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Abstract

In network industries with complementary or sequential consumption, decentralised pricing can generate inefficient cross-firm markups on bundled consumption. Motivated by regulatory provisions for coordinated pricing that have received little formal analysis, we study benchmarking rules that tie cross-firm prices to standalone or bundled products. Benchmark design fundamentally shapes equilibrium outcomes by altering strategic interaction and markup propagation across firms. Coordination is not inherently welfare-improving: discount-based benchmarks actually generate equilibrium surcharges. By contrast, a bundled no-discount rule (ND_B) induces strategic complementarity, insulates markups within firms, and improves welfare. However, private and social incentives can diverge, so welfare-improving coordination may not arise endogenously. The analysis extends beyond small networks to general n-firm settings. A transport calibration indicates substantial consumer-surplus gains (around 20%), alongside external benefits comparable in magnitude to existing operating subsidies.

Keywords: network pricing; benchmarking; complementary goods; network industries; competition policy.
JEL: L13; L41; L51; D43; D62.

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1 Introduction

When firms supply complementary components, decentralised pricing generates inefficient markups on composite consumption. This is particularly pronounced in network industries with sequential or multi-part consumption, where independently set component prices create cross-firm price premia on bundled consumption. Such pricing distortions arise in settings including telecommunications, payment and platform networks, modular supply chains, and public transport.[1]

Regulators in these industries often seek to mitigate cross-network premia through access or transaction-price constraints, which typically attenuate rather than eliminate markups (e.g., Shabgard and Asensio, 2023). Existing UK regulatory provisions, alongside broader EU competition guidance permitting such arrangements, suggest an alternative and largely unanalysed approach: benchmarking rules that tie cross-firm prices to existing products, albeit with some ambiguity about how benchmarks may be anchored.[2] Benchmarking rules do not simply restrict prices: they determine how markups are transmitted across firms and thereby shape the strategic environment in which competition takes place. More broadly, the paper relates to a growing literature showing how platform and market-design rules shape equilibrium behaviour and welfare (e.g., Johnen and Somogyi, 2024). We therefore ask how alternative benchmarking rules affect prices, strategic interaction, and welfare in networks with complementary components. The paper's central contribution is to show that benchmark design itself shapes strategic interaction by determining how cross-firm markups are transmitted across the network.

We study four coordination regimes corresponding to alternative interpretations of such benchmarking rules. Benchmarking may anchor cross-firm prices either to standalone components or to own-firm bundled products, and may operate either with or without explicit

[1] See, e.g., Armstrong (1998) and Rochet and Tirole (2002).

[2] The UK Public Transport Ticketing Schemes Block Exemption (PTTSBE) permits coordination through constraints on final prices rather than inter-firm access charges (Department for Transport, 2013). Comparable provisions appear in EU competition guidance (European Commission, 2023).

discounts. Two regimes are therefore discount-based: D_B, where cross-firm bundle prices are discounted relative to own-firm bundles, and D_S, where they are discounted relative to the sum of standalone component prices. The corresponding no-discount benchmarks, ND_B and ND_S, are obtained by setting the discount factor equal to unity.

We nest these benchmark regimes alongside the free market in a model with both standalone and bundled demand, since standalone demand is required to define D_S/ND_S and bundled demand to define D_B/ND_B. This structure arises naturally in network industries with sequential or multi-part consumption.[3] Existing analyses typically focus on composite demand alone or consider only small finite-n environments without modelling explicit benchmarking rules (e.g., Lin, 2004; McHardy, 2024). Building on Economides and Salop (1992) (henceforth ES92), we provide a tractable framework that jointly incorporates benchmarking and mixed demand while preserving analytical transparency. The framework yields closed-form equilibria for the free-market and no-discount regimes in the general n-firm case.[4] By contrast, discount-based regimes admit explicit solutions only in duopoly, require numerical analysis in triopoly, and do not extend to general n in closed form. The underlying mechanism depends on whether benchmarking rules propagate or insulate markups across firms, and therefore extends beyond the specific functional form.

Three main results follow. First, under both discount-based interpretations of benchmarking, equilibrium "discounts" become surcharges relative to the benchmark, undermining the intended coordination objective. This aligns with observed pricing patterns: even where coordinated pricing is permitted, multi-firm prices often exceed comparable own-firm bundles.

Second, the bundled no-discount benchmark (ND_B) is welfare-dominant in duopoly, outperforming the free market and all alternative coordination rules. This advantage extends

[3]Examples include platform ecosystems (core services and add-ons), payment systems, modular supply chains, and durable goods with aftermarkets. In settings with only bundled usage, the model reduces to the case without standalone demand.

[4]More elaborate network formulations (e.g., Hotelling, logit) typically become analytically intractable beyond duopoly and do not admit closed-form characterisations for general n (e.g., Zhou, 2021).

to larger networks and across demand compositions and calibrated elasticity ranges. The results are particularly robust in small-n environments, which are most common in practice. A transport calibration indicates economically meaningful effects: ND_B raises consumer surplus by around 20%.[5] It also increases network usage by around 10%, with associated external benefits offsetting a substantial share of operating subsidies. These results continue to hold under endogenous network size: even when the free market supports larger networks, ND_B delivers higher welfare, reflecting the dominance of pricing efficiency over network scale.

Third, private and social incentives can diverge: the welfare-dominant benchmark need not arise endogenously, while alternative coordination rules may be privately attractive even when welfare-inferior.

These results apply broadly to industries with complementary components and decentralised pricing, and speak directly to the design of benchmarking provisions that tie cross-firm prices to existing products. Existing guidance points toward discount-based benchmarks, yet such rules have not been analysed formally. By contrast, the own-firm bundled no-discount benchmark (ND_B) has appeared only as a modelling convenience in a small literature (e.g., McHardy et al., 2023; McHardy, 2024), and without standalone demand.

When standalone and bundled demand coexist, benchmark choice has first-order implications for equilibrium surcharges, deviation incentives, and welfare. Unlike price caps, ND_B is self-anchoring: it links cross-firm prices to firms' own prices rather than regulator-chosen parameters and performs robustly across empirically relevant environments.

The paper proceeds as follows. Section 2 introduces the model. Section 3 analyses duopoly equilibria. Section 4 considers triopoly both as a robustness check for the non-generalisable discount regimes and as a representative extension for empirically relevant small-n market structures. Section 5 examines the general n-firm case. Section 6 studies robustness to demand composition. Section 7 endogenises network size. Section 8 concludes.

[5] Transport provides a useful calibration environment because realistic demand-elasticity bounds are empirically well established and broadly comparable across countries.

2 Base Model

Two firms $i \in \{1, 2\}$ each supply an x- and a y-component. Consumers may purchase components individually or combine one x- and one y-component into a composite bundle. In the duopoly case,

$$\boldsymbol{Q} = (Q_1, Q_2, Q_3, Q_4), \qquad \boldsymbol{X} = (X_1, X_2), \qquad \boldsymbol{Y} = (Y_1, Y_2),$$

where Q_t indexes the bundle demands Q_{ij} in lexicographic order, with Q_{ij} denoting demand for bundle (x_i, y_j). Let X_i and Y_i denote demand for firm i's standalone x- and y-components. The general n-firm case involves n standalone and n^2 bundle types.

Following Häckner (2000), we adopt a quasi-linear quadratic utility specification in general n-firm form and extend the ES92 framework to allow both bundled (Q_t) and standalone (X_i, Y_i) consumption. Although the analysis below focuses on duopoly, the general formulation allows several results to extend naturally to larger networks.

$$\begin{aligned} U(\mathbf{Q}, \mathbf{X}, \mathbf{Y}, M_0) = \alpha \sum_{t=1}^{n^2} Q_t + \mu\alpha \sum_{i=1}^{n} (X_i + Y_i) - \frac{1}{2}\sum_{t=1}^{n^2} Q_t^2 + \gamma \sum_{1 \le t < q \le n^2} Q_t Q_q \\ - \frac{1}{2}\sum_{i=1}^{n} (X_i^2 + Y_i^2) + \gamma \sum_{1 \le i < j \le n} (X_i X_j + Y_i Y_j) + M_0. \end{aligned} \tag{1}$$

M_0 denotes consumption of the numeraire good. Substitution across composite bundles and across standalone components is governed by $\gamma \in (0, 1)$. Composite demand has baseline intensity $\alpha > 0$. Standalone demand is scaled by μ, which governs the relative valuation of individual components. Our analytical benchmark takes $\mu = 0.5$, corresponding to a standalone component being valued at half of a composite bundle.[6]

Denote P_{ij} the price of bundle (x_i, y_j), and p_i firm i's standalone price (common across x_i and y_i by symmetry). Bundles may be priced directly or via separately priced components,

[6] Section 6 examines robustness over $\mu \in [\frac{1}{2}, 1]$. Values below $\frac{1}{2}$ imply relatively weak standalone demand, while values above 1 imply standalone components are valued more highly than the corresponding composite bundle.

depending on the pricing regime. Marginal cost is normalised to zero.[7]

Under the mapping $Q_t \equiv Q_{ij}$, demand in the n-firm case is given by

$$Q_{ij} = a - bP_{ij} + d \sum_{(k,\ell)\neq(i,j)} P_{k\ell}, \qquad X_i = Y_i = \hat{a} - \hat{b}\,p_i + \hat{d}\sum_{j\neq i} p_j,$$

where the coefficients $(a, b, d, \hat{a}, \hat{b}, \hat{d})$ are functions of (α, μ, γ, n):

$$\begin{aligned} a &= \frac{\alpha}{1+\gamma(n^2-1)}, \quad & b &= \frac{\gamma(n^2-2)+1}{(1-\gamma)(1+\gamma(n^2-1))}, \quad & d &= \frac{\gamma}{(1-\gamma)(1+\gamma(n^2-1))}, \\ \hat{a} &= \frac{\mu\alpha}{1+(n-1)\gamma}, \quad & \hat{b} &= \frac{1+\gamma(n-2)}{(1-\gamma)(1+\gamma(n-1))}, \quad & \hat{d} &= \frac{\gamma}{(1-\gamma)(1+\gamma(n-1))}. \end{aligned} \tag{2}$$

We begin with the duopoly ($n = 2$) case, the minimal structure in which strategic interaction and coordination across complementary components arise, and representative of many empirically relevant small-n environments. The duopoly benchmark allows pricing mechanisms to be characterised analytically before extending the analysis to richer market structures in Sections 4 and 5.

