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Abstract

Network oligopolies with sequential or multi-part consumption face double marginalisation across complementary components, motivating constraints on inter-firm pricing. Building on regulatory provisions permitting coordinated pricing for composite or multi-firm products, we study pricing rules that benchmark cross-firm prices against firms' standalone or bundled prices. Coordination is not inherently welfare improving: discount-based benchmarks can generate equilibrium surcharges. By contrast, a no-discount rule, ND_B, ties cross-firm pricing to own-firm bundles, internalising complementarities without propagating markups and raising welfare across a wide range of market sizes and demand parameterisations. However, private and social incentives need not align, so welfare-improving coordination need not arise endogenously. Whilst these results apply broadly to coordinated pricing in network industries, a calibration to the UK bus market illustrates quantitative relevance. ND_B delivers substantial consumer-surplus gains (around 20%) and increases ridership, generating external benefits comparable in magnitude to current operating subsidies, up to £0.5 billion p.a.

Keywords: network pricing; coordination regimes; complementary components; pricing benchmarks; competition policy; network industries.
JEL: L13; L51; D43; D62; R48.

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1 Introduction

When multiple firms supply complementary components, uncoordinated pricing generates distortionary double marginalisation. Network industries with sequential or multi-part consumption are particularly exposed. Public transport provides a salient example: independently set leg-level fares imply cross-network premia on multi-operator journeys, reducing connectivity and network use.[1]

Regulators in network industries often seek to mitigate cross-network double marginalisation through access or transaction-price constraints. Such rules typically cap, rather than eliminate, markups. Evidence from payment-card networks shows incomplete pass-through remains even under regulation (e.g., Shabgard and Asensio, 2023). This raises the question of how benchmark design shapes prices and welfare in networks with such interlinked demands.

The UK Public Transport Ticketing Schemes Block Exemption (PTTSBE) provides a setting in which coordinated multi-operator fares are permitted, operating through constraints on final fares rather than inter-firm access charges. UK guidance (Department for Transport, 2013) refers to benchmarking such fares to "single" tickets - a notion that remains analytically underspecified and may encompass both standalone and through-journey products. Comparable provisions exist in EU competition law (European Commission, 2023).[2]

We study four coordination regimes corresponding to natural interpretations of UK guidance on benchmarking prices to "single" tickets. Two are discount-based: D_B, where cross-firm bundle prices are discounted relative to own-firm bundles, and D_S, where they are discounted relative to the sum of standalone component prices. The corresponding no-discount benchmarks, ND_B and ND_S, are obtained by setting the discount to unity. Combining standalone and bundled demand is essential here: without the former (latter), D_S/ND_S (D_B/ND_B) are not defined. This structure arises naturally in network industries with se-

[1] With implications for congestion, emissions, and social inclusion (Vickerman et al., 1999; Lucas, 2012).

[2] International evidence shows that coordinated pricing remains limited even in overlapping transport networks, despite potential efficiency gains (OECD / ITF, 2008).

quential or multi-part consumption.[3] Our contribution is to show that, when standalone and bundled demand coexist, the choice of benchmarking rule alone can fundamentally alter equilibrium pricing: different benchmarks propagate or insulate markups across firms, altering strategic interaction and welfare rankings across coordination regimes.

Methodologically, we extend Economides and Salop (1992) (henceforth ES92) to allow for both standalone and bundled demand and to nest all four benchmark-based coordination regimes within a unified framework. Existing ES92-based analyses tend to restrict attention to composite demand alone or, when standalone demand is also present, consider only duopoly and do not model explicit benchmarking rules (e.g., Lin, 2004; McHardy, 2024). We therefore, to the best of our knowledge, provide the first ES92-type model with both demand modes and coordinated benchmarking.

The ES92 formulation preserves tractability, yielding closed-form results for the free market and no-discount regimes in the general n-firm case.[4] In contrast, discount-based regimes do not scale: admitting explicit solutions only in duopoly, with triopoly requiring numerical analysis as a minimal robustness check, and do not extend tractably to general n.

Three main results follow. First, under both discount-based interpretations of benchmarking, equilibrium "discounts" become surcharges relative to the benchmark, undermining the intended coordination objective (consistent with evidence, discussed below, that even in settings where coordinated pricing is permitted under benchmark-based guidance, multi-operator prices often exceed comparable own-firm bundled prices).

Second, the bundled no-discount benchmark (ND_B) is welfare-dominant in duopoly, outperforming the free market and all alternative coordination rules. This advantage proves robust as market structure becomes richer: ND_B continues to perform strongly under triopoly and across larger networks, demand compositions, and empirically relevant elasticities.

[3]Examples include platforms (core services vs. add-ons), payment systems (single-side vs. issuer-acquirer services), durables with aftermarkets, and modular supply chains. In settings with only bundled usage (e.g., telecoms interconnection), the model reduces to the case without standalone demand.

[4]More elaborate network formulations (e.g., Hotelling, logit) typically become analytically intractable beyond duopoly and do not admit closed-form characterisations for general n (e.g., Zhou, 2021).

These effects are particularly relevant in small-n environments, which are empirically common in network industries with complementary components. A UK bus-market calibration suggests economically meaningful effects: ND_B raises consumer surplus by around 20% and ridership by around 10%, with induced externality benefits offsetting roughly three-quarters to almost twice current sector operating subsidies.

Third, private and social incentives diverge, so the welfare-dominant benchmark need not arise endogenously, as firms do not fully internalise the consumer-surplus gains from coordinated pricing.

These results speak directly to benchmark choice under schemes such as the PTTSBE. Existing guidance points toward discount-based benchmarks, yet such rules have not been analysed formally. By contrast, the own-firm bundled no-discount benchmark (ND_B) has appeared only as a modelling convenience in a small literature (e.g., McHardy et al., 2023; McHardy, 2024), and only without standalone demand. When standalone and bundled demand coexist, benchmark choice has first-order implications for equilibrium surcharges, deviation incentives, and welfare. Unlike price caps, ND_B is self-anchoring: it links cross-firm prices to firms' own fares rather than to regulator-chosen parameters, and performs robustly across empirically relevant environments. In particular, ND_B reduces surcharges and improves welfare relative to discount-based benchmarks implied by current guidance, as well as the uncoordinated free-market outcome.

The paper proceeds as follows. Section 2 introduces the model. Section 3 analyses duopoly equilibria. Section 4 considers triopoly as a robustness check for the otherwise non-generalisable discount regimes, and as a representative extension within empirically relevant small-n market structures. Section 5 examines the general n-firm case for the tractable classes of regimes. Section 6 endogenises network size. Section 7 concludes.

2 Base Model

Two firms $i \in \{1, 2\}$ each supply an x- and a y-component. Consumers may purchase components standalone or combine one x- and one y-component into a composite bundle. Hence, in the duopoly case ($n = 2$):

$$\boldsymbol{Q} = (Q_1, \dots, Q_4), \qquad \boldsymbol{X} = (X_1, X_2), \qquad \boldsymbol{Y} = (Y_1, Y_2),$$

where Q_t lexicographically indexes the four bundle demands Q_{ij}, with Q_{ij} denoting demand for bundle (x_i, y_j) (more generally n^2 bundles), and X_i and Y_i denote demand for firm i's standalone x- and y-components.

