Integrating Problem Analysis and Algorithmic Development in MCDA

Theodor Stewart

Department of Statistical Sciences, University of Cape Town
and
Manchester Business School, University of Manchester

7th International Conference on Evolutionary Multi-Criterion Optimization, Sheffield, 19–22 March 2013

Personal Experiences on MCDA

- Experience largely based and natural resource management and public sector planning applications, often with multiple stakeholders
- Some examples:
  - Regional water resources planning
  - Fisheries rights allocations
  - Project prioritization and portfolio construction
  - land-use planning
  - Operation of "food banks"

Characteristics of Experienced MCDA

- Decision Makers (DMs) are typically not technical experts, and seek decision support in integrating value judgements (often qualitative) with technical performance measures
- . . . Somewhat different to other more technical multiobjective optimization studies in (for example) engineering design applications
- Typically gives rise to large numbers (10–30) of objectives
- At least some objectives qualitative in nature
- Multiple stakeholders the norm!

Discrete Choice vs. Multiobjective Optimization

- Many natural resource / public sector /multi-stakeholder planning problems are structured in a discrete choice framework
- . . . especially when many qualitative criteria are present
- Multi-objective optimization may well precede this discrete analysis phase
  - As part of the structuring process to generate a shortlist of policy scenarios for detailed decision analysis
  - Typically quite large numbers of criteria/objectives (7–20 or more)
  - With minimal preference inputs from the “real” decision maker
  - And a need to generate widely dispersed but potentially optimal solutions
- This context provides opportunities for synergy and learning between EMO and other MCDA approaches

Prior/Progressive/Posterior Preference Expression

- Prior preference expression: Construct a substantially complete preference model (e.g. value or utility function) which is applied to the full decision space
- Posterior preference expression: Summary presentation of the efficient set presented to the decision for direct holistic evaluation
- Progressive preference expression ("interactive methods"): Partial preference information used to identify part of the decision space; local preferences used to guide search to another part.

Question: How do we cope with diverse stakeholders?

Prior Preference Elicitation and Discrete Choice Problems

- Discrete choice formulations often arise from a strategic choice context
- Prior preference elicitation then often favoured:
  - Aids in learning and structuring towards value focussed thinking
  - Useful in responding to new alternatives
  - Provides a clear audit trail
- . . . But can be time and resource consuming to do properly (and misleading otherwise) . . . multi-day workshops
- Probably not justified at stage of generating the shortlist (by multiobjective optimization)

Multiobjective Optimization Context

- Superficial prior preference elicitation (e.g. weighted sums) inappropriate and potentially dangerous
- . . . but comprehensive prior preference elicitation may be too costly in time and effort
- Conventional EMO is typically based on posterior preference expression
- . . . Clearly advantageous (even in the short-list generation context) for 2 or perhaps 3 objectives
- But beyond that . . . ?

- How do we ensure comprehensive and effective (in achieving decision support aims) exploration of the efficient frontier?
- . . . Especially in higher dimensionality contexts
- Concern that graphical representations for posterior preference elicitation for higher dimensionality spaces may bias perceptions in subtle ways.
- Multiobjective optimization problems in higher dimensionality problems for short-list generation in natural resources / public sector planning will require some level of progressive interaction
- . . . especially when involving multiple stakeholders

Reference Points and Scalarizing Functions

- Partial preferences expressed in terms of a reference point, or aspiration levels for each criterion/objective
  - Let $\mathbf{x} \in \mathbb{X}$ be the decision vector
  - Let $z_i = f_i(\mathbf{x})$ be the corresponding function value for i-th objective (to be *minimized*, say)
  - Let a_i the aspiration level for objective i
- Solution found which minimizes a scalarizing function $S(\mathbf{z}, \mathbf{a})$
- Conventional form: $S(\mathbf{z}, \mathbf{a}) = \max_i \{f_i(\mathbf{x}) - a_i\} + \epsilon \sum_i f_i(\mathbf{z})$
- Useful alternative form $S(\mathbf{z}, \mathbf{a}) = \left[\dfrac{f_i(\mathbf{x}) - z_i^*}{a_i - z_i^*}\right]^\gamma$, where z_i^* is the ideal level for objective i
- Finds “closest” efficient solution to the reference point

- Important advantage of the reference point approach is low sensitivity to preferential dependencies (unlike additive value functions)
- Typically, the reference point is varied interactively to explore the efficient set
- But questions around how reference points are modified in exploratory mode . . . evidence of sensitivity to known cognitive biases
  - Anchoring to previously seen solutions and/or to previous goals
  - Resistance to sure loss
- Such cognitive biases may lead to
  - Incomplete search of the efficient set
  - Insufficient diversity when seeking a shortlist

