Optimization of Adaptation:

A Multi-objective Approach for Optimizing Changes to Design Parameters

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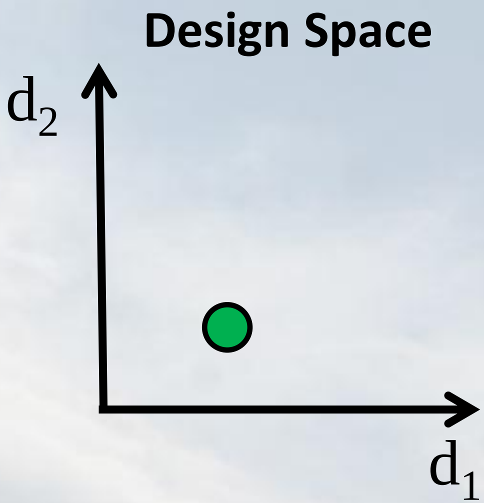
This research is supported by the Marie Curie Fellowship program

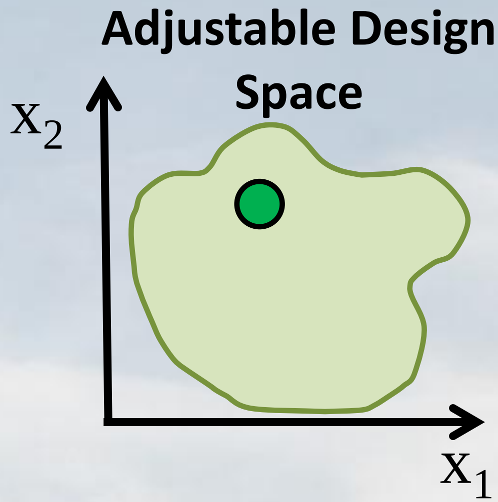
Optimization of Adaptive Products

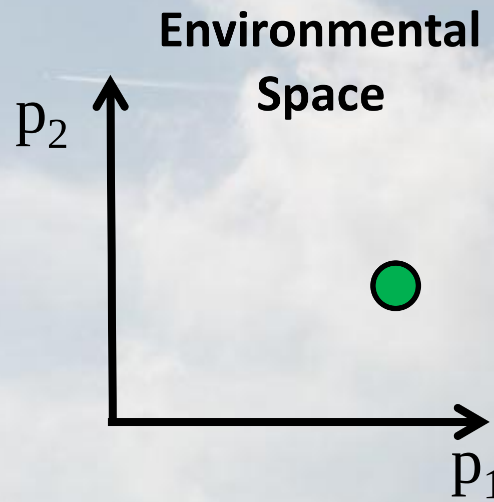
The Context of this Study

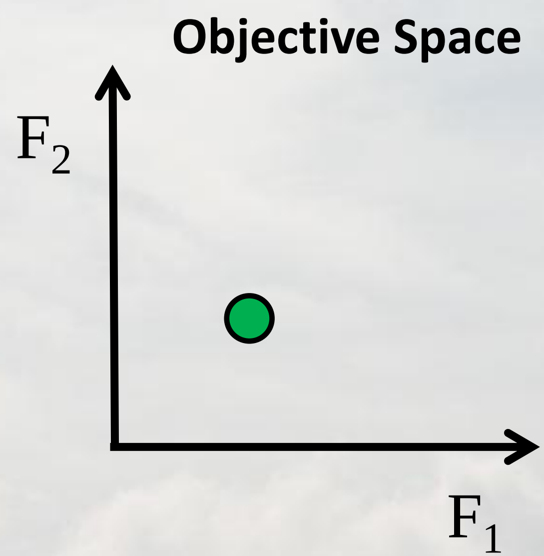
- Can react to changes in environmental conditions
- Include adjustable variables for late decision

