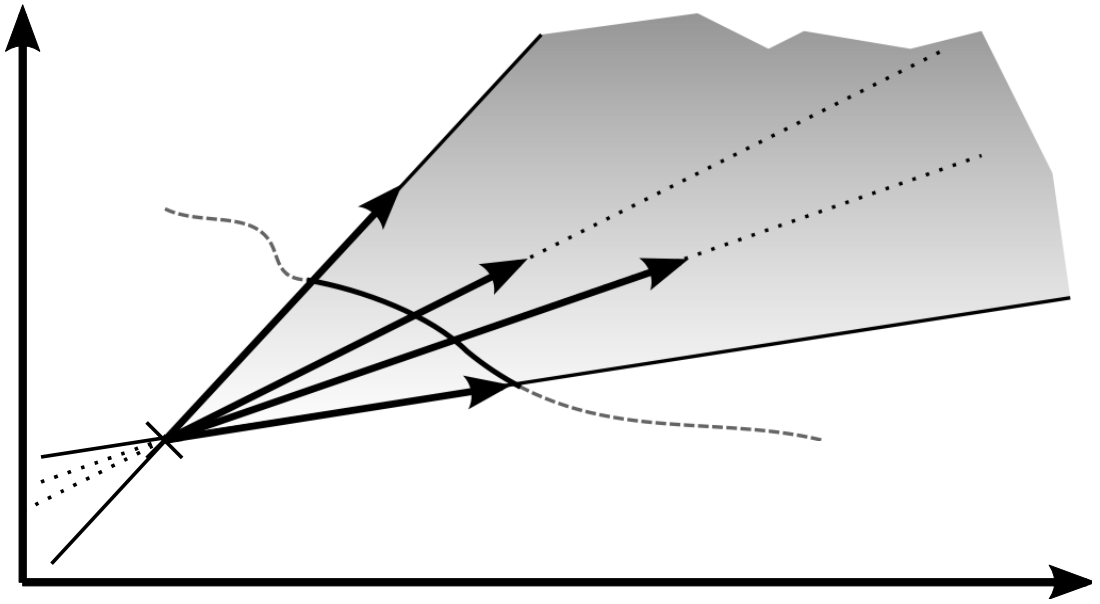
Preference Articulation by Means of the R2 Indicator

Tobias Wagner, Heike Trautmann, Dimo Brockhoff



Motivation

Set-Based Multi-Objective Optimization

- Consideration of sets of solutions (e.g., EMOA population)
- Fixed set size μ
- Evaluation based on a performance indicator PI (e.g., hypervolume)
  → Implicit expression of preferences

What are the preferences actually encoded by the choice of the PI?

Research Questions

Focus on the unary R2 indicator (*Brockhoff et al.*, 2012)

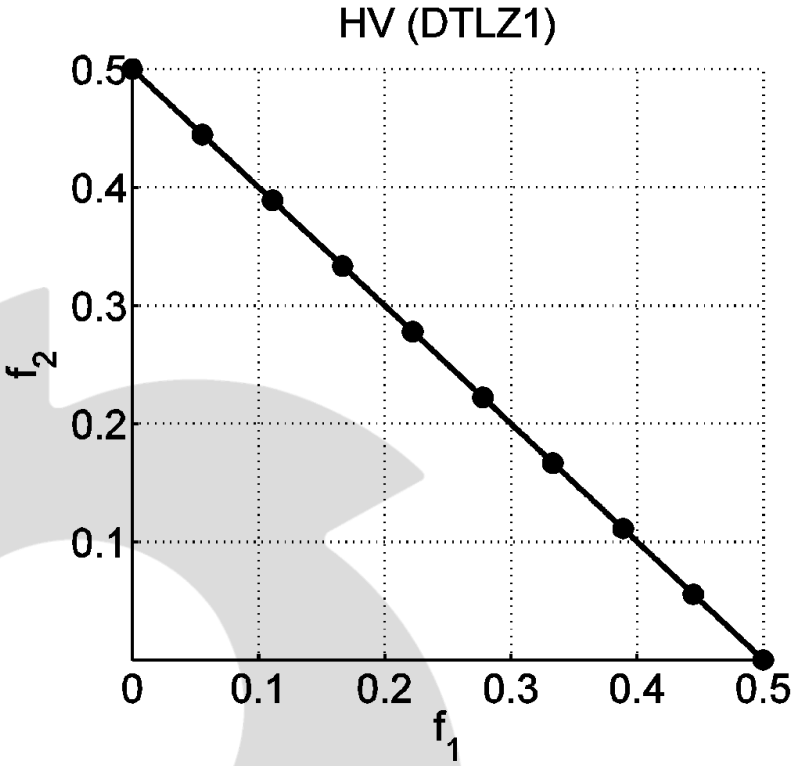
$$R2^*(A, \Lambda, z^*) = \frac{\sum_{\lambda \in \Lambda} \min_{a \in A} \left(\max_{j=1,\ldots,m} \lambda_j |z_j^* - a_j| \right)}{|\Lambda|}$$

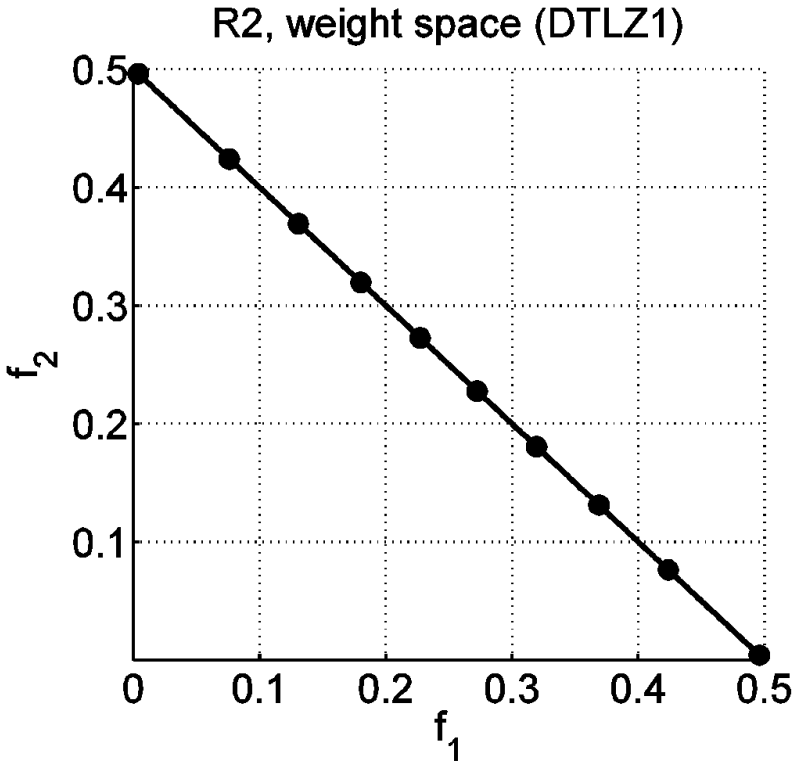
Effect of the parameters

- Position of the ideal point z^*
- Restriction of the weight space $\mathcal{W}$ ($\Lambda \subset \mathcal{W}$)
- Density of the weight vector distribution

