Innovization:
Discovery of Innovative Solution Principles Using Multi-Objective Optimization

Kalyanmoy Deb

Gurmukh and Veena Mehta Chair Professor

Indian Institute of Technology Kanpur

Email: deb@iitk.ac.in

http://www.iitk.ac.in/kangal/deb.htm

Adjunct Professor, Aalto University, Helsinki, Finland

Visiting Professor, University of Skövde, Sweden

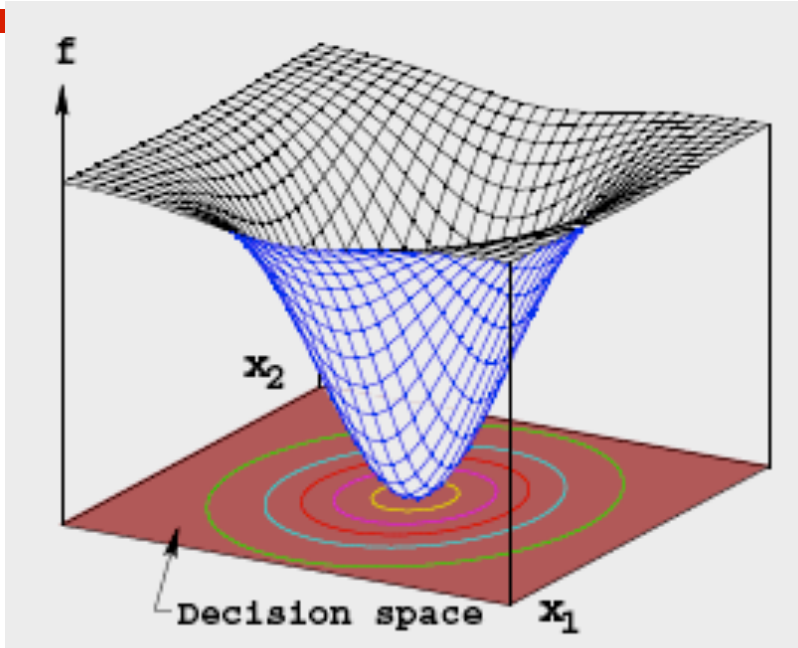
Currently at Michigan State University, East Lansing, USA

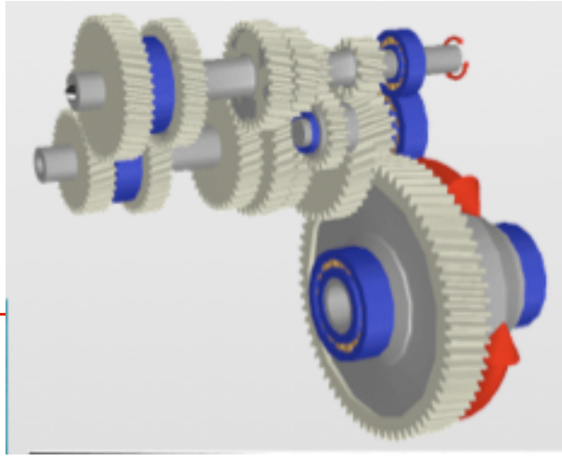
Overview

- *Innovization:* *Innov*ation through optim*ization*
- Knowledge discovery through optimization:
  - "Beyond optimization"
- Evolutionary multi-objective optimization (EMO)
- Case studies
- Recent extensions
  - Higher and lower-level *innovizations*
  - Automated *innovization*
  - *Innovization* for faster EMO convergence
  - Temporal *innovization*
- Conclusions

How Much Can Be Learned From A Single-Objective Optimization?

- Often, one optimum x*
- x* minimizes f(x) subject to satisfaction of some constraints
- *Sensitivity analysis* provides neighborhood information
- Not much can be gathered from one solution





The Very Idea of *Innovization*

Innovation through Optimization

- Common features hidden in multiple "high-performing" solutions

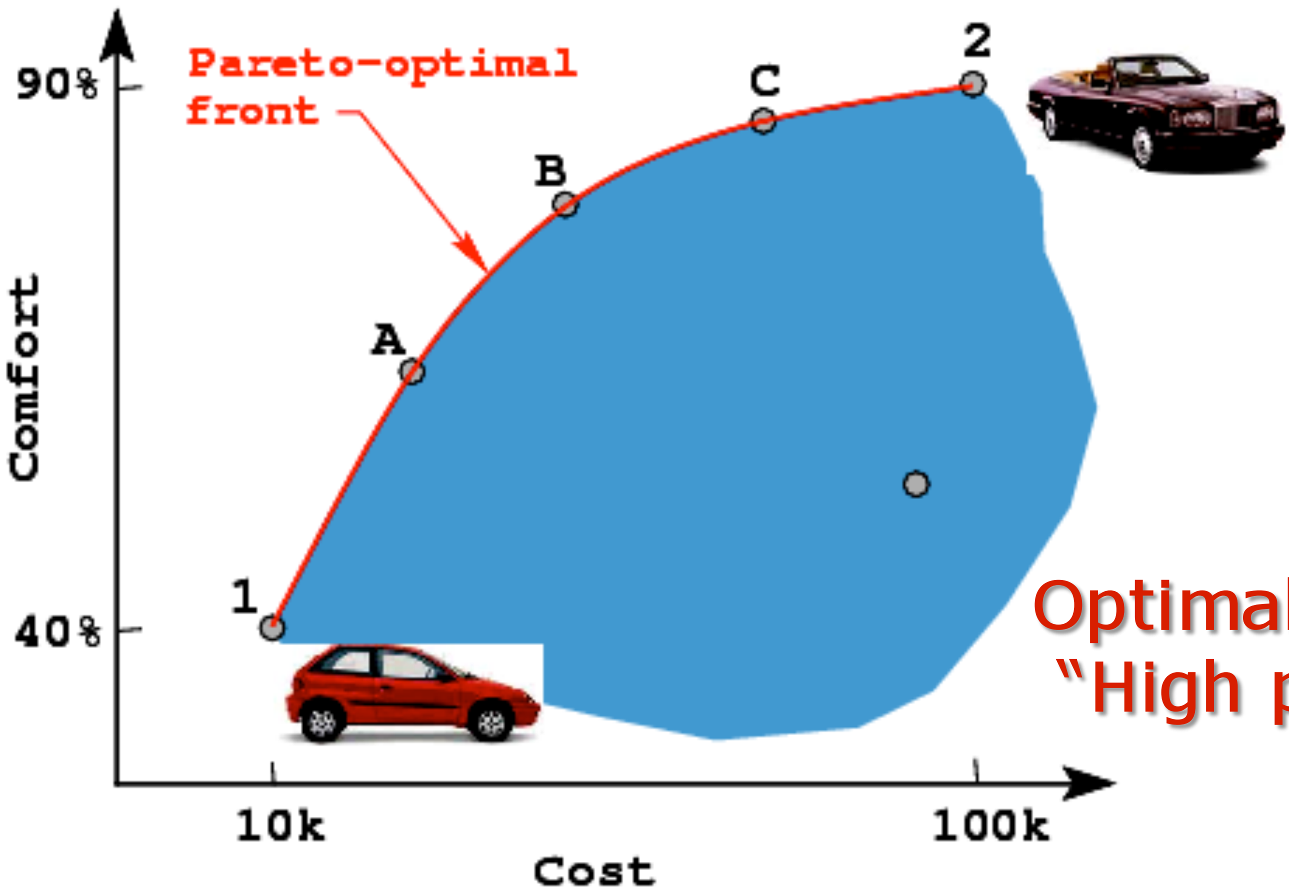
www.askmen.com/women/top50/index.html

What is common in these pretty faces?

Two Questions:

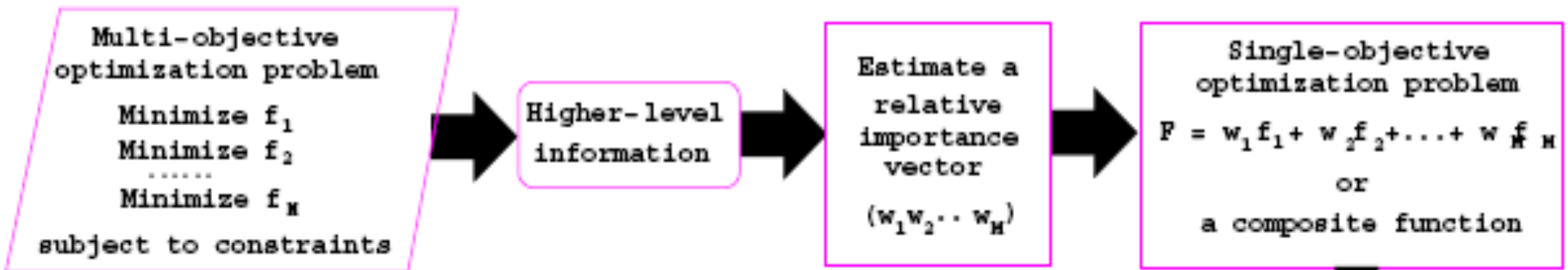
- How to find/locate multiple high-performing solutions?
- How to reveal common features?

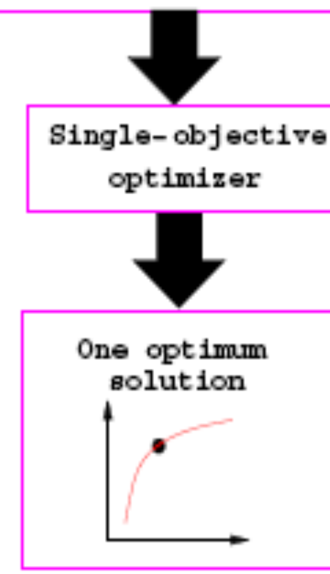
Answer to the First Question:
Multi-Objective Optimization to Find Multiple Trade-off Optimal Solutions



Optimal = "High performing"

Why Evolutionary?





Difficulty:

- Classical methods find a single solution at a time
- Evolutionary methods can find multiple solutions in a single run

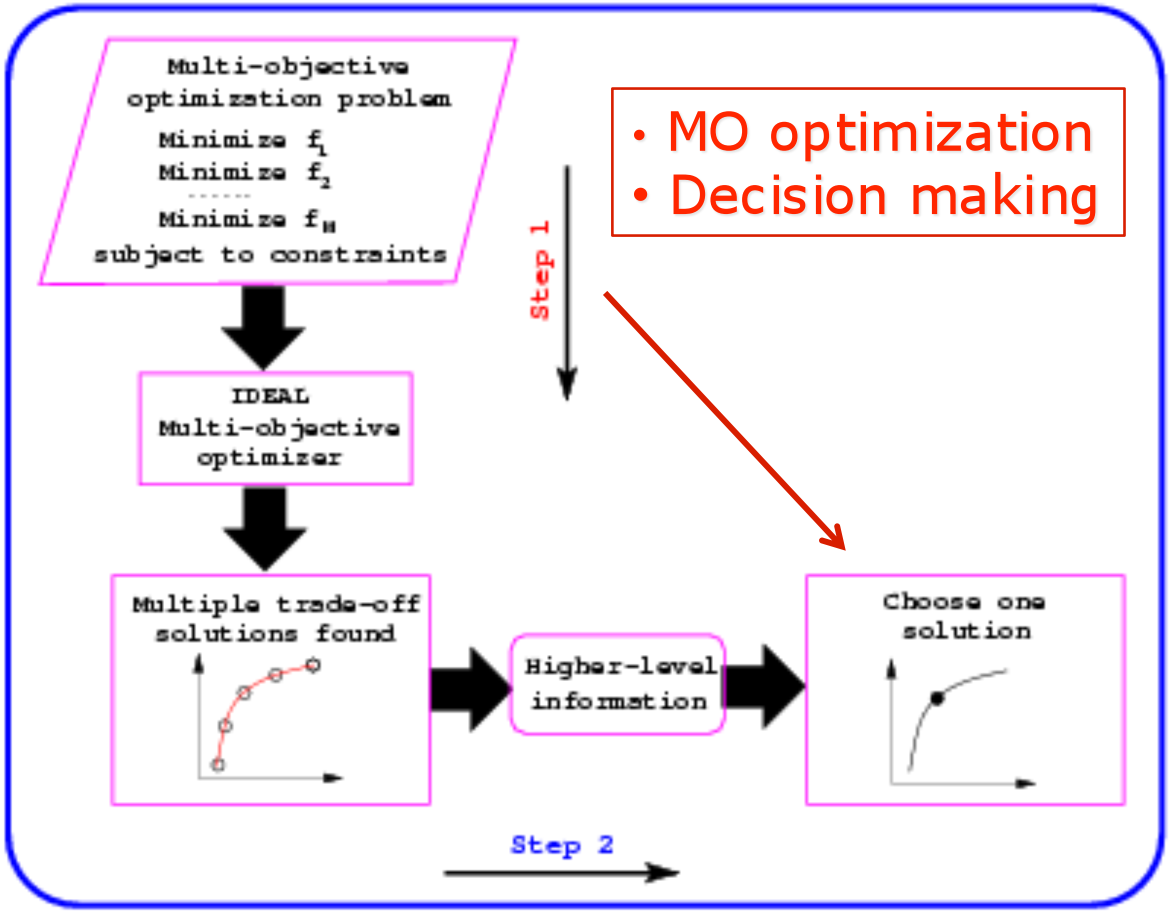
Evolutionary Multi-Objective Optimization (EMO)

Step 1 :

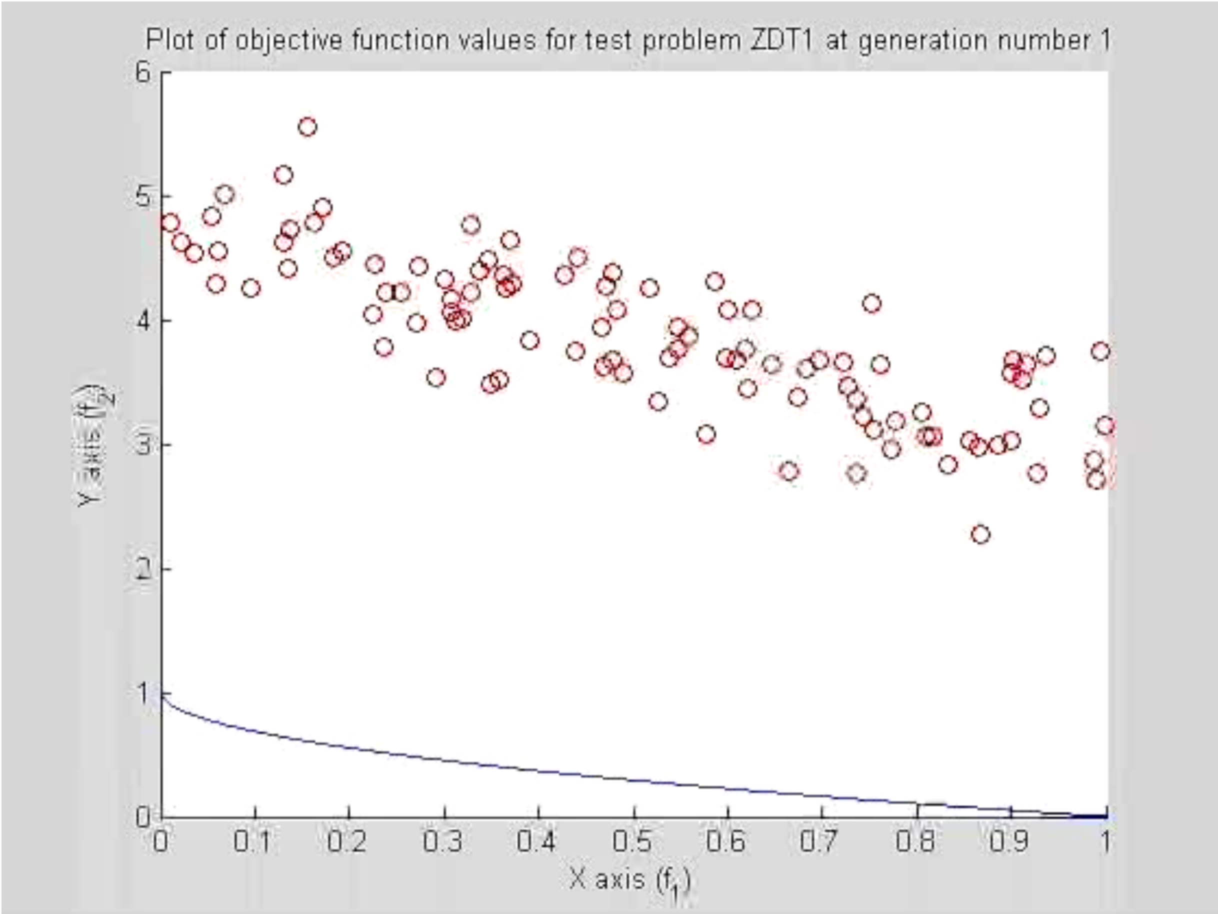
Find a set of Pareto-optimal solutions

Step 2 :