Following Hackner (2000) and McHardy (2024), we adopt a quasi-linear quadratic utility specification, consistent with the linear-demand ES92 framework we extend, here defined over bundled (Q_t) and standalone (X_i, Y_i) consumption. While the analysis in this section focuses on duopoly ($n = 2$), we present the model in its general n-firm form to avoid repetition, with $n = 3$ and the general case considered in Sections 4 and 5, respectively.

$$\begin{aligned} U(\mathbf{Q}, \mathbf{X}, \mathbf{Y}, M_0) = \alpha \sum_{t=1}^{n^2} Q_t + \mu\alpha \sum_{i=1}^{n} (X_i + Y_i) - \frac{1}{2}\sum_{t=1}^{n^2} Q_t^2 + \gamma \sum_{1 \le t < q \le n^2} Q_t Q_q \\ - \frac{1}{2}\sum_{i=1}^{n} (X_i^2 + Y_i^2) + \gamma \sum_{1 \le i < j \le n} (X_i X_j + Y_i Y_j) + M_0. \end{aligned} \tag{1}$$

M_0 denotes consumption of the numeraire good. Substitutability across products is governed by $\gamma \in (0, 1)$, capturing substitution across composite bundles and across standalone components.[5] Composite demand has baseline intensity $\alpha > 0$. Standalone demand is scaled by $\mu \in [0.5, 1]$, which measures the value of a single component relative to a two-component bundle.[6] A value $\mu = 0.5$ implies that a standalone component is worth half of a composite bundle, while $\mu = 1$ implies equal valuation. We use $\mu = 0.5$ as the analytical benchmark.

[5] We restrict attention to $\gamma \in (0, 1)$, which under duopoly ($n = 2$) ensures strict second-order conditions and well-behaved substitution. At the boundaries $\gamma \in \{0, 1\}$ cross-price effects become degenerate.

[6] The restriction $\mu \in [0.5, 1]$ ensures interior solutions in the symmetric model.

Results that hold for general $\mu \in [0.5, 1]$ are stated explicitly; otherwise, results refer to the benchmark case. Numerical robustness for $\mu \in \{0.75, 1\}$ are reported in Appendix A.4.

Denote P_{ij} the price of bundle (x_i, y_j), and p_i firm i's standalone price (common across x_i and y_i by symmetry). Bundles may be priced directly or via separately priced components, depending on the pricing regime. Marginal cost per passenger is normalised to zero.[7]

Solving the representative consumer's problem yields linear demands that generalise to the n-firm case:

$$Q_{ij} = a - bP_{ij} + d \sum_{(k,\ell)\neq(i,j)} P_{k\ell}, \qquad X_i = Y_i = \hat{a} - \hat{b}\, p_i + \hat{d} \sum_{j\neq i} p_j,$$

where the coefficients $(a, b, d, \hat{a}, \hat{b}, \hat{d})$ are functions of (α, μ, γ, n):

$$\begin{aligned} a &= \frac{\alpha}{1+\gamma(n^2-1)}, \quad b = \frac{\gamma(n^2-2)+1}{(1-\gamma)(1+\gamma(n^2-1))}, \quad d = \frac{\gamma}{(1-\gamma)(1+\gamma(n^2-1))}, \\ \hat{a} &= \frac{\mu\alpha}{1+(n-1)\gamma}, \quad \hat{b} = \frac{1+\gamma(n-2)}{(1-\gamma)(1+\gamma(n-1))}, \quad \hat{d} = \frac{\gamma}{(1-\gamma)(1+\gamma(n-1))}. \end{aligned} \tag{2}$$

We begin with the duopoly ($n = 2$) case, the minimal structure in which strategic interaction and coordination across complementary components arise, and representative of many empirically relevant small-n settings.[8] The duopoly benchmark allows pricing mechanisms to be characterised analytically before extending the analysis to richer market structures in Sections 4 and 5.

2.1 Pricing Regimes and Strategic Environment

The pricing regimes differ only in how the cross-firm bundle price P_{ij} ($i \neq j$) is determined. Equilibrium prices are symmetric within each regime R: standalone price p^R, within-firm

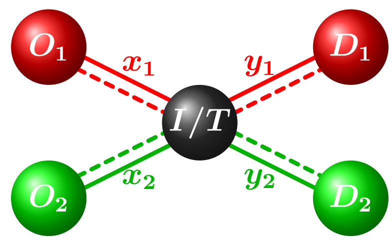
[7]Normalising marginal cost to zero is standard in models with quasi-linear preferences, where only relative prices matter. In transport applications, this is also empirically relevant when defined with respect to an additional passenger holding service plans fixed (Mohring, 1972; Parry and Small, 2009).

[8]For $n = 2$, Appendix A.4 shows that, in transport applications, the implied share of single-leg demand lies within empirically plausible ranges at the benchmark $\mu = 0.5$.

bundle price P^R, and cross-firm bundle price P^R_{cro}.[9]

2.1.1 Free Market (FM)

Figure 1: Network Structure: free market



Note: Solid: own bundled journeys (P_{ii}); Dashed: standalone components (p_i per component).

Figure 1 provides a transport interpretation of the model under the free market. Firm i supplies a two-leg journey $O_i \to D_i$ via (x_i, y_i), with own bundled journeys (solid lines) priced at P_{ii}. All two-leg journeys pass through a common interchange/terminal node (I/T), with interchange costs normalised to zero. Cross-firm bundles (x_i, y_j) are therefore priced as the sum of component prices, $P_{ij} = p_i + p_j$ for $i \neq j$. Consumers may also purchase individual components separately (dashed lines connecting O_i or D_i to the I/T node), generating standalone demands X_i and Y_i at price p_i.

Firm i chooses (p_i, P_{ii}), taking the rival's prices as given, to maximise

$$\pi_i = P_{ii}Q_{ii} + p_i(Q_{ij} + Q_{ji} + X_i + Y_i).$$

The symmetric equilibrium prices are:

$$P^{FM} = \frac{5\alpha(\gamma^2 - 1)}{6\gamma^2 - 7\gamma - 10}, \qquad p^{FM} = \frac{\alpha(7\gamma^2 - \gamma - 6)}{2(6\gamma^2 - 7\gamma - 10)}, \qquad P^{FM}_{\text{cro}} = 2p^{FM}.$$

Before turning to the coordinated regimes, recall a standard feature of complementary-component markets with network structure. When products are weak substitutes, decentralised pricing performs poorly because firms do not internalise cross-network pricing externalities. Internalising these externalities can therefore raise welfare, even at the cost of weaker price competition. As substitutability increases - and, more generally, as competitive

[9] For FM, ND_B, and ND_S, each firm's profit is strictly concave in own prices at the symmetric equilibrium, so the pricing subgame is a concave game in the sense of Rosen (1965), implying a locally unique symmetric Nash equilibrium. This property extends to the n-firm case.

pressure intensifies - the importance of the externality declines, a pattern we formalise below.