2.1 Pricing Regimes and Strategic Environment

The pricing regimes differ only in how the cross-firm bundle price P_{ij} ($i \neq j$) is determined. Within each regime R, equilibrium standalone prices are symmetric across firms, as are within-firm bundle prices and cross-firm bundle prices. We denote these by p^R, P^R, and P^R_{cro}, respectively.

2.1.1 Free Market (FM)

Figure 1 illustrates the structure of products and pricing under the free market for a representative three-firm network. Although the analysis in this section begins with duopoly, the figure highlights the more general n-firm structure developed throughout the paper.

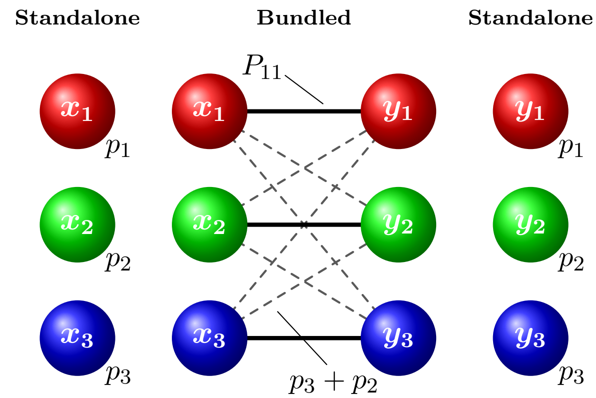
[7] This normalisation is standard in models with quasi-linear preferences, where only relative prices matter.

Solid (dashed) links denote within-firm (cross-firm) bundles. Within-firm bundles Q_{ii} are priced directly at P_{ii}, while under the free market cross-firm bundles Q_{ij} are priced additively from standalone component prices, $p_i + p_j$.

Under duopoly, firm i chooses (p_i, P_{ii}) to maximise profit π_i

$$\max_{p_i, P_{ii}} P_{ii}Q_{ii} + p_i(X_i + Y_i + Q_{ij} + Q_{ji}),$$

Figure 1: Standalone and bundled products under the free market



Note: Solid (dashed) links denote within-firm (cross-firm) bundles.

taking the rival's prices as given. The symmetric equilibrium prices are:

$$P^{FM} \equiv P_{ii}^{FM} = \frac{5\alpha(1-\gamma^2)}{10 - 6\gamma^2 + 7\gamma}, \quad p^{FM} \equiv p_i^{FM} = \frac{\alpha(6 - 7\gamma^2 + \gamma)}{2(10 - 6\gamma^2 + 7\gamma)}, \quad P_{\text{cro}}^{FM} \equiv P_{ij}^{FM} = 2p^{FM}.$$

Before turning to the coordinated regimes, note a standard feature of complementary-component markets with network structure. When products are weak substitutes, decentralised pricing performs poorly because firms fail to account for the effect of their prices on cross-network consumption. Coordinating these pricing incentives can therefore raise welfare, even at the cost of weaker price competition. As substitutability increases, the importance of this mechanism declines, a pattern we formalise below.

2.1.2 Discount-Based Regimes (D_B and D_S)

Turning to the discount regimes, UK guidance under the PTTSBE can be interpreted as setting cross-firm fares by applying a discount relative to a benchmark "single-ticket" fare, scaled by typical usage (see Department for Transport, 2013, p. 22) The notion of a "single-ticket" benchmark is left undefined. It may refer either to the sum of standalone component fares or to a bundled through-journey fare. We model these as discount-on-standalone pricing (D_S) and discount-on-bundled pricing (D_B), respectively. In both cases, the pricing rule can be written

$$\text{multi-operator fare} = \text{benchmark fare} \times \text{usage} \times \delta, \tag{3}$$

where $\delta > 0$ is a common discount factor, with $\delta < 1$ corresponding to a discount and $\delta > 1$ to a surcharge.[8]

The network representation in Figure 1 extends naturally to the benchmarking environments considered below. Unlike the free-market case, in which cross-firm prices coincide only in equilibrium, the dashed cross-bundle links now denote a common benchmarked cross-firm price determined by the relevant coordination regime.

We now formalise the discount-based coordination regimes and solve by backward induction. At Stage 1, under regime D_r ($r \in \{B, S\}$), firms jointly choose a common discount factor δ_r. At Stage 2, firms independently choose (p_i, P_{ii}).

For the discount-based coordination rules, the cross-firm bundle price is defined as a proportional scaling of a network-wide benchmark. Although the analysis in this section focuses on duopoly, we state the pricing rule for general n to avoid repetition. Specifically, for $i \neq j$:

$$P_{ij}^{D_S} = \delta_S \frac{2\sum_{k=1}^{n} p_k(X_k + Y_k)}{\sum_{k=1}^{n}(X_k + Y_k)}, \qquad P_{ij}^{D_B} = \delta_B \frac{\sum_{k=1}^{n} P_{kk}Q_{kk}}{\sum_{k=1}^{n} Q_{kk}}. \tag{4}$$

At Stage 2, each firm i solves

$$\max_{p_i, P_{ii}} \; P_{ii}Q_{ii} + p_i(X_i + Y_i) + \frac{P_{ij}^{D_r}}{2}(Q_{ij} + Q_{ji}),$$

taking rivals' prices as given and treating $P_{ij}^{D_r}$ as an endogenous function of prices via the benchmark.

Substituting the Stage 2 best replies into profits yields a Stage 1 problem:

$$\max_{\delta > 0} \; \Pi_r(\delta; \gamma),$$

[8] The "usage" term in Eq. (3) is normalised, without loss of generality.

where $\Pi_r(\delta;\gamma)$ denotes aggregate profit. The first-order condition reduces to a quartic equation $K_r(\delta;\gamma) = 0$, yielding multiple algebraic candidate solutions for δ_r.

Substituting the locally unique symmetric Stage 2 equilibrium into joint profit $\Pi_r(\delta;\gamma)$ yields a Stage 1 objective that is strictly concave in δ on the economically relevant region.[9]

Although the first-order condition admits multiple algebraic solutions, we focus on the economically relevant branch defined by the anchoring condition $\delta_r^*(0) = 1$, corresponding to the limit of independent demands in which no discount applies. The resulting solution $\delta_r^*(\gamma)$ is therefore unique and interior on the admissible region. We include $\gamma = 0$ to define this anchoring condition, although the main analysis concerns $\gamma \in (0, 1)$.

Proposition 1 (Equilibrium surcharge)**.** *For both discount regimes D_r, $r \in \{B, S\}$, the anchored equilibrium satisfies $\delta_r^*(\gamma) \geq 1$ for all $\gamma \in [0, 1)$, with strict inequality for $\gamma > 0$.*

Thus, discount-based benchmarking yields equilibrium surcharges rather than discounts. Lowering cross-firm bundle prices expands demand for cross-firm bundles, from which a firm derives only partial revenue, while reallocating demand away from own-bundle consumption, on which it earns the full margin. This composition effect weakens firms' incentives to reduce cross-firm prices. Anticipating this, firms soften competition at Stage 1 by raising the benchmarked price, yielding $\delta_r^*(\gamma) \geq 1$.

Under bundled benchmarking (D_B), the mechanism is intuitive: cross-firm bundle prices are benchmarked to average own-firm bundle prices, so lowering the benchmark brings cross-firm prices closer to (or below) own-bundle prices, intensifying business stealing. The standalone-based case (D_S) is less straightforward. Here, standalone prices p_i affect both standalone demand and, indirectly, cross-firm prices. Although this partially corrects the free-market distortion in standalone prices, it also reallocates demand away from Q_{ii} toward cross-firm bundles. The business-stealing channel nevertheless dominates, yielding the surcharge result.

[9] These properties justify the use of the Implicit Function Theorem (IFT) and underpin Propositions 1 and 2.

This prediction is broadly consistent with observed pricing patterns. Despite widespread availability in the UK, multi-operator tickets are typically priced above comparable single-operator fares, with little evidence of systematic discounting (see Department for Transport, 2013; Urban Transport Group, 2019, p. 43).

The next lemma shows that the anchored root can be tracked continuously in γ, ensuring continuity of the economically relevant equilibrium branch.

Lemma 1 (Continuity)**.** *For both discount regimes D_r, $r \in \{B, S\}$, the anchored solution $\delta_r^*(\gamma)$ with $\delta_r^*(0) = 1$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus G_r$, where G_r is a finite set of parameter values at which the polynomial has a repeated root.*

It follows that all equilibrium outcomes are continuous in γ on $[0, 1)$ (by closed form in FM and ND_r, and by continuity of the optimal discount rule in D_r). Moreover, the Stage-1 objective is strictly concave in δ_r on the admissible set and satisfies $\frac{\partial^2 \Pi^r}{\partial \delta \partial \gamma} > 0$ along the optimal branch. These imply:

Proposition 2 (Monotonicity)**.** *For both discount regimes D_r, $r \in \{B, S\}$, the optimal discount $\delta_r^*(\gamma)$ is non-decreasing in γ on $[0, 1)$.*

Hence, although the PTTSBE guidance is framed in terms of discounts, the profit-maximising solution on the admissible branch yields surcharges that increase monotonically with substitutability. As bundles become closer substitutes (higher γ), competitive pressure intensifies, and firms respond by raising cross-firm prices to insulate against this effect, as illustrated in Figure 2.