Optimization of Adaptive Products



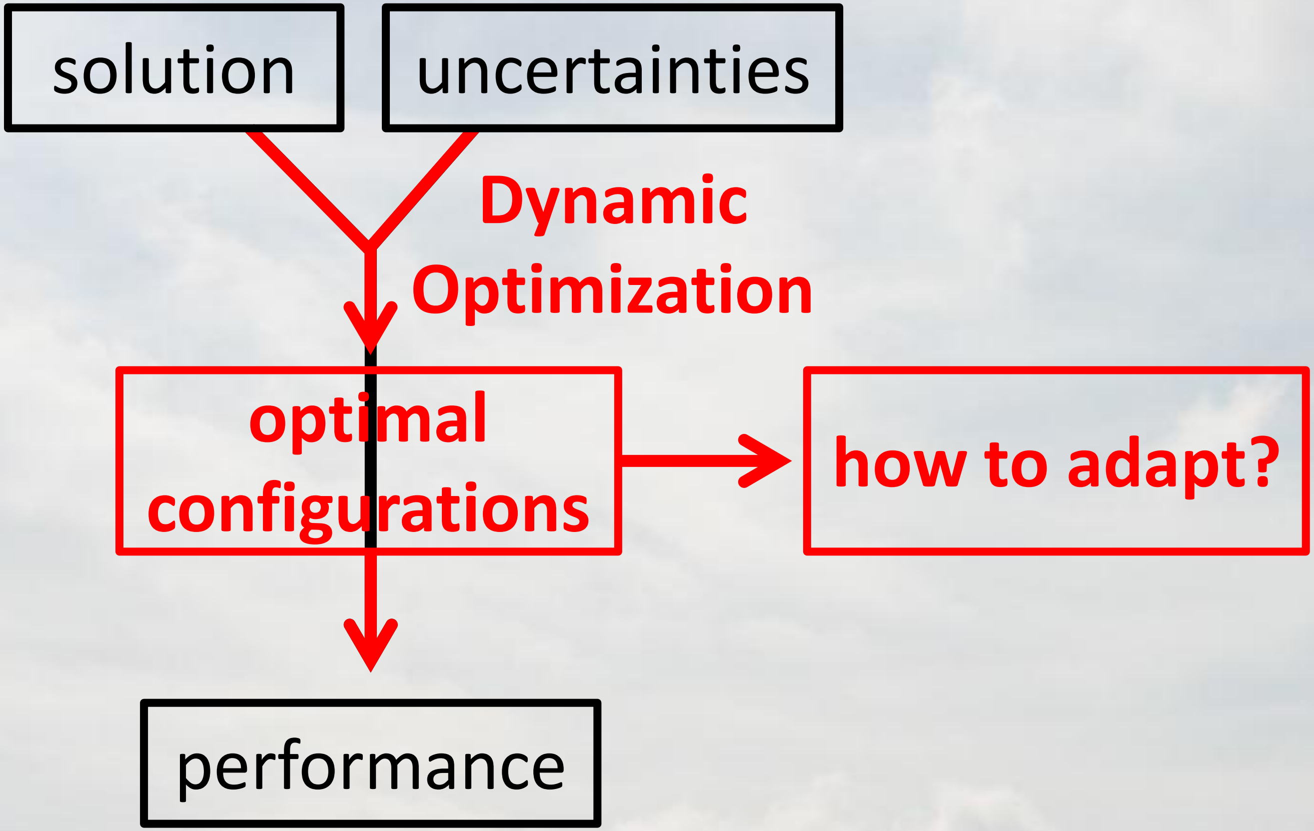






Optimization of Adaptive Products

Active Robust Optimization Problem

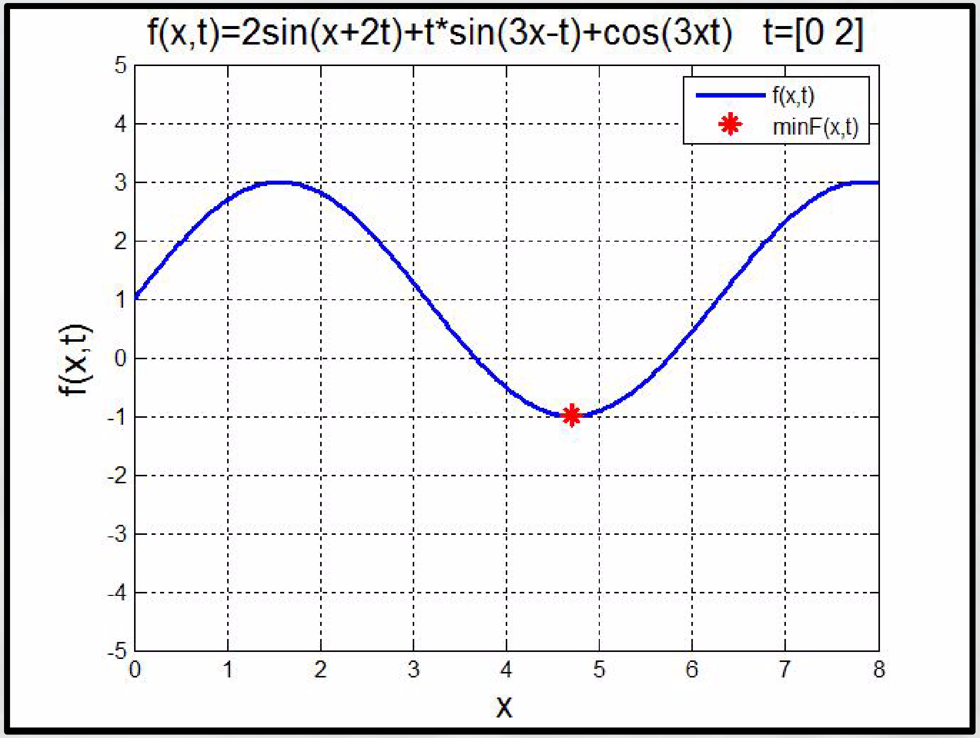


Dynamic Optimization

$$\min_{\boldsymbol{x} \in Q} f(\boldsymbol{x}, t)$$

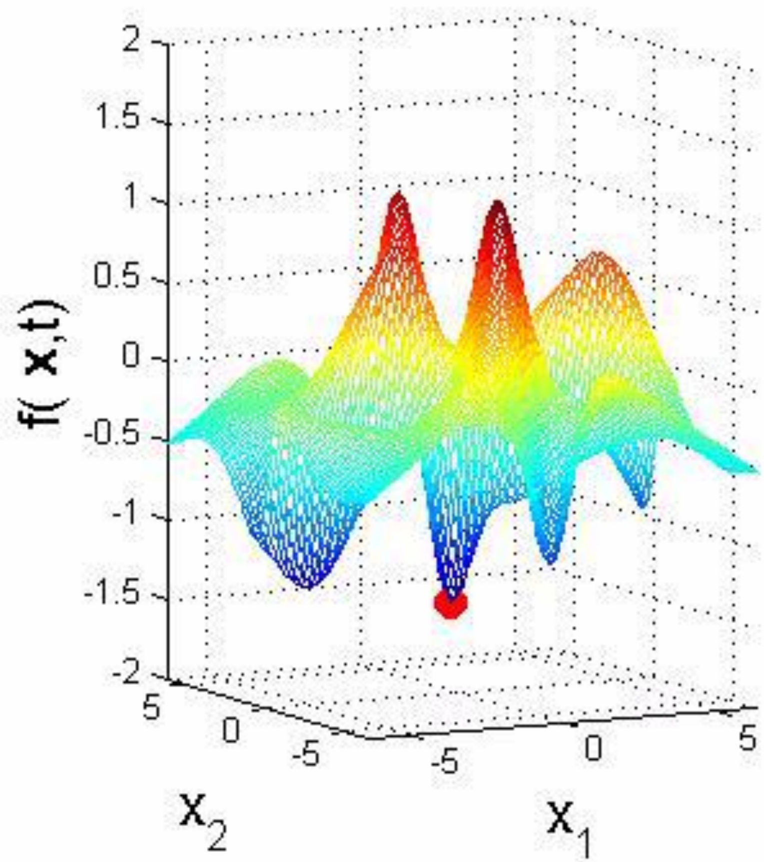
$$s.t. \quad g_i(\boldsymbol{x}, t) \geq 0 \;, \quad (i = 1, \ldots, I)$$

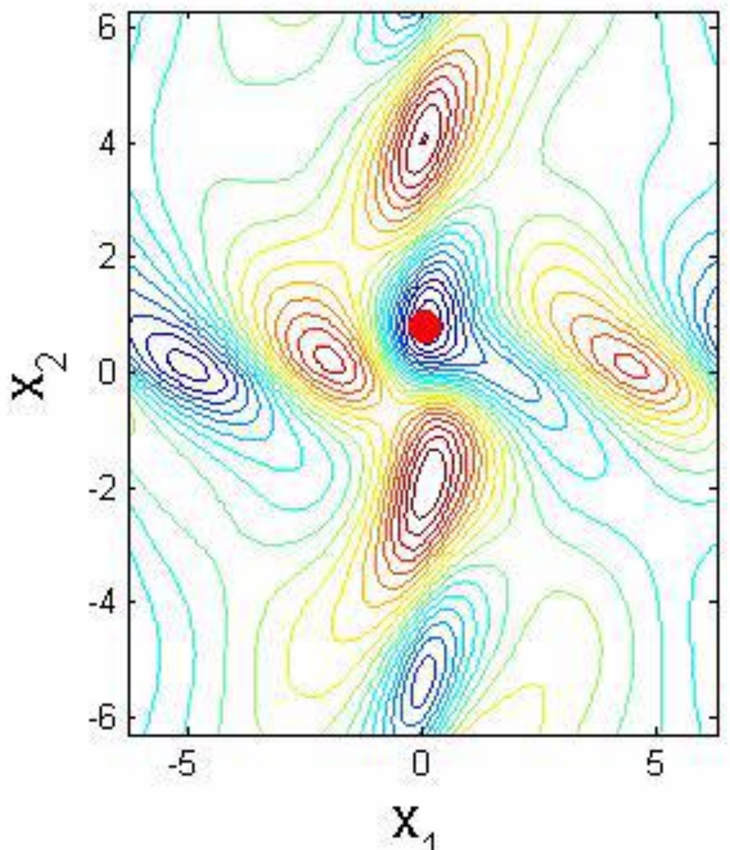
$$h_j(\boldsymbol{x}, t) = 0 \;, \quad (j = 1, \ldots, J)$$



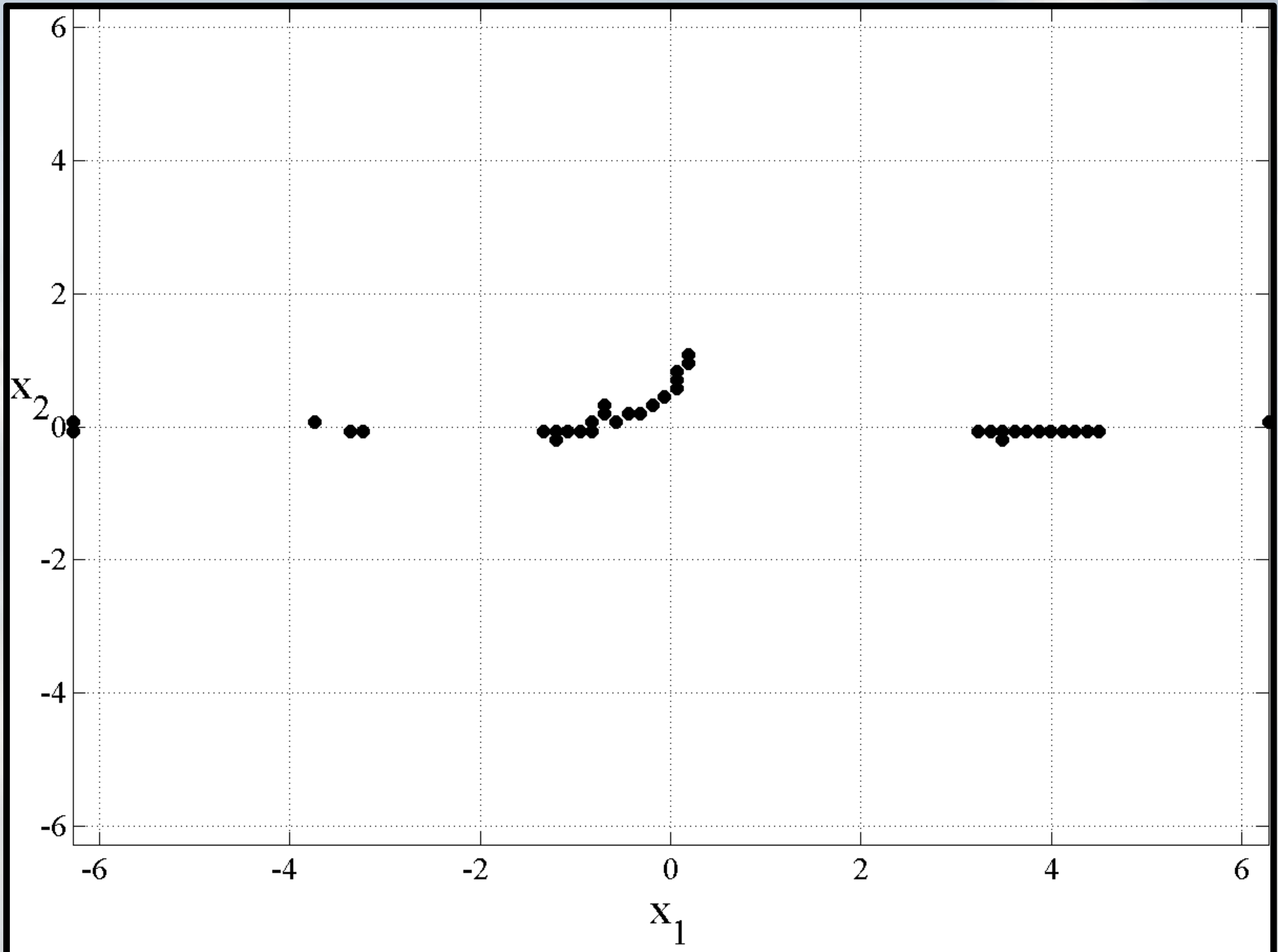
Dynamic Optimization

$$\min_{x \in Q} f(\boldsymbol{x}, t) = \frac{\sin(x_1 + x_2 + t/20) - \cos\left(x_1 - x_2 + \left(\frac{t}{10000}\right)^2 + 4\right)}{{x_2}^2 + 1} + \frac{{x_1}^2}{80}$$