2.1.2 Discount-Based Regimes (D_B and D_S)

Turning to the discount regimes, UK guidance can be interpreted as setting cross-firm fares by applying a discount to a benchmark "single-ticket" fare, scaled by typical usage (see Department for Transport, 2013, p. 22). The notion of a "single-ticket" benchmark is left undefined. It may refer either to (i) the sum of standalone component fares or to (ii) a bundled through-journey fare. We model these as discount-on-standalone pricing (D_S) and discount-on-bundled pricing (D_B), respectively. In both cases, the pricing rule can be written

$$\text{multi-operator fare} = \text{benchmark fare} \times \text{usage} \times \delta, \tag{3}$$

where δ is a common discount factor chosen by participating firms, with $\delta < 1$ corresponding to a discount and $\delta > 1$ to a surcharge.[10]

We now formalise the discount-based coordination regimes and solve by backward induction. At Stage 1, under regime D_r ($r \in S, B$), firms jointly choose a common discount factor δ_r. At Stage 2, firms independently choose (p_i, P_{ii}).

For duopoly, $n = 2$, we present the rules for general n to avoid repetition and facilitate the later extension. Under the discount-based coordination rules, the cross-firm bundle price is defined as a proportional scaling of a network-wide benchmark. Specifically, for $i \neq j$:

$$P_{ij}^{D_S} = \delta_S \frac{2\sum_{k=1}^{n} p_k(X_k + Y_k)}{\sum_{k=1}^{n}(X_k + Y_k)}, \qquad P_{ij}^{D_B} = \delta_B \frac{\sum_{k=1}^{n} P_{kk}Q_{kk}}{\sum_{k=1}^{n} Q_{kk}}. \tag{4}$$

At Stage 2, each firm i solves

$$\max_{p_i, P_{ii}} \; P_{ii}Q_{ii} + p_i(X_i + Y_i) + \frac{P_{ij}^{D_r}}{2}(Q_{ij} + Q_{ji}),$$

[10] Usage is normalised, without loss of generality, to 1 under bundled benchmarking and to 2 under standalone benchmarking, reflecting the number of components in the journey.

taking rivals' prices as given and treating $P_{ij}^{D_r}$ as an endogenous function of prices via the benchmark. Cross-firm bundle revenue is allocated symmetrically across firms.[11]

Substituting the Stage 2 best replies into profits yields a Stage 1 problem:

$$\max_{\delta \in (0,\infty)} \Pi_r(\delta; \gamma),$$

where $\Pi_r(\delta; \gamma)$ denotes aggregate profit. The first-order condition reduces to a quartic equation $K_r(\delta; \gamma) = 0$.[12] We focus on the anchored solution branch $\delta_r(\gamma)$ defined by $\delta_r(0) = 1$, corresponding to the limit of independent services, in which no discount applies.

Proposition 1 (Equilibrium surcharge)**.** *For both discount regimes D_r, $r \in \{S, B\}$, the anchored equilibrium satisfies $\delta_r^*(\gamma) \geq 1$ for all $\gamma \in [0, 1)$, with strict inequality for $\gamma > 0$.*

Thus, discount-based benchmarking yields equilibrium surcharges rather than discounts. Lowering cross-firm bundle prices expands demand for cross-firm bundles, from which a firm derives only partial revenue, but also reallocates demand away from standalone consumption Q_{ii}, on which it earns the full margin. Anticipating this composition effect, firms soften price competition at Stage 1 by raising the benchmarked price, yielding $\delta_r^*(\gamma) \geq 1$.

Under bundled benchmarking (D_B), the mechanism is intuitive: cross-firm bundle prices are benchmarked to average own-firm bundle prices, so lowering the benchmark brings cross-firm prices closer to (or below) own-bundle prices, intensifying business stealing. Under standalone benchmarking (D_S), the mechanism is more subtle. The benchmark operates through standalone prices p_i, which affect both standalone demand and, indirectly, cross-firm prices. While this partially corrects the free-market distortion in standalone prices, it also intensifies substitution away from Q_{ii} toward cross-firm bundles. The net effect is therefore a

[11]Under symmetry, this coincides with quantity-based apportionment.

[12]For each discount regime D_r ($r \in \{S, B\}$), Stage 2 is a concave pricing game: each firm's profit is strictly concave in own prices on the admissible parameter set, implying a locally unique symmetric Nash equilibrium in prices. Substituting this Stage 2 equilibrium into joint profit $\Pi^r(\delta_r, \gamma)$ yields a Stage 1 objective that is strictly concave in δ_r on the economically relevant region, so the optimal discount $\delta_r^*(\gamma)$ is unique and interior. These properties justify the use of the Implicit Function Theorem (IFT) in the main text and underlie Propositions 1 and 2.

priori ambiguous, but the business-stealing channel dominates, yielding the surcharge result.

Consistent with the model, despite widespread availability in the UK (see McHardy et al., 2023), multi-operator tickets are typically priced above comparable single-operator fares, with little evidence of systematic discounting (see Department for Transport, 2013; Urban Transport Group, 2019, p. 43).

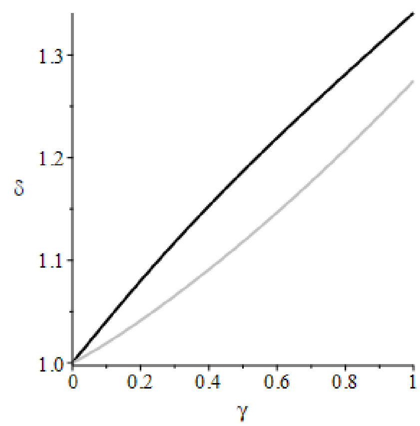
The next lemma shows that the anchored root can be tracked continuously in γ, ensuring continuity of the economically relevant equilibrium branch.

Lemma 1 (Continuity). *For both discount regimes D_r, $r \in \{S, B\}$, the anchored solution $\delta_r^*(\gamma)$ with $\delta_r^*(0) = 1$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus S_r$, where S_r is a finite set of parameter values at which the polynomial admits multiple roots.*

It follows that all equilibrium outcomes are continuous in γ on $[0, 1)$ (by closed form in FM and ND$_r$, and by continuity of the optimal discount rule in D_r). Moreover, the Stage-1 objective is strictly concave in δ_r on the admissible set and satisfies $\frac{\partial^2 \Pi^r}{\partial \delta \partial \gamma} > 0$ along the optimal branch. These imply:

Proposition 2 (Monotonicity). *For both discount regimes D_r, $r \in \{S, B\}$, the optimal discount $\delta_r^*(\gamma)$ is non-decreasing in γ on $[0, 1)$.*

Figure 2: Optimal discount factors $\delta_r^*(\gamma)$ for D$_r$ regimes ($r \in \{S, B\}$).