R2 Indicator vs. Hypervolume

- Given a uniform density of weight vectors, the R2 indicator prefers balanced over extreme trade-offs (*Brockhoff et al.,* 2012)
- About N = 50μ weight vectors are required for stable results (CMA-ES)



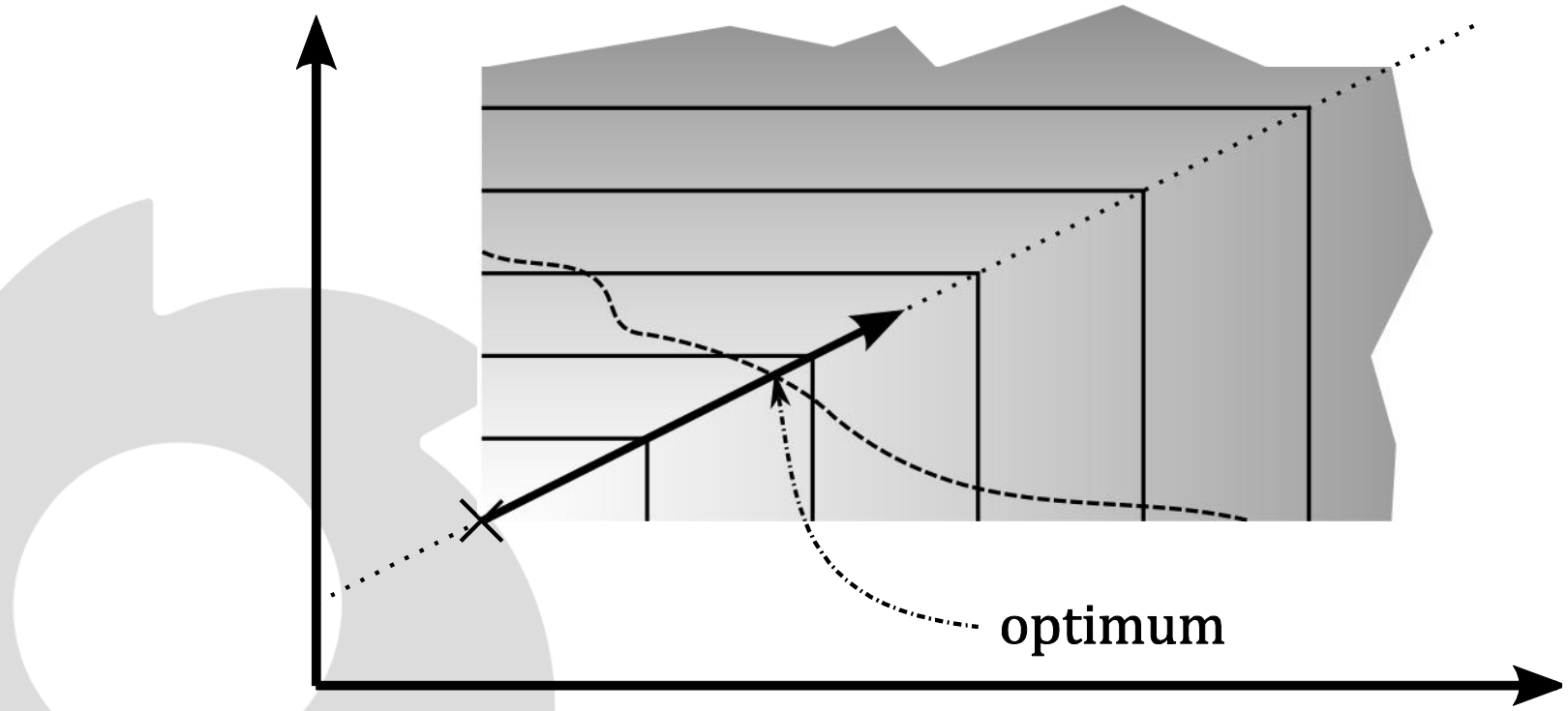


Approximated μ-optimal sets for a linear Pareto front

Target Vector

Definition 1

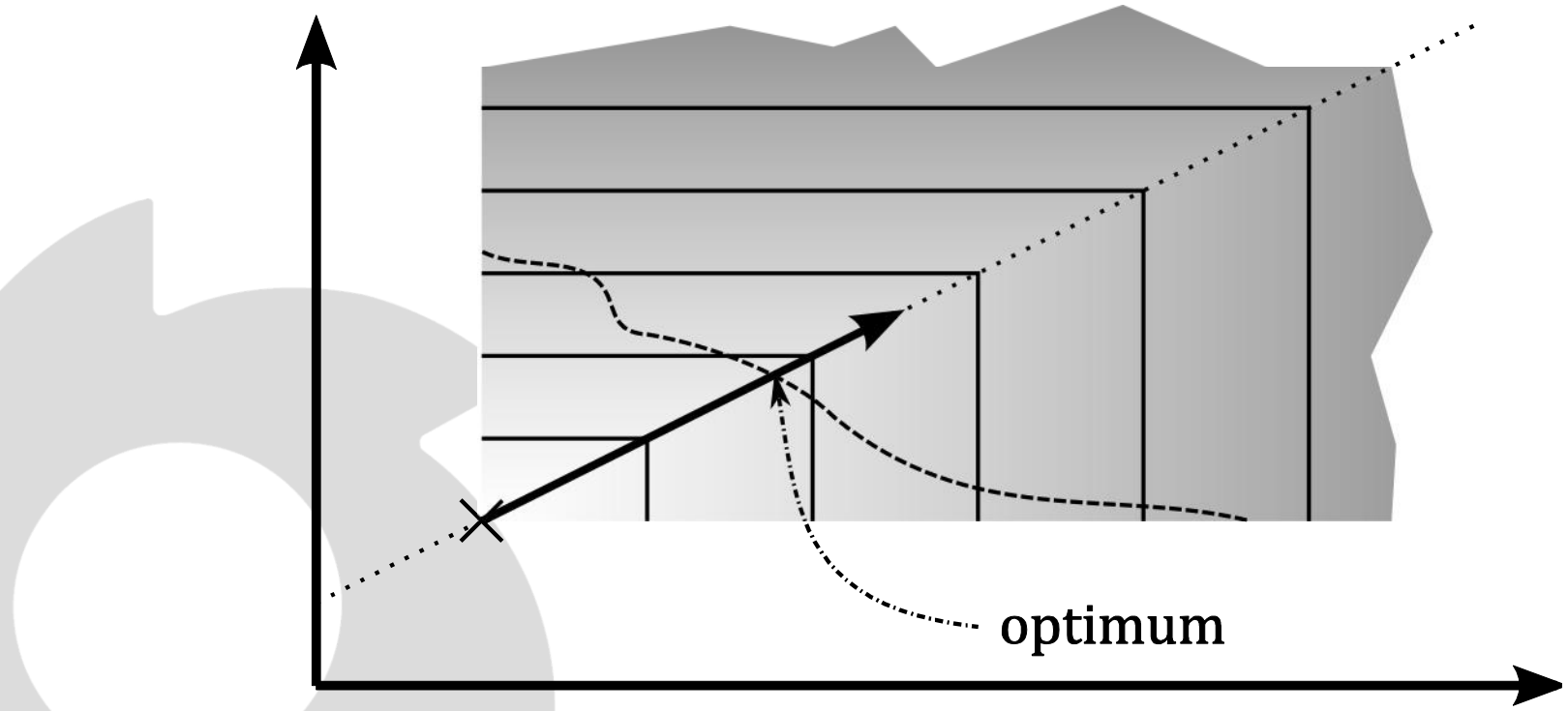
Given a weight vector λ, we call the feasible solution mapped to the minimum of the ASF $\min_{a} \lambda_i(a_i - z^*_i)$ the corresponding optimum solution