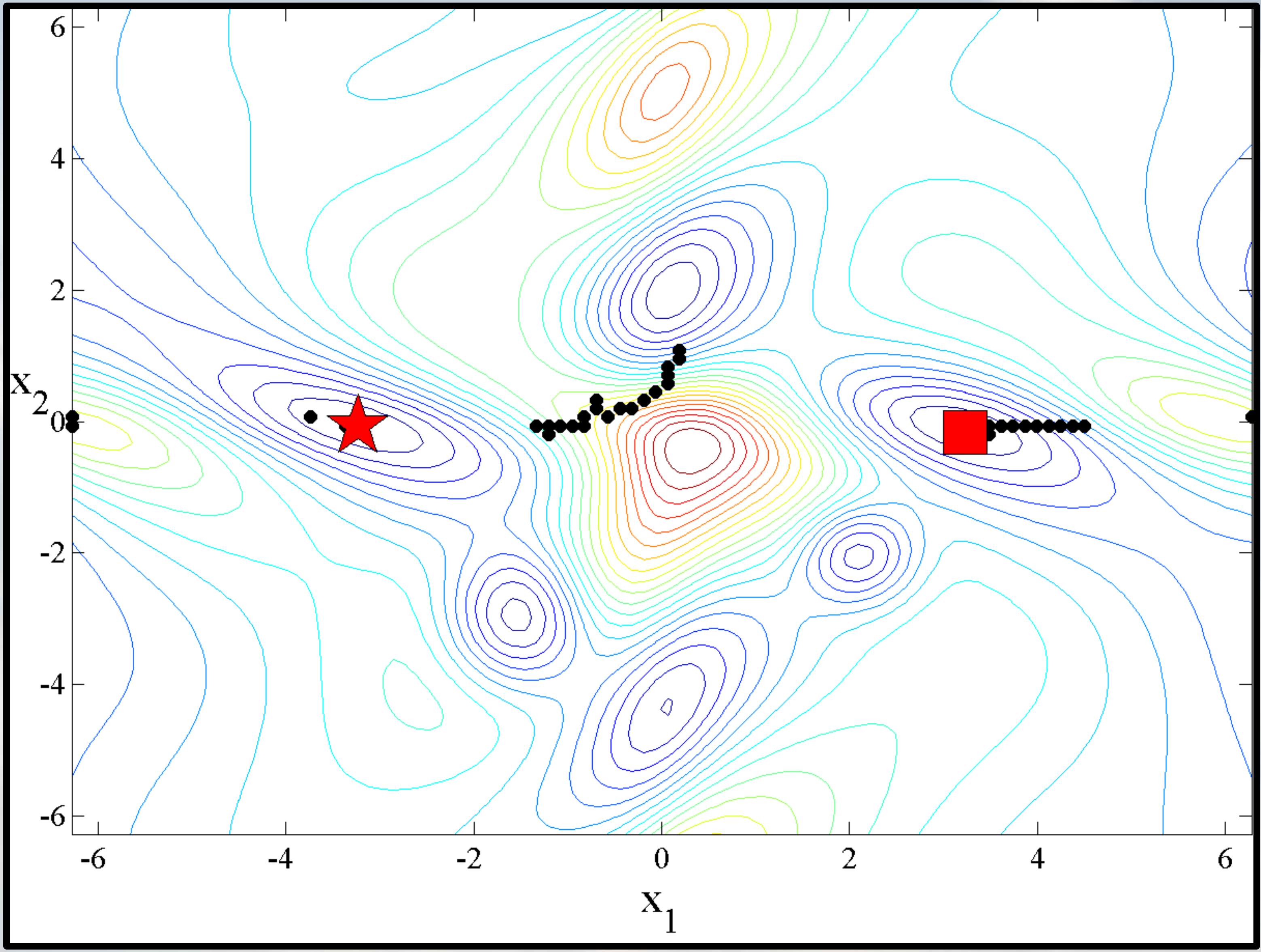




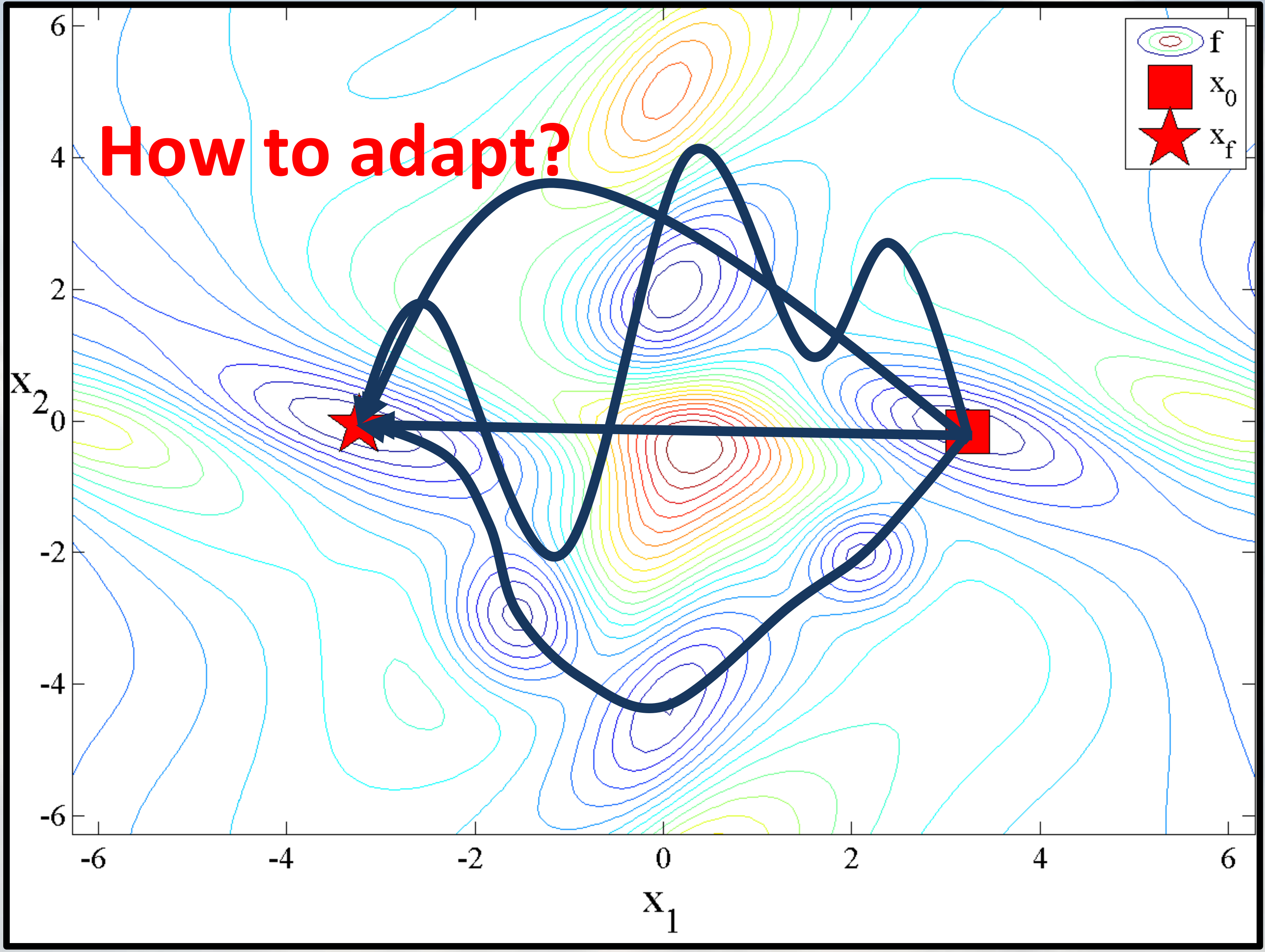
Dynamic Optimization



Dynamic Optimization



Dynamic Optimization



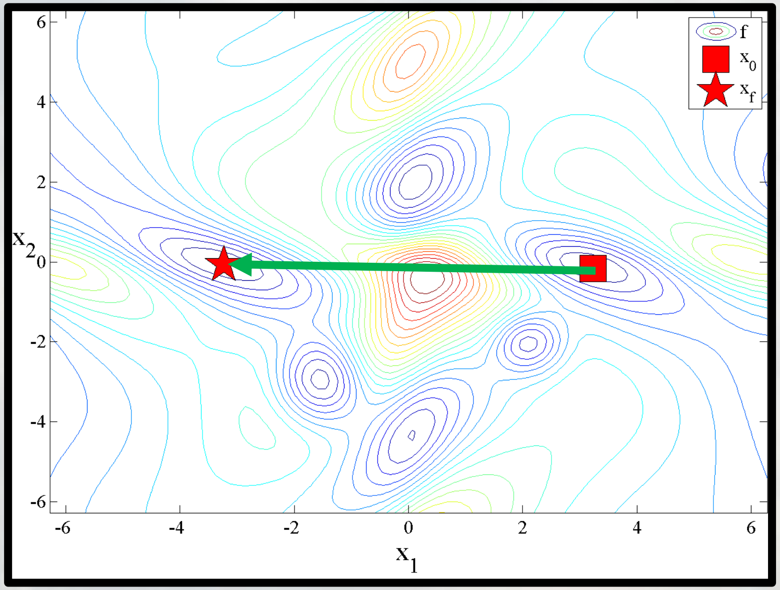
How to Adapt?

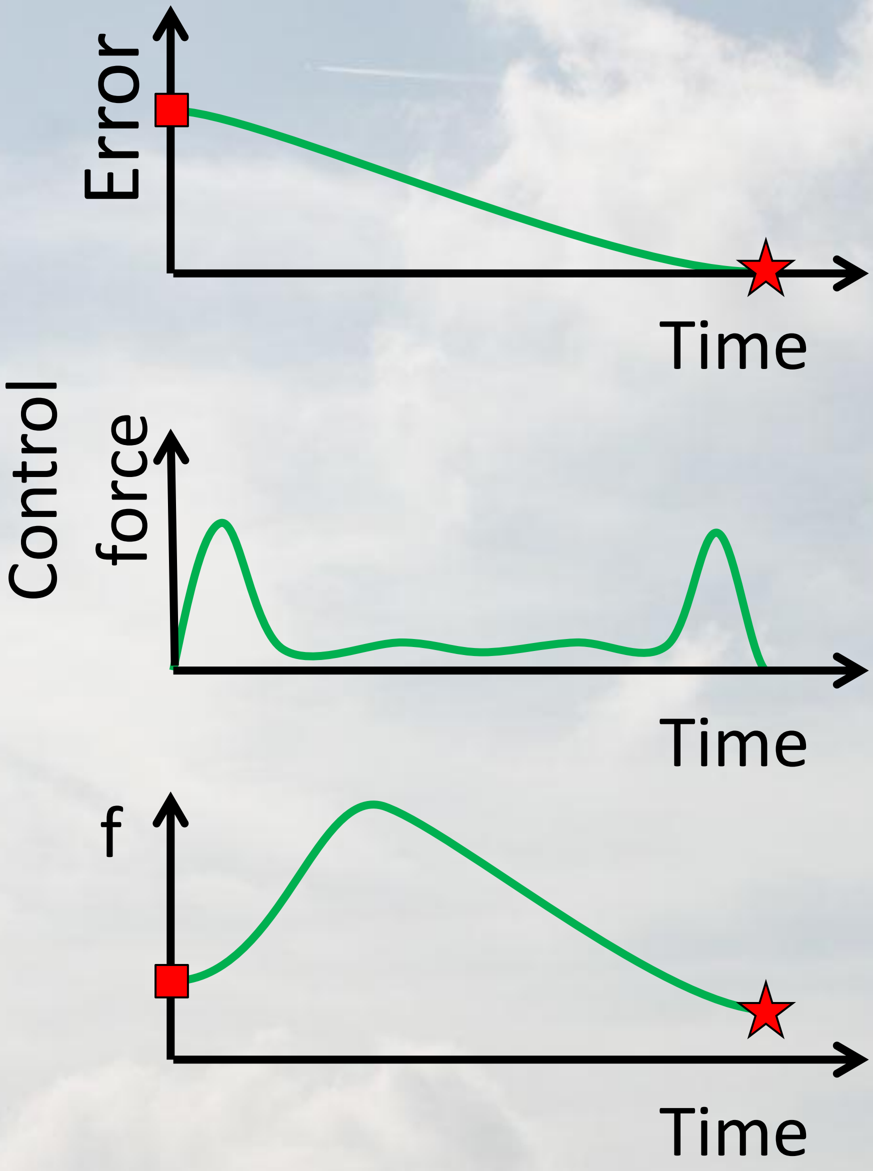
Optimal Control

$$\min_{\boldsymbol{u}} J(\boldsymbol{x}, \boldsymbol{u})$$

$$s.t.\quad \boldsymbol{x}(t_1) = \boldsymbol{x}_0, \boldsymbol{x}(t_K) = \boldsymbol{x}_f$$

$$J = \sum_{k=1}^{K} \boldsymbol{e}^T Q \boldsymbol{e} + \boldsymbol{u}^T R \boldsymbol{u}$$