Note. Black/grey: δ_B/ δ_S.

Hence, although the block exemption envisages discounts, the profit-maximising solution on the admissible branch yields surcharges that increase monotonically with substitutability, as illustrated in Figure 2.

2.1.3 No-Discount Regimes (ND$_B$ and ND$_S$)

Finally, the no-discount regimes set $\delta_r = 1$ in (4), yielding two transparent benchmarks, ND$_S$ and ND$_B$. These rules eliminate the Stage 1 choice and admit closed-form equilibria.

Under ND_S and ND_B, symmetric equilibrium prices are, respectively:

$$P^{ND_S} = \frac{(11\gamma^3 + 4\gamma^2 - 11\gamma - 4)\alpha}{(5\gamma^2 - 6\gamma - 8)(3\gamma + 1)}, \qquad p^{ND_S} = \frac{(16\gamma^3 + 2\gamma^2 - 14\gamma - 4)\alpha}{2(3\gamma + 1)(5\gamma^2 - 6\gamma - 8)}, \qquad P^{ND_S}_{\text{cro}} = 2p^{ND_S},$$
$$P^{ND_B} = P^{ND_B}_{\text{cro}} = \frac{3(1-\gamma)\alpha}{2(3-\gamma)}, \qquad p^{ND_B} = \frac{(1-\gamma)\alpha}{2(2-\gamma)}.$$

Strategic interaction in standalone pricing differs across the benchmark regimes.[13]

Lemma 2 (Strategic interaction)**.** *In symmetric equilibrium, p_i is a strategic complement to p_j under ND_B and a strategic substitute under FM and ND_S.*

Under ND_B, cross-firm bundle prices are pinned to own-bundle prices, insulating standalone pricing from cross-firm price determination. Best responses are upward-sloping, yielding strategic complementarity and, as in standard Bertrand competition, downward pressure on equilibrium prices. Under FM and ND_S, standalone prices enter additively into cross-firm bundle prices, as in complementary monopoly. This induces strategic substitutability, overturning the baseline Bertrand complementarity with upward pressure on prices.

3 Analysis of Welfare, Prices, and Incentives

We examine how welfare, consumer surplus, and aggregate bundle-equivalent quantity, Q_{tot}, vary with γ. Here, Q_{tot} captures overall network usage (ridership), which in transport settings is associated with external benefits. In symmetric equilibrium:

$$Q_{\text{tot}} = 2(Q_{ii} + Q_{ij}) + (X_i + Y_i).$$

We begin by establishing a welfare and ridership ranking across the benchmark regimes that holds for all $\mu \in [1/2, 1]$.

[13]This result is independent of μ.

Proposition 3 (Duopoly ranking for all μ)**.** *For all* $\gamma \in (0, 1)$ *and* $\mu \in [1/2, 1]$*,*

$$H^{ND_B} > H^{ND_S} > H^{FM}, \qquad \forall H \in \{W, Q_{\text{tot}}\}.$$

ND_B internalises cross-network pricing externalitiesacross composite journeys without introducing the cross-price linkage present in FM and ND_S. By Lemma 2, standalone prices remain strategic complements under ND_B, while the additive linkage in FM and ND_S induces strategic substitutability and weakens competitive discipline, driving the welfare ranking.

To obtain a complete ranking across all regimes - including consumer surplus and the discount-based cases - we turn to the benchmark $\mu = 0.5$.

Proposition 4 (Benchmark duopoly dominance)**.** *For all* $\gamma \in (0, 1)$*,*

$$H^{ND_B}(\gamma) > H^R(\gamma) > H^{FM}(\gamma), \qquad \forall H \in \{W, \text{CS}, Q_{\text{tot}}\}, \quad \forall R \in \{ND_S, D_S, D_B\}.$$

This result delivers a complete ordering across all regimes at the benchmark parameterisation. In particular, ND_B dominates not only the free market and ND_S, but also the discount-based regimes, in welfare, consumer surplus, and ridership. Together with Proposition 3, this shows that the welfare and usage advantages of ND_B are robust across μ, while a full ordering including consumer surplus can be established at empirically relevant benchmark values.

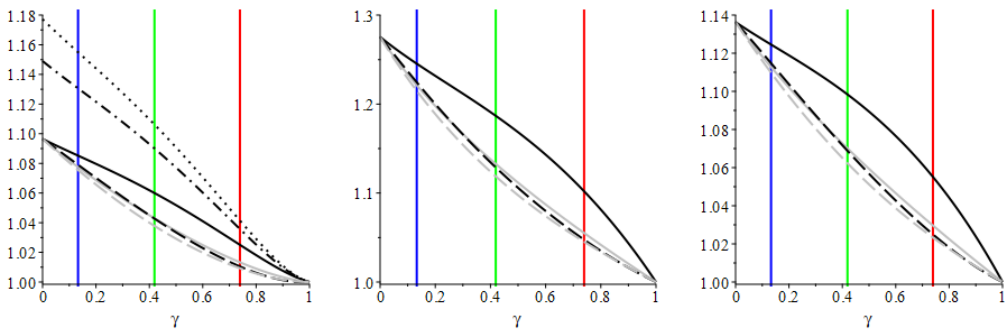
Because higher ridership is associated with external benefits in transport settings, and ND_B delivers the largest increase in aggregate ridership, we report augmented welfare that incorporates avoided external costs associated with induced trips. This is defined as

$$\widetilde{W}^R = W^R + \beta\, \Delta Q^R_{tot} f_{\text{avg}}, \tag{5}$$

where ΔQ^R_{tot} denotes the change in aggregate ridership relative to the free market, f_{avg} is the free-market average fare, and β captures marginal external costs (see Appendix A.5).

Figure 3 reports welfare, consumer surplus, and aggregate ridership relative to the free market under $\mu = 0.5$, illustrating the magnitude of gains under Proposition 4.

Figure 3: Duopoly welfare, consumer surplus, and aggregate ridership (Q_{tot}) relative to the free market. Augmented welfare is reported for ND_B.



(a) Welfare (b) Consumer Surplus (c) Ridership

Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

To relate these profiles to empirical settings, we map substitutability (γ) to standard public-transport own-price elasticities $\eta \in \{-0.3, -0.7, -1.1\}$, spanning short-, medium-, and long-run responses.[14] For $n = 2$, this mapping yields $\gamma \approx \{0.74, 0.42, 0.13\}$ (Appendix A.3). At the medium-run calibration ($\eta = -0.7$), ND_B raises consumer surplus by around 20%, welfare by about 6%, and ridership by roughly 10%.[15] Augmenting welfare to account for avoided external costs increases the welfare gain by a further 3-7 percentage points.

A monetary calibration to England's pre-COVID bus market outside London implies gains of approximately £186-£285m per year, rising to £238-£363m under higher elasticities (Appendix A.5). These gains amount to roughly 75-146% of annual operating support and correspond to reductions of 1.9-2.4% in national bus-sector CO_2 emissions.