2.1.3 No-Discount Regimes (ND_B and ND_S)

Finally, the no-discount regimes, ND_r for $r \in \{B, S\}$, set $\delta_r = 1$ in Eq. (4), yielding two transparent benchmark regimes. These eliminate the Stage 1 choice and admit closed-form equilibria, with symmetric equilibrium prices given by:[10]

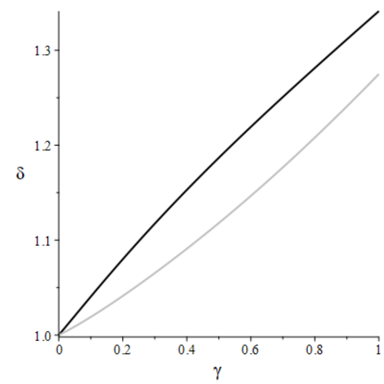
$$P^{ND_S} = \frac{(11\gamma^3 + 4\gamma^2 - 11\gamma - 4)\alpha}{(5\gamma^2 - 6\gamma - 8)(3\gamma + 1)}, \quad p^{ND_S} = \frac{(16\gamma^3 + 2\gamma^2 - 14\gamma - 4)\alpha}{2(3\gamma + 1)(5\gamma^2 - 6\gamma - 8)}, \quad P_{\text{cro}}^{ND_S} = 2p^{ND_S},$$
$$P^{ND_B} = P_{\text{cro}}^{ND_B} = \frac{3(1-\gamma)\alpha}{2(3-\gamma)}, \quad p^{ND_B} = \frac{(1-\gamma)\alpha}{2(2-\gamma)}.$$

Focusing on the analytically tractable free-market and no-discount regimes, we can characterise how benchmark design alters strategic interaction.

Strategic interaction in standalone pricing differs across benchmark regimes.

Lemma 2 (Strategic interaction)**.** *In symmetric equilibrium, p_i is a strategic complement to p_j under ND_B and a strategic substitute under FM and ND_S.*

Figure 2: Equilibrium discounts in duopoly



Note. Black/grey: δ_B^*/δ_S^*.

Under ND_B, cross-firm bundle prices are pinned to own-bundle prices, insulating standalone pricing from cross-firm price determination. This preserves the standard Bertrand logic in which prices are strategic complements, yielding downward pressure on equilibrium prices.

Under FM and ND_S, by contrast, standalone prices enter additively into cross-firm bundle prices, as in complementary monopoly. This introduces a countervailing force: firms internalise the impact of their prices on the profitability of cross-firm bundles, leading to strategic substitutability. In this setting, the complementary-monopoly effect dominates the baseline Bertrand complementarity, generating upward pressure on prices.

Pricing rules therefore determine not only price levels, but the nature of strategic interaction itself.

[10] For FM, ND_B, and ND_S, each firm's profit is strictly concave in own prices at the symmetric equilibrium, so the pricing subgame is a concave game in the sense of Rosen (1965), implying a locally unique symmetric Nash equilibrium. This property extends to the n-firm case.

3 Analysis of Welfare, Prices, and Incentives

We examine how welfare, consumer surplus, and aggregate bundle-equivalent quantity Q_{tot} vary with γ. The latter captures overall network usage and, in transport applications, corresponds to network usage associated with external benefits. It is defined as

$$Q_{\text{tot}} = \sum_{t=1}^{n^2} Q_t + \tfrac{1}{2}\sum_{i=1}^{n}(X_i + Y_i).$$

In duopoly, the bundled no-discount regime (ND_B) strictly dominates all alternative regimes in terms of welfare (W), consumer surplus (CS), and aggregate usage (Q_{tot}).

Proposition 3 (Benchmark duopoly dominance)**.** *For all* $\gamma \in (0, 1)$*,*

$$H^{ND_B}(\gamma) > H^R(\gamma) > H^{FM}(\gamma), \qquad \forall H \in \{W, \text{CS}, Q_{\text{tot}}\}, \quad \forall R \in \{ND_S, D_S, D_B\}.$$

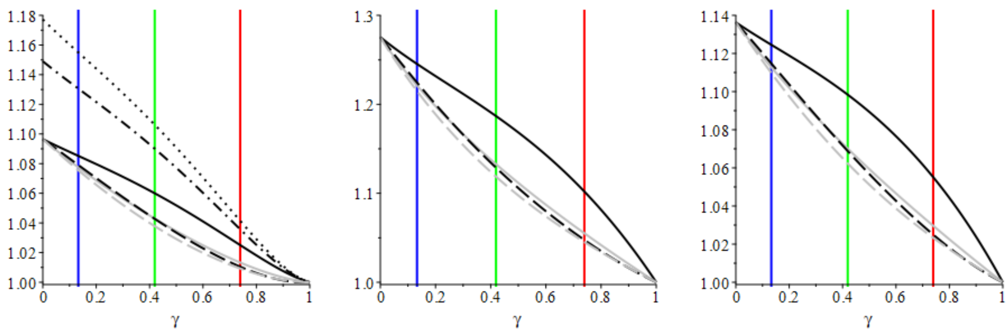
This ranking highlights that the design of pricing rules, rather than coordination per se, determines whether coordination improves or worsens outcomes. The dominance of ND_B reflects its ability to coordinate cross-network pricing incentives while avoiding the cross-price linkage present under FM and ND_S. By Lemma 2, standalone prices remain strategic complements under ND_B, whereas the additive linkage in FM and ND_S induces strategic substitutability and weakens competitive discipline. This difference in strategic interaction drives the ranking.

Figure 3 illustrates these differences across regimes relative to the free market. To relate the model to empirical settings, we map substitutability (γ) to standard public-transport own-price elasticities $\eta \in \{-0.3, -0.7, -1.1\}$, spanning short-, medium-, and long-run responses.[11] Transport provides a convenient benchmark because elasticity estimates are well established and complementary network demand is central to many transport settings. For

[11] Public transport demand estimates typically place own-price elasticities around -0.3 in the short run and -1.0 in the long run, with intermediate medium-run responses (e.g., Paulley et al., 2006; Holmgren, 2007).

$n = 2$, this mapping yields $\gamma \approx \{0.74, 0.42, 0.13\}$ (Appendix B). At the medium-run calibration ($\eta = -0.7$), ND_B raises consumer surplus by around 20%, welfare by about 6%, and network usage by roughly 10%.[12]

Figure 3: Duopoly welfare, consumer surplus, and network usage under coordinated regimes relative to the free market. Augmented welfare is reported for ND_B.



(a) Welfare (b) Consumer Surplus (c) Network Usage

Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Because higher network usage generates external benefits in our motivating transport application, we also report augmented welfare incorporating avoided external costs from induced trips.[13] This is defined as

$$\widetilde{W}^R = W^R + \beta\, \Delta Q^R_{tot} f_{\text{avg}}, \tag{5}$$

where ΔQ^R_{tot} denotes the change in aggregate usage relative to the free market, f_{avg} is the free-market average fare, and β captures marginal external costs (Appendix D provides details of the construction and calibration).

Accounting for avoided external costs increases welfare under ND_B at the medium-run calibration by a further 3-7 percentage points, corresponding to the dash-dot and dotted welfare profiles in Figure 3(a). Calibrated to England's pre-COVID bus market outside

[12]The implied welfare and usage changes are comparable in magnitude to those associated with major transport-policy and infrastructure interventions, such as dedicated bus lanes.

[13]Public transport usage is commonly associated with wider effects on congestion, emissions, and social inclusion (e.g., Vickerman et al., 1999; Lucas, 2012).

London, this corresponds to additional gains of approximately £186-£285m annually, rising to £238-£363m under higher elasticities. These gains amount to roughly 75-146% of annual operating support and correspond to reductions of 1.9-2.4% in national bus-sector CO_2 emissions.

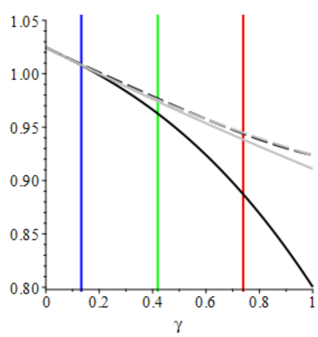
The welfare ranking is supported by the equilibrium price structure:

Proposition 4 (Prices)**.** *For all $\gamma \in (0,1)$, although no coordinated regime uniformly minimises prices across all services, the welfare-dominant regime ND_B sets lower standalone and cross-network prices than ND_S and the free market:*

$$p^{ND_B} < p^{ND_S} < p^{FM}, \qquad P_{cro}^{ND_B} < P_{cro}^{ND_S} < P_{cro}^{FM}.$$

These price rankings reflect the strategic mechanism in Lemma 2. Under FM and ND_S, standalone prices are strategic substitutes and enter cross-network prices additively, sustaining double marginalisation. Under ND_B, standalone prices are strategic complements and do not enter cross-network prices directly, implying a single effective mark-up.

Figure 4: Profit under coordinated regimes relative to the free market



Note: Black/grey: bundled/standalone benchmark;. solid/dashed: no-discount/discounted. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

We now turn to the alignment between private and social incentives, focusing on the choice between coordinated regimes and the free market.

Proposition 5 (Incentive misalignment)**.** *For each regime $R \in \{ND_B, ND_S, D_B, D_S\}$, firm profits admit a unique threshold $\tilde{\gamma}_R \in (0,1)$ such that $\pi^R(\gamma) > (<)\pi^{FM}(\gamma)$ for $\gamma < (>)\tilde{\gamma}_R$, with equality at $\gamma = \tilde{\gamma}_R$.*

Consistent with Proposition 5, Figure 4 shows that coordinated regimes raise profits relative to the free market only at low levels of substitutability.