Choose one from the set



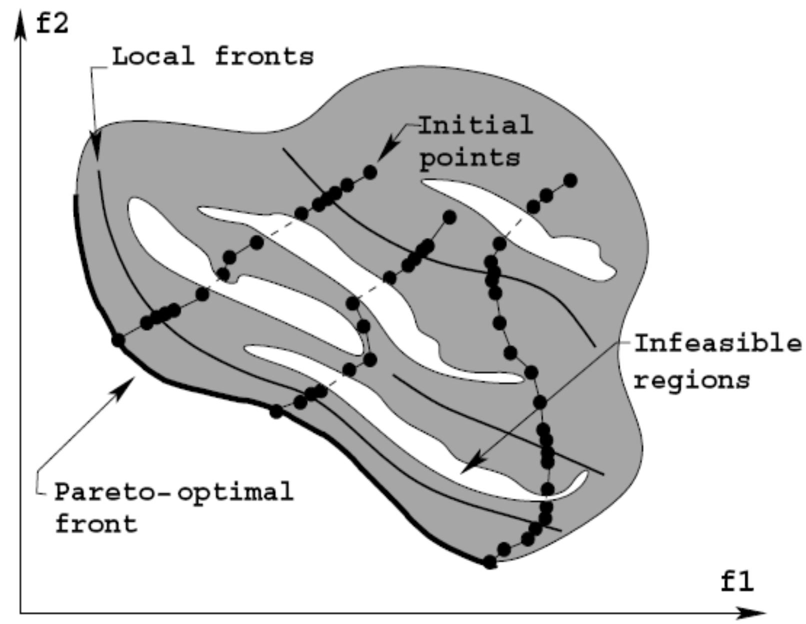
Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

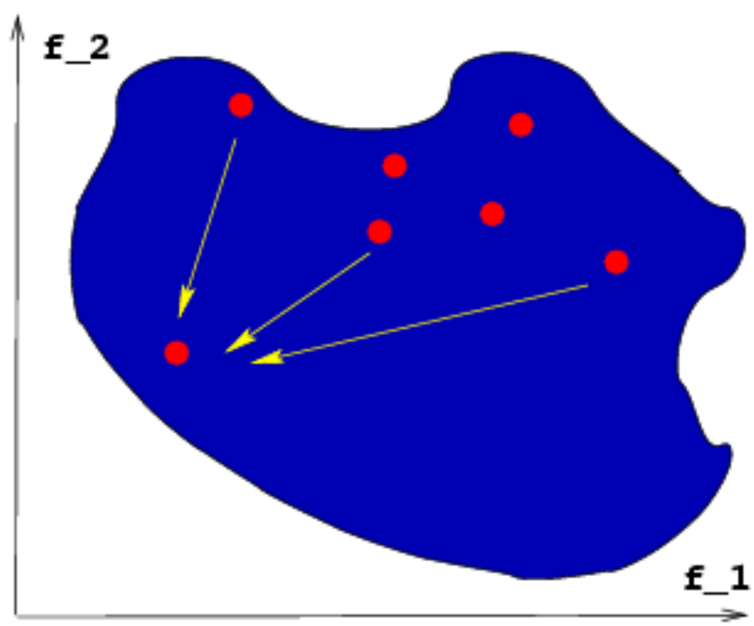


EMO Constitutes a Parallel Search

- Population approach suits well to find multiple solutions
- Implicit parallelism helps provide a parallel search

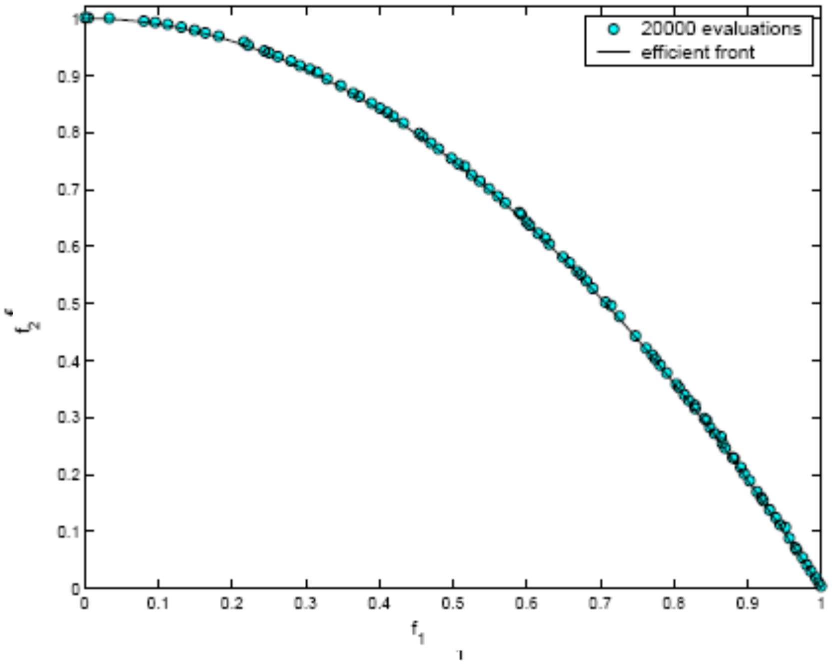
- Multiple applications of classical methods do not constitute a parallel search



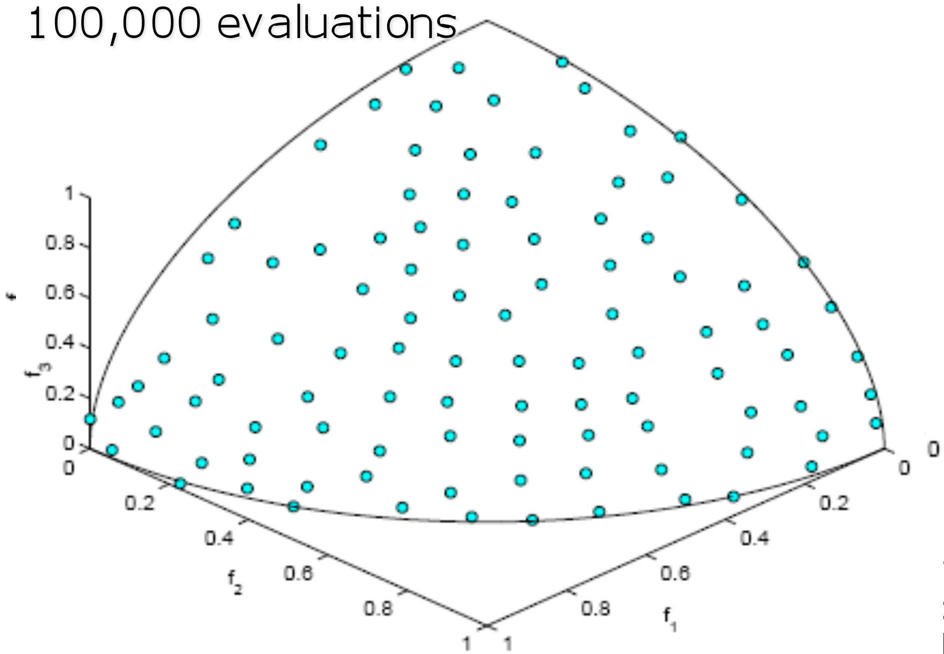


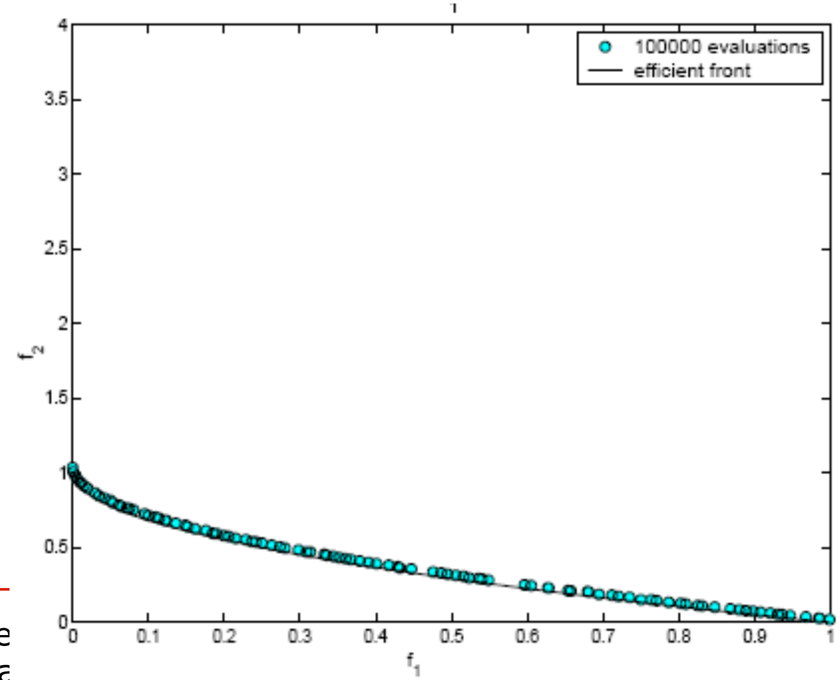
NSGA-II Versus NCM Results

- Population size=100
- Standard parameter setting
- Seems to work well



100,000 evaluations



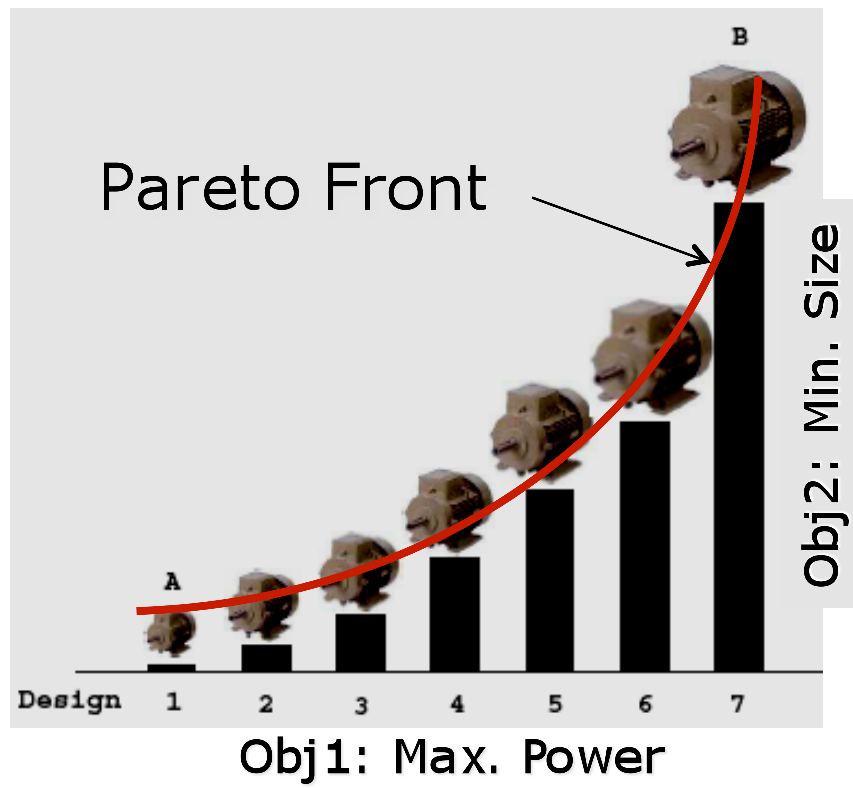


Answer to the Second Question:

Innovization Task Through an Example

Unveil important design principles in a routine design scenario

- Example: Electric motor design with varying ratings, say 1 to 10 kW
  - Each will vary in size and power
  - Armature size, number of turns etc.
- How do solutions vary?
  - Any common principles!



Pareto-optimal Solutions Must Satisfy Optimality Conditions

Fritz-John Necessary Condition:

Solution x^* satisfy

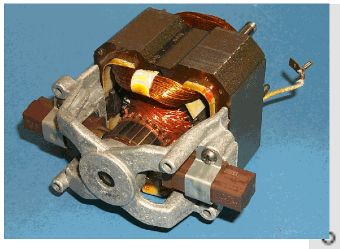
1. $\sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0,$ and
2. $u_j g_j(x^*) = 0$ for all $j = 1, 2, 3, \ldots\ldots, J$
3. $u_j \geq 0$, $\lambda_j \geq 0$, for all j and $\lambda_j > 0$ for at least one j

- To use above conditions requires differentiable objectives and constraints
- Yet, it lurks existence of some properties among Pareto-optimal solutions

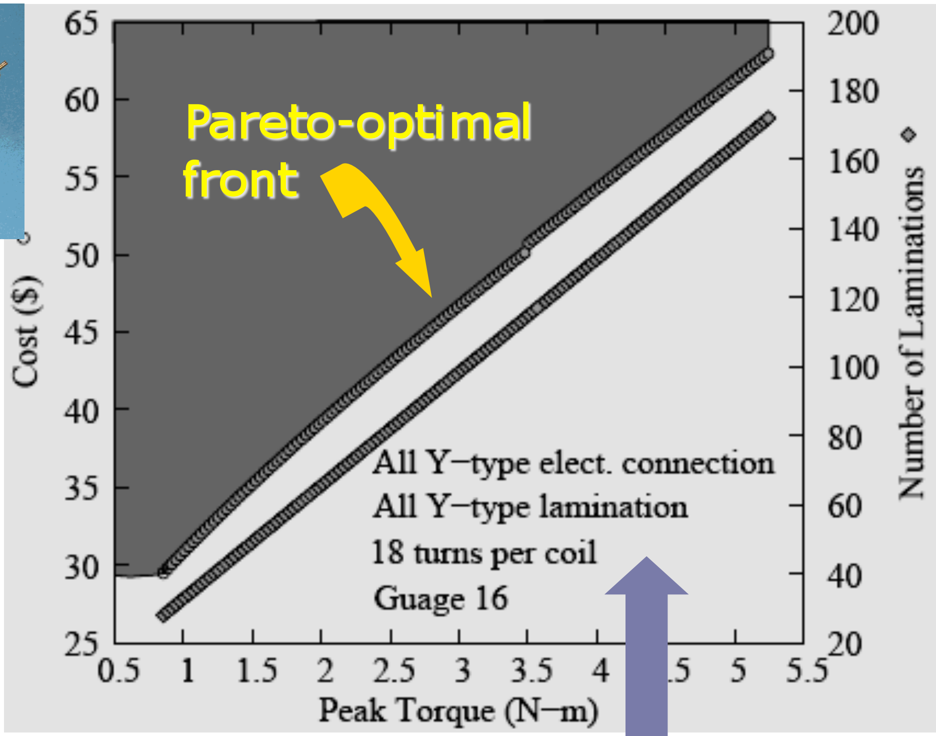
Similar Other Studies

- Rules (Papalambros, 1984)
- Graphs or influence diagrams (Michelena and Agogino, 1993)
- SVD based approach (Sarkar et al., 2008)
- Clustering in design space (Obayashi and Sasaki, 2003); MODE (MO design exploration) (Obayashi et al., 2005)
- Heatmap (Pryke et al., 2007
- Dendogram grouping (Ulrich et al., 2008))
- *None can provide explicit math. relationship*

Case Studies of *Innovization*: *Brushless DC Permanent Magnet Motor Design for Cost and Peak Torque*



- Five variables (all discrete), three constraints
- Non-convex, disconnected P-O fronts