The welfare ranking is mirrored in the equilibrium price structure.

[14]Public transport demand estimates typically place own-price elasticities around −0.3 in the short run and −1.0 in the long run, with intermediate medium-run responses (e.g., Paulley et al., 2006; Holmgren, 2007). We use transport as a benchmark given the availability of well-established estimates and its prominence in the literature on complementary network demand. Comparable elasticity evidence for other network industries is more limited and often context-specific, reflecting differences in market structure and identification. The calibrated values should therefore be interpreted as spanning a plausible range, with further empirical work needed to assess industry-specific magnitudes.

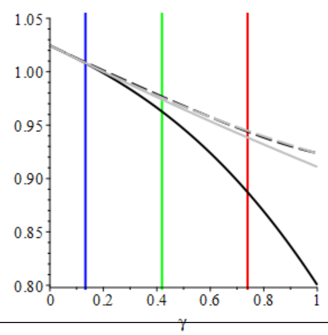
[15]Implied welfare and ridership changes compare to those for major operational or demand-side transport interventions (e.g., dedicated bus lanes, see Paulley et al., 2006; Litman, 2022).

Proposition 5 (Prices)**.** *For all $\gamma \in (0,1)$, although no coordinated regime uniformly minimises prices across all services, the welfare-dominant regime ND_B sets lower standalone and cross-network prices than ND_S and the free market:*

$$p^{ND_B} < p^{ND_S} < p^{FM}, \qquad P_{cro}^{ND_B} < P_{cro}^{ND_S} < P_{cro}^{FM}.$$

These price rankings reflect the mechanism in Lemma 2. Under FM and ND$_S$, standalone prices are strategic substitutes and enter cross-network fares additively, sustaining double marginalisation. Under ND$_B$, standalone prices are strategic complements and do not enter cross-network fares directly, so through-journey prices carry only a single effective mark-up. This reduction in cross-price distortions at the level of composite journeys underlies the welfare and ridership gains documented above.

Figure 4: Coordinated-regime profit relative to free market



Note: Black/grey: bundled/standalone benchmark;. solid/dashed: no-discount/discounted. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

We now turn to the alignment between private and social incentives.

Proposition 6 (Incentive misalignment)**.** *For each regime $R \in \{ND_B, ND_S, D_B, D_S\}$, firm profits admit a unique threshold $\tilde{\gamma}_R \in (0,1)$ such that $\pi^R(\gamma) > (<)\pi^{FM}(\gamma)$ for $\gamma < (>)\tilde{\gamma}_R$, with equality at $\gamma = \tilde{\gamma}_R$.*

Figure 4 shows that coordinated regimes raise profits relative to the free market only at low levels of substitutability, showing the incentive misalignment characterised in Proposition 6.

Corollary 1 (Policy implication)**.** *Although firms may earn lower profits than under laissez-faire, since $W^{ND_B} > W^{FM}$ for all $\gamma \in (0,1)$, coordination can be implemented with transfers financed from the consumer-surplus gain.*

In transport applications, the case for coordination is further strengthened by external benefits associated with increased ridership.

The welfare and quantity orderings are robust to variation in the demand weight on standalone services. For $\mu = 0.75$ and $\mu = 1$, Appendix A.4 (Figure 10) shows that ND_B remains the clear leader in welfare and aggregate ridership, while ND_S, D_S, and D_B lose ground and fall below the free market at moderate and high elasticities.[16]

4 Extension to Triopoly

We now test whether the welfare dominance of ND_B and the surcharge and underperformance results for the discount regimes extend beyond duopoly. Triopoly is the first multi-firm setting in which all five regimes can be examined, while the discount regimes no longer admit closed-form solutions and are therefore analysed numerically. We then turn in Section 5 to the general n-firm case for the analytically tractable free-market and no-discount regimes.[17]

Cross-firm bundle prices under coordination are now benchmarked to firm-level prices averaged across three firms, taking the form:

$$P_{ij}^{D_S} = \frac{2\delta_S}{3}\sum_{k=1}^{3} p_k, \qquad P_{ij}^{D_B} = \frac{\delta_B}{3}\sum_{k=1}^{3} P_{kk},$$

with $P_{ij}^{ND_S}$ and $P_{ij}^{ND_B}$ obtained under $\delta_r = 1$. Closed-form solutions for the free-market and no-discount regimes follow from evaluating the n-firm expressions at $n = 3$ (Appendix A.1).

We first establish that the dominance of ND_B over the free market and ND_S in welfare and aggregate ridership extends to triopoly.

Proposition 7 (Triopoly dominance for all μ)**.** *For all $\gamma \in (0,1)$, all $\mu \in [1/2, 1]$, and for each $H \in \{W, Q_{\text{tot}}\}$,*

$$H^{ND_B} > H^{FM}, H^{ND_S}.$$

[16] At $\mu = 1$, consumer surplus under ND_S exceeds ND_B in the duopoly case. This reflects a compositional effect: ND_S shifts demand away from bundled travel, especially cross-firm journeys Q_{ij}, and toward standalone usage X_i. Consumer surplus therefore rises through stronger standalone consumption rather than improved coordination of cross-firm travel, so the gain does not carry over to welfare or bundle-equivalent ridership and disappears as n rises.

[17] Appendix A.4 shows that, for transport applications, the implied share of single-leg demand remains within plausible ranges at $n = 3$ under the benchmark $\mu = 0.5$, and for moderate n.

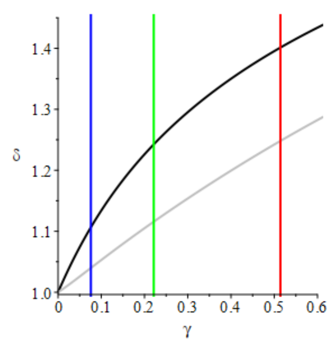
However, unlike in duopoly, we no longer have a complete ordering between ND_S and FM at this level of generality. As we show below, this reflects a change in the underlying strategic interaction as the number of firms increases. The intuition for ND_B's dominance remains unchanged: it internalises complementarities without propagating standalone mark-ups across the network.

Remark 1 (Strategic interaction)**.** *At $n = 3$, as characterised in Lemma 3 and illustrated in Figure 7 (in the following Section), standalone prices under FM switch from strategic substitutes to complements as substitutability increases, strengthening competitive pressure. By contrast, ND_S remains in the substitutes region except for a narrow high-γ interval exhibiting complementarity.*

This explains why a uniform ranking between ND_S and FM no longer holds across $\mu \in [0.5, 1]$ in triopoly: the relative performance of the two regimes becomes sensitive to substitutability.

We now turn to the discount-based coordination regimes, which do not admit closed-form solutions in triopoly.

Figure 5: Equilibrium triopoly discounts ($n = 3$).