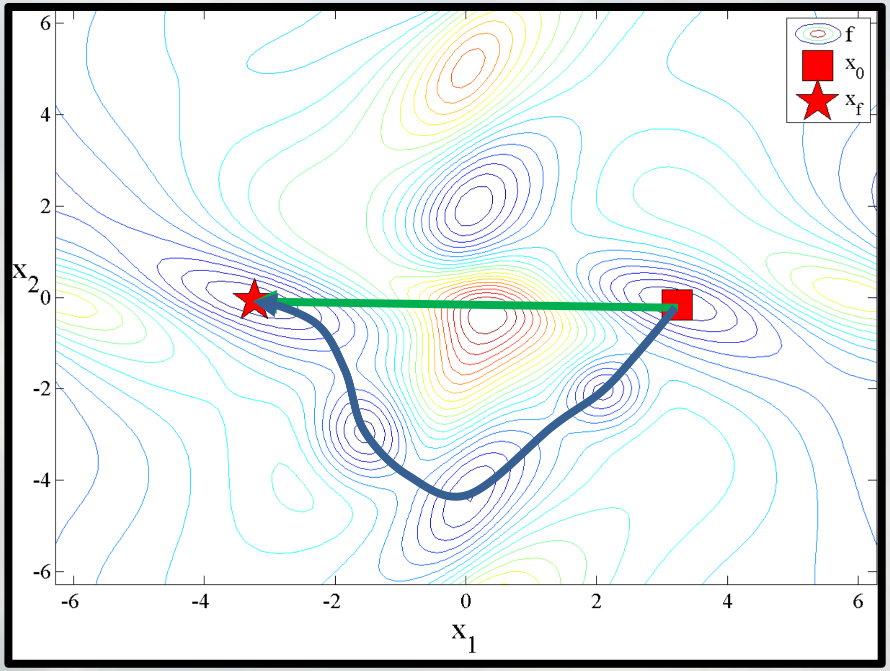


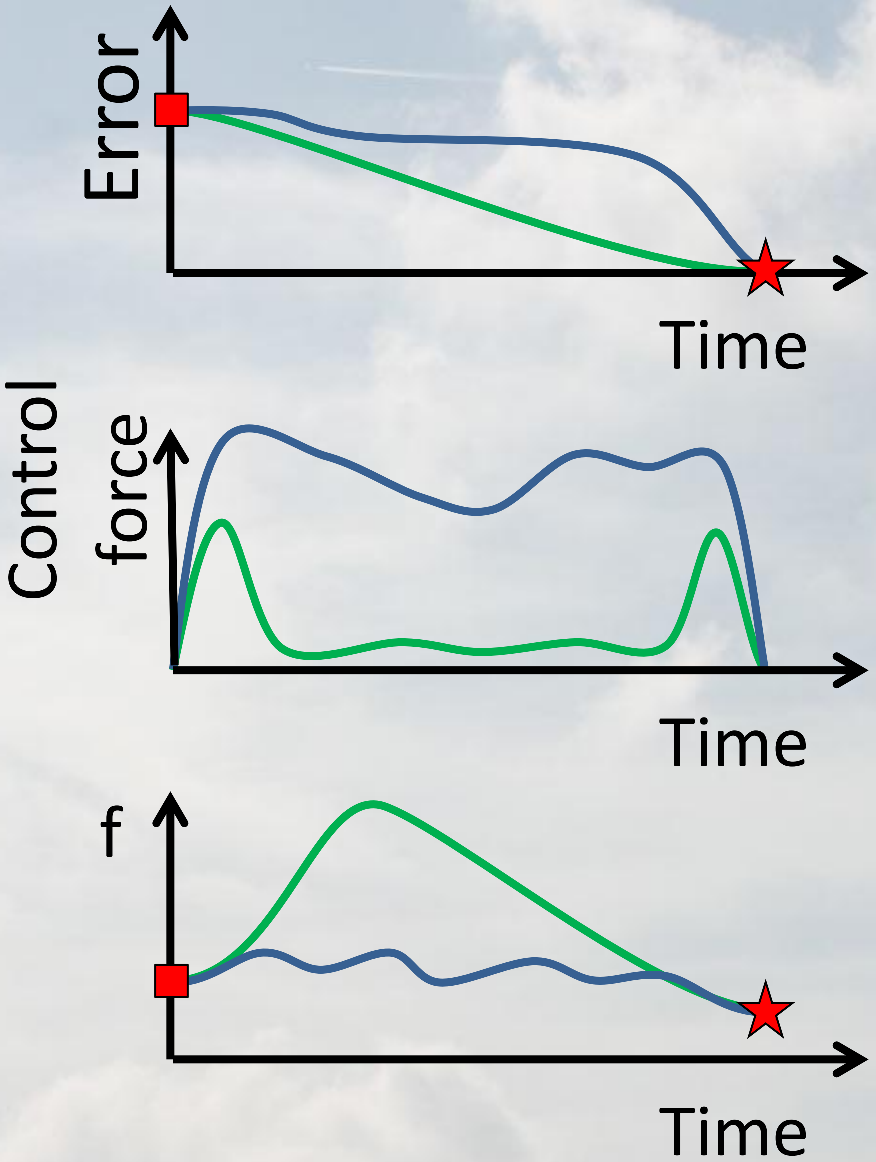
How to Adapt?

Optimal Adaptation

$$\min_{\boldsymbol{x}}(\boldsymbol{f}, \boldsymbol{cost})$$

$$s.t. \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0, \boldsymbol{x}(t_f) = \boldsymbol{x}_f$$





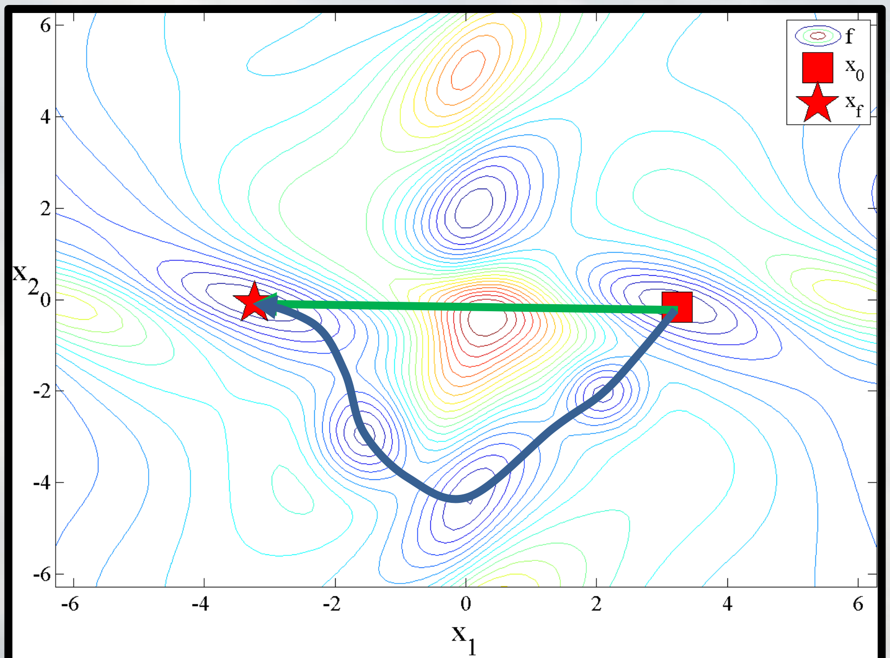
How to Adapt?

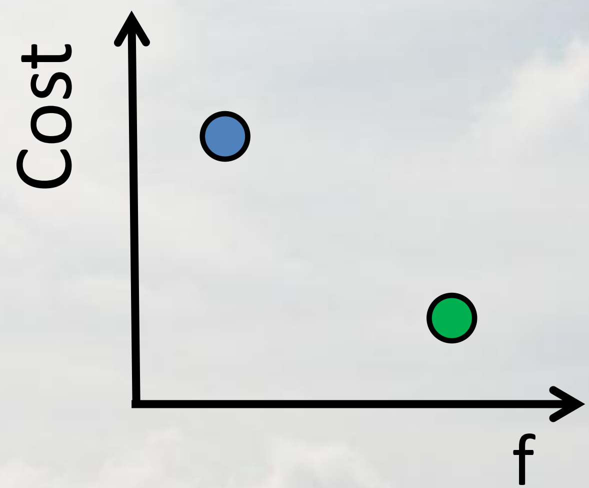
Optimal Control

$$\min_{\boldsymbol{u}}\{\boldsymbol{error}, \boldsymbol{cost}\}$$

Optimal Adaptation

$$\min_{\boldsymbol{x}}\{\boldsymbol{f}, \boldsymbol{cost}\}$$





The Optimal Adaptation Problem

$$\min_{\boldsymbol{x}(t)\in Q}\left\{f\left(\boldsymbol{x}(t)\right), cost\left(\boldsymbol{x}(t)\right)\right\}, \qquad t \in \left[t_0, t_f\right]$$

$$s.t. \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \quad , \qquad \boldsymbol{x}\left(t_f\right) = \boldsymbol{x}_f$$