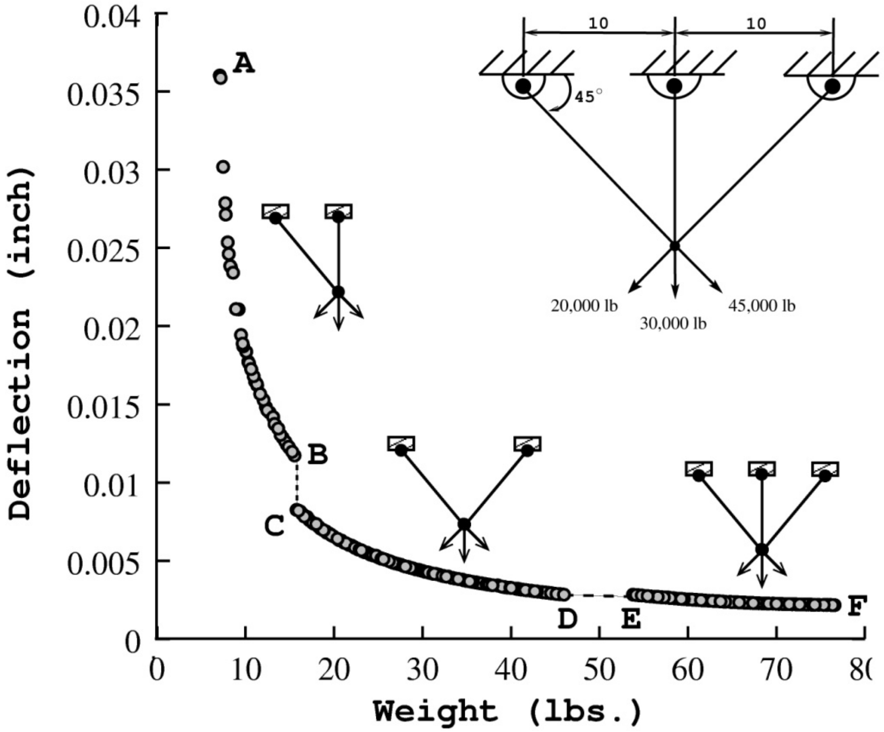


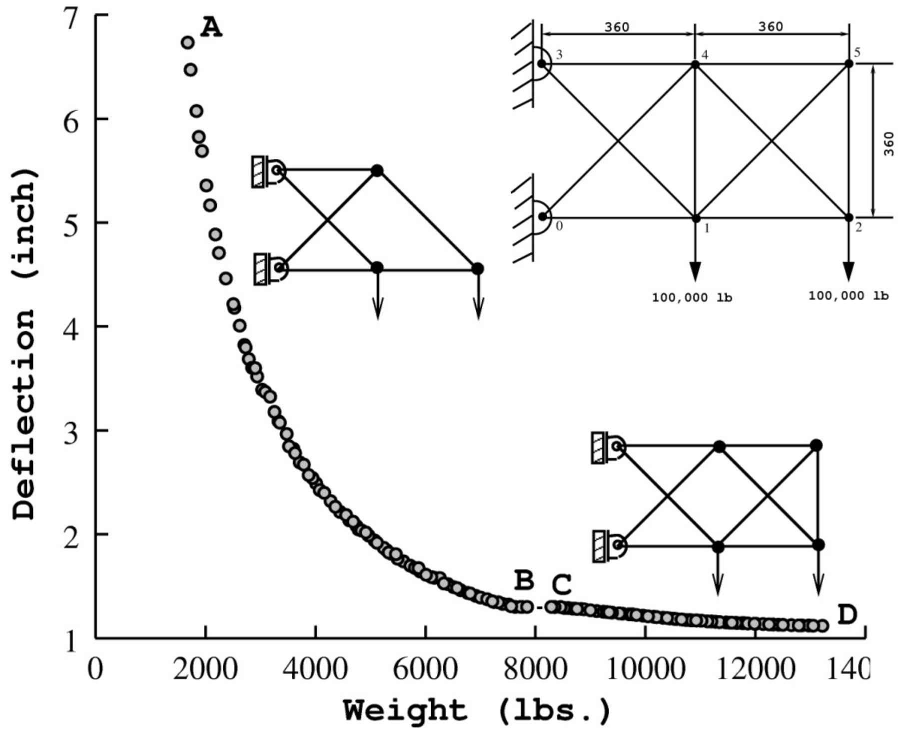
- *Innovization*:
  - Connection: Y (betn. Y & Δ)
  - Lamination Type: Y (X, Y, Z)
  - 1 out of 16 wire guages
  - 18 turns per coil (10,80)
  - *More peak torque by adding linearly more laminations*

Design Innovation

Truss Structure Design

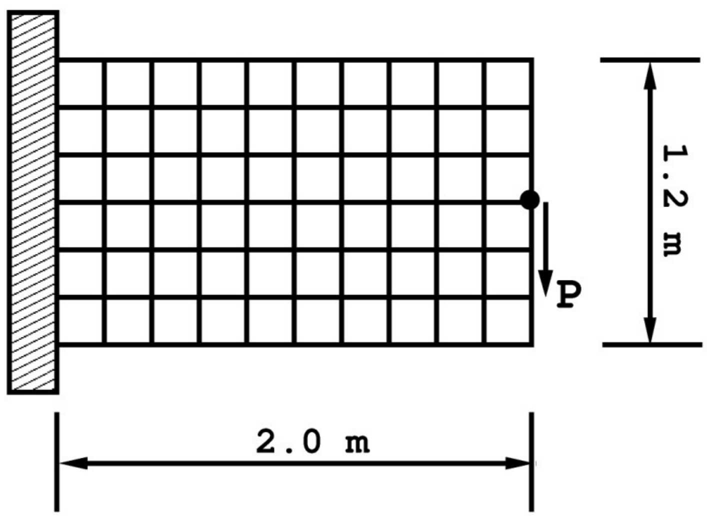
- Variables: Member size and connectivity
- Objectives: Weight and deflection



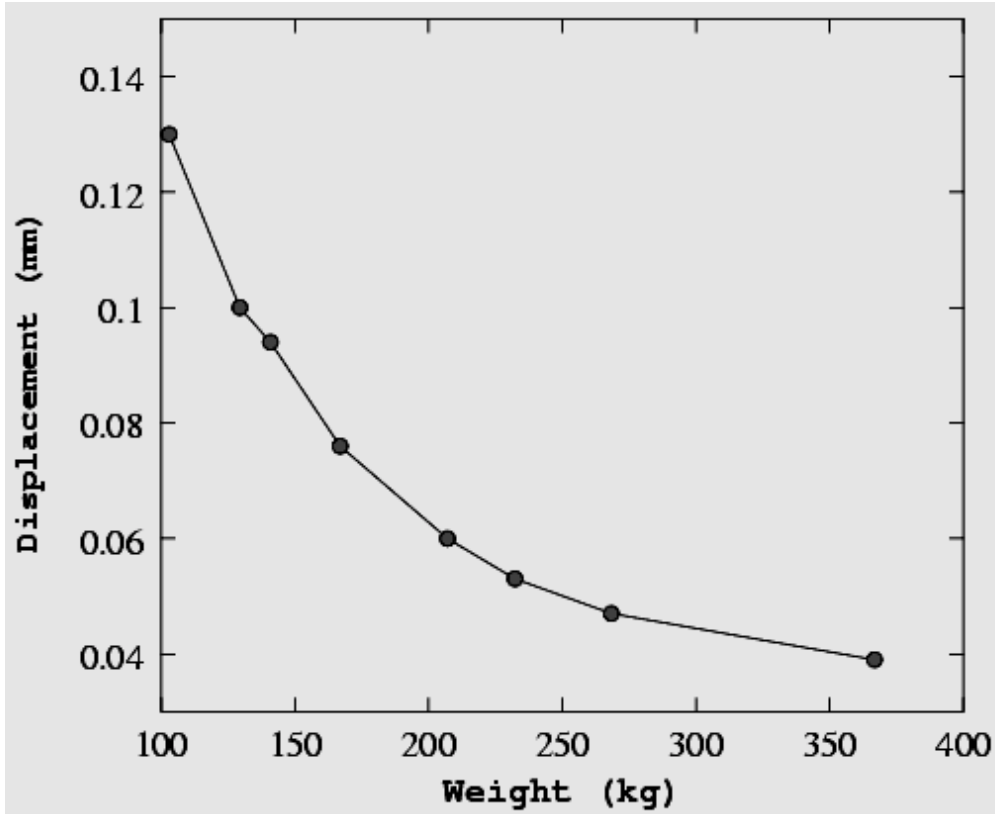


A Cantilever Plate Design

Base Plate



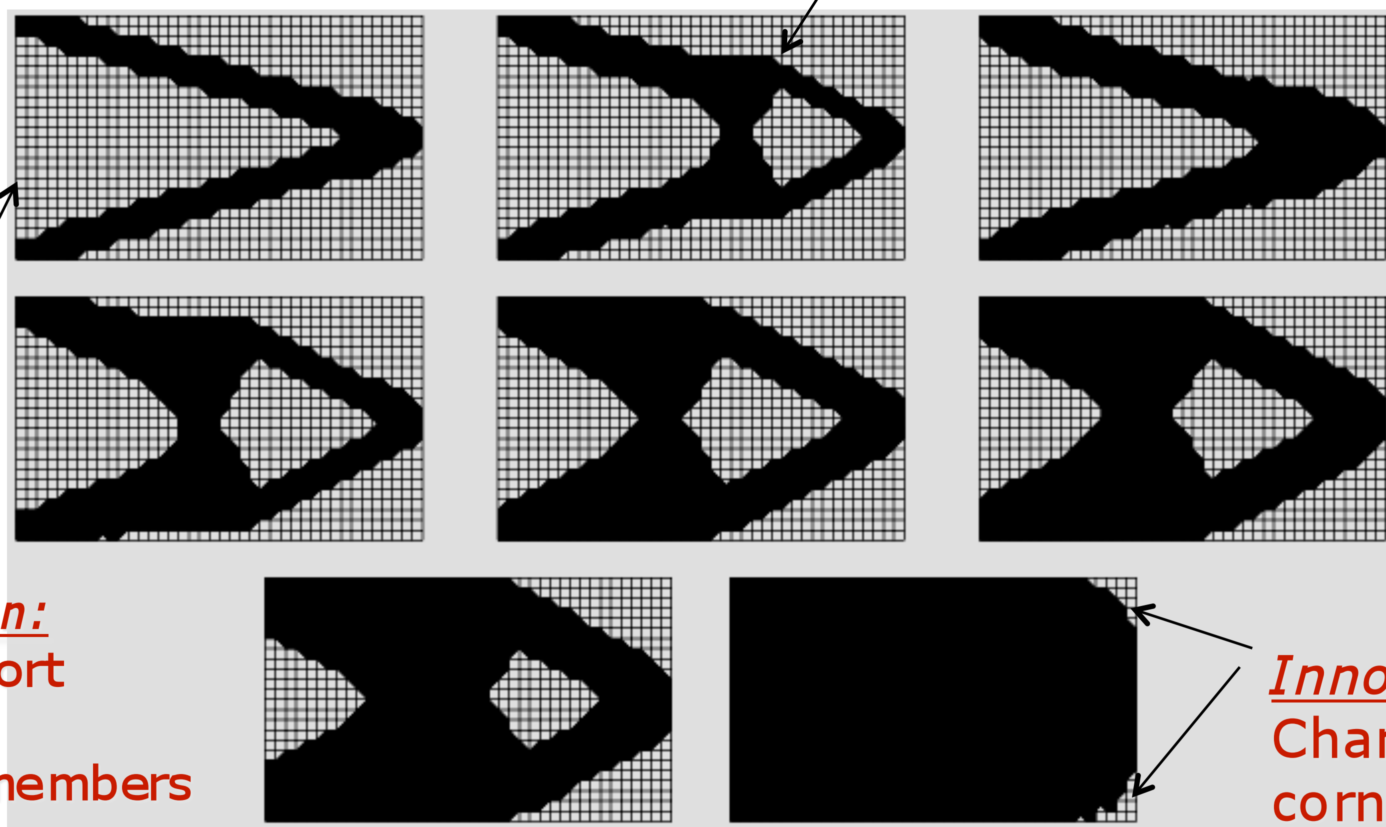
Eight trade-off solutions are chosen



Trade-Off Solutions

<u>Innovation:</u>
A stiffener

- Symmetry in solutions about mid-plane, discovery of stiffener

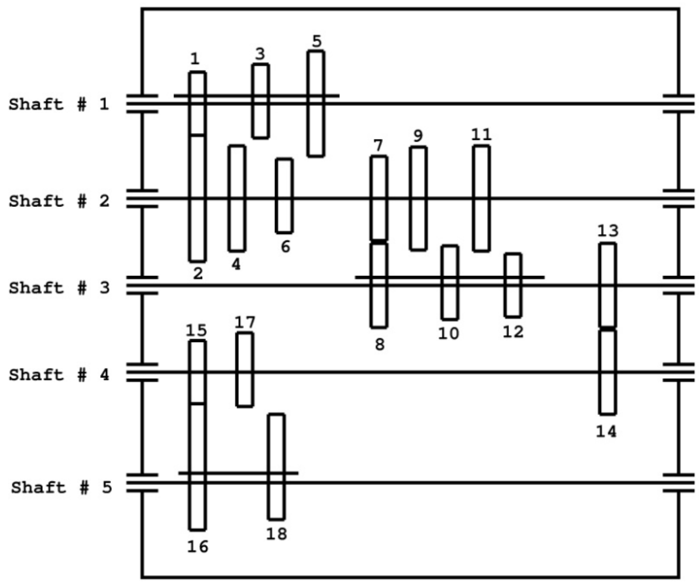


<u>Innovation:</u>
Join support
& load by
straight members

<u>Innovation:</u>
Chamfering
corners

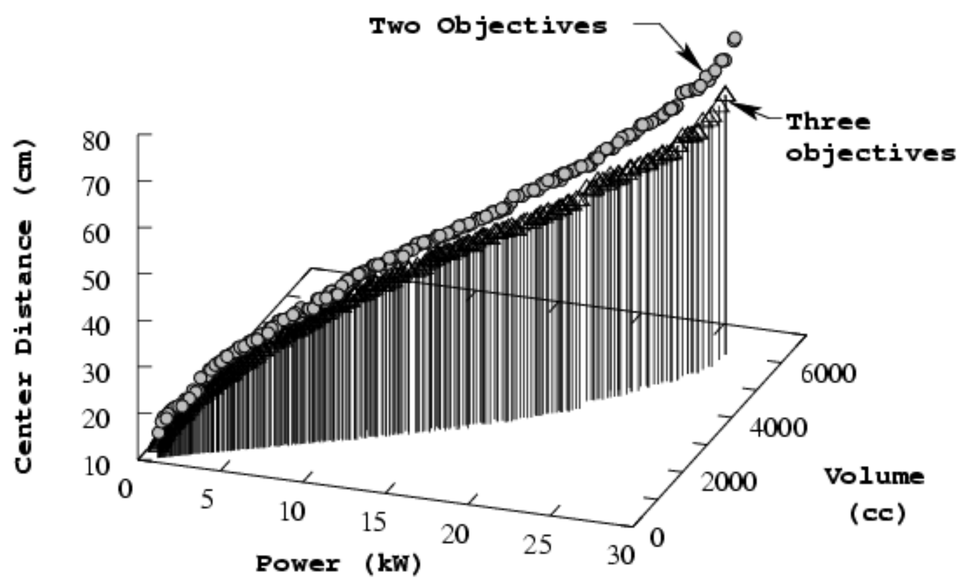
Gear-box Design

- A multi-spindle gear-box design
- 29 variables (integer, discrete, real-valued)
- 101 non-linear constraints