Note. Black/grey: bundled/standalone benchmark. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Remark 2 (Triopoly discount regimes)**.** *In triopoly, the coordinated discount regimes are solved numerically. For all $\gamma \in [0, 1)$, we obtain a unique smooth equilibrium discount $\delta_r^*(\gamma)$, $r \in \{S, B\}$, which is weakly above one and increasing over the calibrated range. Figure 5 illustrates the equilibrium paths.*

To obtain a complete ranking across all regimes - including consumer surplus and the discount-based cases - we restrict attention to the benchmark $\mu = \frac{1}{2}$.

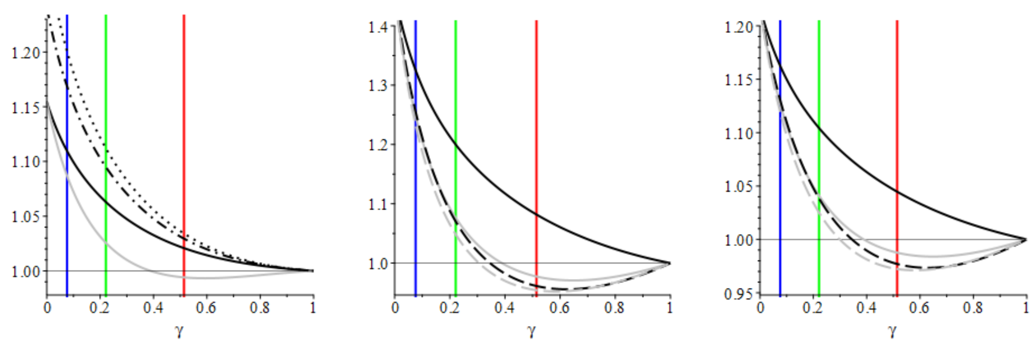
Proposition 8 (ND_B triopoly dominance)**.** *For triopoly, and all* $\gamma \in (0,1)$*,*

$$H^{ND_B}(\gamma) > H^R(\gamma), \qquad \forall H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}, \quad \forall R \in \{FM, ND_S, D_S, D_B\}.$$

Figure 6 illustrates these results, reporting welfare, augmented welfare, consumer surplus, and ridership at $n = 3$. ND_B remains the clear performance leader, dominating all other regimes across all measures for all $\gamma \in (0,1)$. At the medium-run elasticity, welfare, consumer surplus, and ridership exceed the free-market benchmark by roughly 6.5%, 20%, and 10%, respectively, with augmented welfare adding a further 3-9 percentage points depending on the externality calibration.

In marked contrast, ND_S and the discount regimes lose ground relative to the free market as substitutability increases. While they outperform the free market at low and medium γ, they fall below it beyond the mid-range calibration (around $\gamma \simeq 0.22$). For ND_S, this reversal is consistent with the change in strategic interaction described above: as competitive pressure strengthens in triopoly, price-linking mechanisms amplify, rather than mitigate, distortions relative to the free market.

Figure 6: Triopoly welfare, consumer surplus, and aggregate ridership relative to the free market. Augmented welfare is shown for ND_B.



(a) Welfare (b) Consumer Surplus (c) Ridership

Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Consistent with Proposition 8, ND_B delivers the largest ridership gains of all regimes, exceeding 10% over the free market at the medium-run elasticity ($\eta = -0.7$). Mapping the percentage uplifts in augmented welfare shown in Figure 6(a) into monetary values using Appendix A.5 yields external benefits of approximately £198-£301m per year under medium-run elasticities, rising to £308-£469m when demand is more elastic. These gains are comparable to, and in some cases exceed, annual operating support and imply CO_2 reductions of about 2.0-3.1% of bus-sector emissions (Table 1).

Turning to incentives, under duopoly, participation depended only on whether both firms preferred the regime to the free market, since coordination cannot operate with a single firm. In triopoly, participation additionally depends on deviation incentives: given full participation (a grand coalition), does any one firm gain by opting out, leaving a two-firm partial coalition alongside one free-market firm?

Under partial coordination, only participating firms internalise cross-network pricing: for any $i \neq j$, P_{ij}^R is determined by the pricing rule $R \in \{ND_B, ND_S, D_B, D_S\}$ when both firms participate, while bundles involving a non-participating firm remain priced additively.

Because the discount regimes do not admit tractable solutions, we evaluate coalition outcomes numerically over the calibrated range.

Remark 3 (Triopoly coalition incentives). *At low substitutability, all coordination regimes sustain grand and partial coalitions that dominate the free market in welfare, consumer surplus, and ridership (with deviation incentives under D_B). Partial coalitions persist into slightly higher substitutability. At high substitutability, grand coalitions (except under ND_B) are again sustainable but underperform the free market on all three measures.*

The triopoly results separate coalition feasibility from coalition desirability. Coordination is jointly sustainable and beneficial at low substitutability, breaks down at intermediate levels, and may re-emerge at high substitutability despite reducing welfare. Thus, privately stable coalitions need not align with socially preferred outcomes.

Across all cases, ND_B delivers the most robust performance. Since $W^{ND_B} > W^{FM}$ throughout the interior, any profit losses from coalition instability can be offset by transfers financed from consumer-surplus gains, leaving consumers better off than under laissez-faire, with further scope for compensation in transport applications due to external benefits.[18]

5 No-Discount n-Firm Case

Markets for composite or sequential goods are often served by a small number of firms supplying partially overlapping components, motivating our focus on $n \in 2, 3$. To assess how these results scale in more integrated or extensive networks, we extend the analysis to arbitrary n, focusing on the regimes that continue to admit closed-form solutions: FM, ND_S, and ND_B.[19]

For all $n \geq 2$, symmetric closed-form equilibria exist under FM, ND_S, and ND_B (Appendix A.1).

Under the free market, taking rivals' prices as given, firm i chooses (p_i, P_{ii}) to maximise

$$\max_{P_{ii}, p_i} \ \pi_i^{FM} = P_{ii}Q_{ii} + p_i\Big[X_i + Y_i + \sum_{\substack{j=1 \\ j \neq i}}^{n}(Q_{ij} + Q_{ji})\Big],$$

In the no-discount regimes ND_r ($r \in \{S, B\}$), the objective is identical except that cross-firm bundle revenue is pinned to the benchmark price $P_{ij}^{ND_r}$ (given in Eq. (4)):

$$\max_{P_{ii}, p_i} \ \pi_i = P_{ii}Q_{ii} + p_i(X_i + Y_i) + \frac{P_{ij}^{ND_r}}{2}\sum_{\substack{j=1 \\ j \neq i}}^{n}(Q_{ij} + Q_{ji}), \tag{6}$$

We proceed in three steps. First, we establish analytical dominance results that hold globally for all $\mu \in [\frac{1}{2}, 1]$. Second, we characterise the strategic interaction underlying those rankings. Third, to establish the remaining results, we return to the benchmark $\mu = \frac{1}{2}$ and, where necessary, use calibrated comparisons.