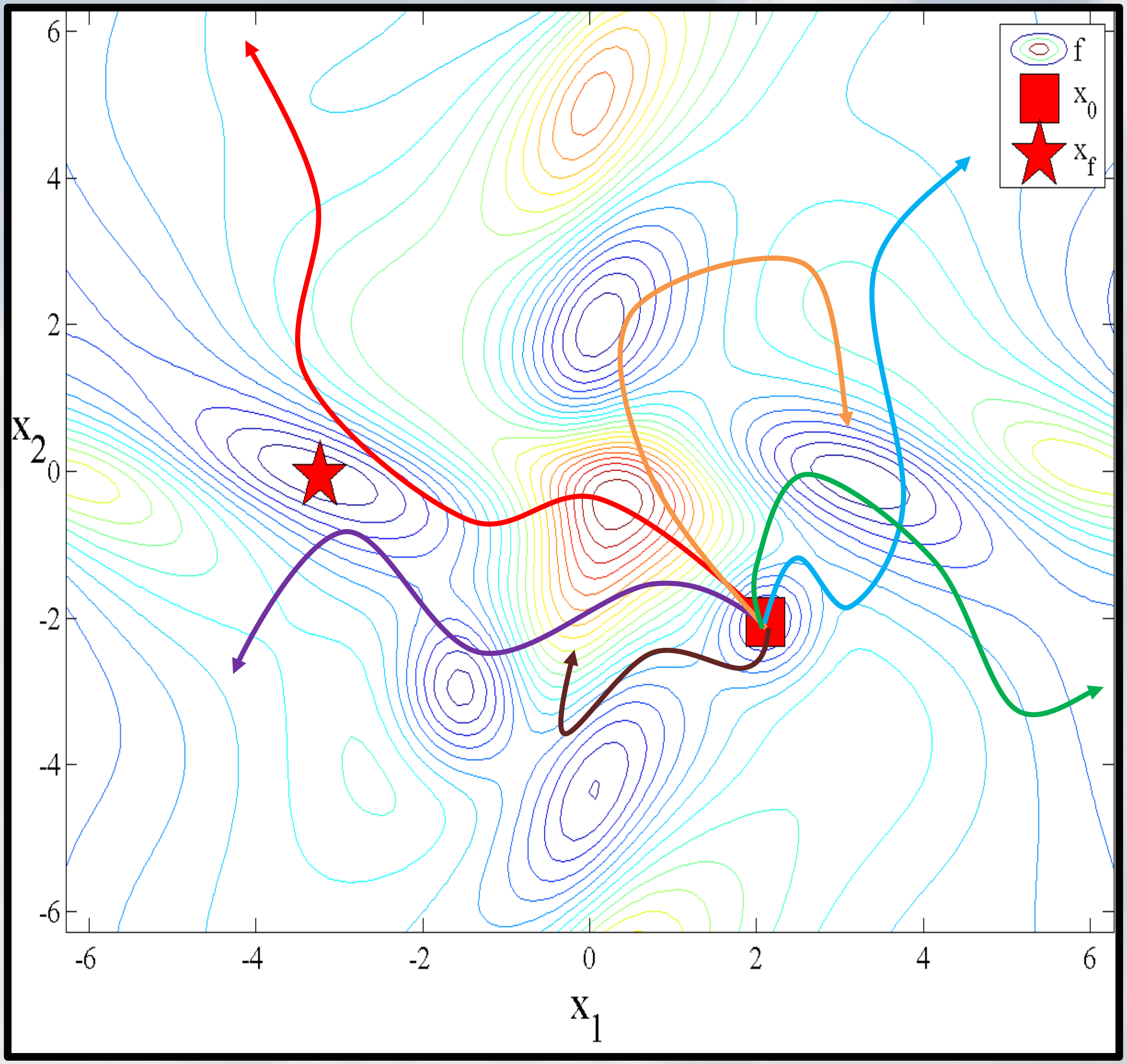
$$u_{i,l} \leq u_i \leq u_{i,u}, \qquad (i = 1, \ldots, I)$$

$$g_j(\boldsymbol{x}, t) \geq 0, \qquad (j = 1, \ldots, J)$$

$$h_s(\boldsymbol{x}, t) = 0, \qquad (s = 1, \ldots, S)$$

The Optimization Procedure

- **Trajectories generation**
- Repair method
- Evaluation
- Evolution



The Optimization Procedure

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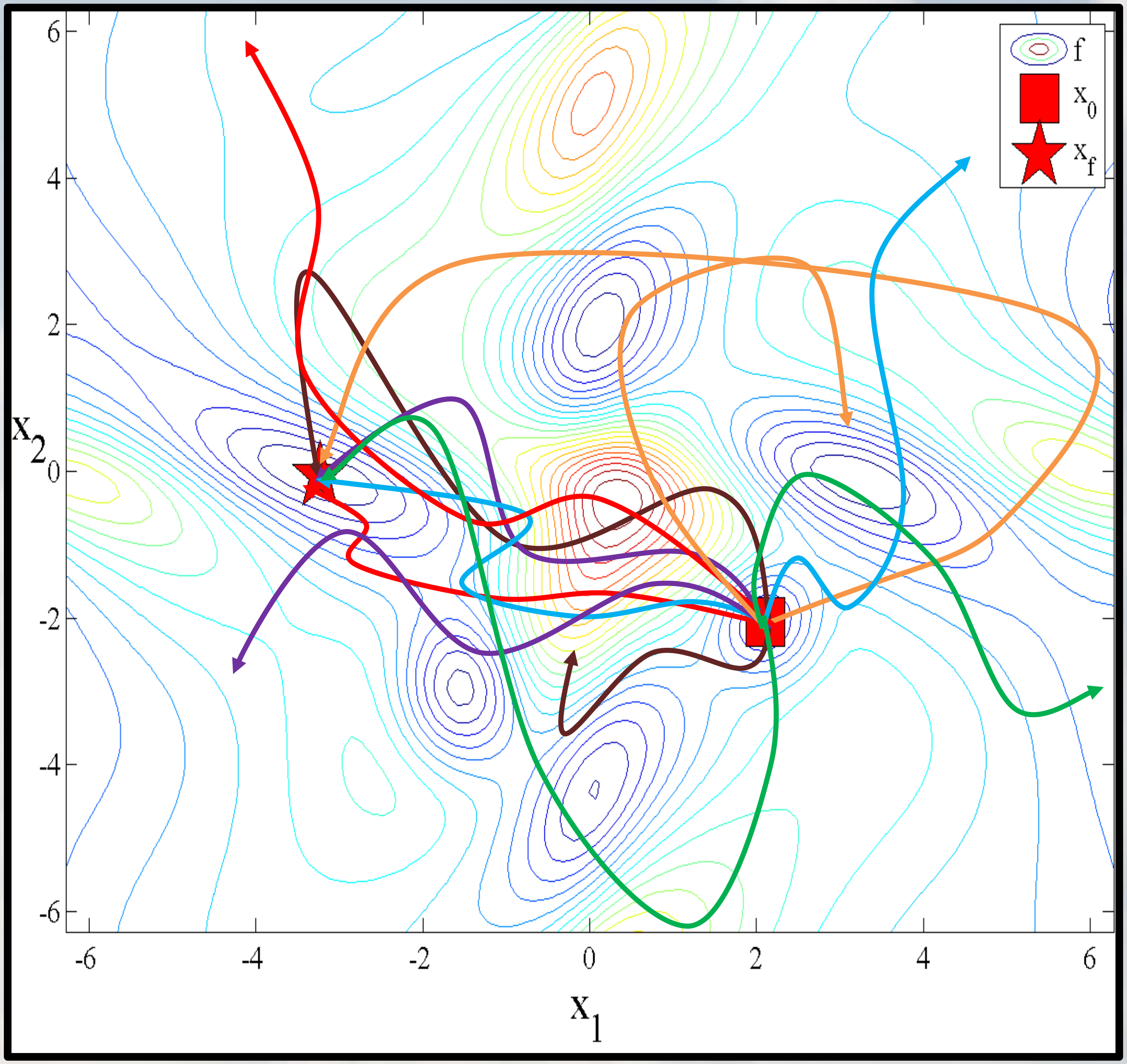

The Optimization Procedure

- Trajectories generation
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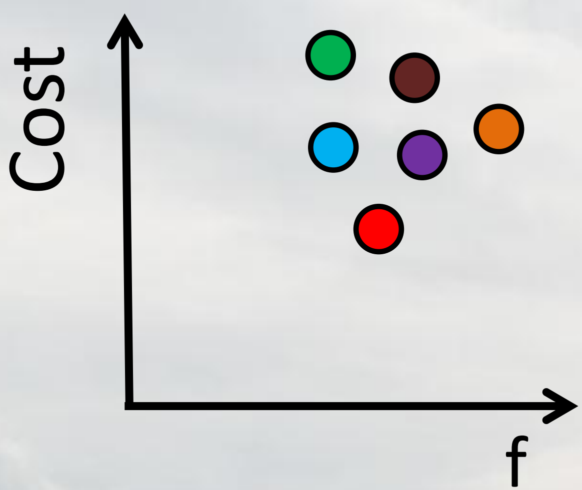