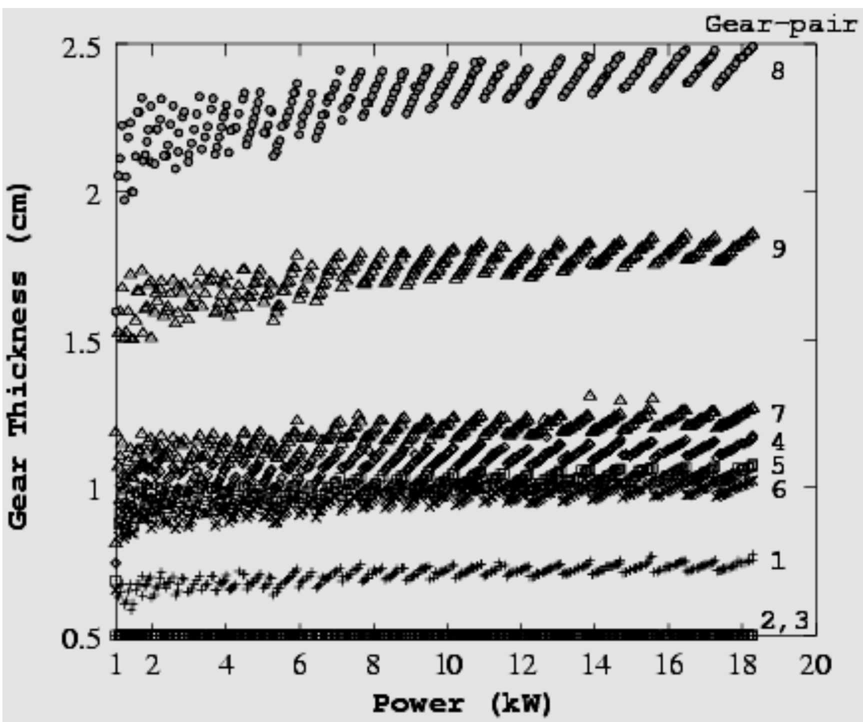
- Important insights obtained

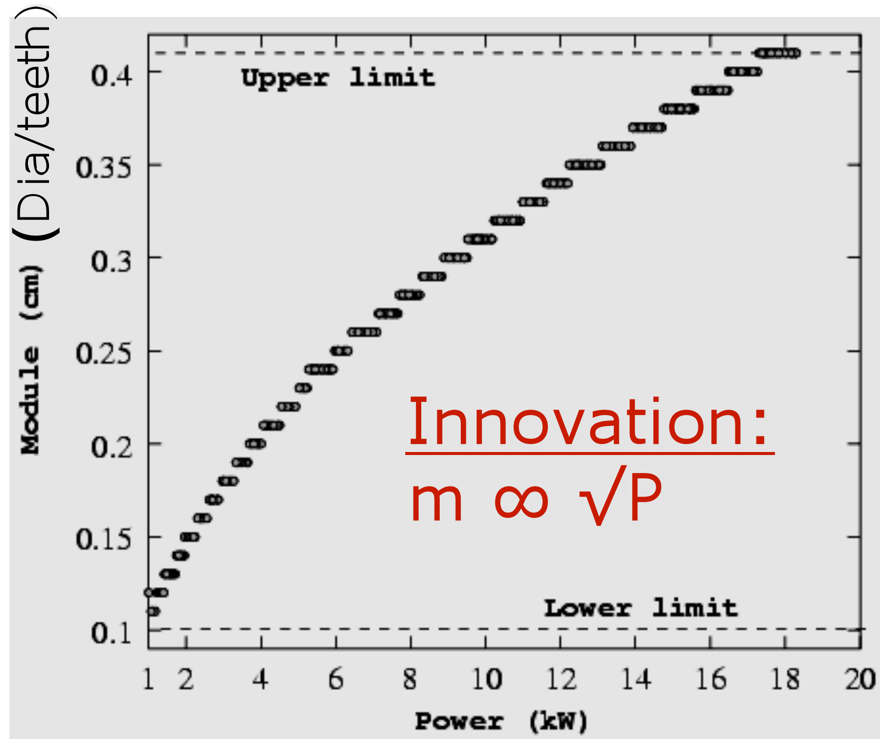
(larger module for more power)



Design Principles

- Module (discrete) varies proportional to square-root of power
- Keep other 28 variables more or less the same

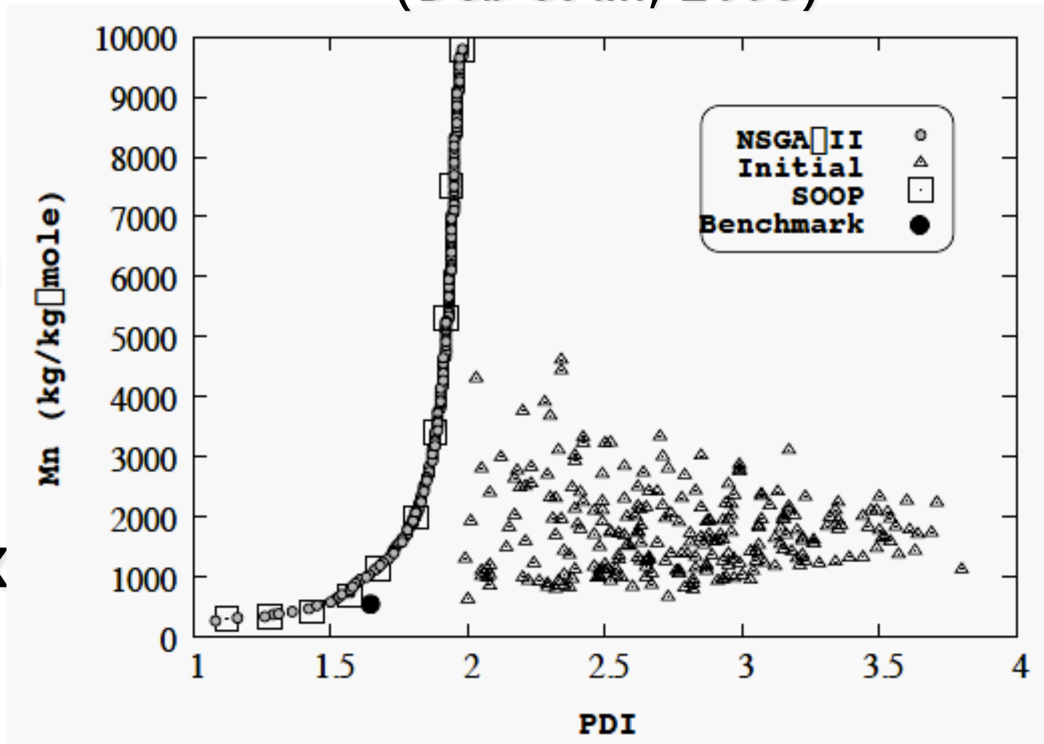




Epoxy Polymerization

- Three ingredients added hourly
- 54 ODEs solved for a 7-hour simulation
- Maximize chain length (Mn)
- Minimize polydispersity index (PDI)
- Total 3x7 or 21 variables

(Deb et al., 2003)

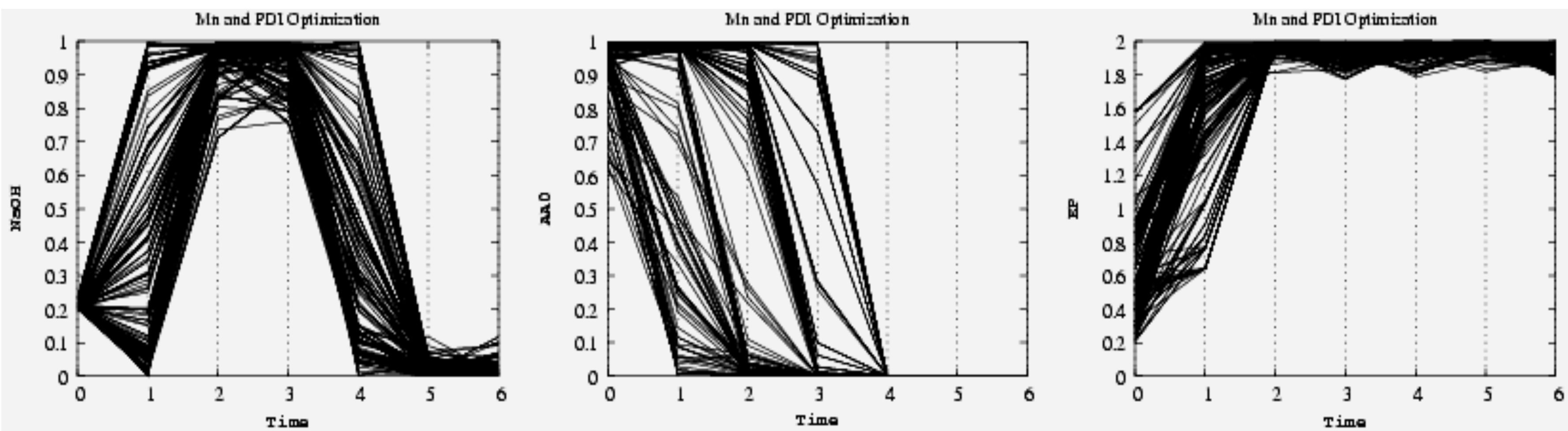


A non-convex frontier

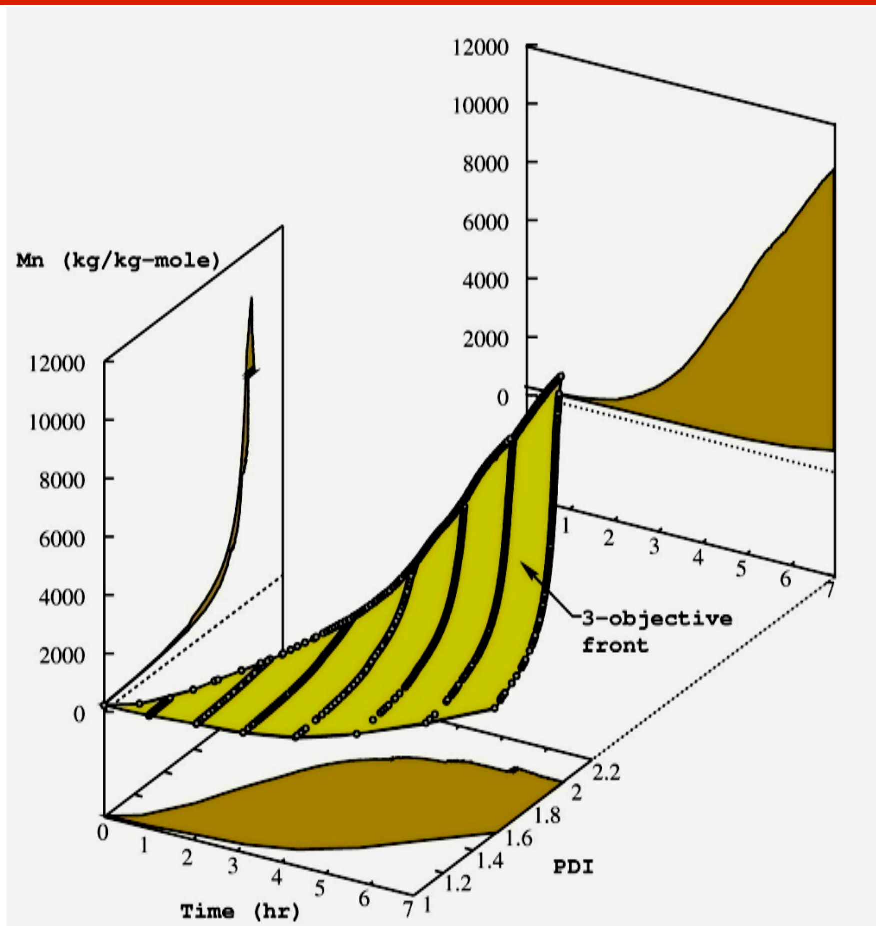
Epoxy Polymerization (cont.)

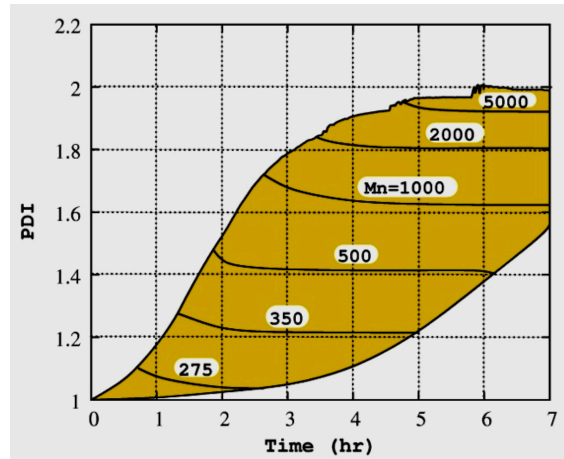
Innovative Operating Principles Revealed:

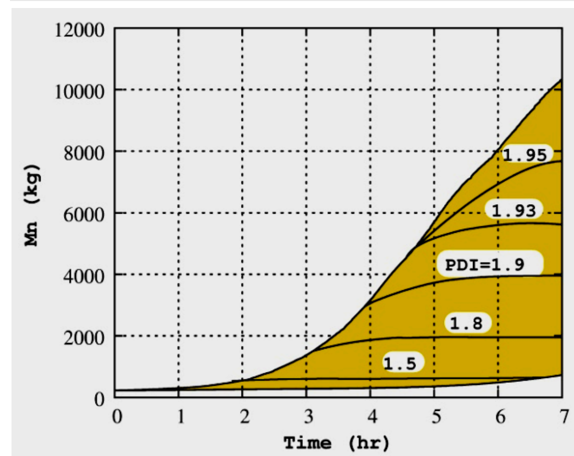
- Some patterns emerge among obtained solutions
- Chemical significance unveiled



Innovative Principles in Use:
An Operating Chart

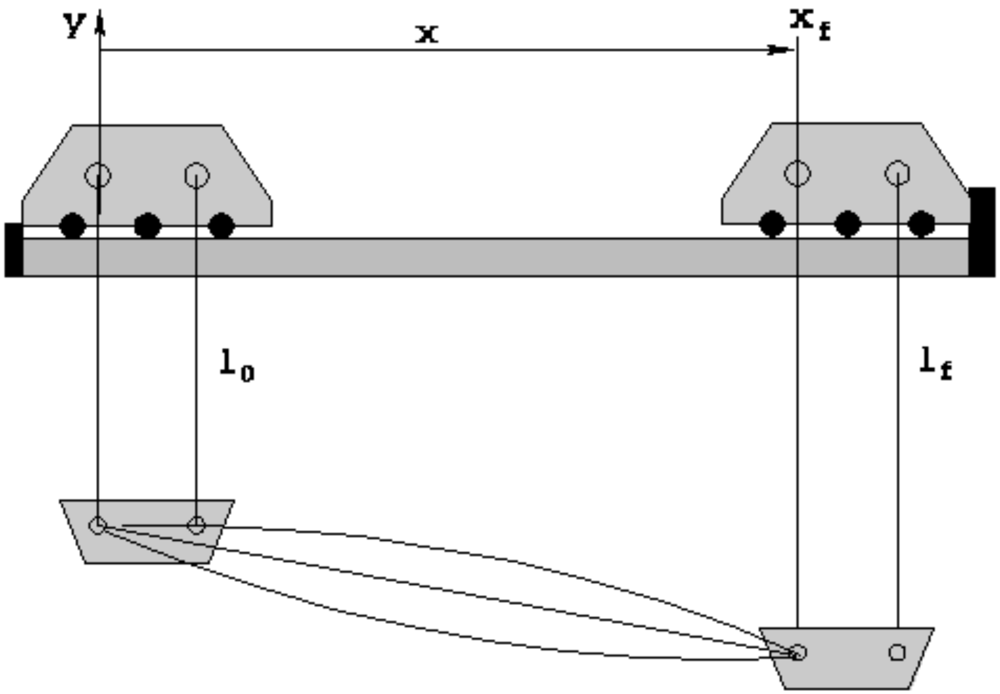




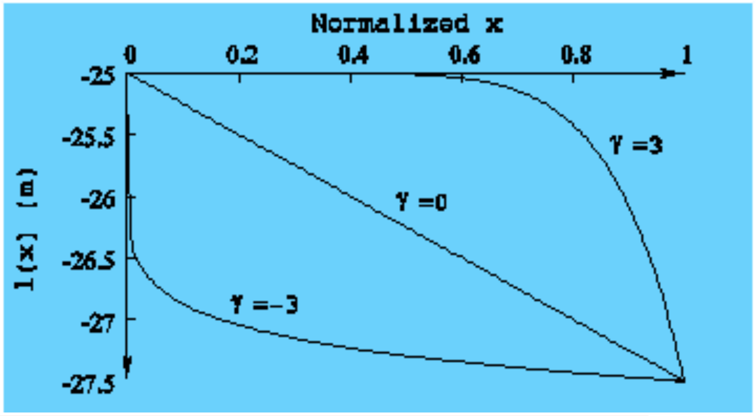


Overhead Crane Maneuvering

- Minimize Operation Time
- Minimize Operating Energy

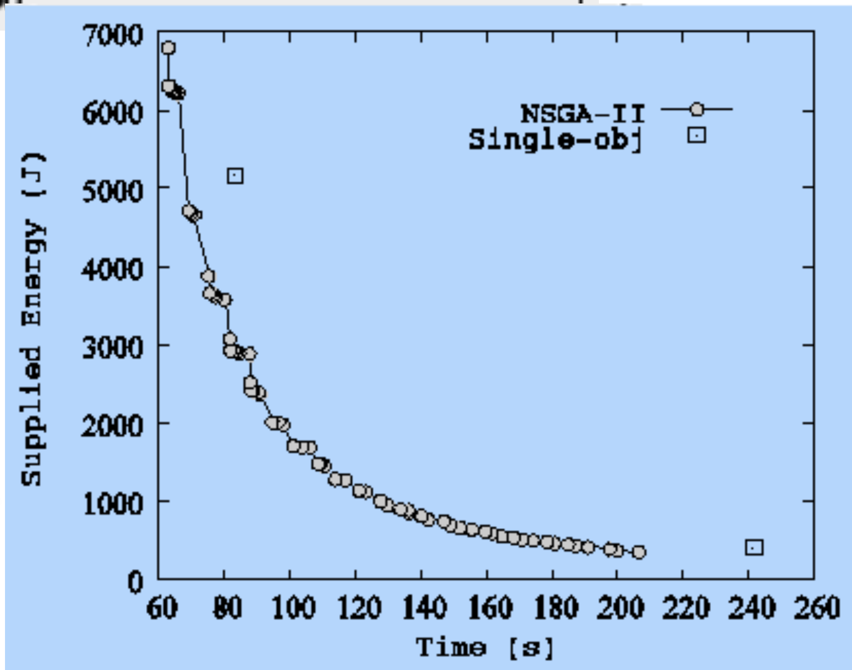


(Deb and Gupta, 2004)