Target Vector

Definition 2

Given the target direction $z^* + m\,t$ to the associated weight vector $\lambda = (\beta/t_1, \ldots, \beta/t_k)$, we call any vector $m\,t$ $(m > 0)$ a target vector for the corresponding target direction



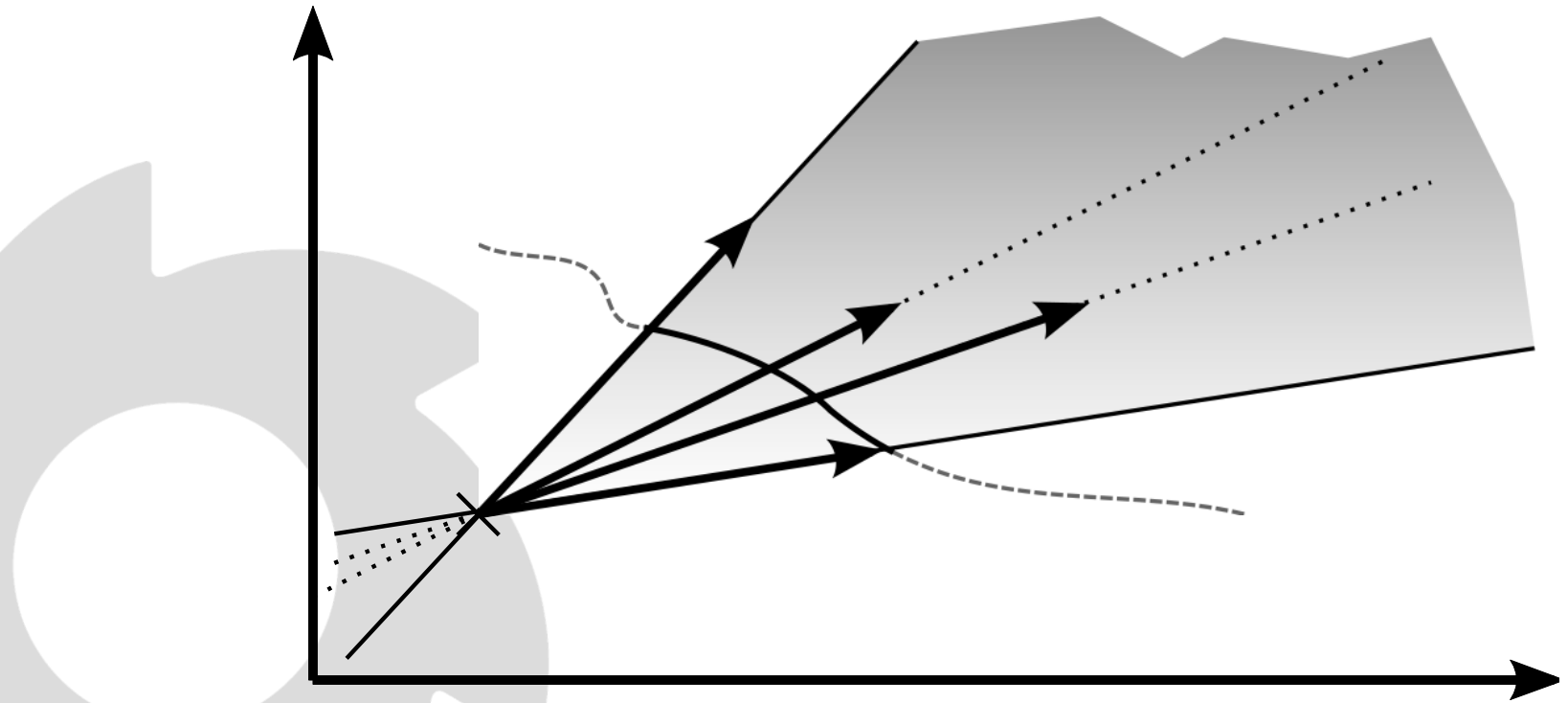
Target Cone

Definition 4

The minimum cone including the target directions t related to all weight vectors $\lambda \in \Lambda$, is denoted as target cone

Theorem 1

The solutions of the μ-optimal set lie within the target cone



Variation of the Weight Vector Density

Algorithm 1 Generate Weight Vectors

function GENERATEWEIGHTVECTORS(γ, n)

▷ γ: *skew factor,* n: *number of vectors, must be odd*

weights.x $\leftarrow$ sequence$(0, 0.5$,stepsize$= 1/(n-1))$ ▷ *distribute uniformly*

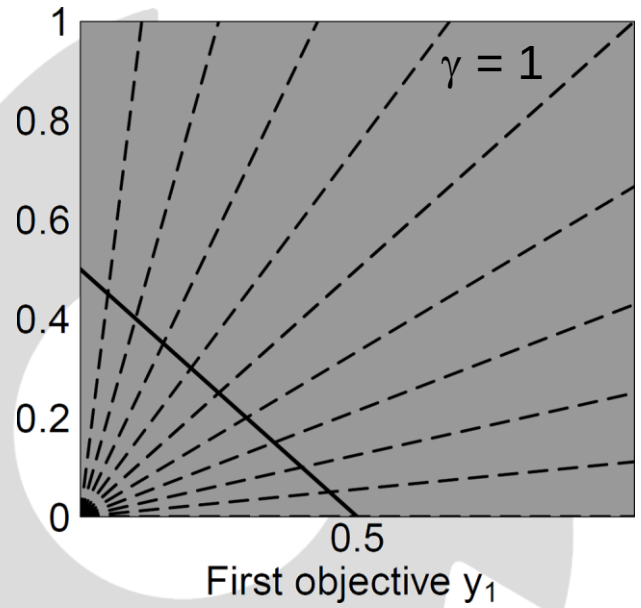
weights.x $\leftarrow 0.5^{-\gamma+1}$(weights.x)$^{\gamma}$ ▷ *skew and rescale weights*