Corollary 1 (Policy implication)**.** *Although firms may earn lower profits than under laissez-faire, the welfare gain under ND_B implies that coordination can be implemented with transfers* *financed from the increase in consumer surplus.*

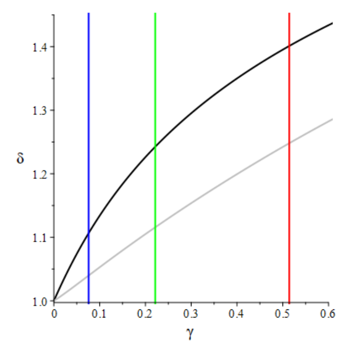
In transport applications, the case for coordination is further strengthened by external benefits associated with increased network usage.

4 Extension to Triopoly

We extend the analysis to triopoly, where the discount regimes no longer admit closed-form solutions and are therefore analysed numerically. This provides a test of whether the duopoly results reflect a special case or a more general property of the pricing rules. We then turn in Section 5 to the general n-firm case for the analytically tractable free-market and no-discount regimes.

Under coordination, cross-firm bundle prices follow the pricing rule $P_{ij}^{D_r}$ ($r \in \{B, S\}$) defined in Eq. (4) evaluated at $n = 3$, with no-discount counterparts obtained by setting $\delta_r = 1$. While the discount regimes are solved numerically, the free-market and no-discount regimes admit closed-form solutions obtained by evaluating the general n-firm expressions at $n = 3$ (Eqs. (7)-(9) in Section 5).

Figure 5: Equilibrium triopoly discounts.



Note. Black/grey: bundled/standalone benchmark. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Remark 1 (Triopoly discount regimes)**.** *For all $\gamma \in [0, 1)$, there exists a unique smooth equilibrium discount $\delta_r^*(\gamma)$, $r \in \{B, S\}$, which is weakly above one (a surcharge) and increasing over the calibrated range, as illustrated in Figure 5. Thus, the surcharge result extends beyond duopoly.*

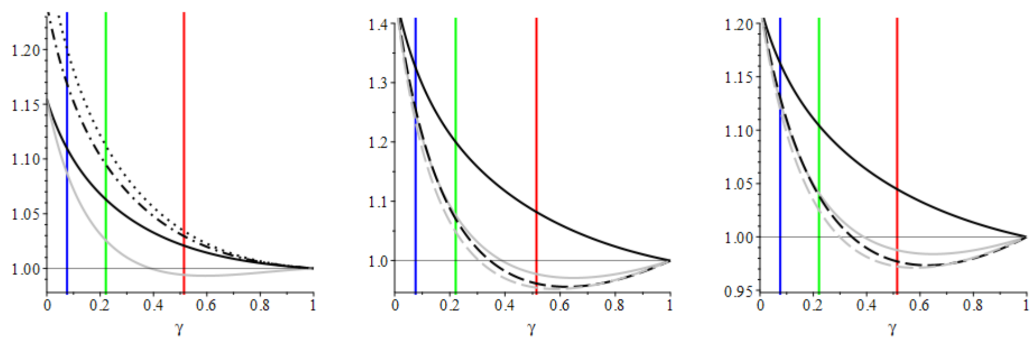
We next examine whether the welfare ranking from duopoly survives in triopoly.

Proposition 6 (ND$_B$ triopoly dominance)**.** *For triopoly, and all $\gamma \in (0,1)$,*

$$H^{ND_B}(\gamma) > H^R(\gamma), \qquad \forall H \in \{W, \text{CS}, Q_{\text{tot}}\}, \quad \forall R \in \{FM, ND_S, D_S, D_B\}.$$

ND$_B$ remains the welfare-dominant regime. However, the complete ordering established for duopoly in Proposition 3 no longer holds in triopoly: in particular, the ranking between ND$_S$ and the free market becomes sensitive to substitutability.

Figure 6: Triopoly welfare, consumer surplus, and network usage under coordinated regimes relative to the free market. Augmented welfare is reported for ND$_B$.



(a) Welfare (b) Consumer Surplus (c) Network Usage

Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Figure 6 illustrates these results. ND$_B$ dominates all alternative regimes across all measures for all $\gamma \in (0,1)$. At the medium-run elasticity, welfare, consumer surplus, and network usage exceed the free-market benchmark by roughly 6.5%, 20%, and 10%, respectively, with augmented welfare adding a further 3-9 percentage points depending on the externality calibration.

By contrast, ND$_S$ and the discount regimes deteriorate relative to the free market as substitutability increases. Although they outperform the free market at low and medium γ, they fall below it beyond the mid-range calibration (around $\gamma \simeq 0.22$). Triopoly therefore shows that while the complete ordering across regimes need not persist beyond duopoly, the dominance of ND$_B$ remains robust.

Remark 2 (Strategic interaction)**.** *In triopoly, as characterised by Lemma 3 evaluated at $n = 3$ and illustrated in Figure 7 (see Section 5), standalone prices under FM switch from strategic substitutes to complements as substitutability increases, strengthening competitive pressure. By contrast, ND_S remains in the strategic-substitutes region except over a narrow high-γ interval, helping explain its deterioration relative to the free market.*

Consistent with Proposition 6, ND_B delivers the largest network-usage gains, exceeding 10% over the free market at the medium-run elasticity. Mapping the augmented welfare gains into monetary terms yields £198-£301m annually, rising to £308-£469m under higher elasticities, corresponding to CO_2 reductions of approximately 2.0-3.1% of bus-sector emissions (Table 3; Appendix D).

We now examine whether privately stable coordination aligns with welfare improvements by turning to incentives and coalition formation. Under duopoly, participation depends only on whether both firms prefer coordination to the free market, since coordination cannot operate with a single firm. In triopoly, we distinguish between grand coalitions (full participation) and partial coalitions. Participation then also depends on deviation incentives: given a grand coalition, does any firm gain by opting out?

Under partial coalitions, only participating firms internalise cross-network pricing: for any $i \neq j$, P_{ij}^R is determined by the pricing rule $R \in \{ND_B, ND_S, D_B, D_S\}$ when both firms participate; cross-firm bundles involving a non-participating firm remain priced additively.

Because the discount regimes do not admit tractable algebraic solutions, coalition outcomes are characterised numerically over the calibrated range, with the resulting patterns summarised in the following remark.

Remark 3 (Triopoly coalition incentives)**.** *At low substitutability, all coordination regimes sustain welfare-improving grand and partial coalitions. Partial coalitions persist at intermediate substitutability. At high substitutability, grand coalitions (except under ND_B) remain privately sustainable but become welfare-inferior to the free market.*

Coalition feasibility need not align with welfare. As substitutability increases, all coordination regimes except ND_B sustain privately stable grand coalitions even after they become welfare-inferior to the free market. ND_B, by contrast, remains welfare-dominant throughout the interior.

Because $W^{ND_B} > W^{FM}$ throughout the interior, any profit losses from coordination can in principle be offset through transfers financed from consumer-surplus gains, while still leaving consumers better off than under laissez-faire. In transport applications, external benefits further strengthen the case for coordination.

5 General n-Firm Case

Markets for composite or sequential goods are often served by a small number of firms, motivating our focus on $n \in \{2, 3\}$. To assess how the underlying mechanisms extend to larger networks, we now consider the general n-firm case. We focus on the regimes that continue to admit closed-form equilibria - the free market and the no-discount regimes (ND_B, ND_S). This allows us to isolate how pricing rules shape outcomes as market size increases.

Under the free market, taking rivals' prices as given, firm i solves

$$\max_{P_{ii}, p_i} P_{ii}Q_{ii} + p_i\Big[X_i + Y_i + \sum_{\substack{j=1 \\ j\neq i}}^{n}(Q_{ij} + Q_{ji})\Big],$$

In the no-discount regimes ND_r ($r \in \{B, S\}$), the objective is identical except that cross-firm bundle revenue is pinned to the benchmark price $P_{ij}^{ND_r}$, determined endogenously by the pricing rule in Eq. (4) evaluated at $\delta_r = 1$:

$$\max_{P_{ii}, p_i} \; P_{ii}Q_{ii} + p_i(X_i + Y_i) + \frac{P_{ij}^{ND_r}}{2}\sum_{\substack{j=1 \\ j\neq i}}^{n}(Q_{ij} + Q_{ji}), \tag{6}$$

Equilibrium bundled, standalone, and cross-network prices are given by the following expressions under each regime. For the free market:

$$P^{FM} = \nabla_1(4\gamma n^2 - 5\gamma n - \gamma + 3n - 1), \quad p^{FM} = \frac{1}{2}\nabla_1(6\gamma n^2 - 9\gamma n + \gamma + 4n - 2), \quad P_{cro}^{FM} = 2p^{FM}, \tag{7}$$

where $\nabla_1 \equiv \frac{\alpha(1-\gamma)}{(2n^4-5n^3-4n^2+7n+4)\gamma^2+(2n^3+5n^2-14n-1)\gamma+6n-2}$.

For the bundled no-discount regime:

$$P^{ND_B} = \frac{\alpha(2n-1)(1-\gamma)}{(n^3-n^2-4n+2)\gamma+4n-2}, \quad p^{ND_B} = \frac{\alpha(1-\gamma)}{4+(2n-6)\gamma}, \quad P_{cro}^{ND_B} = P^{ND_B}. \tag{8}$$

For the standalone-based no-discount regime:

$$\begin{aligned} P^{ND_S} &= \nabla_2\Big[(n^5+3n^4-5n^3-13n^2+20n-6)\gamma^2+(5n^3+7n^2-24n+10)\gamma+6n-4\Big], \\ p^{ND_S} &= \nabla_2\Big[(n-1)(n^4+\tfrac{5}{2}n^3-3n^2-\tfrac{11}{2}n+3)\gamma^2+(4n^3+\tfrac{1}{2}n^2-\tfrac{21}{2}n+5)\gamma-2+3n\Big], \\ &P_{cro}^{ND_S} = 2p^{ND_S}, \end{aligned} \tag{9}$$

where $\nabla_2 \equiv \frac{\alpha(1-\gamma)}{(2+(n-3)\gamma)(1+\gamma(n^2-1))[(2n^3+3n^2-11n+4)\gamma-4+6n]}$.