Innovation in Crane Operation

γ	Force (N)	Pattern
2.98	149.26	111111110010001000000000000000000000000000000000000
3.00	187.76	11111111001000100000000000000000000000000
3.00	238.56	111111110010001000000000000000000
3.00	327.86	11111011001001100000000000
2.98	427.93	111110110010011000000
3.00	544.94	11111111001000100
2.87	629.62	111111110010001
2.88	818.99	110111010010001

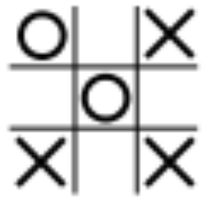


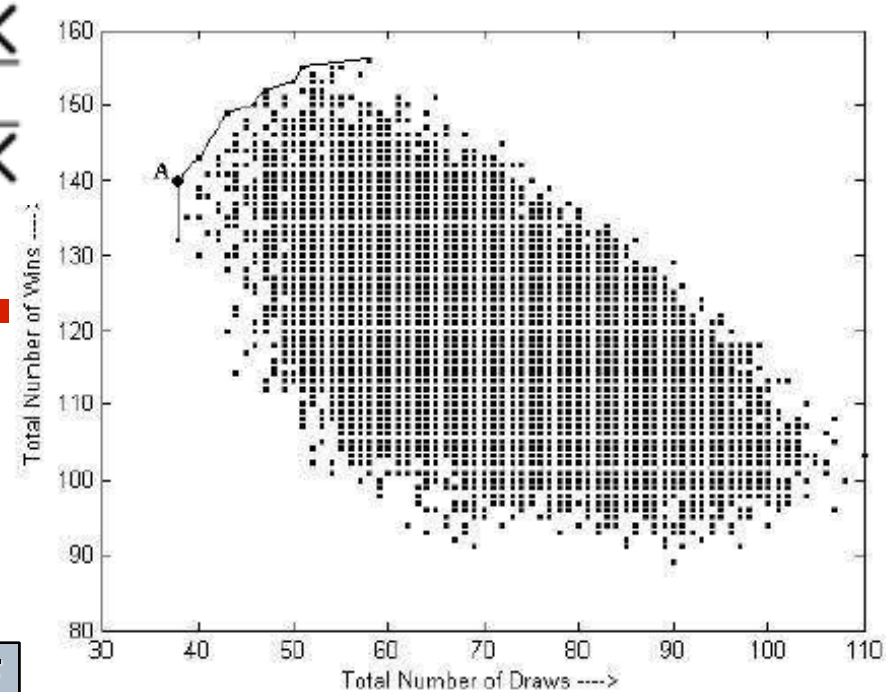
Innovation:

γ≈3, means

'Move laterally first, then drop'

Tic-Tac-Toe Playing Strategy





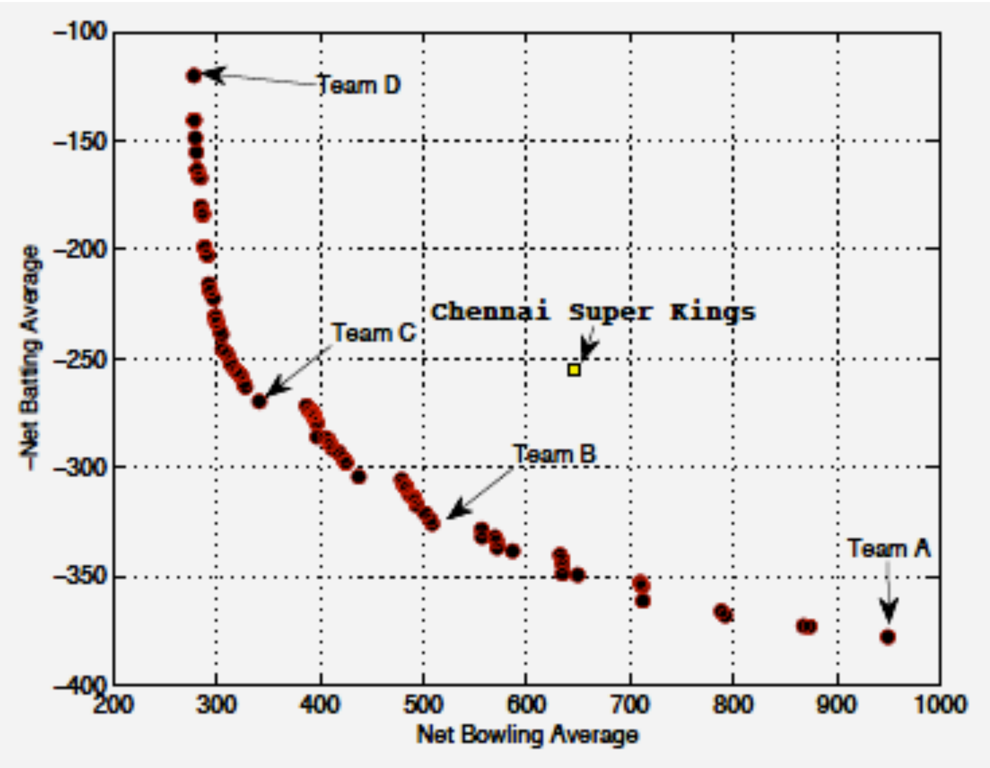
- Widely varying solutions are analyzed and following properties are discovered:
  - If opponent is one short of winning, block it
  - If center is empty, occupy it
  - If center is filled, occupy corner and edge-center, in this order
- All 72,657 solutions are split in #draws and #wins
- Agree with what a human will come up with experience

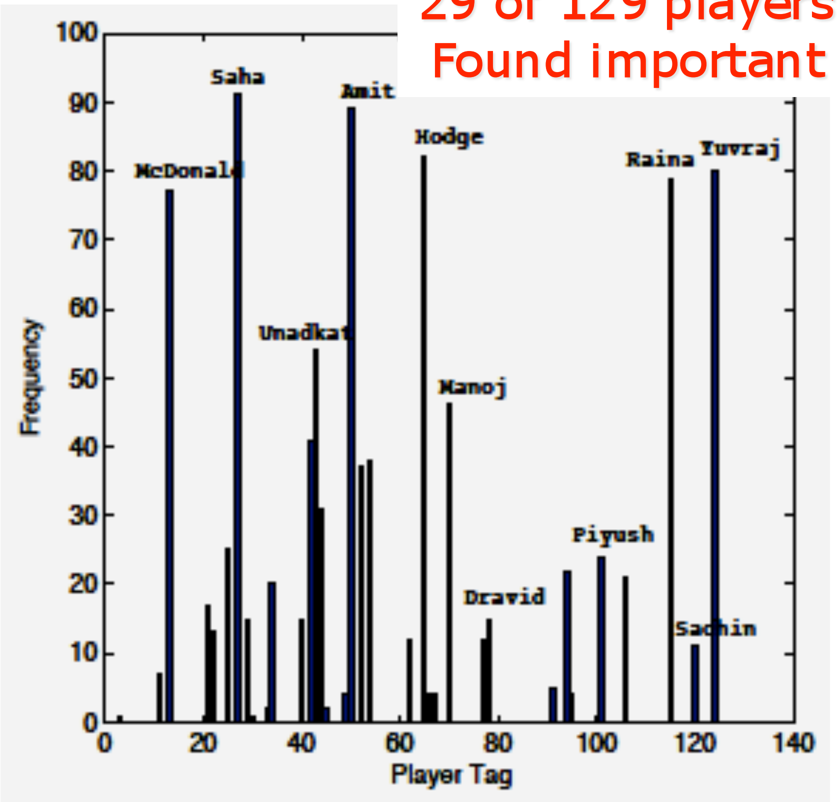
Done this on an Iterated Prisioner's Dilemma problem

Player Selection in the Game of Twenty20 Cricket

- Compute frequency of players and choose from them

29 of 129 players Found important





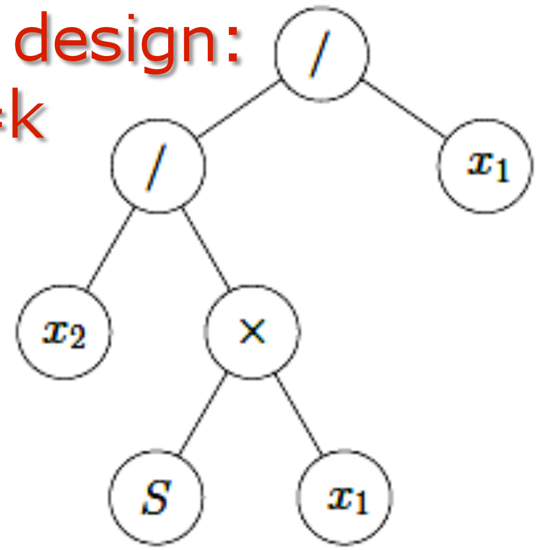
Recent Extensions of *Innovization* Concept

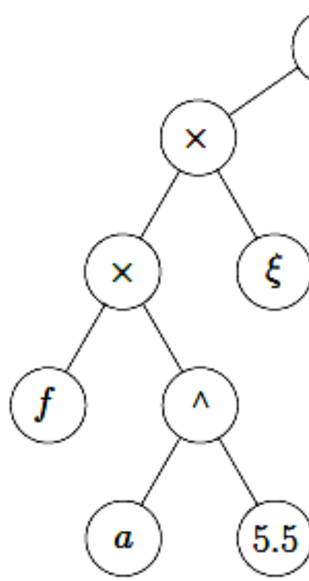
- More generic *innovization*
- Higher-level *innovization*
- Lower-level *innovization*
- Automated *innovization*
- Temporal *innovization*
- *Innovization* to speed-up EMO's convergence

Generic *Innovization* Using Genetic Programming

- Dimensionally-aware GP
  - Meaningful rules
- Any structure rules can be deciphered
- Details in our EMO-13 pape

Two-bar truss design:
$x_2/(S*x_1*x_1)=k$





Metal-cutting problem:
$f\ a^{5.5}\ \xi/T_p = k$
(a new property)

Higher-Level *Innovization:*
Common Principles Among Multiple Fronts

- Over different parameter settings
  - Material properties, load, bounds, resources
- Over different variable sizes and types
  - Continuous to discrete
- Over multi-modal solutions
- Over different constraint combinations
- More possibilities

Procedure:

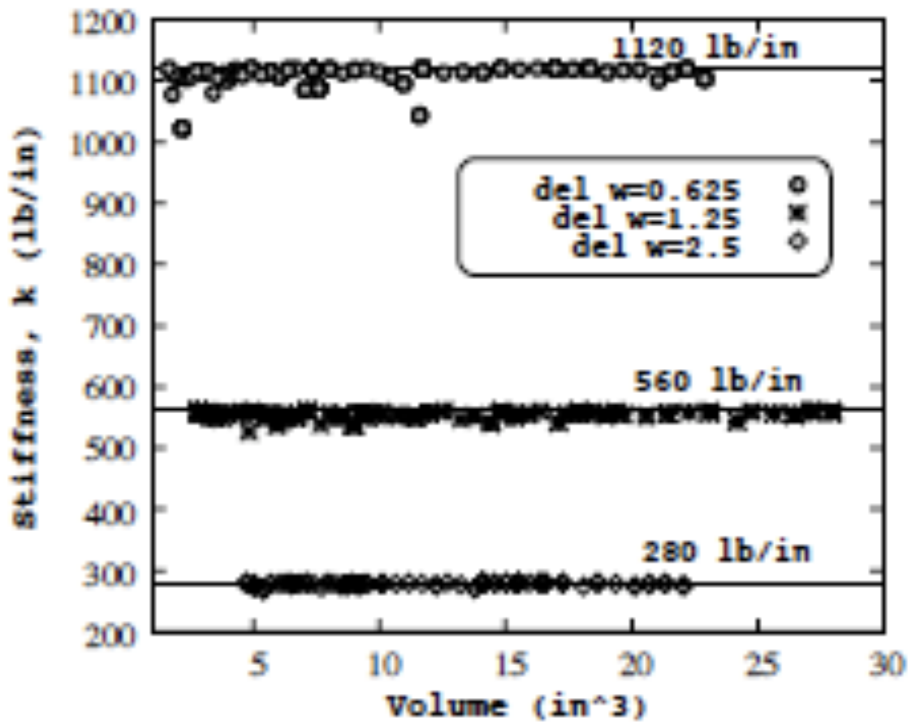
• Multiple fronts put together -> Extension of *innovization* task

Higher-Level *Innovization* Results on Spring Design Problem

- δ_w varied
- Multiple fronts combined
- Innovization performed

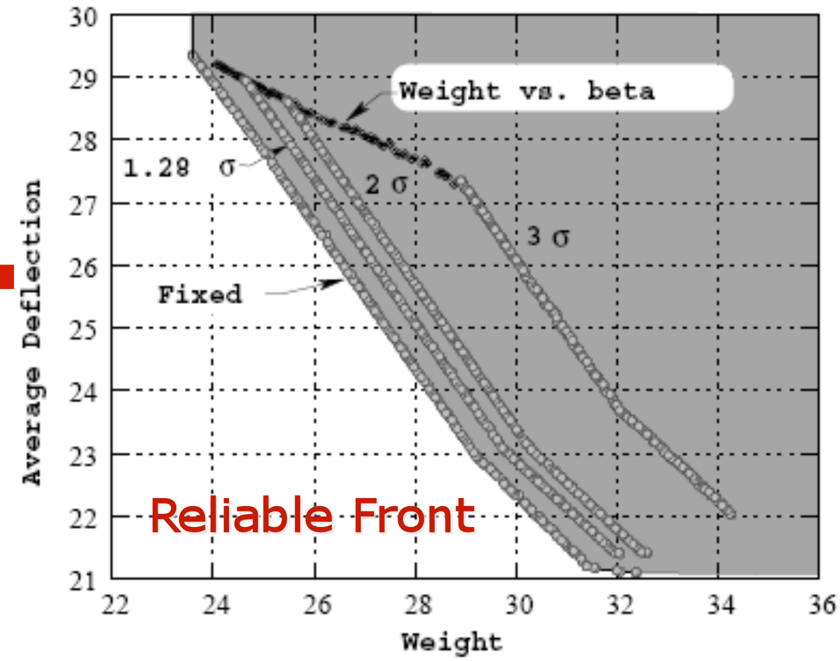
Higher-level *Innovization*:

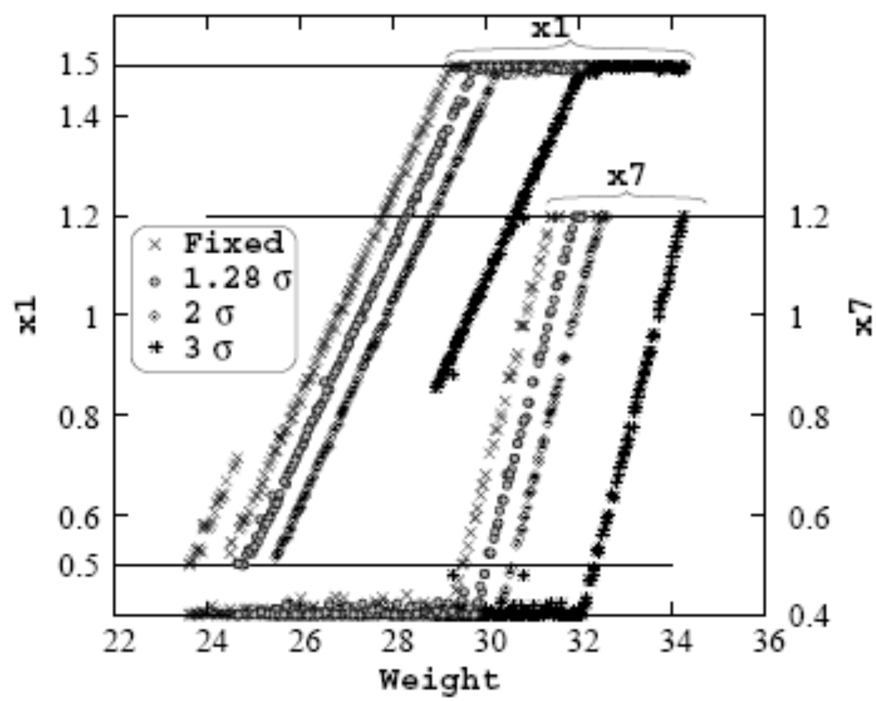
- Stiffness= $700/\delta_w$



Reliability Considerations

- A car side-impact design problem
- How common principles change with reliability value? (Deb et al., 2010)



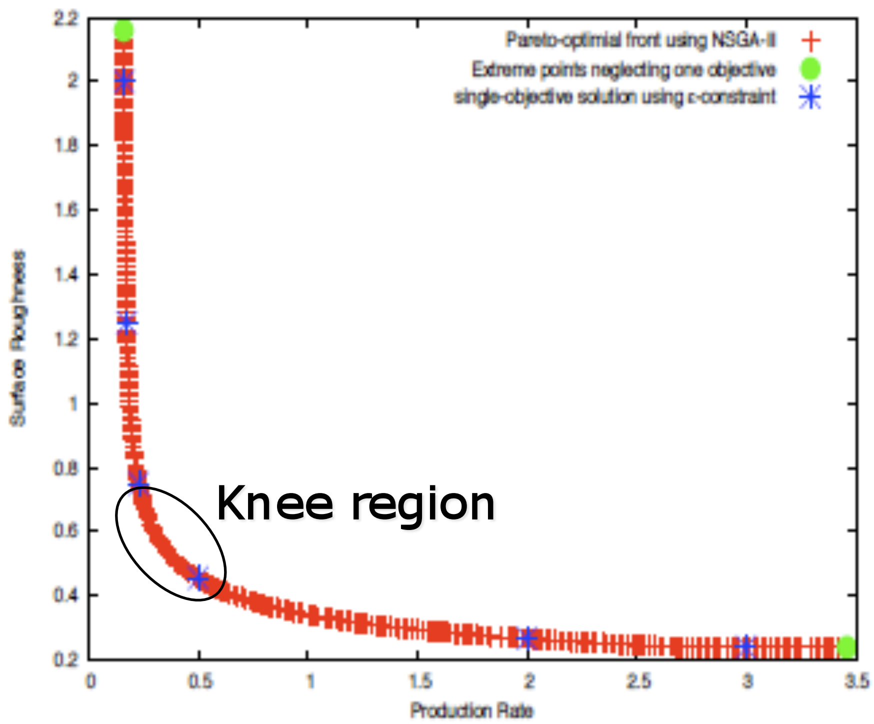


x1: thk. of B-Pillar inner
x7: thk. of roof rail

Lower-Level *Innovization:*

Common Principles Within a Part of PO Front

- Knee region
- Over Preferred PO solutions
  - Reference point based solutions, weight based solutions
- Over specified values of objective or constraints or variable boundaries
- Over some fixed values of variables



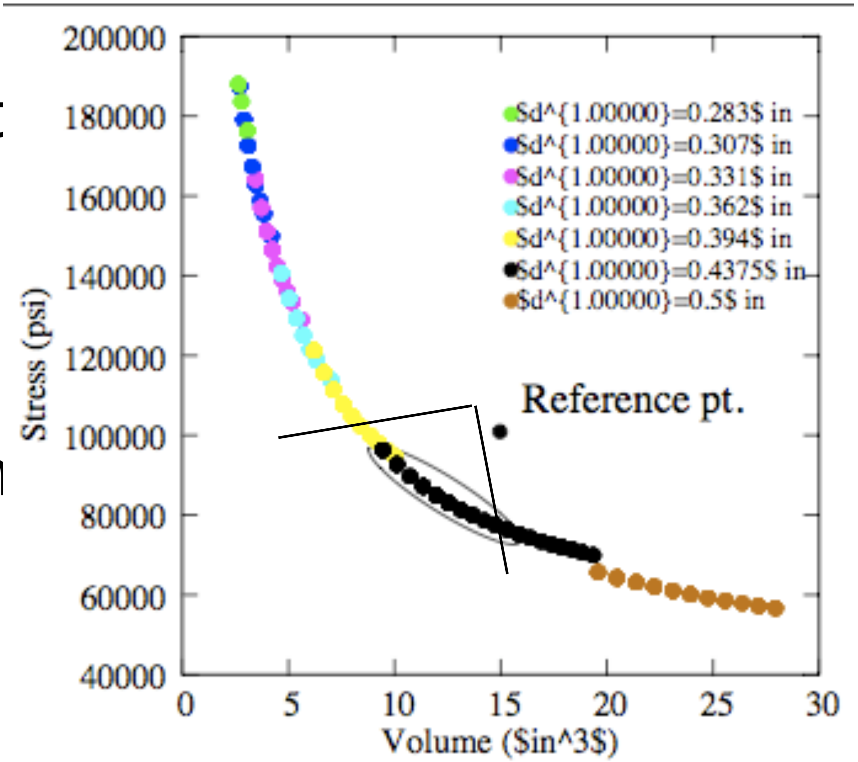
Lower-Level Innovization

- Two sets:
  - What is common in one **and** what does not exist in another?

$S_i \geq S_{high}$ for set1,
$S_i \leq S_{low}$ for set2

- Spring design problem with a reference pt.
  - d=0.43755 in

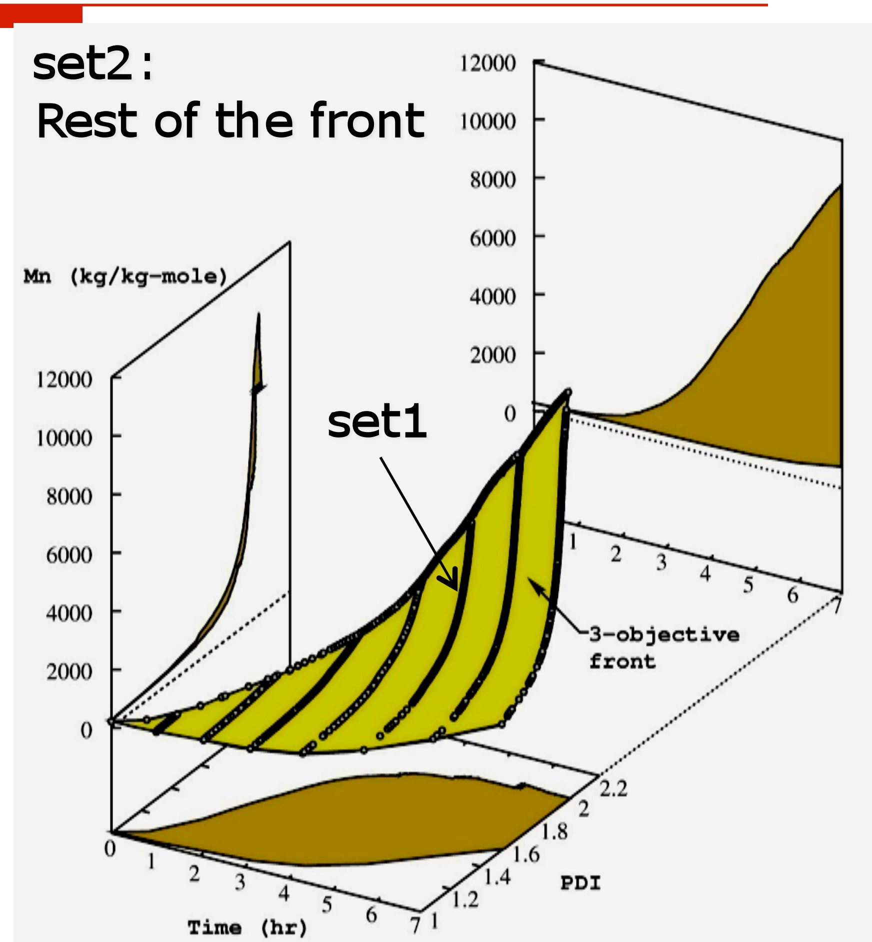
Spring Design Problem



More Lower-Level *Innovizations*

- Three-objective front
- What principles are common for f1=f1*, but not common with rest of the front?

- Epoxy polymerization process

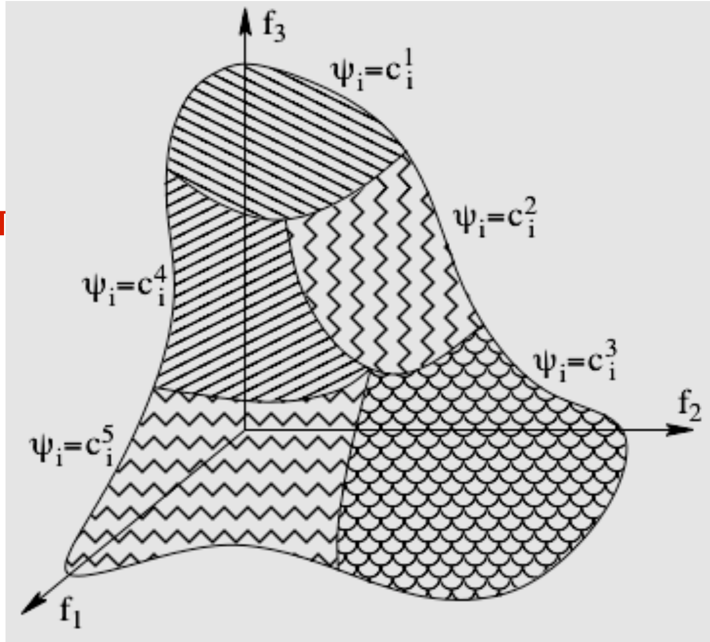


Automated *Innovization*

- Find principles from Pareto-optimal data
  - Objectives and decision variables
- A complex data-mining task
  - Clustering cum concept learning
  - Rule extraction
- Difficulties
  - Multiple relationships
  - Relationships span over a partial set
  - Mathematical forms not known a-priori
  - Dealing with inexact data

First Study

(Bandaru and Deb, 2010, EO)



- Rules of type:

$$\psi_i(\mathbf{x}, \mathbf{f}(\mathbf{x}), \mathbf{g}(\mathbf{x})) = c_i.$$

- Currently, limited to

$$\psi_i(\boldsymbol{\phi}(\mathbf{x})) \equiv \Pi_{j=1}^{N} \phi_j(\mathbf{x})^{b_{ij}} = c_i.$$

- Solve (*m* data points):

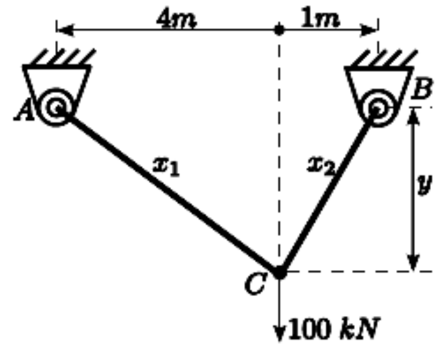
$$\begin{aligned}
\text{Minimize} \quad & \left(number\ of\ clusters + unclustered\ points + \sum_{clusters} c_v \right), \\
\text{Subject to} \quad & 1 \leq d_i \leq m, \\
& unclustered\ points \leq 0, \\
& -1 \leq b_{ij} \leq 1\ \forall j, \\
& |b_{ij}| \geq 0.1\ \forall j, \\
& d_i \text{ is an integer and } b_{ij}\text{'s are real.}
\end{aligned}$$

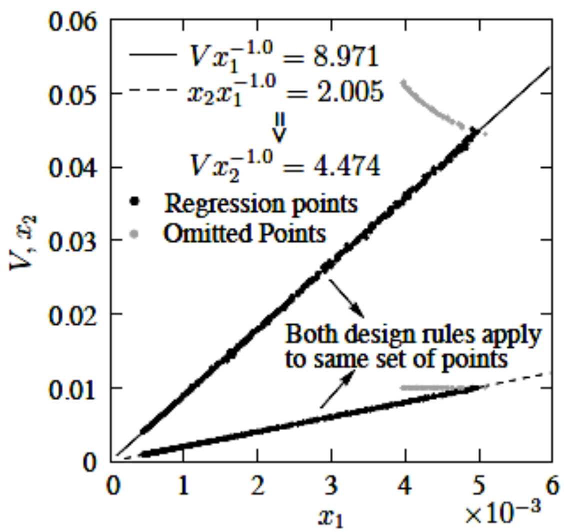
Coeff. of variance

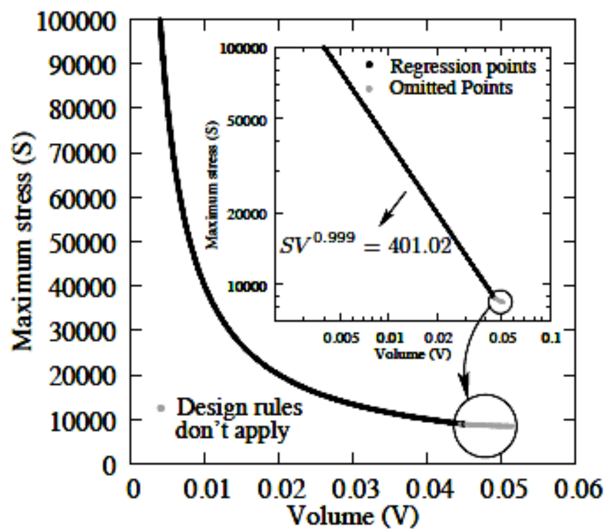
A Truss Design Problem

Optimization formulation

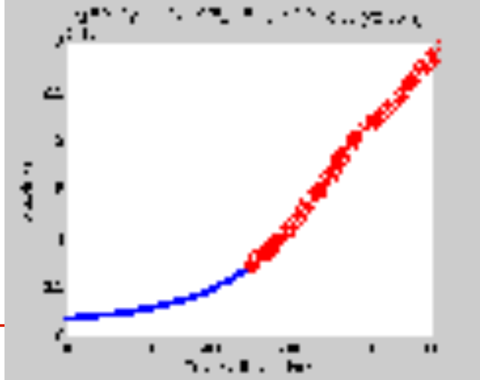
$$\begin{array}{ll} \text{Min.} & f_1(\vec{x}) = V = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2}, \\ \text{Min.} & f_2(\vec{x}) = S = \max(\sigma_{AC}, \sigma_{BC}), \\ \text{s.t.} & \max(\sigma_{AC}, \sigma_{BC}) \leq 10^5 \text{ kPa}, \\ & 0 \leq x_1, x_2 \leq 0.01 \text{ m}^2, \\ & 1 \leq y \leq 3 \text{ m}. \end{array}$$