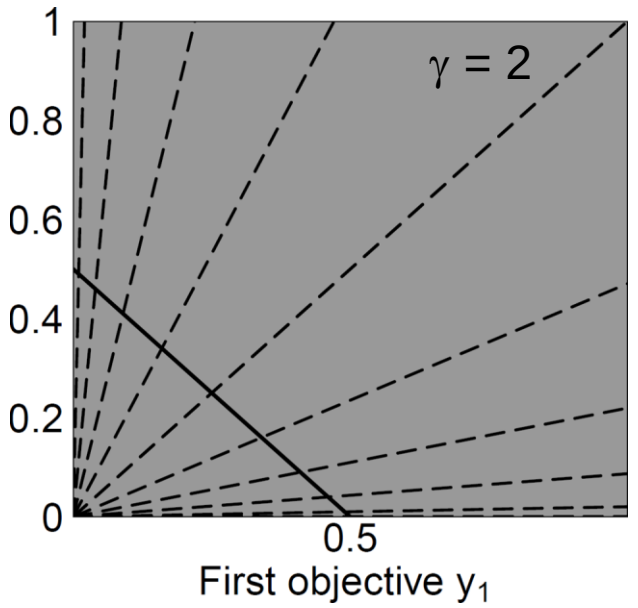
diffs $\leftarrow 1 -$ reverse(weights.x$[1, \ldots, (n-1)/2]$) ▷ *Mirroring differences along* $w = 0.5$

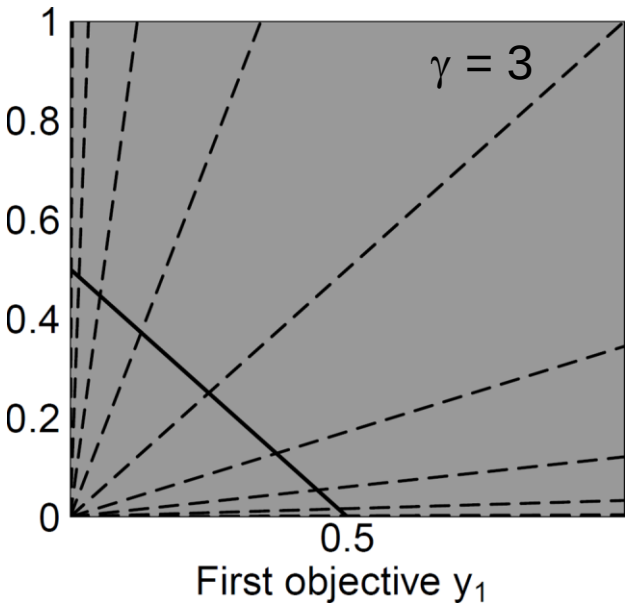
weights.x $\leftarrow$ concatenate(weights.x, diffs) ▷ *Build symmetric distribution*