Despite the richer pricing environment, a sharp ranking emerges between the no-discount regimes.

Proposition 7 (Global benchmark dominance)**.** *For all integers $n \geq 2$ and all $\gamma \in (0,1)$,*

$$H^{ND_B}(n,\gamma) > H^{ND_S}(n,\gamma), \qquad \forall H \in \{W, CS, Q_{\text{tot}}\}.$$

This establishes that bundled benchmarking (ND_B) strictly dominates standalone-based benchmarking (ND_S) for all n. As in triopoly, however, ND_S and the free market are not uniformly ranked, with their relative performance depending on market conditions.

This dependence reflects differences in strategic interaction across regimes.

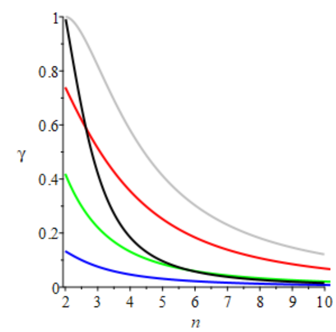
Lemma 3 (Strategic interaction for general n)**.** *In symmetric n-firm equilibrium, standalone prices are strategic complements under ND_B for all $(\gamma, n) \in (0,1) \times [2,\infty)$. By contrast, under FM and ND_S the nature of strategic interaction varies with n.*

Figure 7 illustrates these results. The contours map equilibrium (n, γ) pairs corresponding to the calibrated elasticities. As n rises, the FM zero locus (black) - which separates regions of strategic substitutability and complementarity - shifts downward and is crossed by the contours corresponding to lower elasticities (green and red). This implies that equilibrium pricing transitions from strategic substitutes to complements, first for low (red) and then for medium (green) elasticities.

By contrast, the ND_S zero locus (grey) lies above all calibrated contours over $\gamma \in (0, 1)$, so standalone prices remain strategic substitutes under these elasticities. This helps explain why ND_S can underperform the free market as market size grows. Under ND_B, standalone prices are strategic complements throughout, consistent with its stronger performance.

This variation in strategic interaction across regimes explains the divergence in performance as market size increases.

Figure 7: Strategic interaction in standalone prices under FM and ND_S.



Note. Black/grey: ND_B/ND_S. Lines show the zero loci of $\partial^2\pi_i/\partial p_i \partial p_j$ at the symmetric equilibrium; regions above and to the right of each curve indicate strategic complements. Calibrations: $\eta = -0.3$, $\eta = -0.7$, and $\eta = -1.1$.

To assess the quantitative implications of these mechanisms, we turn to calibrated elasticities.

Proposition 8 (Calibrated dominance). [14] *For each calibrated elasticity* $\eta \in \{-0.3, -0.7, -1.1\}$ *and all integers* $n \geq 2$,

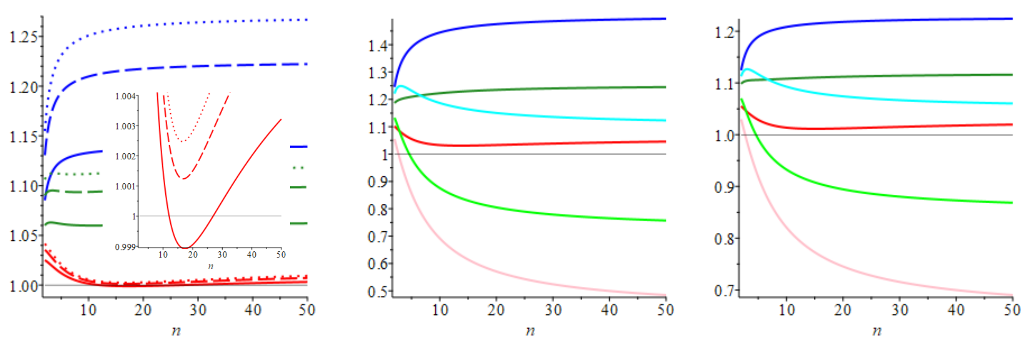
$$H^{ND_B} > H^{FM}, \qquad H \in \{\text{W}^*, \text{CS}, Q_{\text{tot}}, \widetilde{W}\}.$$

Thus, for empirically plausible elasticity calibrations, ND_B continues to outperform the free market as market size increases.

[14] *Note:* * At $\eta = -0.3$, the welfare inequality fails only on the finite set $n \in \{13, \ldots, 26\}$, where W^{ND_B}/W^{FM} is arbitrarily close to one.

Figure 8 reports the calibrated magnitudes corresponding to these results, plotting welfare, consumer surplus, and network usage under ND_B and ND_S relative to the free market for $n \in [2, 50]$ and $\eta \in \{-0.3, -0.7, -1.1\}$. Consistent with the analytical results, ND_B dominates the free market in consumer surplus and network usage across all elasticities. In welfare, it dominates at medium and high elasticities and for small n at low elasticity, and is otherwise nearly indistinguishable, while augmented welfare remains strictly higher throughout. At medium to high elasticities, this corresponds to welfare gains of 6-13%, augmented welfare gains of 8-21%, consumer-surplus gains of 20-45%, and network usage increases of 10-20%.

Figure 8: Calibrated no-discount regimes: welfare, augmented welfare, consumer surplus, and network usage relative to the free market for $n \in [2, 50]$.



(a) Welfare (b) Consumer Surplus (c) Network Usage

Note: Blue/DarkGreen/Red: H^{ND_B}/H^{FM}; Cyan/LightGreen/Pink: H^{ND_S}/H^{FM}; $H \in \{W, CS, Q_{tot}\}$.
Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

By contrast, gains under ND_S are fragile and vanish as n increases.

These results extend the duopoly and triopoly findings. ND_B is the only regime that dominates ND_S analytically for all n, and continues to perform favourably relative to the free market across empirically relevant market sizes, with any departures confined to low-elasticity cases.

The advantage of bundled benchmarking therefore does not rely on small market size, but reflects a general property of how pricing rules shape strategic interaction.

6 Robustness to Demand Composition

Although the main analysis focuses on the benchmark case $\mu = 0.5$, several key dominance results extend to $\mu \in [0.5, 1]$, with additional calibrated evidence at $\mu \in \{0.75, 1\}$.[15] The benchmark facilitates analytical characterisation, but the results below show that the main economic insights are not sensitive to this restriction.

We proceed from low-dimensional environments to the general n-firm case, and then relate these results to the calibrated evidence reported in Appendix C. The key question is whether the dominance of bundled benchmarking (ND_B) depends on the relative importance of standalone demand.

The following duopoly dominance result extends to all $\mu \in [\frac{1}{2}, 1]$ for welfare and network usage.

Proposition 9 (Duopoly dominance for all μ)**.** *For all $\gamma \in (0, 1)$ and $\mu \in [\frac{1}{2}, 1]$,*

$$H^{ND_B} > H^{ND_S} > H^{FM}, \qquad \forall H \in \{W, Q_{\text{tot}}\}.$$

Thus, the duopoly ranking is preserved for all $\mu \in [\frac{1}{2}, 1]$ in welfare and network usage, with ND_B dominating ND_S, which in turn dominates the free market.

Because the discount regimes do not admit tractable analytical characterisation over the full range $\mu \in [\frac{1}{2}, 1]$, we evaluate them at the limiting case $\mu = 1$. Figure 10 in Appendix Appendix C shows that ND_B continues to dominate the other regimes in welfare and network usage throughout $\gamma \in (0, 1)$, while ND_S and the discount regimes continue to dominate the free market on these measures.

Consumer-surplus rankings are more sensitive to demand composition. At $\mu = 1$, ND_S exceeds the free market throughout $\gamma \in (0, 1)$, while ND_B falls below both ND_S and the

[15] The interval $\mu \in [\frac{1}{2}, 1]$ captures economically relevant environments in which bundled consumption is at least as valuable as its individual components, while ruling out implausibly weak standalone demand relative to the bundle. For empirically relevant small-n transport calibrations, Appendix C shows that the benchmark value $\mu = 0.5$ implies plausible shares of standalone (single-leg) demand.

free market at low levels of substitutability. This reflects a compositional effect: ND_S shifts demand away from bundled consumption toward standalone demand, increasing consumer surplus through greater standalone usage rather than improved coordination. The consumer-surplus dominance of ND_S over ND_B does not carry over to welfare or bundle-equivalent network usage, is absent at intermediate values (e.g., $\mu = 0.75$), and disappears at empirically plausible levels of substitutability as n increases. Any consumer-surplus shortfall under ND_B is more than offset by higher profits and aggregate welfare. By contrast, consumer surplus under the discount regimes lies below ND_S, ND_B, and the free market throughout $\gamma \in (0, 1)$.

The triopoly results are similarly robust.

Proposition 10 (Triopoly dominance for all μ)**.** *For all* $\gamma \in (0, 1)$*, all* $\mu \in [\frac{1}{2}, 1]$,

$$H^{ND_B} > H^{FM},\ H^{ND_S}, \qquad \forall H \in \{W, Q_{\text{tot}}\}.$$

ND_B continues to dominate both ND_S and the free market in welfare and network usage for all $\mu \in [\frac{1}{2}, 1]$. As in the benchmark case, the ranking between ND_S and the free market is not uniform, and the consumer-surplus ordering between ND_B and ND_S need not hold throughout. These differences reflect changes in strategic interaction as the number of firms increases, while the mechanism underlying ND_B's dominance remains unchanged.

