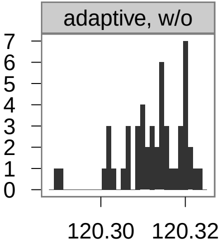
Presentational setup

Factors to consider

- test function
- reference point handling
- number of fitness function evaluations (FFE)

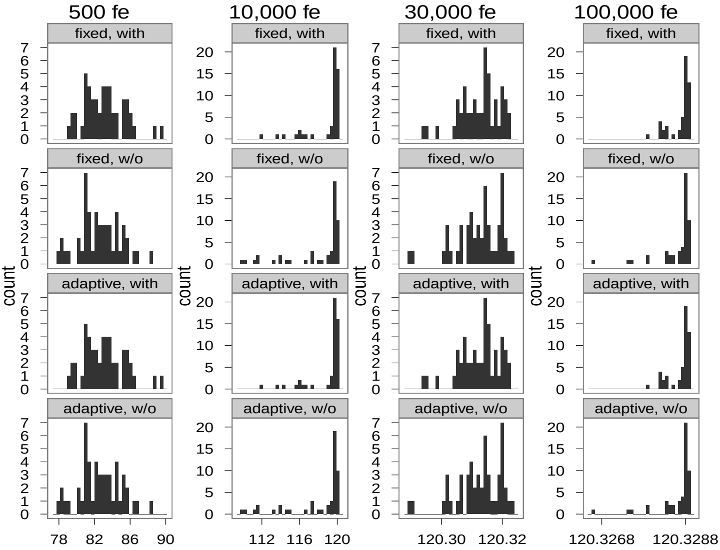
Histograms of 50 independent runs

1. progression over FFE for one representative instance
2. results for all instances at 30 000 FFE (recommended number in literature)

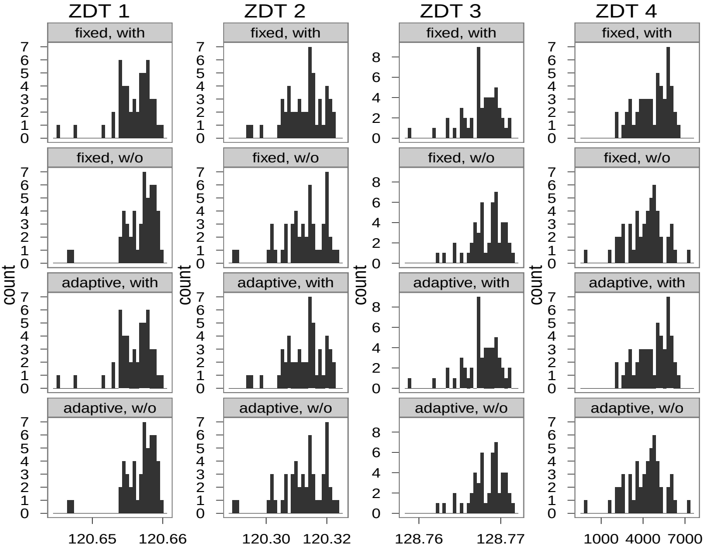


Coming next: Grid of 4x4 such histograms ...

Results: 2D, ZDT2 progression



Results: 2D, all test functions at 30 000 FFE



(ZDT4: value of 990 000 subtracted from actual hypervolume value!)

Results: 2D

- Two with and two without schemes provide identical results
- Results solely depending on acceptance of decreases

In line with expectations! In 2D:

- Decreases can arise for fixed strategies
- Decreases independent of reference point handling

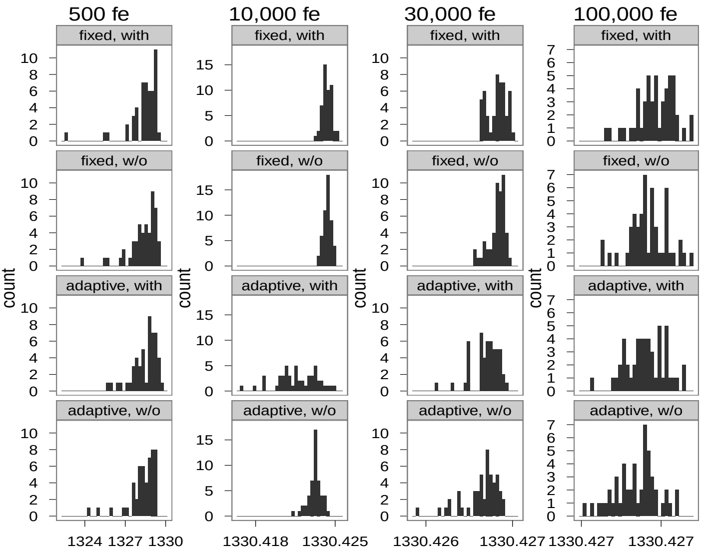
However

- more and more hypervolume gained on progression
- similar distributions at the end

Best variant? No clear evidence!

Advise: Continue using standard implementation

Results: 3D, DTLZ2, progression



Results: 3D, all test functions at 30 000 FFE

Most striking

- variant `fixed/without` differs from variant `fixed/with`

- Decreases (in 3D) have not been expected due to invoking same reference point for selection as for performance measure
- Moreover, they were thought to be impossible!