return weights.x

end function

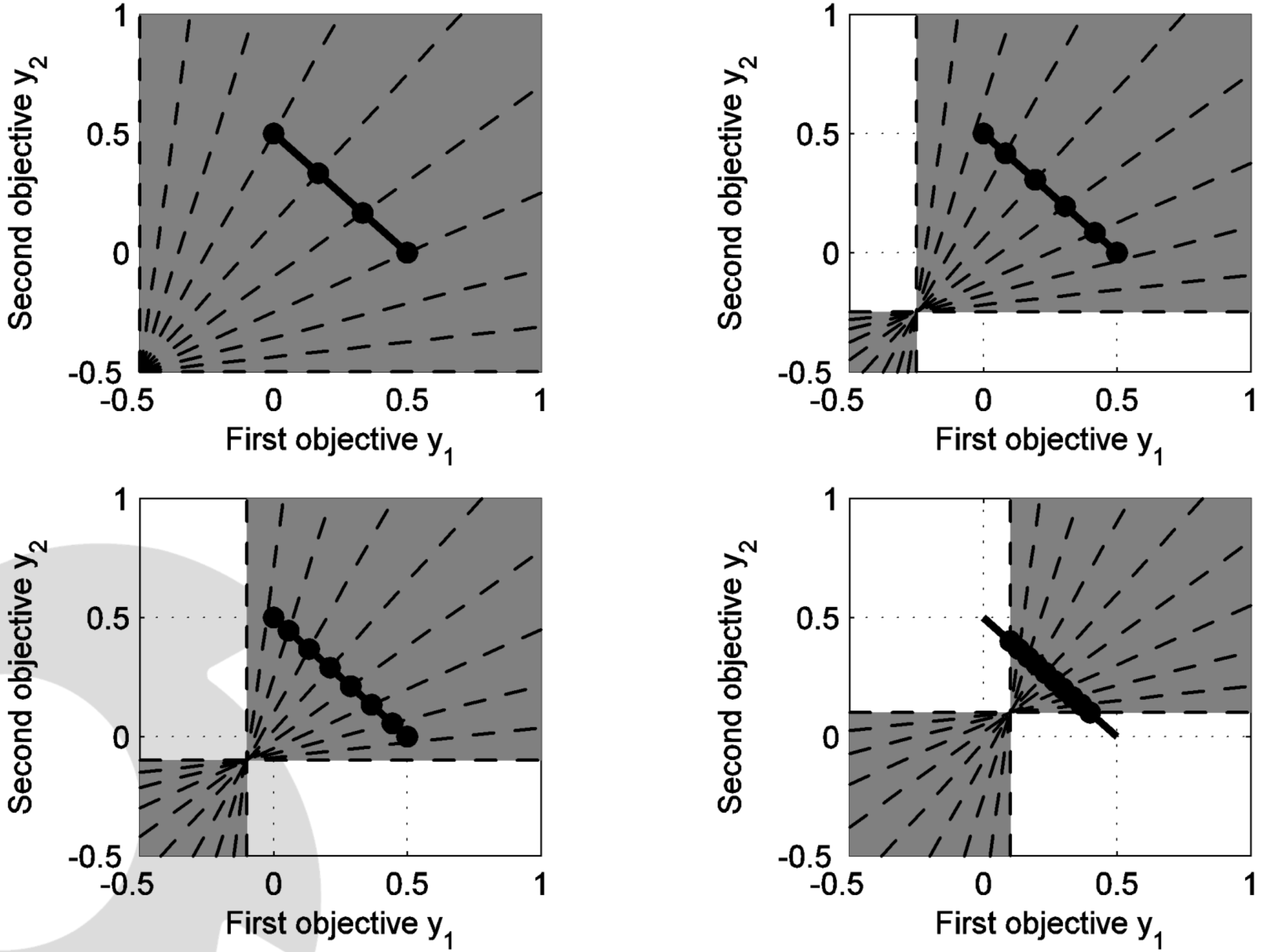




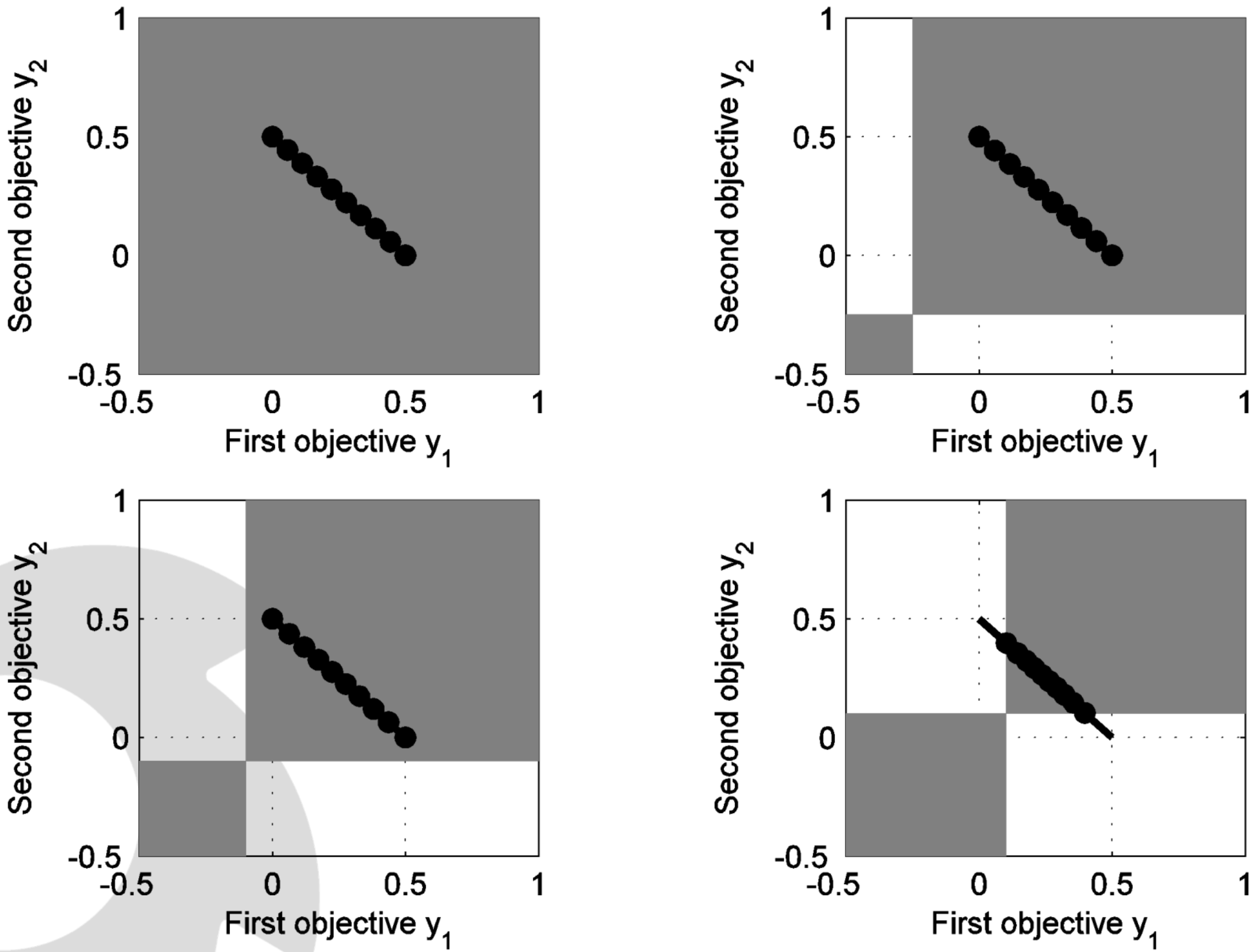


Results

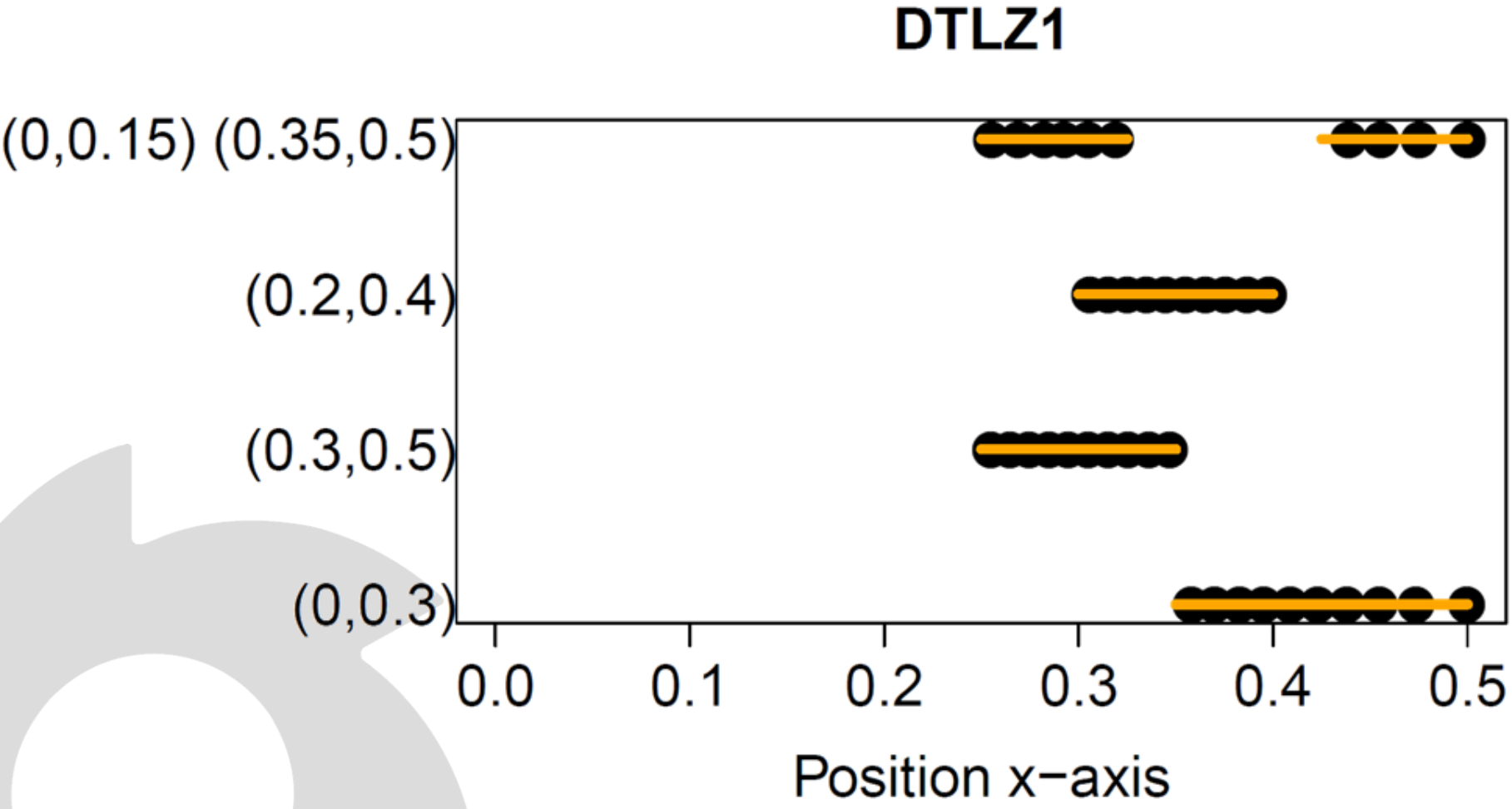
Position of the Ideal Point (N = 10)