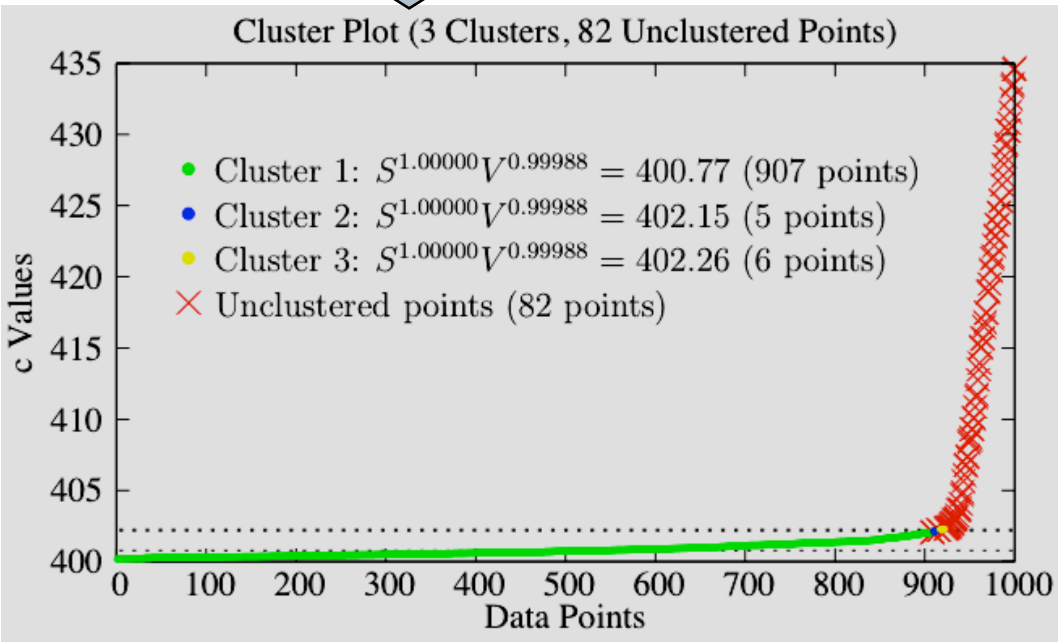


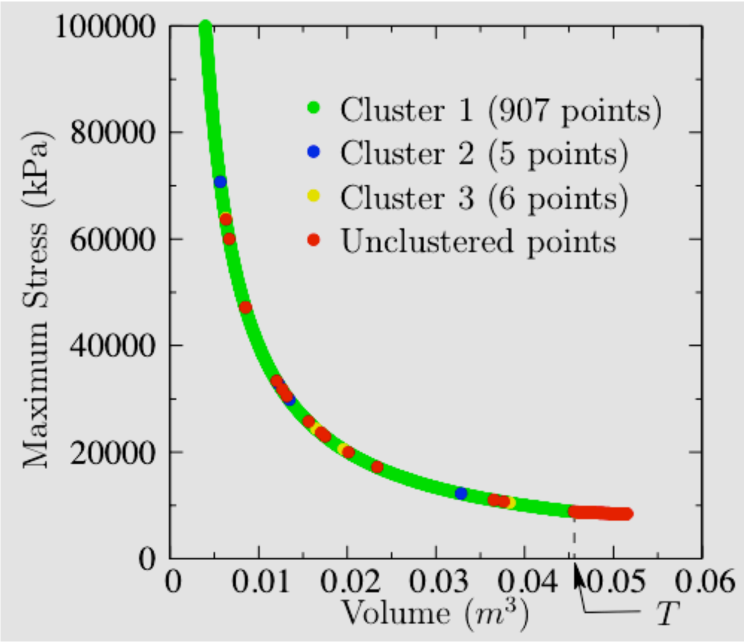
Truss Design (Automated Innovization)



- Obtained Rules (independent applications):

$$SV = 400.77, \quad \frac{x_1}{V} = 0.1105, \quad \frac{x_2}{V} = 0.2236, \quad \frac{x_2}{x_1} = 1.9838$$



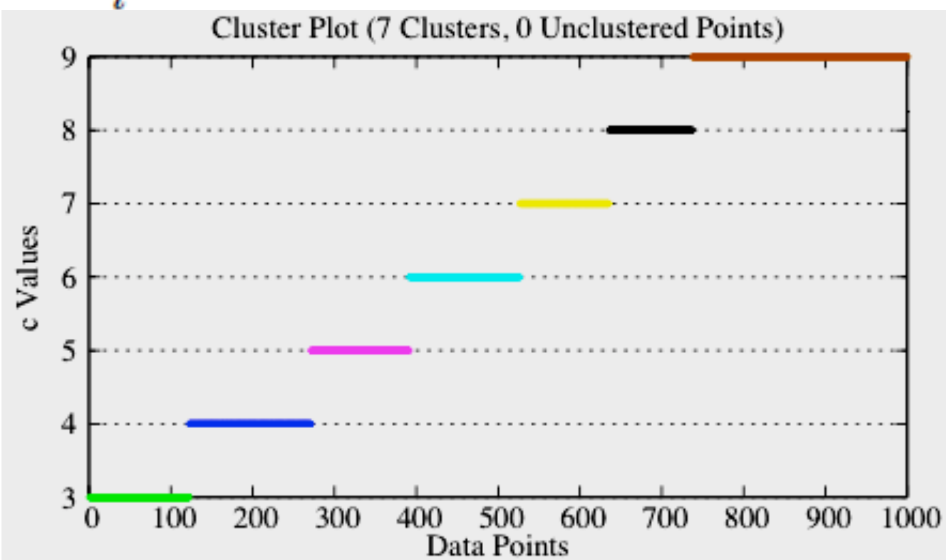


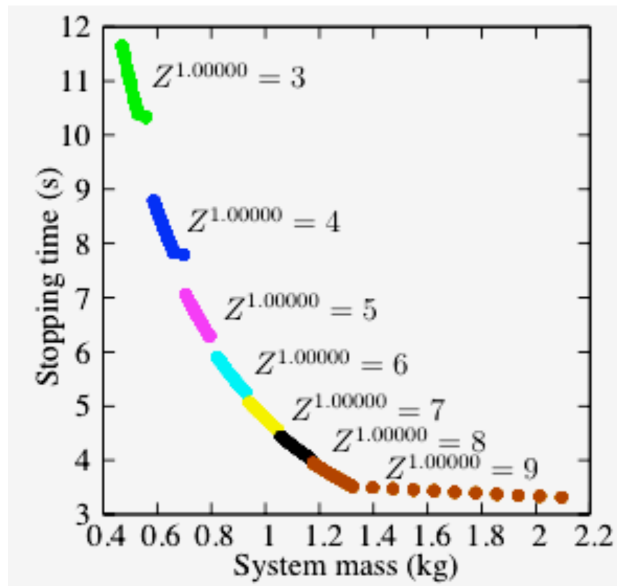
Clutch Design (Automated Innovization)

- Obtained Rules (independent applications):

$$t = 1.5 \text{ mm}, \quad F = 1000 \text{ N}, \quad Z = \{3, 4, 5, 6, 7, 8, 9\},$$

$$\frac{r_o}{r_i^{0.78885}} = 3.1524, \quad TS^{0.98635} = 305885.84,$$





Multiple Rules Simultaneously

(Bandaru and Deb, 2010)

- Use of a *niching* operator to find multiple rules
- Row-echelon operation to get a minimal set

i	$a^*_{i1}b^*_{i1}$	$a^*_{i2}b^*_{i2}$	$a^*_{i3}b^*_{i3}$	$a^*_{i4}b^*_{i4}$	$a^*_{i5}b^*_{i5}$	d^*_i	S_i
DR1	1.0000000	0.0000000	0.0000000	-1.0006158	0.0000000	520	88.2%
DR2	0.0000000	1.0000000	0.0000000	1.0005781	0.0000000	508	80.8%
DR3	0.0000000	0.0000000	1.0000000	-1.0009661	0.0000000	507	86.8%
DR4	0.0000000	0.0000000	0.0000000	0.0000000	1.0000000	511	87.2%

10	10101	0.8388214	0.0000000	[illegible]1667	0.0000000	1.0000000	869
11	10111	0.8813441	0.0000000	[illegible]007	-0.3013669	1.0000000	869
17	10100	1.0000000	0.0000000	-0.9971116	0.0000000	0.0000000	869
18	10110	1.0000000	0.0000000	-0.7353538	-0.2647701	0.0000000	869
19	11010	1.0000000	0.6702390	0.0000000	-0.3300355	0.0000000	869
20	11100	1.0000000	0.7911332	-0.2102126	0.0000000	0.0000000	860

Truss Design

$$\text{DR1: } \frac{V}{x_1} = c_{\text{DR1}}, \quad \text{DR2: } Sx_2 = c_{\text{DR2}}, \quad \text{DR3: } \frac{x_1}{x_2} = c_{\text{DR3}}, \quad \text{DR4: } y = c_{\text{DR4}}$$

Temporal Evolution

Step 1: Perform multiple runs of an EMO

- store all P_t of non-dominated solutions of gen. t
- remove dominated solutions

Step 2: Perform automated *innovization* for final pop and identify design principles (DP)

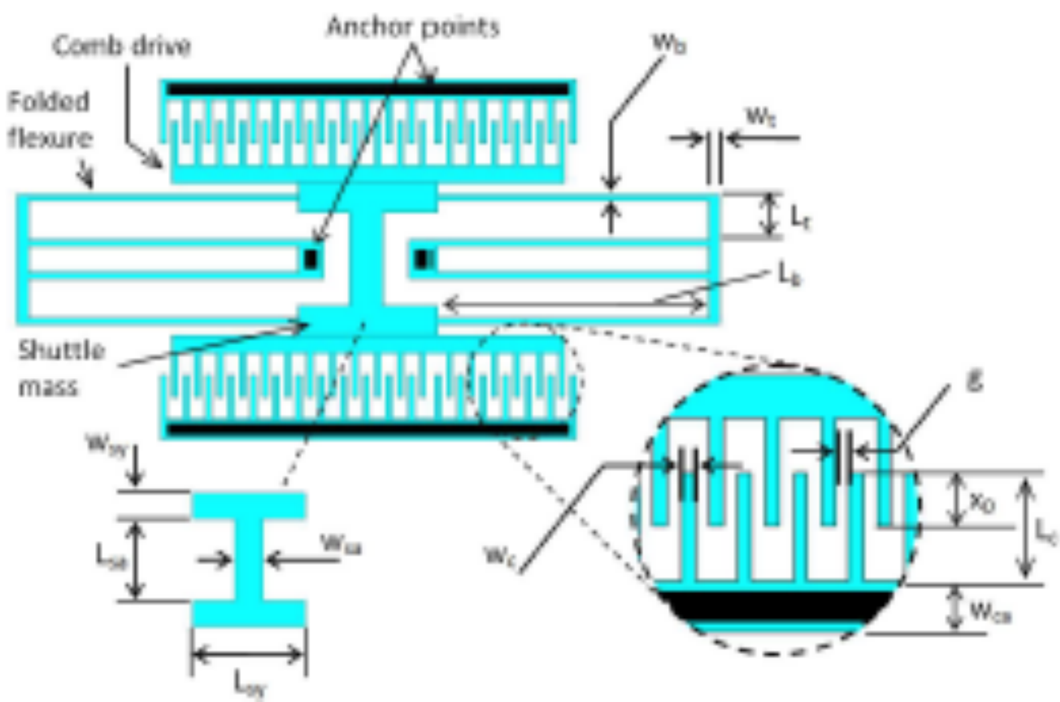
Step 3: Identify DPs that exist with a statistical significance in all earlier generations

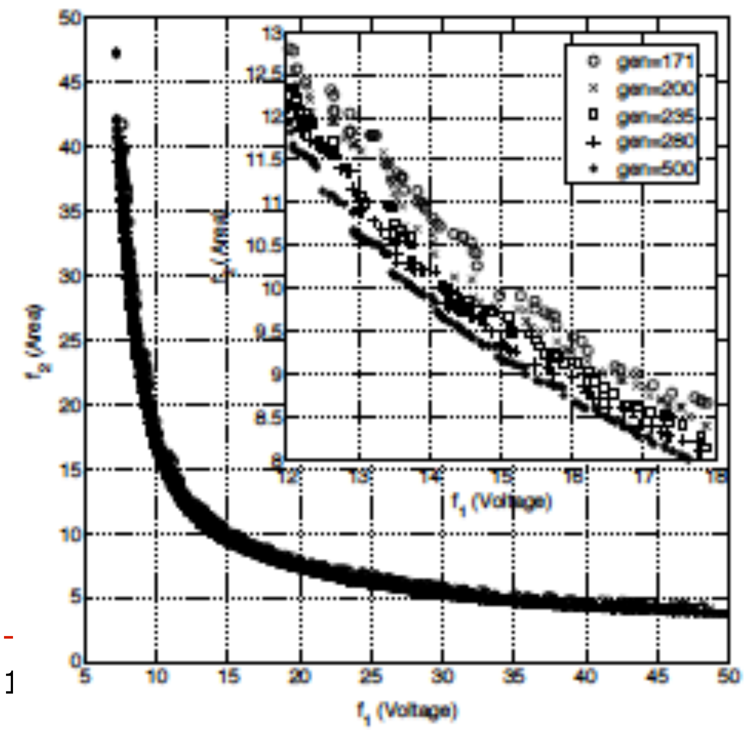
Step 4: Make a time-line genesis of DPs and decipher the hierarchy of evolution of DPs

An Example:
A MEMS Design Problem

- 14 design variables, 24 constraints
- Obj: (i) min power consumption (ii) min area

Step 1





Automated *Innovization* Step 2

- 15 design principles (DPs) obtained

Notation	Design principle	Cluster average ($\mu_{largest}$)	Significance
DP1	$w_c^{1.0000} = c$	2.000231E-06	98.50 %
DP2	$w_{sy}^{1.0000} = c$	1.000441E-05	97.16 %
DP3	$L_{sa}^{1.0000} = c$	1.169490E-05	88.23 %
DP4	$w_t^{1.0000} = c$	2.001497E-06	87.65 %
DP5	$L_t^{1.0000} = c$	6.873649E-06	87.40 %
DP6	$L_{sy}^{1.0000} = c$	3.605399E-05	86.56 %
DP7	$w_{sa}^{1.0000} = c$	1.000482E-05	86.06 %
DP8	$w_b^{1.0000} = c$	2.000028E-06	84.72 %
DP9	$w_{cy}^{1.0000} = c$	1.000088e-05	79.63 %
DP10	$f_1^{1.0000} L_b^{0.6470} = c$	1.078929E-01	78.46 %
DP11	$f_2^{1.0000} L_b^{-0.4888} = c$	3.671301E+02	74.12 %
DP12	$f_1^{0.2546} f_2^{1.0000} L_b^{-0.3563} = c$	2.812855E+02	73.79 %
DP13	$f_2^{1.0000} L_b^{-0.4800} L_c^{-0.1160} = c$	1.258088E+03	72.70 %
DP14	$f_1^{1.0000} L_b^{0.6490} L_c^{0.1429} = c$	2.112050E-02	72.70 %
DP15	$f_1^{0.7737} f_2^{1.0000} = c$	7.301285E+01	70.45 %

Chronology of Evolution of DPs

Step 3:

- Set a threshold on confidence level (8% used here)
- Identify chronology of evolution of DPs

	Notation	Design principle	Cluster average (μ ...)	Significance
5	DP1	$w_c^{1.0000} = c$		
1	DP2	$w_{sy}^{1.0000} = c$		
	DP3	$L_{sa}^{1.0000} = c$		
	DP4	$w_t^{1.0000} = c$		
	DP5	$L_t^{1.0000} = c$		
15	DP6	$L_{sy}^{1.0000} = c$		
	DP7	$w_{sa}^{1.0000} = c$		
13	DP8	$w_b^{1.0000} = c$		
14	DP9	$w_{cy}^{1.0000} = c$		
	DP10	$f_1^{1.0000} L_b^{0.6470} = c$		
3	DP11	$f_2^{1.0000} L_b^{-0.4888} = c$		
4	DP12	$f_1^{0.2546} f_2^{1.0000} L_b^{-0.3563} =$		
2	DP13	$f_2^{1.0000} L_b^{-0.4800} L_c^{-0.1160}$		
	DP14	$f_1^{1.0000} L_b^{0.6490} L_c^{0.1429} =$		
	DP15	$f_1^{0.7737} f_2^{1.0000} = c$		