[18] Appendix A.4 (Figure 11) shows that ND$_B$ continues to dominate the discount-based regimes across all $\gamma \in (0, 1)$ in welfare, consumer surplus, and aggregate ridership for $\mu \in \{0.75, 1\}$. Comparisons with ND$_S$ and the free market at higher μ are examined in the next section.

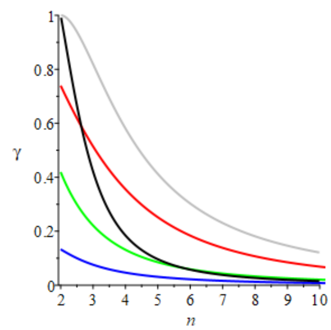
[19] While McHardy et al. (2023) and McHardy (2024) extend the FM and ND$_B$ regimes to arbitrary n, their analyses are confined to bundled-only demand. Allowing for independent standalone demand at general n appears to be novel.

Proposition 9 (Global dominance for all μ)**.** [20] *For all integers $n \geq 4$, all $\gamma \in (0,1)$, and all $\mu \in [1/2, 1]$,*

$$H^{ND_B}(\gamma,\mu,n) > H^{ND_S}(\gamma,\mu,n), \qquad \forall H \in \{W, Q_{\text{tot}}\}.$$

Proposition 9 shows that the welfare advantage of bundled coordination does not depend on low-dimensional market structure. Once cross-firm prices are benchmarked to bundled rather than standalone prices, the resulting reduction in effective mark-ups dominates for all admissible μ and all degrees of substitutability.

Figure 7: Strategic interaction in standalone prices: zero loci of $\partial^2\pi_i/\partial p_i \partial p_j$ under FM and ND$_S$ in (γ, n) space.



Note. Black/grey: ND$_B$/ND$_S$. Lines show the zero loci of $\partial^2\pi_i/\partial p_i \partial p_j$ at the symmetric equilibrium; regions above/right indicate strategic complements. Calibrations: $\eta = -0.3$, $\eta = -0.7$, and $\eta = -1.1$.

The mechanism again lies in strategic interaction. As seen in the triopoly case, the nature of strategic interaction becomes sensitive to market structure as n increases. The next lemma shows that the key distinction persists: standalone prices remain strategic complements under ND_B, whereas under FM and ND_S the sign of interaction may vary with n.[21]

Lemma 3 (Strategic interaction for general n)**.** *In symmetric n-firm equilibrium, standalone prices are strategic complements under ND_B for all $(\gamma, n) \in (0,1) \times [2, \infty)$. By contrast, under FM and ND_S the nature of strategic interaction may vary with n.*

As seen in the triopoly case, moving beyond duopoly breaks uniform substitutability, with strategic complementarity arising under the free market at lower (n, γ) than under ND_S. This pattern persists as n increases, so standalone-anchored coordination lags behind the free market.

[20] The cases $n = 2, 3$ are covered in Sections 3 and 4.
[21] As in duopoly, this result is independent of μ.

Figure 7 characterises and quantifies the strategic interaction results. The coloured contours map the equilibrium (γ, n) pairs corresponding to the calibrated elasticities. As n rises, the FM zero locus (black) - above and to the right of which standalone prices are strategic complements - falls and is crossed by the red and green contours, implying that equilibrium pricing transitions from strategic substitutes to complements at low and then medium elasticities. By contrast, the ND_S zero locus (grey) lies above all three calibrated contours over $\gamma \in (0, 1)$, so standalone prices remain strategic substitutes under these elasticities. This helps explain why ND_S can underperform the free market as market size grows. Under ND_B, standalone prices are strategic complements throughout, consistent with its stronger performance.

To establish a complete ranking between ND_B and ND_S, including consumer surplus, we return to the benchmark case $\mu = \frac{1}{2}$.

Proposition 10 (Global benchmark dominance)**.** *For all integers $n \geq 2$ and all $\gamma \in (0, 1)$,*

$$H^{ND_B}(n, \gamma) > H^{ND_S}(n, \gamma), \qquad \forall H \in \{W, CS, Q_{\text{tot}}\}.$$

The dominance of ND_S over the free market does not extend beyond duopoly over the full range $\gamma \in (0, 1)$. As seen in the triopoly case, once $n > 2$ strategic complementarity arises under the free market at lower (n, γ) than under ND_S, so standalone-anchored coordination can underperform the free market.

Because the comparison between ND_B and the free market becomes analytically intractable at general n, we establish the ordering using calibrated elasticities.

Proposition 11 (Calibrated dominance)**.** *For each calibrated elasticity $\eta \in \{-0.3, -0.7, -1.1\}$ and all integers $n \geq 2$,*

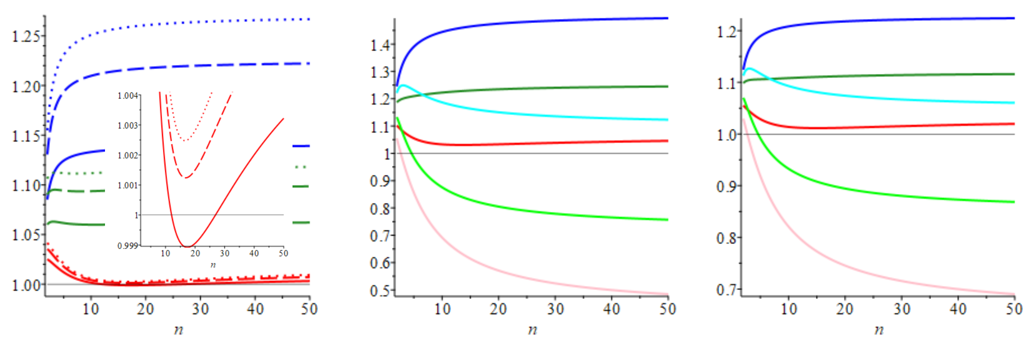
$$H^{ND_B} > H^{FM}, \qquad H \in \{\text{W}^*, \text{CS}, Q_{\text{tot}}, \widetilde{W}\}.$$

Note: * At $\eta = -0.3$, the welfare inequality fails only on the finite set $n \in \{13, \ldots, 26\}$, where W^{ND_B}/W^{FM} is arbitrarily close to one.

Thus, across empirically relevant demand conditions, bundling-based coordination (ND_B) outperforms the free market, even as market size increases.

Figure 8 illustrates these results by plotting welfare, consumer surplus, and ridership under ND_B and ND_S, each relative to the free market, for $n \in [2, 50]$ and the calibrated elasticity values $\eta \in \{-0.3, -0.7, -1.1\}$.