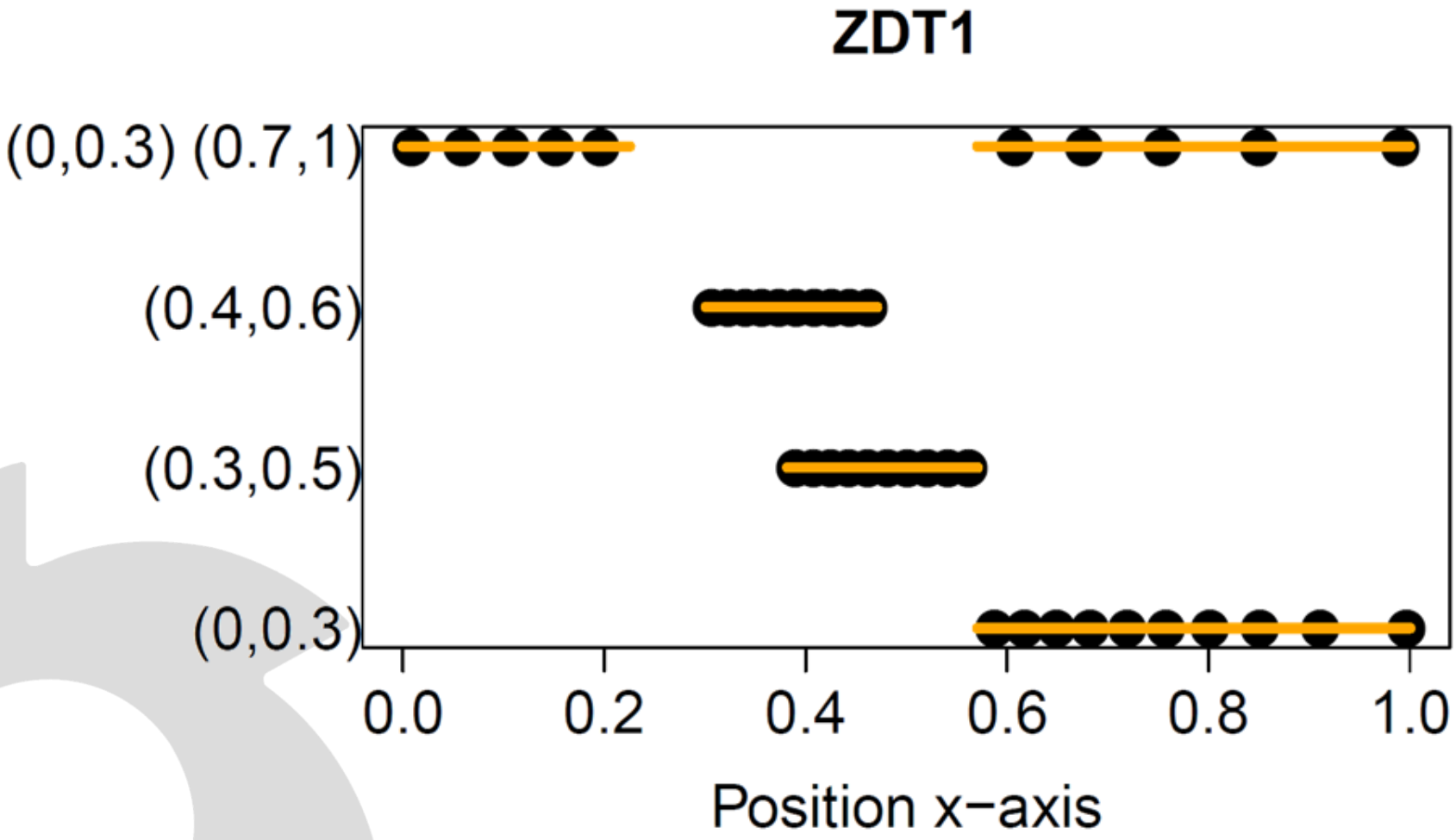
Position of the Ideal Point (N = 1000)