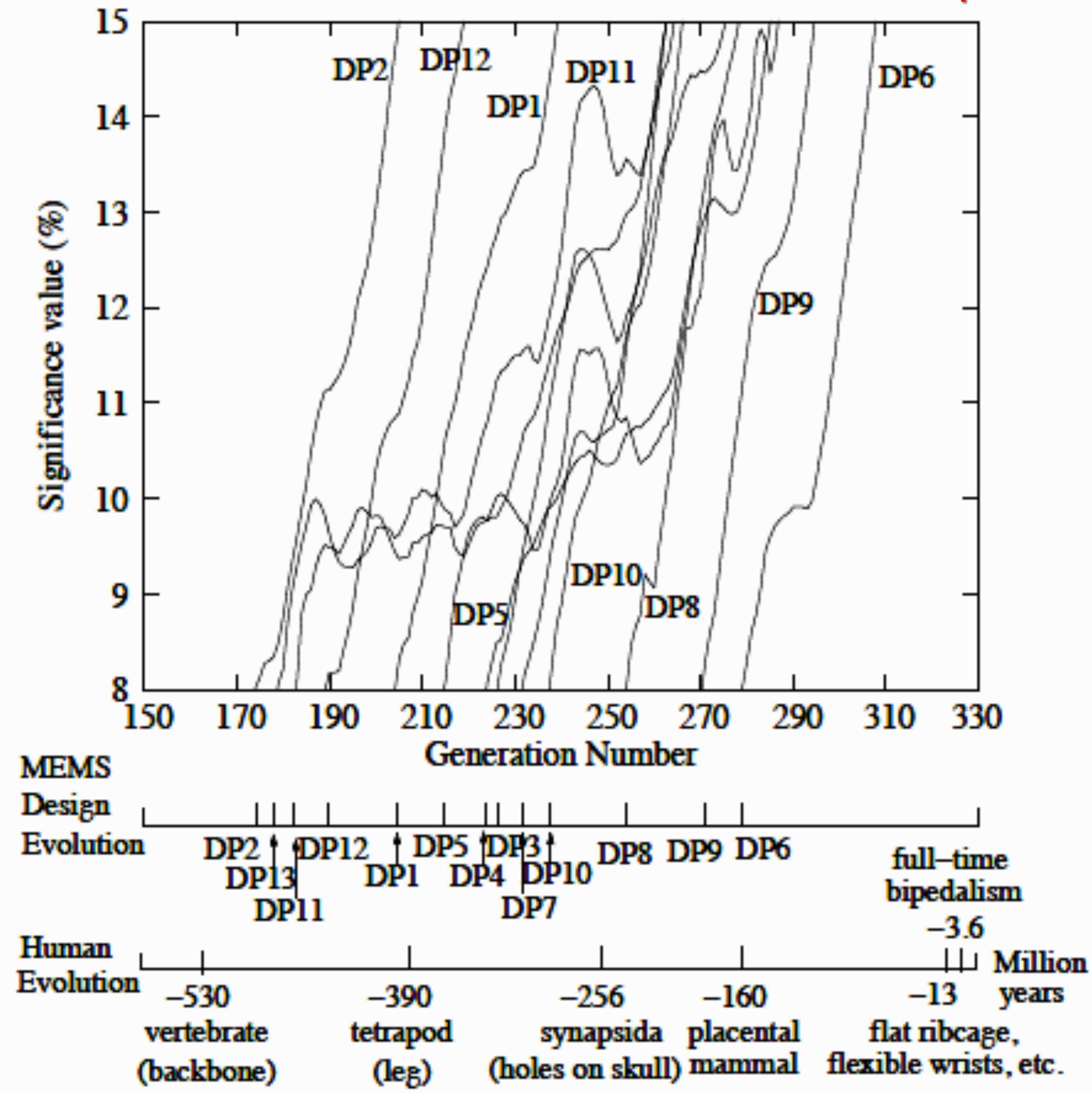
Chronology of DPs:

1: Vert. thk. of I-beam flange
2-4: Horz. Lt. of folded flexure
5: Thickness of folded flexure
6: Thickness of comb
7: Height of I-beam
8: Width of I-beam
…
14: Width of comb base
15: Lsy: width of I-beam

Step 4

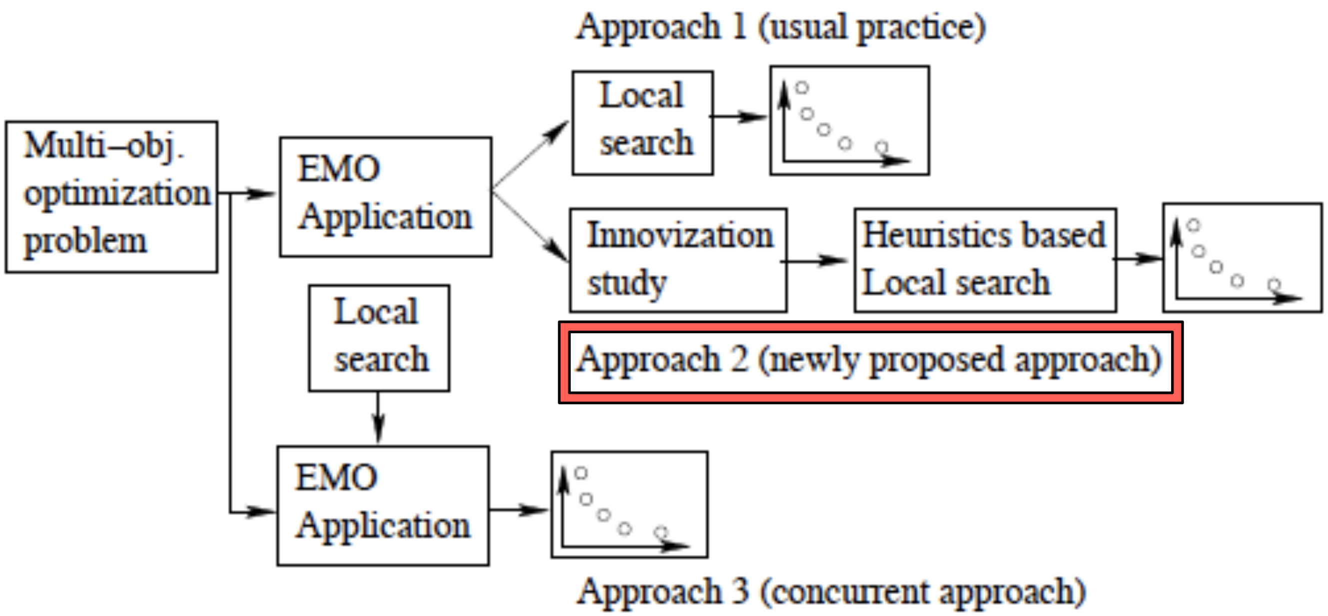
Temporal Evolution of DPs

- Time-line of DPs
- Similar to human evolution
- $w_{sy}=c$ (DP2) after 171 gen.
- L_b and L_c rel. (DP13) evolves



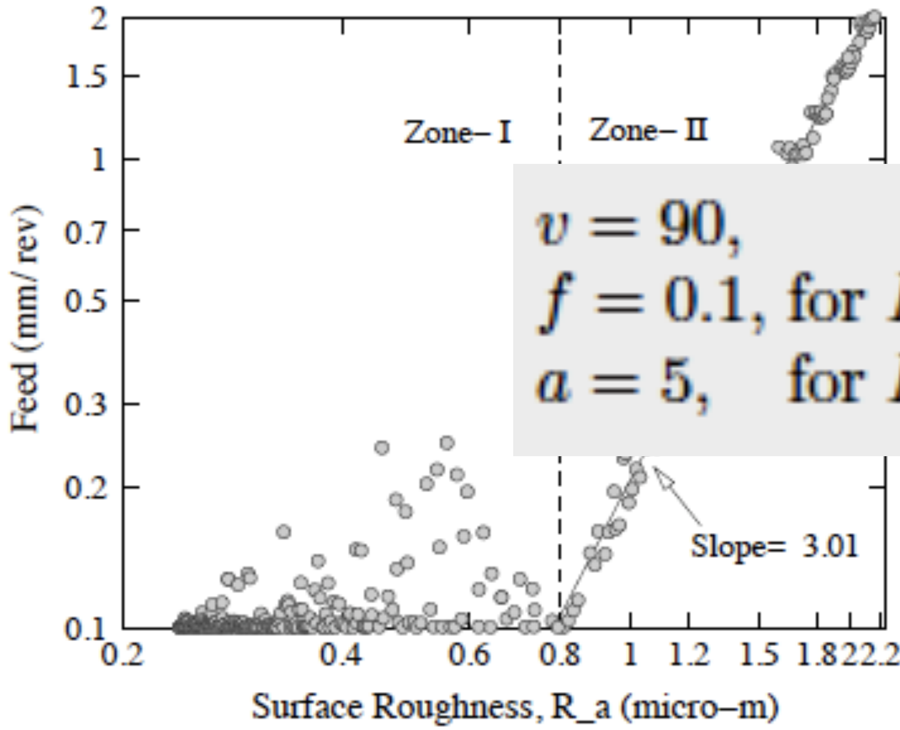
Innovization to Speed-up MO Optimization

- Innovized principles as heuristics for local searches for a further EMO run
- A metal-cutting problem

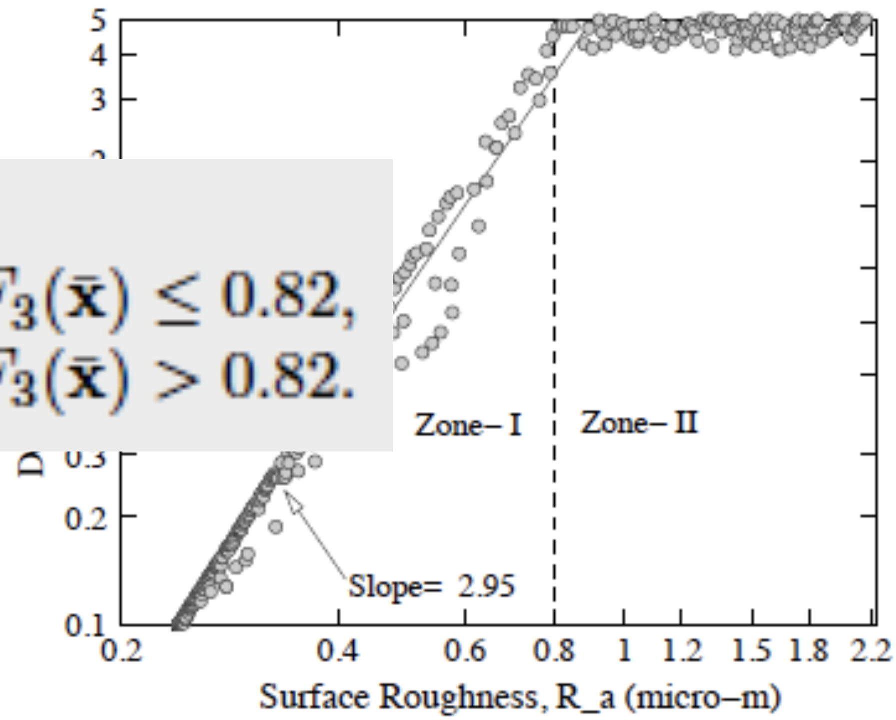


Feed and Depth of Cut

After a few generations of NSGA-II:

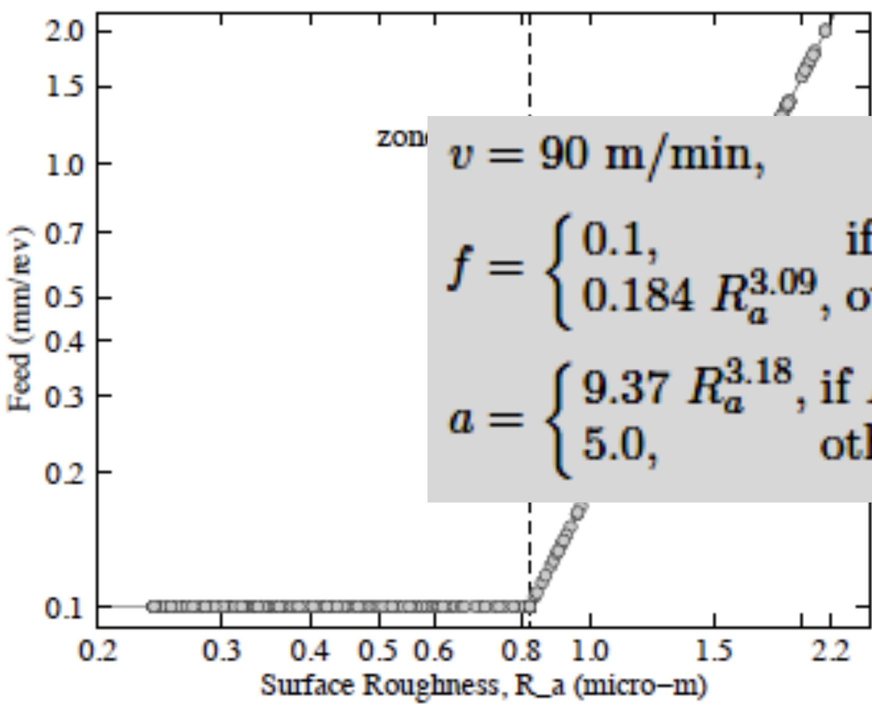


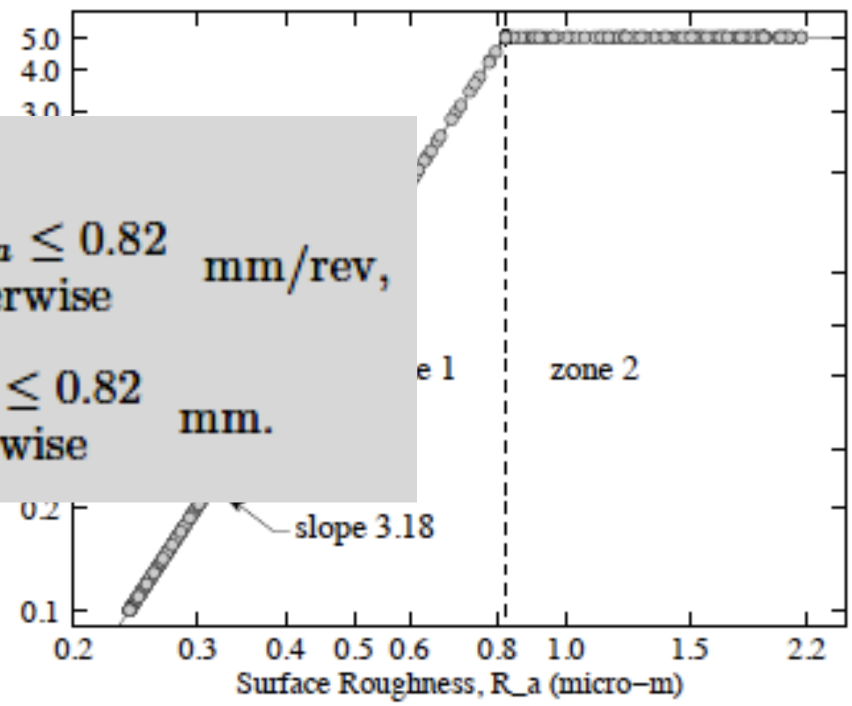
$$v = 90,$$
$$f = 0.1, \text{ for } F_3(\bar{\mathbf{x}}) \le 0.82,$$
$$a = 5, \quad \text{for } F_3(\bar{\mathbf{x}}) > 0.82.$$



Feed and Depth of Cut (cont.)

After Heuristics Based EMO:





$$v = 90 \text{ m/min},$$

$$f = \begin{cases} 0.1, & \text{if } R_a \leq 0.82 \\ 0.184\, R_a^{3.09}, & \text{otherwise} \end{cases} \quad \text{mm/rev},$$

$$a = \begin{cases} 9.37\, R_a^{3.18}, & \text{if } R_a \leq 0.82 \\ 5.0, & \text{otherwise} \end{cases} \quad \text{mm}.$$

Further studies: (Ng et al., 2012, SPS-2012)

Conclusions

- EMO's ability to find multiple trade-off solutions put to a bigger cause
  - Better insight to the problem
  - Learn how to solve the problem
  - Often leads to innovative solution principles
  - Difficult to achieve by other means
- Basic idea extended for different levels of information
- Other possibilities exist (Decision tree approach by Amos Ng and his students at Skovde)
- Applications in design, control and modeling

Acknowledgements

- Robin Purshouse and Peter Fleming
- My collaborators: Aravind Srinivisan, Sunith Bandaru, Cem Tutum, A. Gaspar-Cunha, David Griener, Amos Ng
- Beacon Center for study of evolution at Michigan State University, East Lansing, USA