Since the discount regimes do not admit tractable extensions beyond triopoly, we use this setting to benchmark their performance. Consistent with the duopoly results, Figure 11 in Appendix C shows that ND_B dominates the discount-based regimes across all γ in welfare, consumer surplus, and network usage. While the figure reports $\mu = 1$, intermediate values (e.g., $\mu = 0.75$) yield qualitatively similar patterns.

These results indicate no reversal of the dominance patterns established above for the discount regimes. The analysis that follows therefore focuses on the regimes that admit generalisable n-firm characterisations.

We now turn to the general n-firm case.

Proposition 11 (Global dominance for all μ)**.** *For all integers $n \geq 2$, all $\gamma \in (0,1)$, and all $\mu \in [\frac{1}{2}, 1]$,*

$$H^{ND_B}(\gamma, \mu, n) > H^{ND_S}(\gamma, \mu, n), \qquad \forall H \in \{W, Q_{\text{tot}}\}.$$

Proposition 11 shows that the welfare advantage of bundled benchmarking does not depend on market size or the relative importance of standalone demand. Once cross-firm prices are anchored to bundled rather than standalone prices, the resulting reduction in effective mark-ups dominates for all admissible μ and γ. This reflects the strategic mechanism identified in Lemmas 2 and 3, which is invariant to μ.

The calibrated profiles in Appendix C (Figure 12) confirm that the dominance of ND_B over ND_S in welfare and network usage extends to $\mu = 1$. While this dominance does not extend uniformly to consumer surplus, deviations are confined to small n and vanish as market size increases. Any consumer-surplus losses are more than offset by gains in profits and externalities.

Figure 12 also allows comparison across regimes at $\mu = 1$. ND_B continues to outperform the free market at medium and high elasticities. At low elasticities, small deviations from the benchmark ordering arise over an intermediate range of n, but outcomes remain close and small-n dominance is preserved. By contrast, ND_S quickly falls below the free market on all measures as n increases. These results have clear implications for empirically relevant small-n environments.

Proposition 12 (Small-n dominance)**.** *For each $\mu \in \{\frac{1}{2}, \frac{3}{4}, 1\}$ and each calibrated elasticity, ND_B strictly dominates the free market in total network usage for all $n \leq 5$. It also strictly dominates the free market in welfare over this range, except for a knife-edge case under the least-elastic calibration ($\eta = -0.3$), where $W^{ND_B}/W^{FM} \approx 1$ at $n = 4$ for $\mu = 1$.*

Such small-n environments are common in complementary-component industries, platform ecosystems, infrastructure services, and transport corridors, where the number of interacting components is inherently limited.

These results extend the benchmark analysis by showing that the dominance of bundled coordination is robust to variation in demand composition. The advantage of ND_B does not depend on the relative importance of standalone demand, but reflects a general property of how pricing rules shape strategic interaction.

7 Network-Size Choice

Finally, we examine whether the welfare dominance of ND_B persists when firms can expand their networks. This addresses a natural concern: whether the free market can outperform coordinated pricing by sustaining larger networks despite less efficient pricing.

Network size is endogenised by allowing each firm to choose the number of symmetrically differentiated complementary component pairs it supplies, n_i, with total network size $n = n_1 + n_2$. Under symmetry, all within-firm bundles share price P_{ii} and all standalone components share price p_i. Quantities scale with network size through the number of available bundles and components.[16] We focus on the free market and the welfare-leading coordinated regime, ND_B.

At Stage 0, each firm incurs a fixed overhead F_0, ensuring interior participation; at Stage 1, firms choose $n_i \in \{1, 2, 3\}$ before prices are set as in the base model.

Preferences and demand follow Eq. (1). Operating costs (specified in line with Van den Berg et al., 2022; McHardy, 2024) combine a fixed cost per pair with economies of density, $C_i = (k + mZ_i)Z_i + n_i F$, where $Z_i \equiv n_i^2 Q_{ii} + n_i n_j Q_{ij} + \frac{n_i}{2}(X_i + Y_i)$ denotes bundled-equivalent quantities. Here, $k > 0$ is the baseline per dual-leg-equivalent passenger cost and $m < 0$ captures economies of density, with $(k, m) = (0.1, -0.02)$ following McHardy (2024). The fixed cost per pair is $F \in \{0.10, 0.25\}$. Parameters are chosen to restrict equilibrium choices to $n_i \in \{1, 2, 3\}$ and to generate knife-edge cases in which FM can sustain a larger expected network than ND_B.

[16] Holding prices fixed, a larger network raises utility through a standard variety effect.

Under these calibrations, FM does support a larger expected network, but this expansion lowers welfare relative to ND_B (Table 1).

Table 1: Knife-edge equilibria: types, expected profits, welfare, and network size

(η, F)	Regime	Equilibrium type	$E(\pi_i)$	$E(W)$	$E(n)$
$(-0.7, 0.10)$	FM	mixed (support $\{1,2\}$): $\Pr(1) = 0.767$; $\Pr(2) = 0.233$	0.1213	0.7086	2.5
	ND_B	PSNE $(1,1)$	0.1309	0.7278	2.0
$(-1.1, 0.25)$	FM	mixed (support $\{2,3\}$): $\Pr(2) = 0.680$; $\Pr(3) = 0.320$	0.1483	1.4914	4.6
	ND_B	PSNE $(2,2)$	0.1744	1.5165	4.0
		PSNE $(1,3)$ and $(3,1)$	0.2255	1.4748	4.0

At $\eta = -0.7$, FM mixes over $\{1,2\}$ and ND_B yields a unique symmetric equilibrium at $(1,1)$. At $\eta = -1.1$, FM again sustains a larger expected network, mixing over $\{2,3\}$, whereas ND_B admits equilibria with smaller total network size but higher welfare.

This reflects that welfare is broadly increasing in network size within each regime, but improvements in pricing efficiency under ND_B dominate the gains from larger network size under the free market. Thus, the network-size advantage under the free market does not compensate for distorted pricing.

We then increase fixed costs slightly to eliminate the free market's ability to sustain a larger network (Table 2). When both regimes converge to the same symmetric network size, the welfare advantage of ND_B re-emerges clearly, reflecting the pricing mechanism identified in the core model.

Table 2: Nudging F upward ($\eta = -1.1,\ F = 0.28$)

Regime	Equilibrium type	$E(\pi_i)$	$E(W)$	$E(n)$
FM	PSNE $(2,2)$	0.1350	1.2624	4.0
ND_B	PSNE $(2,2)$	0.1144	1.3965	4.0

Allowing firms to choose network size does not overturn the welfare dominance of ND_B. Even when the free market sustains larger networks, ND_B delivers higher welfare, reflecting the dominance of pricing efficiency over network scale.

8 Conclusion

This paper studies how pricing rules governing cross-firm combinations shape competition in markets with complementary components. Motivated by regulatory provisions permitting coordinated pricing, we analyse benchmarking rules that tie cross-firm prices to firms' own products. We show that discount-based interpretations generate equilibrium surcharges and fail to internalise complementarities across firms. By contrast, a no-discount rule, ND_B, anchors cross-firm prices to own-bundled prices and internalises these complementarities without propagating mark-ups. Unlike parameterised price caps, ND_B is self-anchoring and implementable using observable prices.

Across all environments considered, ND_B is the most robust benchmark. It strictly dominates discount-based rules and, in the general analysis, also dominates the standalone-based no-discount regime, ND_S.

Relative to the free market, ND_B delivers higher consumer surplus and network usage for all $n \geq 2$, and higher welfare except in a narrow set of low-elasticity, large-n cases where differences are negligible. These patterns are robust to variation in demand composition.

Allowing firms to choose network size does not overturn these results. Even when the free market sustains larger networks, ND_B delivers higher welfare, reflecting the dominance of pricing efficiency over network scale.

Although firm profits may be lower under ND_B, the resulting welfare gains allow compensation from consumer surplus, leaving users better off.

A transport calibration indicates that these effects are quantitatively meaningful. Under empirically relevant elasticities, ND_B raises consumer surplus by around 20% and increases demand by roughly 10%, with additional gains from reduced external costs that are large relative to existing sector support.

More broadly, the results show that pricing rules are a central determinant of competitive outcomes in network industries. These findings apply beyond transport to settings in which firms supply complementary components and consumers assemble products across firms.

Bundle-based coordination provides a simple and robust benchmark for aligning pricing incentives with welfare, offering a practical alternative to regulator-set price caps. Extending the analysis to richer network topologies and evaluating these mechanisms empirically in other settings are natural directions for future research.

Appendix A Proofs

Verification convention. Where closed forms are available, signs are verified symbolically. Otherwise, inequalities are verified numerically along the anchored branch, with continuity ruling out sign reversals. Replication code is available from the authors upon request.