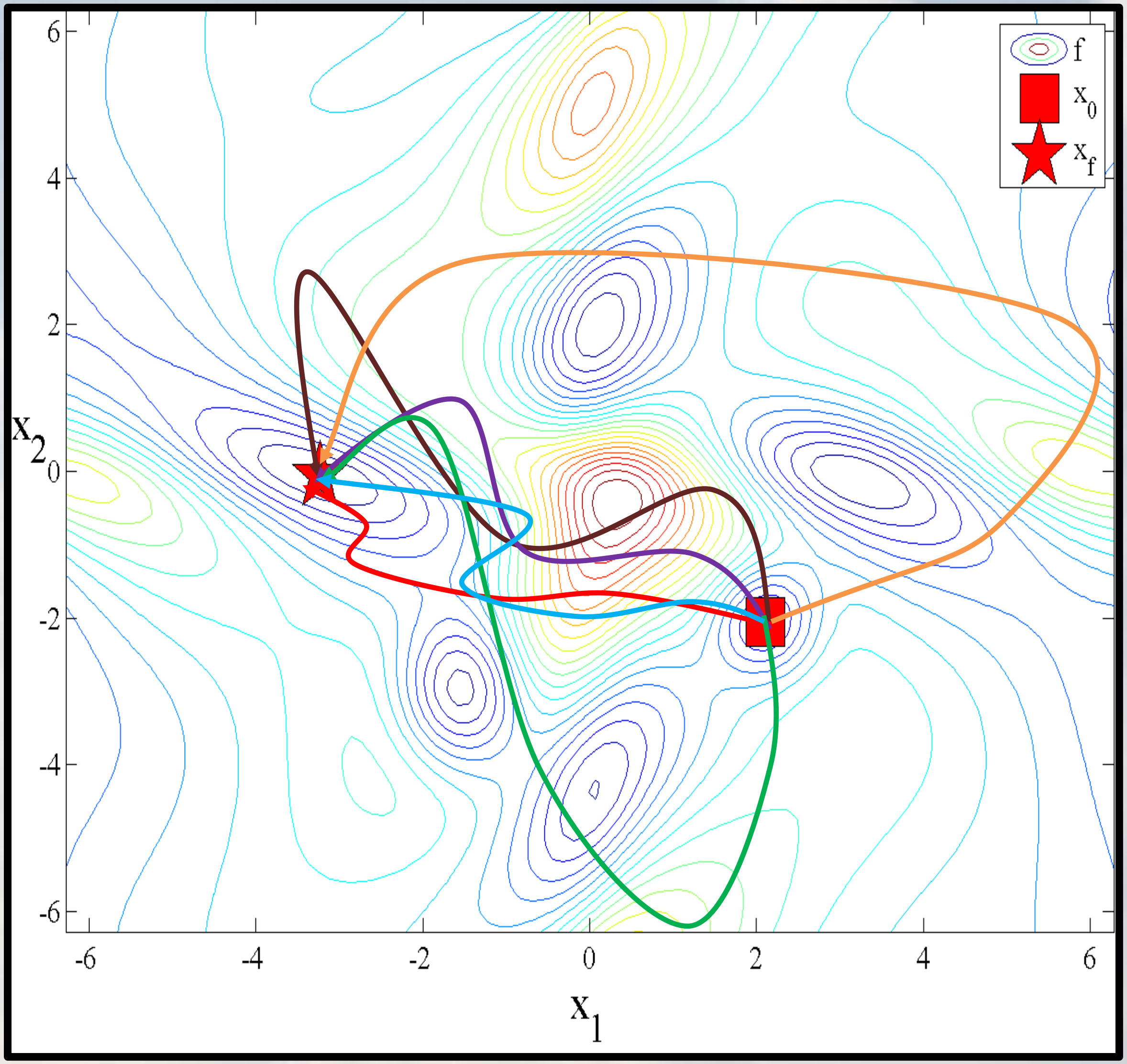

The Optimization Procedure

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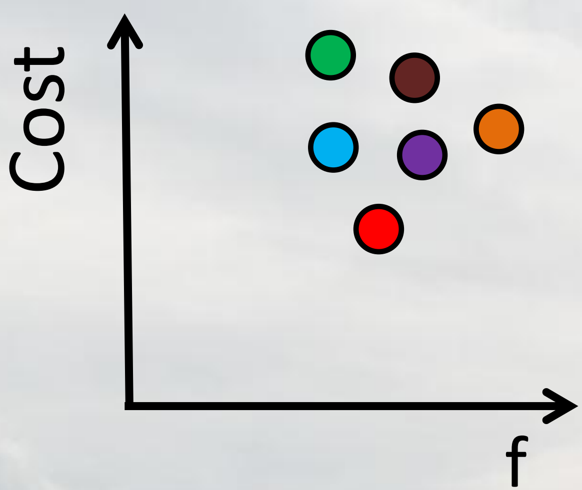
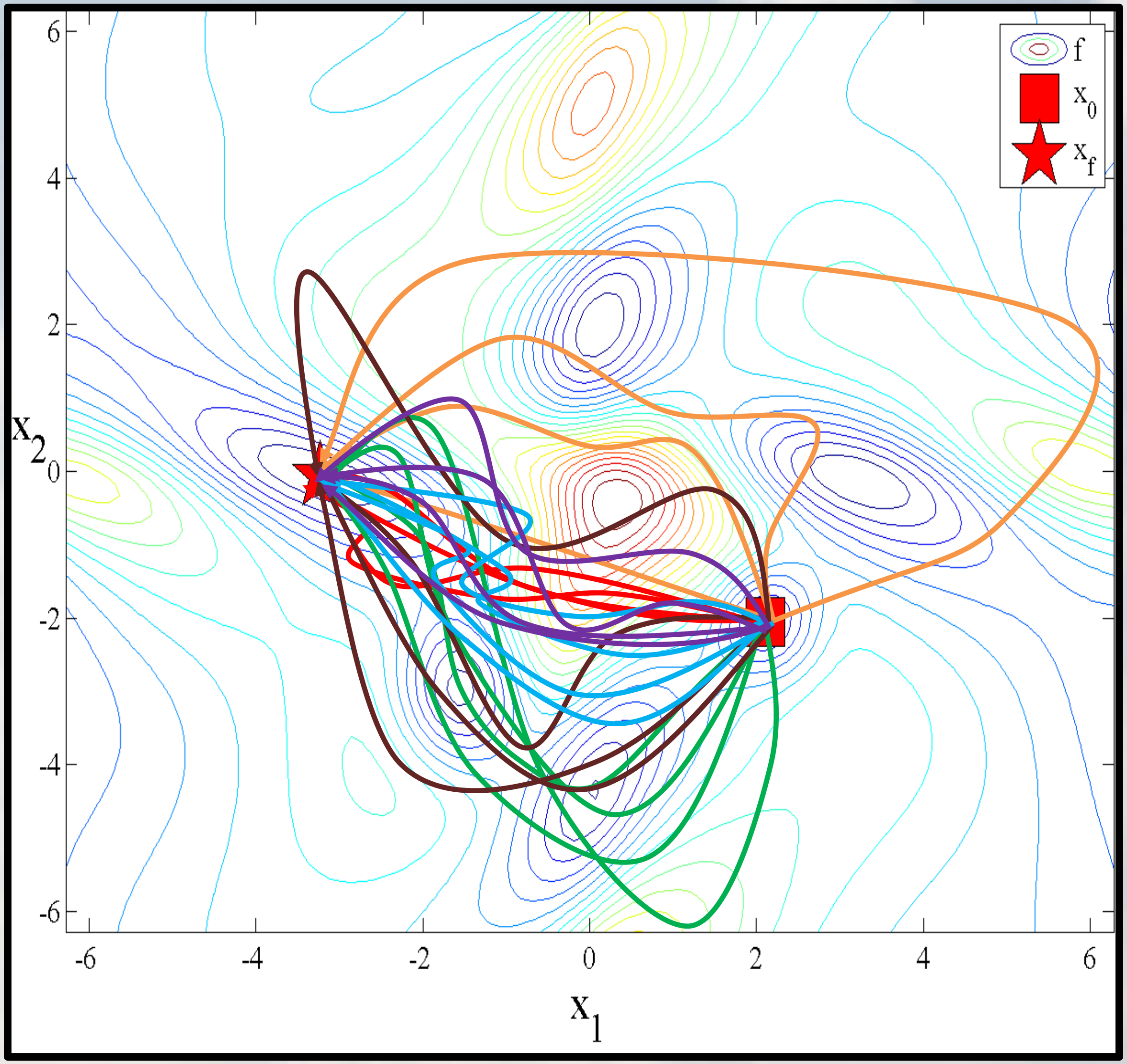




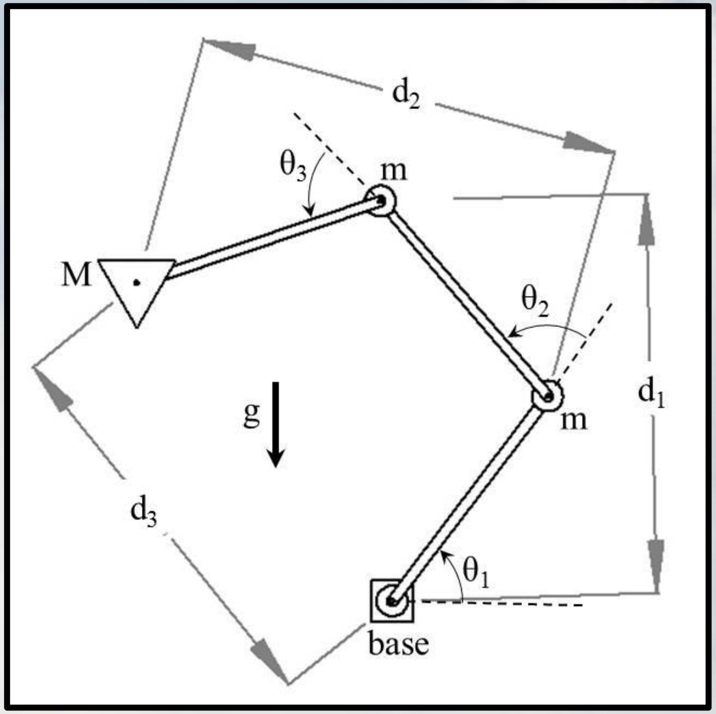
An example of a robotic manipulator

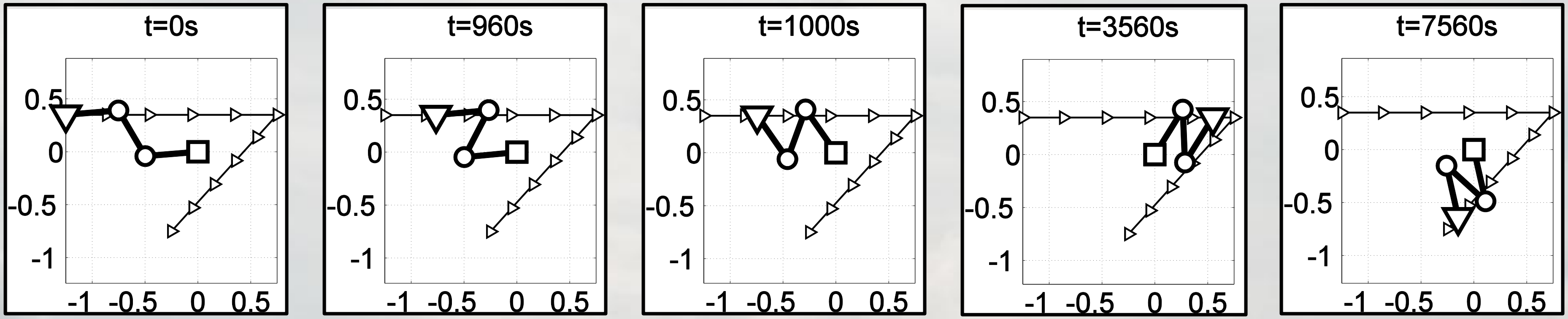
The Dynamic Optimization Problem:

$$\min_{\theta(t)} \; \phi(t)$$

$$\text{s.t.}: \; r_e = P(t)$$

$$\phi(t) = d_1 + d_2 + d_3$$



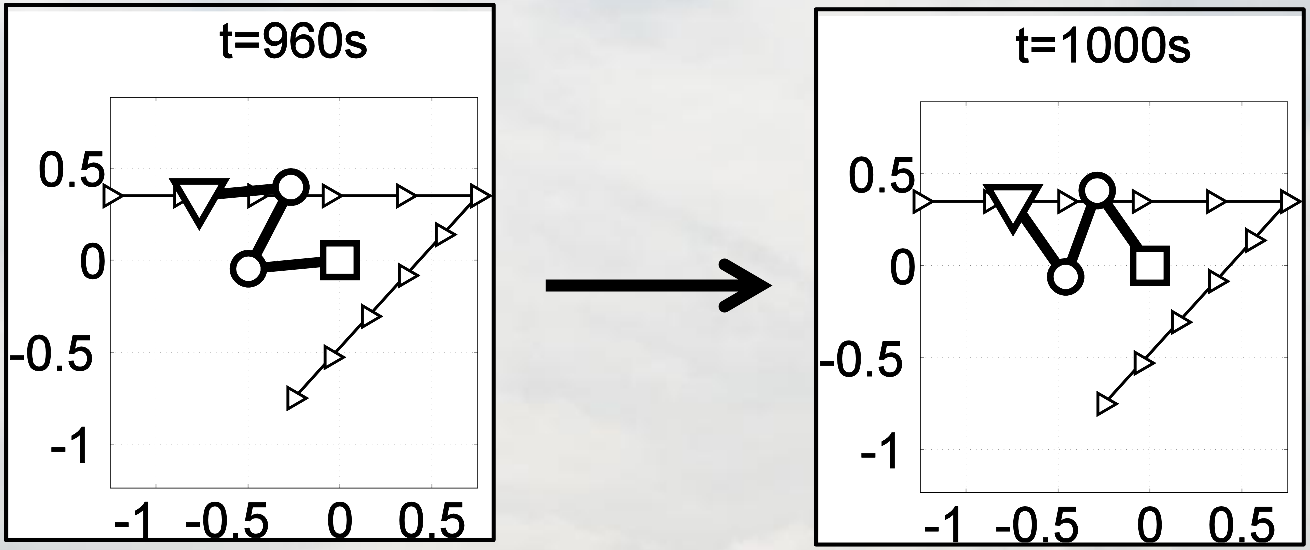


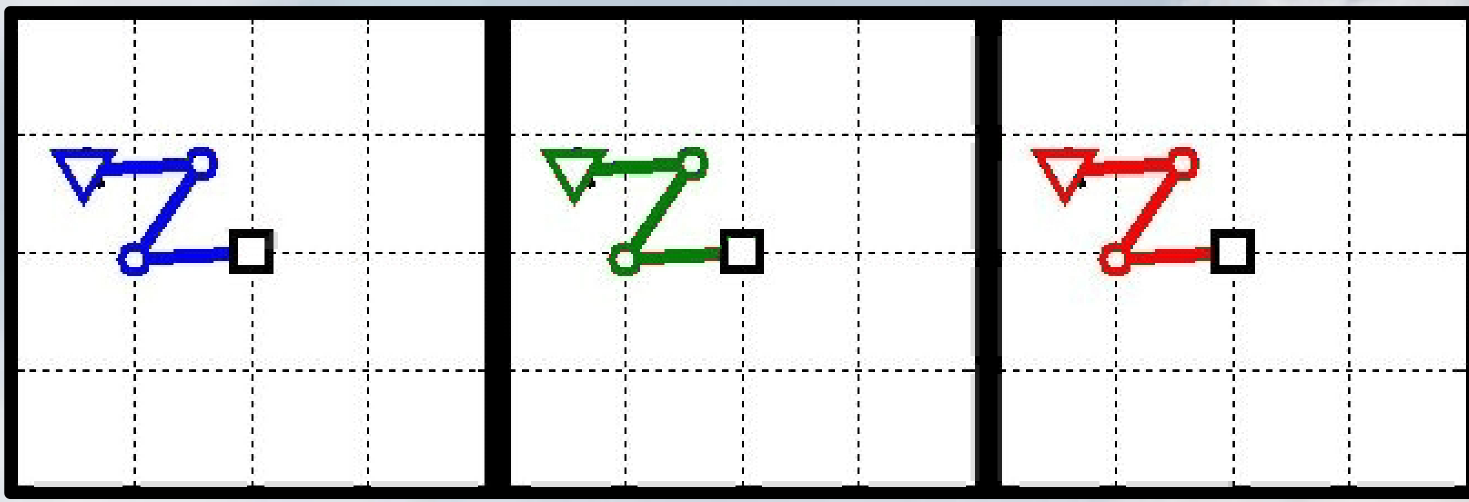
The Optimal Adaptation Problem:

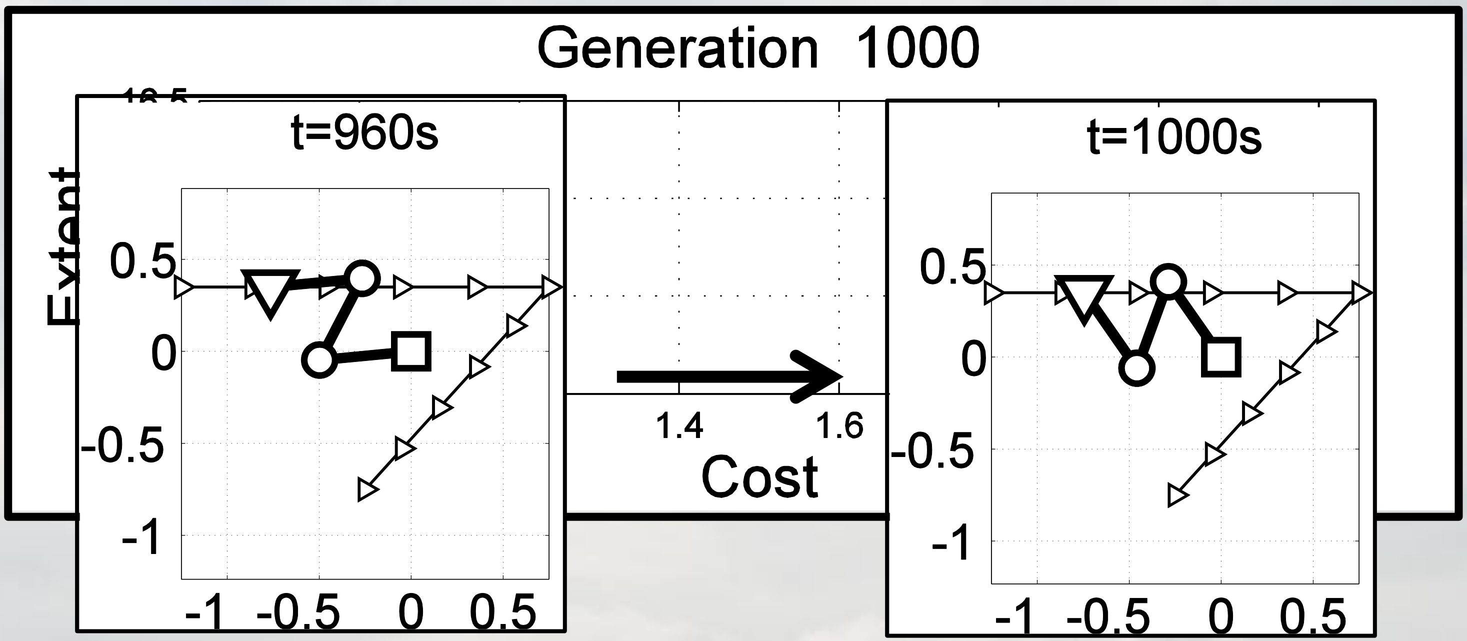
$$\min_{\boldsymbol{\theta}(t)} \{\emptyset(\boldsymbol{\theta}) , \mathrm{T}(\boldsymbol{\theta})\} \;, \qquad t \in [960 \,, 970]$$

$$s.t. \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \quad , \qquad \boldsymbol{x}(t_f) = \boldsymbol{x}_f$$

$$\mathrm{T}_{i,l} \leq \mathrm{T}_i \leq \mathrm{T}_{i,u} \,, \qquad (i = 1,2,3)$$

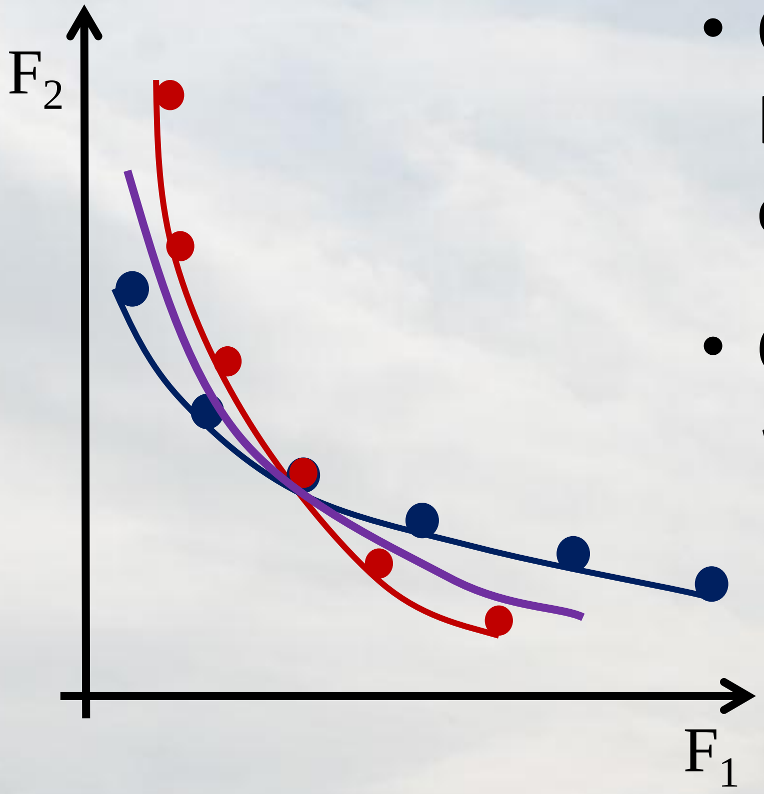





Generation 1000
t=960s
t=1000s
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1.4
1.6
Cost

Future Research Directions

- Optimal adaptation for multi-objective dynamic optimization problems.