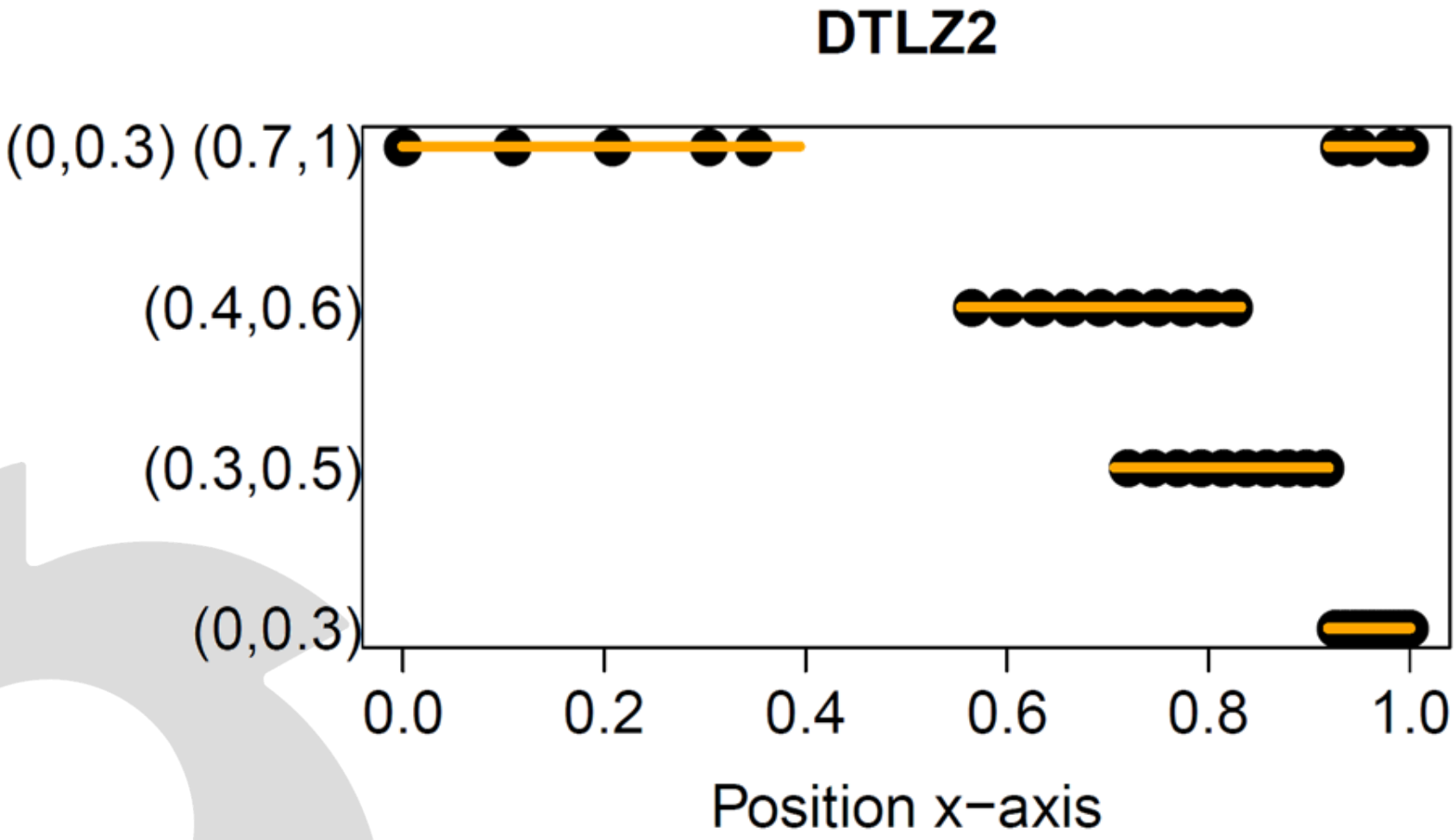
Restriction of the Weight Space



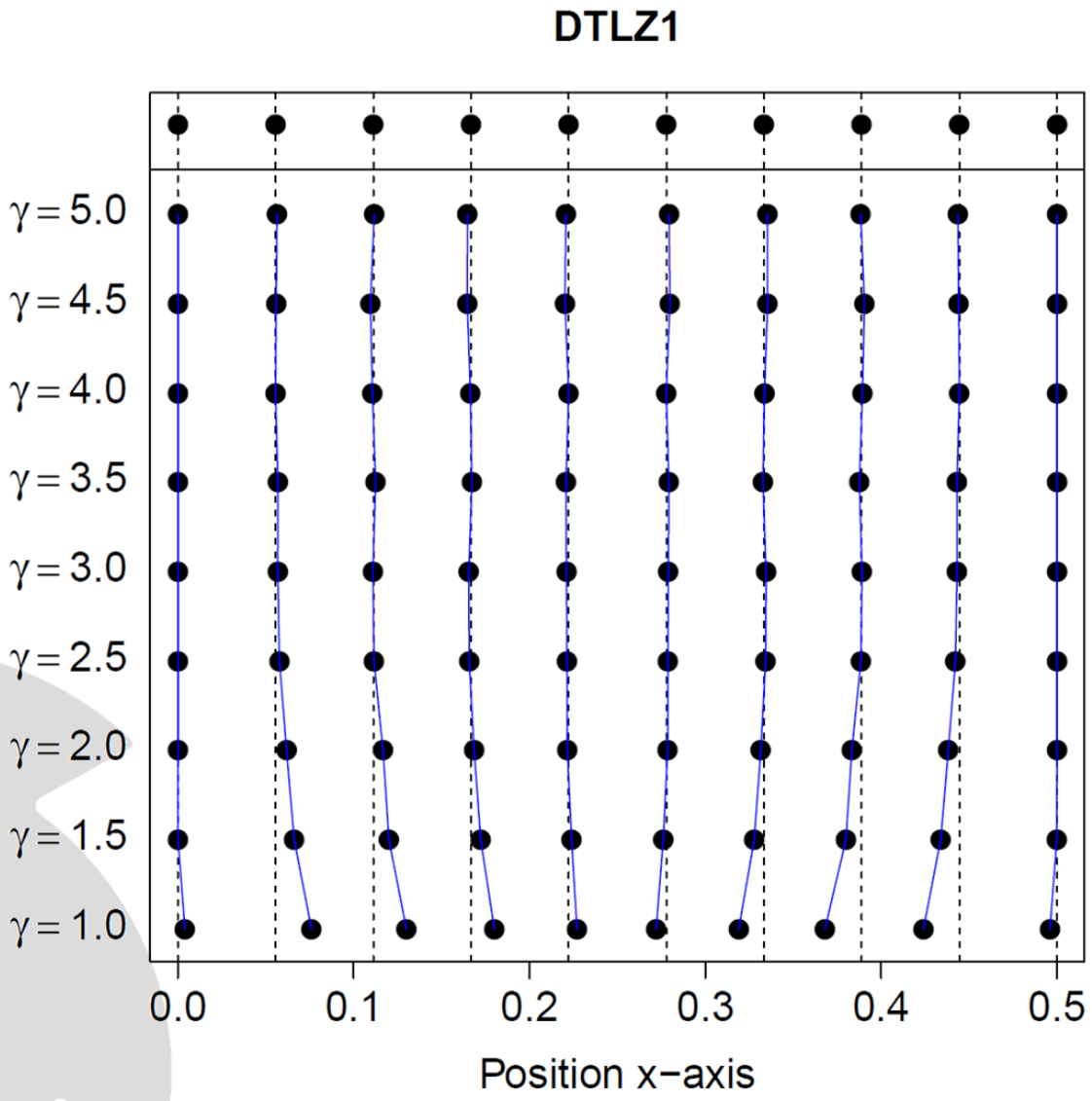
Restriction of the Weight Space



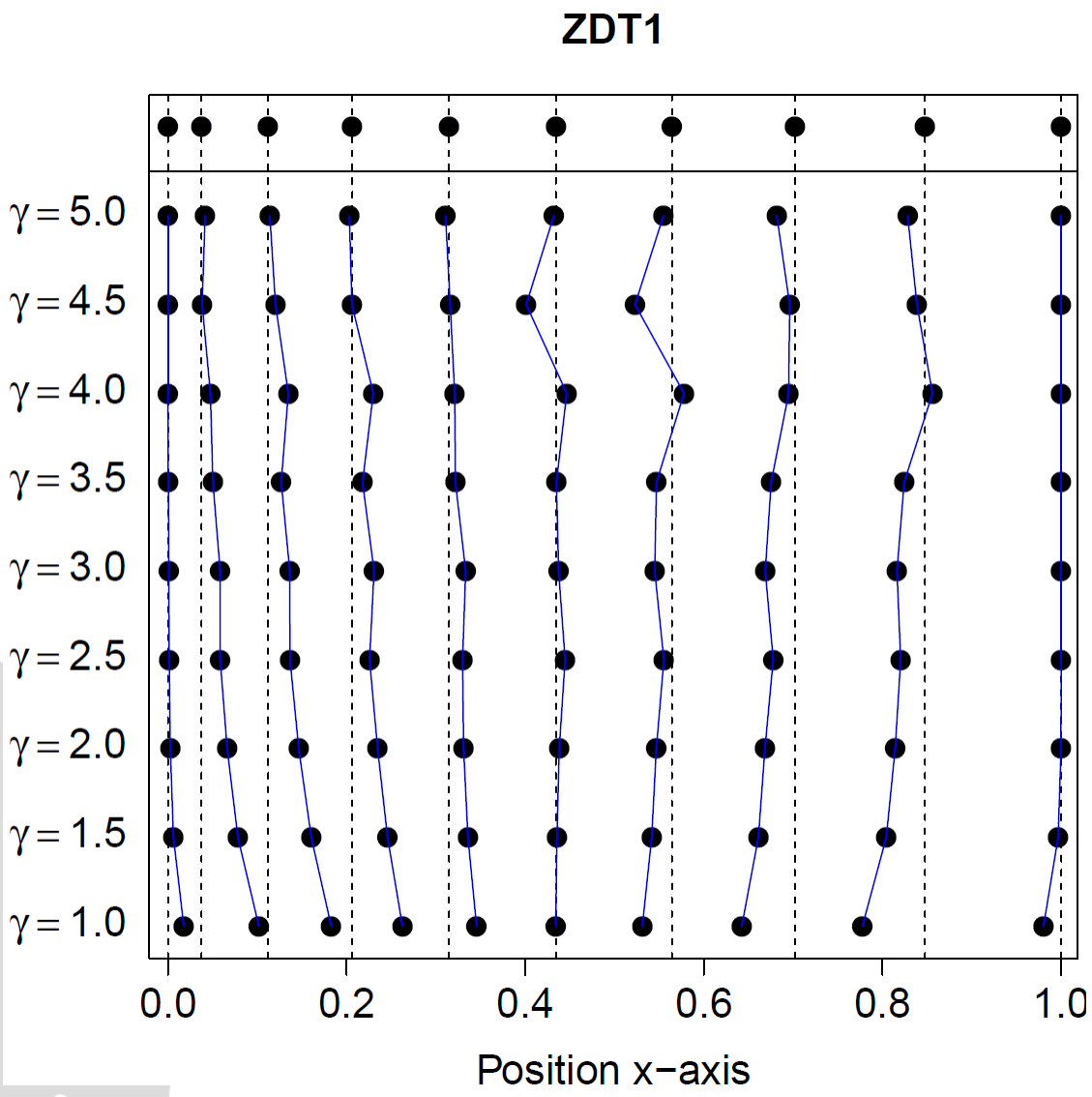
Restriction of the Weight Space



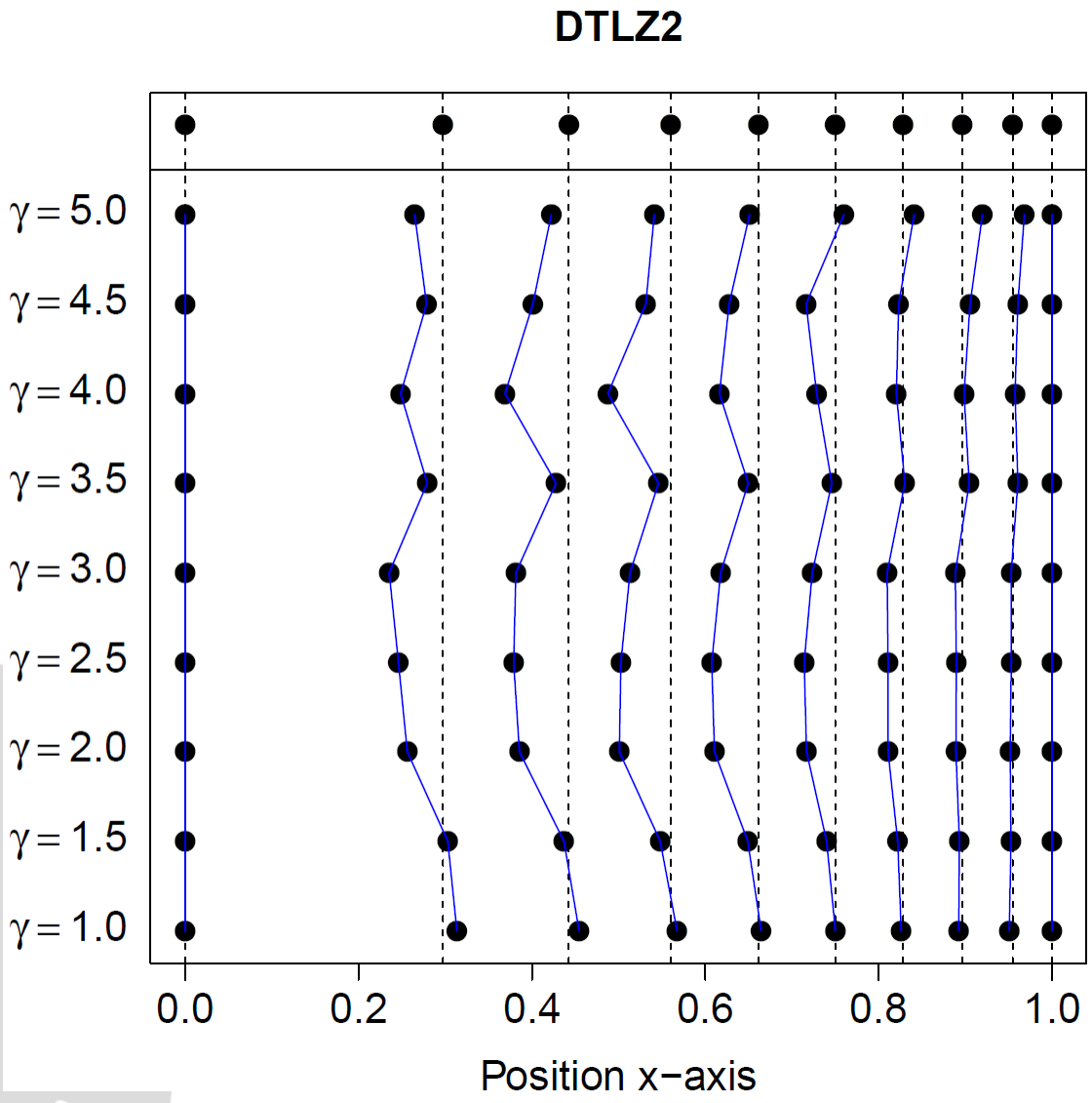
Density of the Weight Vector Distribution



Density of the Weight Vector Distribution



Density of the Weight Vector Distribution



Conclusion and Outlook

The R2 indicator allows...

...a focus on parts of the Pareto front

- Position of the ideal point (simple)
- Restriction of the weight space (more complex)

...the density of the approximation to be controlled

- Transformation of the weight vectors (very complex)

In future...

...the robustness/ approximation of the μ-optimal sets has to be improved

- Analytical approaches
- Problem-oriented heuristics

...the scalability of the R2 indicator has to be analyzed

- Number of weight vectors required for stable results
- Derivation of the μ-optimal sets

Acknowledgement

Thank you for your attention

Deutsche Forschungsgemeinschaft DFG

SFB 823

www.transregio-30.com

The talk is based on investigations of the projects D5 and B4 of the collaborative research centers SFB/TR TRR 30 and SFB 823, respectively, which are kindly supported by the Deutsche Forschungsgemeinschaft (DFG).

In addition, the authors acknowledge support by the French national research agency (ANR) within the Modéles Numérique project “NumBBO - Analysis, Improvement and Evaluation of Numerical Blackbox Optimizers”.