Proof of Proposition 1. Fix $r \in \{B, S\}$. The Stage-1 condition is $K_r(\delta; \gamma) = 0$, where K_r is a polynomial in δ with coefficients continuous in $\gamma \in [0, 1)$. At $\gamma = 0$, $\delta = 1$ is the unique root of $K_r(\cdot; 0)$ in $[1, \infty)$ and it is simple, i.e., $\partial_\delta K_r(1; 0) \neq 0$. Hence, by the IFT, there exists a unique local C^1 anchored branch $\delta_r^*(\gamma)$ with $\delta_r^*(0) = 1$. For all $\gamma \in [0, 1)$, $K_r(1; \gamma) = -\gamma L_r(\gamma)$ with $L_r(\gamma) > 0$, so $K_r(1; \gamma) < 0$ for every $\gamma > 0$. The leading coefficient of K_r is positive, so $K_r(\delta; \gamma) \to +\infty$ as $\delta \to \infty$, and the Intermediate Value Theorem implies for each $\gamma \in (0, 1)$ there exists a root $\hat{\delta}(\gamma) > 1$. By anchored root continuity, $\delta_r^*(\gamma)$ cannot satisfy $\delta_r^*(\gamma) \leq 1$ at any $\gamma > 0$ without crossing $\delta = 1$ at some positive parameter value; but $K_r(1; \gamma) = 0$ only at $\gamma = 0$. Therefore $\delta_r^*(\gamma) \geq 1$ for all $\gamma \in [0, 1)$, with $\delta_r^*(\gamma) > 1$ for $\gamma > 0$. □

Proof of Lemma 1. By Proposition 1, $K_r(1; 0) = 0$ and $\partial_\delta K_r(1; 0) \neq 0$, so the IFT yields a unique local C^1 equilibrium branch $\delta_r^*(\gamma)$ with $\delta_r^*(0) = 1$. Let $\Delta_r(\gamma)$ denote the discriminant of $K_r(\cdot; \gamma)$ with respect to δ. As the root at $\gamma = 0$ is simple, $\Delta_r(0) \neq 0$. Because Δ_r is a polynomial in γ, it can vanish only at finitely many points in $[0, 1)$; let G_r denote this set. For $\gamma \notin G_r$, the anchored root is simple and the IFT implies that $\delta_r^*(\gamma)$ is C^1. At points $\gamma \in G_r$, polynomial roots depend continuously on coefficients, so the anchored root varies continuously with γ. Hence $\delta_r^*(\gamma)$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus G_r$. □

Proof of Proposition 2. Fix $r \in \{B, S\}$ and let $G_r(\delta, \gamma) \equiv \partial_\delta \Pi_r(\delta, \gamma)$. By Lemma 1, the anchored solution $\delta_r^*(\gamma)$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus G_r$, with G_r finite. On the admissible set, Π_r is strictly concave in δ, so $\partial^2_{\delta\delta} \Pi_r < 0$ and the interior maximiser is locally unique. For $\gamma \in (0, 1) \setminus G_r$, differentiating $G_r(\delta_r^*(\gamma), \gamma) = 0$ yields $\delta_r^{*\prime}(\gamma) = -\frac{\partial_{\delta\gamma} \Pi_r(\delta_r^*(\gamma), \gamma)}{\partial^2_{\delta\delta} \Pi_r(\delta_r^*(\gamma), \gamma)}$. Along the anchored branch, $\partial_{\delta\gamma} \Pi_r(\delta_r^*(\gamma), \gamma) \geq 0$, and $\partial^2_{\delta\delta} \Pi_r < 0$, hence $\delta_r^{*\prime}(\gamma) \geq 0$ wherever it exists. Continuity extends the non-decreasing property across G_r. □

Proof of Lemma 2. At a symmetric equilibrium,

$$\frac{\mathrm{d}p_i}{\mathrm{d}p_j} = -\frac{\Delta_{ij}}{\partial^2\pi_i/\partial p_i^2}, \qquad \Delta_{ij} \equiv \frac{\partial^2 \pi_i}{\partial p_i\, \partial p_j}. \tag{10}$$

As $\partial^2\pi_i/\partial p_i^2 < 0$ under all regimes, the sign of the best-response slope equals the sign of Δ_{ij}. Under FM and ND_S, $\Delta_{ij} < 0$ for $\gamma \in (0,1)$, so $\mathrm{d}p_i/\mathrm{d}p_j < 0$: strategic substitutes. Under ND_B, $\Delta_{ij} > 0$ on $(0,1)$, so $\mathrm{d}p_i/\mathrm{d}p_j > 0$: strategic complements. □

Proof of Proposition 3. Fix $H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}$ and $R \in \{ND_S, D_B, D_S\}$. By Lemma 1, equilibrium outcomes are continuous in γ on $(0,1)$, so $\Delta_H^R(\gamma) \equiv H^{ND_B}(\gamma) - H^R(\gamma)$ is continuous. For $R \in \{FM, ND_S\}$, Δ_H^R is rational in γ and its sign follows from symbolic verification. For $R \in \{D_B, D_S\}$, outcomes are evaluated along the anchored equilibrium branch and the sign follows from numerical verification and continuity. Thus $H^{ND_B}(\gamma) > H^R(\gamma) > H^{FM}(\gamma)$, $\forall \gamma \in (0,1)$. □

Proof of Proposition 4. By Lemma 1, equilibrium prices are continuous in γ on $(0,1)$. In FM, ND_B, and ND_S, standalone and cross-network prices admit closed-form expressions, so the differences $p^{ND_S} - p^{ND_B}$, $p^{FM} - p^{ND_S}$ and their cross-network analogues are rational functions of γ with no singularities on $(0,1)$. Symbolic verification yields $p^{ND_B} < p^{ND_S} < p^{FM}$ and $P_{cro}^{ND_B} < P_{cro}^{ND_S} < P_{cro}^{FM}$. The absence of a uniform price minimiser follows from numerical evaluation over the admissible parameter range. □

Proof of Proposition 5. Fix $R \in \{ND_B, ND_S, D_B, D_S\}$ and define $\Delta^R(\gamma) \equiv \pi^R(\gamma) - \pi^{FM}(\gamma)$. By Lemma 1, Δ^R is continuous on $(0,1)$. In the no-discount regimes, Δ^R is rational in γ and symbolic verification shows it has a unique zero $\tilde{\gamma}_R \in (0,1)$, with Δ^R strictly positive for $\gamma < \tilde{\gamma}_R$ and strictly negative for $\gamma > \tilde{\gamma}_R$. In the discount regimes, profits are evaluated along the anchored equilibrium branch. Numerical root search identifies a unique zero on $(0,1)$ and continuity rules out additional crossings, establishing the stated threshold property. □

Proof of Proposition 6. Fix $H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}$ and $R \in \{FM, ND_S, D_S, D_B\}$. Let $\Delta_H^R(\gamma) \equiv$

$H^{ND_B}(\gamma) - H^R(\gamma)$. By Lemma 1, Δ_H^R is continuous on $(0,1)$. For $R \in \{FM, ND_S\}$, Δ_H^R is rational in γ and its sign follows from symbolic verification. For $R \in \{D_B, D_S\}$, outcomes are evaluated along the anchored equilibrium branch and the sign follows from numerical verification and continuity. Thus $\Delta_H^R(\gamma) > 0$ for all $\gamma \in (0,1)$. □

Proof of Proposition 7. Fix $H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}$. For each fixed integer $n \geq 2$, the difference $\Delta_H(n,\gamma) \equiv H^{ND_B}(n,\gamma) - H^{ND_S}(n,\gamma)$ is rational in γ with no singularities on $(0,1)$. Symbolic verification shows $\Delta_H(n,\gamma) > 0$ for all $\gamma \in (0,1)$ and all $n \geq 2$, completing the proof. □

Proof of Lemma 3. From (10), strategic interaction is governed by the sign of Δ_{ij}. Under ND_B, Δ_{ij} admits a closed-form expression satisfying $\Delta_{ij} > 0$ for all $\gamma \in (0,1)$ and all $n \geq 2$, implying strategic complements. Under FM and ND_S, Δ_{ij} is rational in (γ, n) and takes both positive and negative values on the admissible set, so the sign can vary with n. □

Proof of Proposition 8. Fix a calibrated elasticity η and let $\gamma_\eta(n)$ denote the associated parameter path. For each outcome H, define $\Delta_H(n) \equiv H^{ND_B}(\gamma_\eta(n), n) - H^{FM}(\gamma_\eta(n), n)$. Along the calibrated path, write $n = 1/m$ and express the ratio $R_H(n) = H^{ND_B}/H^{FM}$ as a Taylor expansion in m: $R_H(n) = c_0(s) + a_1(s)m + a_2(s)m^2 + a_3(s)m^3 + O(m^4)$, where $s = n^2\gamma_\eta(n)$ lies in a compact interval. For each criterion, $c_0(s)$ is uniformly bounded away from one on this interval (except for the welfare case at $\eta = -0.3$, where it remains arbitrarily close to one). The correction terms admit uniform bounds, so there exists N_η such that $R_H(n) > 1$ for all $n \geq N_\eta$. For the remaining finite set $2 \leq n < N_\eta$, direct evaluation confirms the stated inequalities. □

Proof of Proposition 9. For $n = 2$, equilibrium outcomes under FM, ND_B, and ND_S admit closed-form expressions. For each $H \in \{W, Q_{\mathrm{tot}}\}$, the differences $H^{ND_B} - H^{ND_S}$ and $H^{ND_S} - H^{FM}$ are rational functions of (γ, μ) on $(0,1) \times [\frac{1}{2}, 1]$ with no singularities on the admissible set. Their signs are verified as described in the verification convention, implying $H^{ND_B} > H^{ND_S} > H^{FM}$ whenever the equilibrium is interior. □

Proof of Proposition 10. Fix $n = 3$ and $H \in \{W, Q_{\text{tot}}\}$. Ratios H^{ND_B}/H^{FM} and H^{ND_B}/H^{ND_S} are rational functions of (γ, μ) without singularities on $(0,1) \times [\frac{1}{2}, 1]$. Symbolic verification shows both ratios exceed one throughout the admissible set, implying $H^{ND_B} > H^{FM}$ and $H^{ND_B} > H^{ND_S}$. □

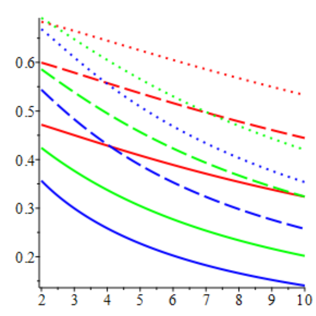
Proof of Proposition 11. Fix an integer $n \geq 4$ and $H \in \{W, Q_{\text{tot}}\}$. For each fixed n, the ratio $R_H(\gamma, \mu, n) \equiv H^{ND_B}(\gamma, \mu, n)/H^{ND_S}(\gamma, \mu, n)$ is rational in (γ, μ) with no singularities on $(0,1) \times [\frac{1}{2}, 1]$. Symbolic verification shows $R_H(\gamma, \mu, n) > 1$ throughout the admissible set, implying $H^{ND_B}(\gamma, \mu, n) > H^{ND_S}(\gamma, \mu, n)$. Together with Propositions 3 and 6, this establishes the result for all $n \geq 2$. □