- Even strict Pareto compliance is violated, i.e.
dominating point accepted, but hypervolume decreases

  Solution sets $\{\mathbf{y}_1, \ldots, \mathbf{y}_n, \mathbf{y}_{n+1}\}$ found with

  $$\widehat{hyp}_{\mathbf{r}}(\{\mathbf{y}_1, \ldots, \mathbf{y}_n\}) < \widehat{hyp}_{\mathbf{r}}(\{\mathbf{y}_1, \ldots, \mathbf{y}_{n-1}, \mathbf{y}_{n+1}\})$$

  even though $\mathbf{y}_n$ dominates $\mathbf{y}_{n+1}$
  ($\widehat{hyp}_{\mathbf{r}}$: numerical approximation of hypervolume w.r.t. reference point $\mathbf{r}$)

Results: 3D, all test functions at 30 000 FFE

Deeper investigation

- Differences very small: in 15th or 16th significant digit (using double precision floating point numbers)
- changes in hypervolume are of same order as observed errors in numerical approximation (of hypervolume)
- Even worth:
  Only able to detect effects if one solution dominates the other but they also occur for incomparable solutions
  $\Rightarrow$ no criterion to decide

Results

Mean number of decreases with `fixed/with`

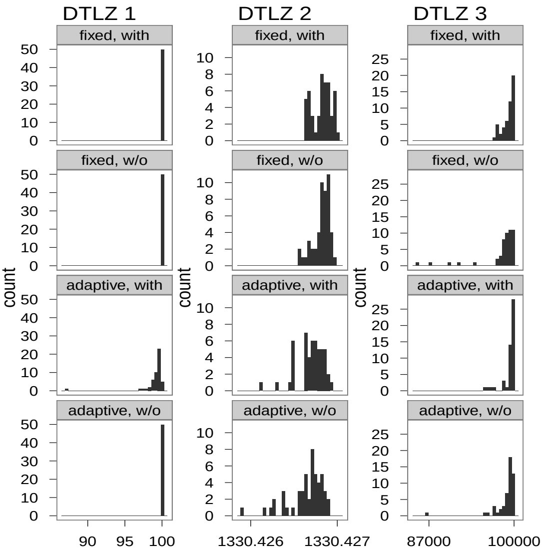
	after 500 fe	after 10 000 fe	after 30 000 fe	after 100 000 fe
DTLZ1	7	34	166	1274
DTLZ2	3	4	4	5
DTLZ3	10	44	200	551
4dim. - DTLZ2	3	3	3	3

Reference points considered:

ZDT1 – ZDT3:	$[11, 11]$	DTLZ1:	$[1000, 1000, 1000]$
ZDT4:	$[1000, 1000]$	DTLZ2:	$[11, 11, 11]$
4-dim. DTLZ2:	$[11, 11, 11, 11]$	DTLZ3:	$[2000, 2000, 2000]$

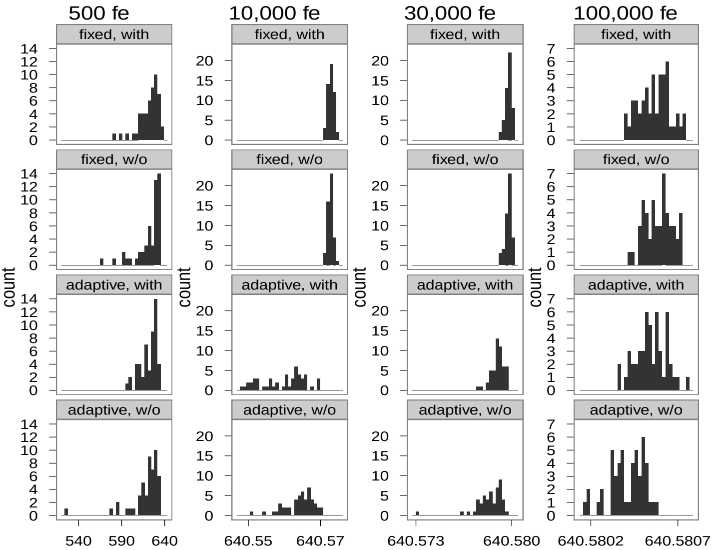
- Avoid reference points far away from Pareto-front?
- Still a question whether decreases matter!

Results: 3D, all test functions at 30 000 FFE



(Subtractions: 999 999 900 for DTLZ1, 7 999 900 000 for DTLZ3!)

Results: 4D, progression



(Subtractions: 14 000 for 4D DTLZ2!)

Summary

- Selection variants perform differently

  But

No significant differences are observed!

- all clear for hypervolume selection
- operator seems to be very reliable and robust

Implications?

- be aware of possible issues from numerics!
  - possible influence on hypervolume approximation techniques?
  - small mistakes in hypervolume will not worsen overall performance
- New reference point adaptation techniques?

Thanks,

questions?

Graduate or undergraduate student?
There is a GECCO students workshop!
Deadline: March, 28th

More info? Contact me or visit
`gecco2013studentws.tiddlyspace.com`
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