Outline of a Simple Search Procedure

- Approximate the ideals (z_i^*) and nadirs (say, z_i^0) for each objective $i = 1, 2, \ldots, m$, perhaps from a payoff table ... In spite of known problems in assessing true nadirs in this way, it does give a broad representation of the outcome space
- Generate $m + 1$ reference points by (i) $0.5z_i^* + 0.5z_i^0$ for all i; and then (ii) $0.9z_i^* + 0.1z_i^0$ for a particular i and $0.1z_k^* + 0.9z_k^0$ for $k \neq i$ ... for each i in turn
- Present the resulting $m + 1$ solutions from the reference point approaches, plus ranges of outcomes for each z_i, to the decision maker (DM)
- DM states acceptable bounds on each z_i, say z_i^{++} and z_i^{--} to replace z_i^* and z_i^0, and repeat previous two steps, eliminating solutions which do not satisfy the worst case bounds given by z_i^{--}

Example – maximizing problem with 11 objectives

Initial Run Set (showing first 6 objectives only):

Run No.	z_1	z_2	z_3	z_4	z_5	z_6
0	6816	7284	7204	0.551	0.578	0.553
1	8338	6969	6840	0.500	0.527	0.531
2	6427	8804	6667	0.525	0.528	0.522
3	6527	7058	8539	0.523	0.527	0.535
4	6252	6957	6512	0.640	0.545	0.531
5	5653	6256	5989	0.474	0.699	0.473
6	6098	6924	6507	0.518	0.530	0.628
7	6686	7298	7025	0.551	0.570	0.551
8	6615	7201	7081	0.554	0.575	0.554
9	6631	7242	7112	0.550	0.566	0.551
10	6674	7368	7003	0.544	0.570	0.553
11	6703	7136	7021	0.547	0.575	0.553
12	6551	7253	7085	0.546	0.580	0.550
Max	8338	8804	8539	0.640	0.699	0.628
Min	5653	6256	5989	0.474	0.527	0.473

Example (Cont.)

	z_1	z_2	z_3	z_4	z_5	z_6
Initial ranges:						
Max	8338	8804	8539	0.640	0.699	0.628
Min	5653	6256	5989	0.474	0.527	0.473
Preference-adjusted acceptable bounds:						
z_i^{++}	7500	8300	7800	0.600	0.600	0.580
z_i^{--}	6300	7800	7200	0.550	0.500	0.500

3 of second set of 12 runs approximately met desired nadirs

Ranges from second set satisfying z_i^{--} bounds:						
Max	6699	8402	7513	0.572	0.560	0.524
Min	6516	7959	7193	0.562	0.500	0.509

Reflections on the Simple Search Procedure

- Some numerical evidence that it can provide meaningful exploration
- Still primarily focussed on generating a "most preferred" solution
- ... Less clear that the range of solutions produced gives an adequately representative shortlist for further evaluation ... perhaps other means of generating reference points need to be investigated
- Possibly not computationally efficient, as a separate optimization required for each reference point
- With multiple stakeholders, the whole procedure might need to be repeated for each ... even less efficient

Let's review where we are at!

- We seek effective and efficient generation of one or more solutions
- In many contexts seek a shortlist for more detailed evaluation
  ... covering diverse preferences (e.g. stakeholders)
- EMO efficient in generating a representative frontier ... But the posterior preference expression presents problems with larger numbers of objectives and/or multiple stakeholders
- Prior preference models time and effort demanding ... especially with multiple stakeholders
- Reference points methods as a tool for progressive preference expression effective with minimal assumptions ... but primarily one solution at a time
- Can we integrate the strengths of e.g., reference point and EMO approaches?

Incorporating reference point and EMO concepts . . .

- Fix on a desired number of solutions to be retained, e.g. "7 ± 2"
- Randomly generate a number of reference points (vectors)
- . . . possibly generating more than needed followed by "filtering" (cf. Steuer's methods for MOLP) to maintain diversity in the retained set
- In principle, minimize the scalarizing function for each reference point to generate a short list (possibly with more "filtering"
- To make this computationally efficient especially used in a iterative manner, heuristics such as genetic algorithms probably needed

- Solution fitness needs to be a function of both ...
  - The scalarizing function value relative to the closest reference point; and
  - The number of other solutions "sharing" this reference point (analogous to the crowding distance concept in EMO)
- Could retain the closest solution to each reference point as the final shortlist...
- OR, as an interactive (progressive elicitation process), select a subset (of more preferred solutions), or partially order the solutions, and repeat with a restricted range of reference points

Research Questions from Proposal

- Do we employ just as many reference points as the desired shortlist, or perhaps more?
- How do we aggregate the scalarizing function and crowding number measures?
- How easily do users (decision makers / stakeholders) evaluate the shortlist globally?
- If the process is interactively repeated, how do we interpret preference information in terms of restrictions on future reference points?

Summary: MCDA and EMO

- Mainstream EMO primarily focusses on posterior preference expression
- Many MCDA applications (e.g. natural resource and public sector planning) involve large numbers of criteria, some qualitative, and multiple stakeholders . . . so that posterior preference expression may not be directly applicable
- However, the different thrusts may be closer when looking at shortlist generation for such problems
- Even here, more of an interactive approach (progressive preference expression) may be needed
- We have suggested potential for a progressive approach that exploits ideas from EMO and reference point methods
- Extension to incorporating other MCDA approaches (e.g. value functions) could be a subject of further research

The University of Manchester
Manchester Business School

Editorial Publicity