Proof of Proposition 12. The result follows from direct evaluation of H^{ND_B}/H^{FM} over integer $n \leq 5$ under the calibrated elasticities and $\mu \in \{\frac{1}{2}, \frac{3}{4}, 1\}$. The ratios exceed one throughout, except at the stated knife-edge case. □

Appendix B Market-wide fare elasticity

Elasticity is computed under a uniform fare scaling $(P, P_{cro}, p) \mapsto (1+\varepsilon)(P, P_{cro}, p)$, where P_{av} denotes the quantity-weighted average fare at the symmetric free market equilibrium: $\eta = \frac{d \ln Q_{\text{tot}}}{d \ln P_{\text{av}}}$.[17] Under symmetry, this yields a closed-form expression $\eta(\gamma, n)$, which is inverted numerically to map empirical elasticities into the corresponding substitutability parameter γ: $\eta(\gamma, n) = \frac{(-7n^3+2n^2+15)\gamma^3+(-13n^3+3n^2+21n+35)\gamma^2+(-6n^3+n^2+25n+25)\gamma+6n+5}{(3n-9)\gamma^3+(-n-30)\gamma^2+(-8n-24)\gamma-4n-5}$.

Figure 9: Free market share of single-leg components, $s = 2 - S_{\text{PT}}$.



Note: Solid/dashed/dotted: $\mu = 0.5/0.75/1$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

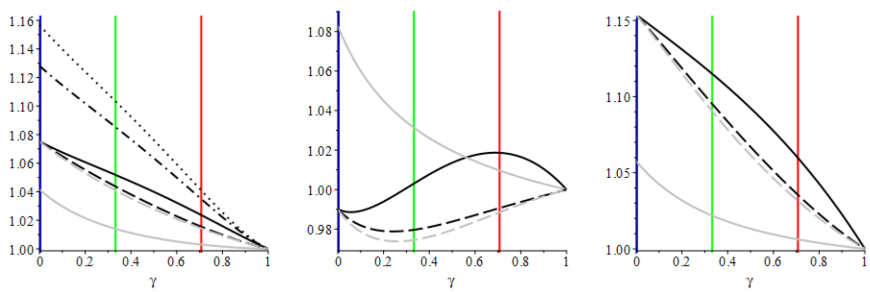
[17]The calibration abstracts from marginal cost, which is empirically reasonable when interpreted relative to an additional passenger holding service frequency and network structure fixed (Parry and Small, 2009).

Appendix C Demand Composition Robustness in μ

FM observed transfer intensity $S_{\text{PT}} \approx 1.5$-$1.7$ (American Public Transportation Association, 2007) implies a single-leg share $s = 2 - S_{\text{PT}} \approx 0.3$-$0.5$, and a corresponding μ. Figure 9 shows that the benchmark $\mu = 0.5$ (solid lines) aligns with this range across calibrated elasticities at small n, supporting its use in the main analysis.

Figures 10 and 12 report welfare, consumer surplus, and aggregate network usage relative to the free market for $\mu = 1$, respectively for $n = 2$ and $n \in [2, 50]$. Figure 11 reports outcomes for $n = 3$ relative to ND_B.

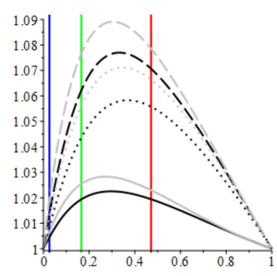
Figure 10: Robustness: welfare, consumer surplus, and network usage under coordinated regimes relative to the free market for $\mu = 1$ in duopoly.



(a) Welfare (b) Consumer Surplus (c) Network Usage

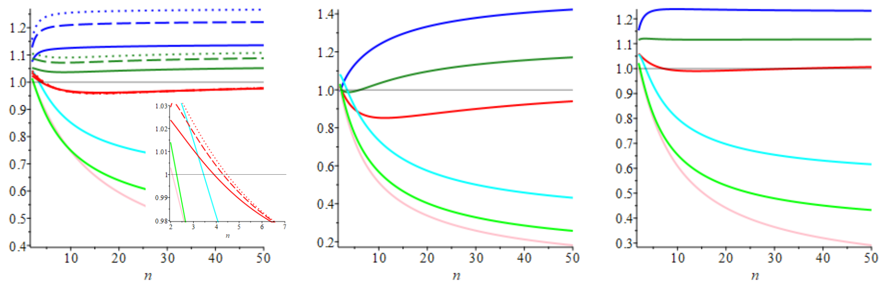
Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Figure 11: Robustness: welfare, consumer surplus, and network usage under discount regimes relative to ND_B in triopoly ($\mu = 1$).



Note: Black/grey: bundled/standalone benchmark; solid/dashed/-dot: $W/CS/Q_{tot}$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Figure 12: Robustness: welfare, consumer surplus, and network usage under ND_B and ND_S relative to the free market for $n \in [2, 50]$ ($\mu = 1$).



(a) Welfare (b) Consumer Surplus (c) Network Usage

Note: Blue/DarkGreen/Red: H^{ND_B}/H^{FM}; Cyan/LightGreen/Pink: H^{ND_S}/H^{FM}; $H \in \{W, CS, Q_{tot}\}$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Appendix D External-cost calibration

Converting induced aggregate usage (ridership) into avoided externalities via marginal external costs (MEC) from UK TAG (Department for Transport, 2023a), let d_c, d_t denote diverted *person* trips from car/taxi per induced bus boarding, with occupancies $\omega_c = \omega_t$.[18] with average trip lengths L_c, L_t. Under TAG diversion rates (A5.4.6), occupancies (A1.3.3), and trip lengths (NTS0303d Department for Transport, 2020), avoided vehicle-km per induced bus boarding are $\kappa = \frac{d_c}{\omega_c} L_c + \frac{d_t}{\omega_t} L_t = \frac{0.24}{1.57} 13.52 + \frac{0.12}{1.57} 8.05 \approx 2.68$ vkm. TAG reports marginal external costs (pence per vkm) by geography (A5.4.2), comprising congestion, safety, air quality, greenhouse gases, noise, and infrastructure.[19] For England (ex London), we use *Other Urban* and *Inner/Outer Conurbations* values, yielding $MEC \in [33.5, 51.1]$ p/vkm and avoided external cost per induced bus boarding $b_{\text{ext}} = \kappa \cdot \frac{\text{MEC}}{100} \in [£0.90, £1.37]$. Total 2019 England (ex London) bus passenger journeys were $J_0 = 2.113$ billion, with revenue £3.506 billion (BUS01;BUS04 Department for Transport, 2025), implying an average fare per boarding of $f_{\text{board}} \approx £1.66$. As observed boardings and model quantities scale proportionally, $\%\Delta Q^R_{\text{tot}}$ also equals the percentage change in boardings. The associated externality benefit is therefore $b_{\text{ext}} \times \%\Delta Q^R_{\text{tot}} \times J_0$. Avoided vkm and CO_2 emissions are reported as

[18] Taxi occupancies are typically higher; using car occupancy is conservative.
[19] TAG marginal external costs exclude fiscal transfers.

Table 3: External benefits of ND_B vs. FM by n and elasticity

		Low		High	
		$\eta = -0.7$	$\eta = -1.1$	$\eta = -0.7$	$\eta = -1.1$
$n = 2$	Welfare add-on (£m/yr) [%]*	186.1 [75.0]	237.3 [95.7]	283.8 [114.4]	362.0 [146.0]
	ΔVKM (m)	555.4	708.4	555.4	708.4
	CO_2 (t) [%]†	55,540 [1.9]	70,840 [2.4]	55,540 [1.9]	70,840 [2.4]
$n = 3$	Welfare add-on (£m/yr) [%]*	197.4 [79.6]	307.6 [124.0]	301.2 [121.4]	469.1 [189.2]
	ΔVKM (m)	589.4	918.1	589.4	918.1
	CO_2 (t) [%]†	58,940 [2.0]	91,810 [3.1]	58,940 [2.0]	91,810 [3.1]

Notes: Low/High correspond to limits MEC $\in [33.5, 51.1]$ p/vkm, implying $\beta \in [0.54, 0.83]$. England (ex-London) operating support £248m is from 2018/19 BUS statistics (Department for Transport, 2025). *Percentages in [.] on the welfare rows express the add-on as a share of operating support. †Percentages in [.] on the CO_2 rows express the saving as a share of UK buses tailpipe CO_2 in 2019 of 3.0 $MtCO_2e$ (ENV0201, Department for Transport, 2023b).

$\Delta \text{VKM}^R = \%\Delta Q^R_{\text{tot}} \times J_0 \times \kappa$, $\text{CO}_{2,R} = \frac{(e_c - e_b)\,\Delta\text{VKM}^R}{10^6}$, using a 100 g/km per-passenger car-bus emissions differential.[20] Augmented welfare is reported applying the proportional uplift Eq. (5), where $\beta = \frac{b_{\text{ext}}}{f_{\text{board}}} \in [0.54, 0.83]$, standardising external benefit per induced boarding in fare-equivalent terms.[21]

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[20] Official UK conversion factors imply average car emissions of about 115 gCO_2e per passenger-km after adjusting for occupancy (Department for Business, Energy & Industrial Strategy, 2019). Marginal bus emissions are smaller when additional passengers are absorbed via higher load factors.

[21] We value avoided car and taxi external costs only; marginal bus external costs are not netted out, consistent with short-run demand absorption through higher load factors.

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