- Changes of preferences:
  How to adapt to a different optimal configuration?
- Changes of objective functions:
  Which configuration to choose?

Future Research Directions

- Optimal adaptation for multi-objective dynamic optimization problems.

Other Issues

- Dealing with uncertainties.
- Active Robust Optimization Problems.

Thank you

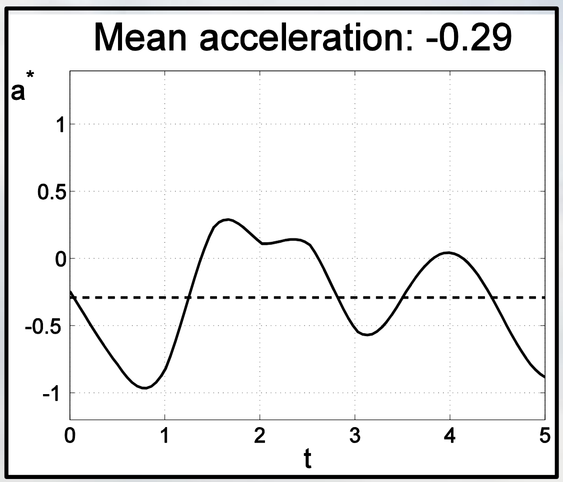
Any questions?

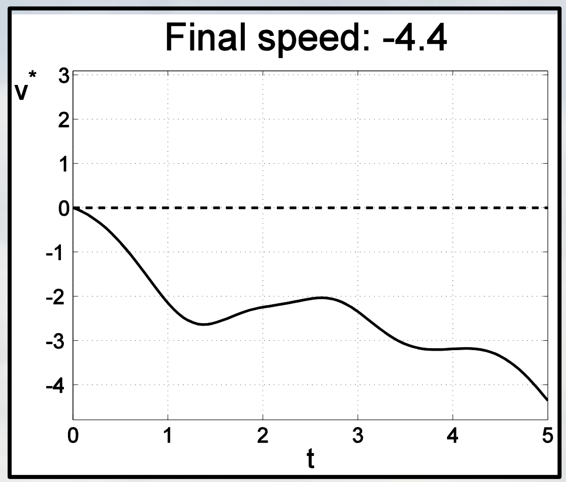
The Repair Method

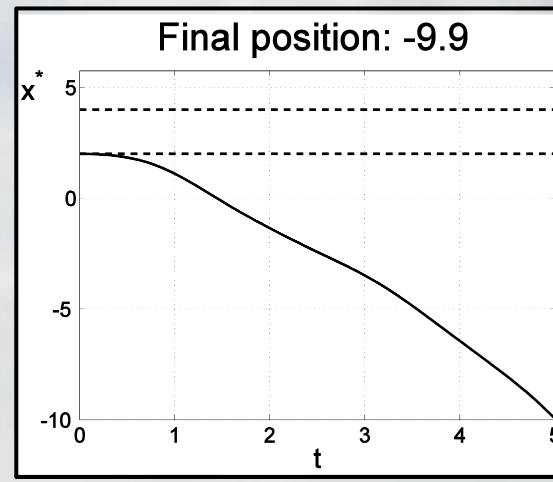
Trajectory Codification

$$v(t_0) = 0 \qquad x(t_0) = x_0$$

$$v(t_f) = 0 \qquad x(t_f) = x_f$$







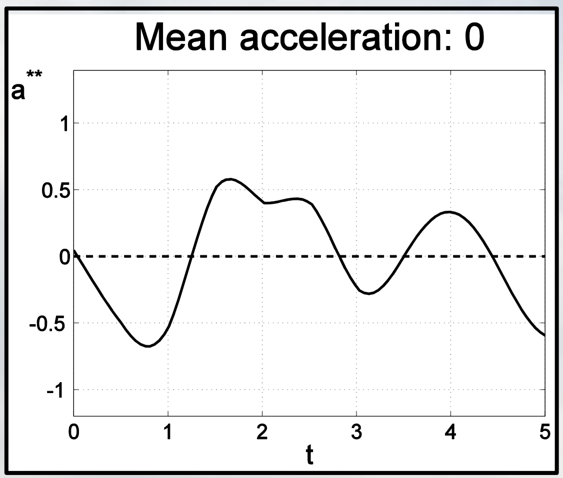
$$a^{**} = a^* - \overline{a^*} = \int_{t_0}^{t_f} a^* dt \qquad x^* = x_0 + \int_{t_0}^{t_f} v^* dt$$

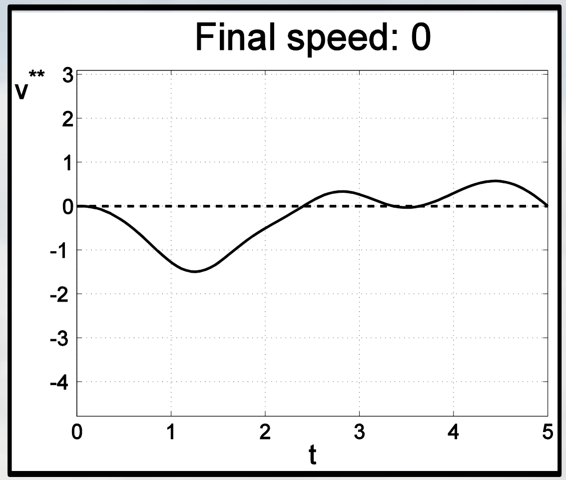
The Repair Method

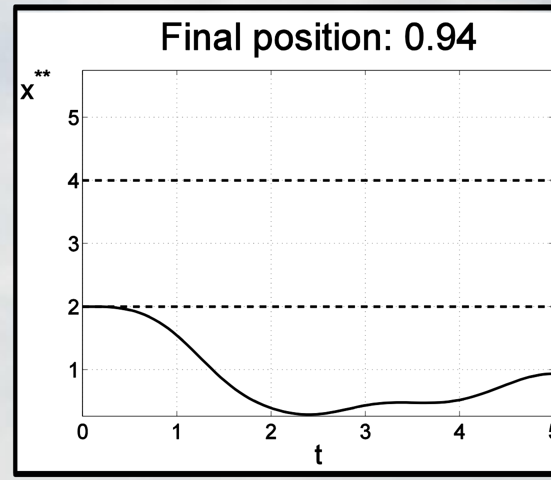
Trajectory Codification

$v(t_0) = 0$ $\quad x(t_0) = x_0$

$v(t_f) = 0$ $\quad x(t_f) = x_f$







$$a^{**} = a^* \frac{x_f - x_0}{x^{**}_f - x_0} = a^* - \bar{a}^{**} = \int_{t_0}^{t_f} a^{**} dt$$

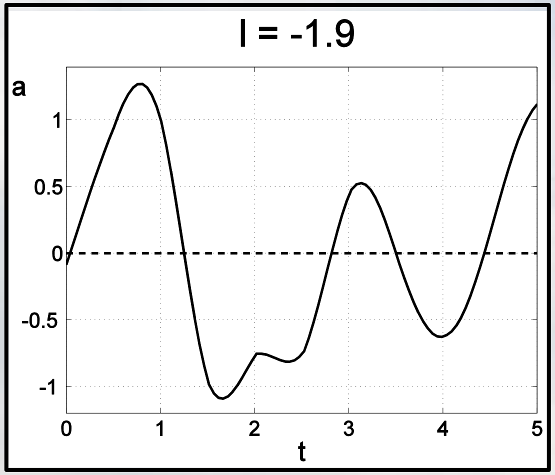
$$x^{**} = x_0 + \int_{t_0}^{t_f} v^{**} dt$$

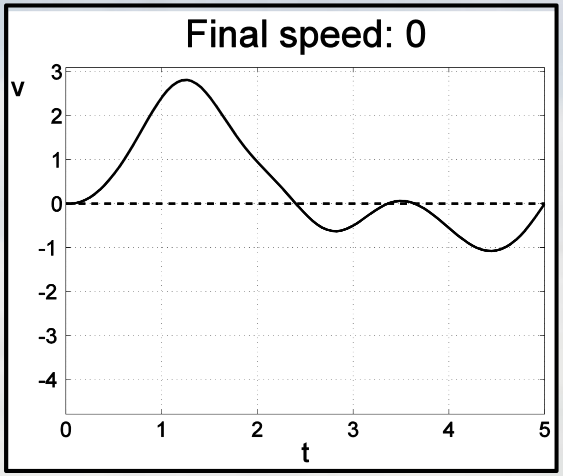
The Repair Method

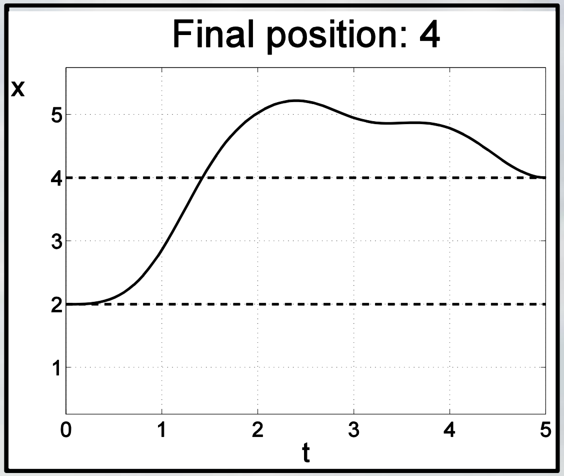
Trajectory Codification

$$v(t_0) = 0 \qquad x(t_0) = x_0$$

$$v(t_f) = 0 \qquad x(t_f) = x_f$$







$$a = a^{**} \cdot \frac{x_f - x_0}{x^{**}_f - x_0}$$

$$v = \int_{t_0}^{t_f} a \, dt$$

$$x = x_0 + \int_{t_0}^{t_f} v \, dt$$

Thank you

Any questions?