Figure 8: Calibrated no-discount ND_B and ND_S: welfare, augmented welfare, consumer surplus, and ridership relative to free market for $n \in [2, 50]$



(a) Welfare (b) Consumer Surplus (c) Ridership

Note: Blue/DarkGreen/Red: H^{ND_B}/H^{FM}; Cyan/LightGreen/Pink: H^{ND_S}/H^{FM}; $H \in \{W, CS, Q_{tot}\}$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

In addition to the dominance of ND_B over ND_S established in Propositions 9 and 10, two further patterns emerge. First, ND_B dominates the free market across all three elasticities in CS and Q_{tot}. On welfare, it dominates at medium and high elasticities and for small n at low elasticity, and is otherwise nearly indistinguishable, while augmented welfare remains above the free market over the range of n shown, consistent with higher Q_{tot}. Quantitatively, at medium to high elasticities this corresponds to welfare gains of around 6-13%, augmented welfare gains of 8-21%, consumer-surplus gains of 20-45%, and ridership increases of 10-20% relative to the free market as n increases. At low elasticities, gains are more modest and welfare is extremely close to the free-market benchmark and exhibits a small finite-range dip below it. Notably, even in the least-elastic calibration where raw welfare exhibits a narrow finite-range dip, augmented welfare (low valued) remains strictly above the free-market benchmark for all n, reflecting the systematic expansion in total ridership under ND_B. Second, gains under ND_S are fragile and vanish quickly as n increases.

Appendix A.4 reports corresponding profiles for $\mu \in \{0.75, 1\}$. The qualitative patterns identified above remain largely intact as the weight on standalone demand rises. In particular, the relative performance of ND_B and the free market is unchanged at medium and high elasticities, while any departures at low elasticity are confined to small n. By contrast, ND_S quickly falls below the free market on welfare, consumer surplus, and ridership as n increases. These robustness patterns underpin the following observation for empirically relevant small-n environments.

Remark 4 (Small-n dominance)**.** *For each $\mu \in \{0.5, 0.75, 1\}$ and each calibrated elasticity, ND_B strictly dominates the free market in total ridership for all $n \leq 5$. It also strictly dominates the free market in welfare over this range, with the exception of a knife-edge case under the least-elastic calibration ($\eta = -0.3$), where $W^{ND_B}/W^{FM} \approx 1$ at $n = 4$ for $\mu = 1$.*

In this knife-edge case, the difference in welfare is negligible, while total ridership remains strictly higher under ND_B. Such small-n environments are common in complementary-component industries, platform ecosystems, infrastructure services, and transport corridors. Taken together, these results generalise the duopoly and triopoly findings while clarifying their scope. Bundled coordination (ND_B) is the only regime that dominates standalone-anchored coordination analytically for all n, and it continues to perform favourably relative to the free market across empirically relevant market sizes and elasticities, with any departures confined to low-elasticity cases.

6 Network-Size Choice

Finally, we test whether the welfare dominance of ND_B persists when firms can expand their networks. Network size is endogenised by allowing each firm to choose the number of complementary component pairs it operates, n_i. We focus on the free market and the welfare-leading coordinated regime, ND_B.

Following McHardy (2024), firms choose $n_i \in \{1, 2, 3\}$ and then set prices as in the base model. Operating costs (specified in line with Van den Berg et al., 2022; McHardy, 2024) combine a fixed cost per component pair with economies of density. Parameters are chosen (i) to restrict equilibrium choices to $n_i \in \{1, 2, 3\}$ and (ii) to generate knife-edge cases in which FM can sustain a larger expected network than ND_B, providing a stringent robustness test.[22] Under these calibrations, FM does support a larger expected network, but this expansion lowers welfare relative to ND_B (Table 2, Appendix A.6). At $\eta = -0.7$, FM mixes over $\{1, 2\}$ while ND_B yields a unique symmetric equilibrium at $(1, 1)$. At $\eta = -1.1$, FM again sustains a larger expected network, mixing over $\{2, 3\}$, whereas ND_B admits equilibria with smaller total network size but higher welfare.

We then raise fixed costs slightly to eliminate FM's ability to sustain a wider network (Table 3, Appendix A.6). Once both regimes converge to the same symmetric network size, the welfare gap in favour of ND_B re-emerges clearly, reflecting the pricing mechanism identified in the core model.

Allowing firms to choose network size therefore does not overturn the welfare dominance of ND_B. Even when scope expansion favours the free market, ND_B can deliver higher welfare by restraining inefficient expansion.

7 Conclusion

This paper is motivated by regulatory guidance permitting coordinated pricing for composite or multi-firm products while benchmarking to firms' own-bundled prices. We show that discount-based interpretations generate equilibrium surcharges and fail to internalise complementarities across firms. By contrast, a no-discount rule, ND_B, anchors cross-firm prices to own-bundled prices and internalises these complementarities without propagating mark-ups. Unlike parameterised price caps, ND_B is self-anchoring and implementable using

[22] Full details of the timing, demand scaling, and cost specification are given in Appendix A.6.

observable prices.

Among the coordination rules considered, ND_B emerges as the most robust benchmark. It strictly dominates discount-based rules and, in the general analysis, also dominates the standalone-based no-discount regime, ND_S, by avoiding distortions toward standalone demand.

For small, empirically relevant network sizes, ND_B uniformly dominates the free market across all criteria and parameter values considered. More generally, calibrated analysis shows that ND_B dominates the free market in consumer surplus and total ridership for all $n \geq 2$. Welfare dominance holds across calibrations except for a narrow finite range under the least-elastic case, where outcomes are nearly indistinguishable, while augmented welfare (accounting for externality benefits from increased ridership) remains strictly higher throughout.

While firm profits may be lower under ND_B than the free market, higher welfare allows compensation from consumer surplus, leaving passengers better off. Overall, the results suggest that bundle-based coordination provides a simple and robust benchmark for aligning pricing incentives with welfare in network industries.

A calibration to England's pre-COVID bus market suggests that these effects are economically meaningful. Under empirically relevant elasticities, ND_B delivers gains in consumer-surplus gains (≈20%) and ridership (≈10%), generating substantial external benefits. More broadly, these findings are not transport-specific: they apply to any network industry in which firms supply complementary but partially substitutable components. Bundle-based coordination provides a simple, implementable, and robust benchmark for aligning pricing incentives with welfare. Extending the analysis to richer network structures and to empirical applications beyond transport is a natural direction for future work.

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Appendix

A.1 n-Firm Equilibrium Prices ($\mu = \frac{1}{2}$)

The equilibrium bundled, standalone, and cross-network prices for the free market and no-discount regimes are given by:

$$P^{FM} = \nabla_1(4\gamma n^2 - 5\gamma n - \gamma + 3n - 1),\ p^{FM} = \frac{1}{2}\nabla_1(6\gamma n^2 - 9\gamma n + \gamma + 4n - 2),\ P^{FM}_{cro} = 2p^{FM},$$

$$\text{where } \nabla_1 \equiv \frac{\alpha(1-\gamma)}{(2n^4 - 5n^3 - 4n^2 + 7n + 4)\gamma^2 + (2n^3 + 5n^2 - 14n - 1)\gamma + 6n - 2}$$

$$P^{ND_B} = \frac{\alpha(2n-1)(1-\gamma)}{(n^3 - n^2 - 4n + 2)\gamma + 4n - 2},\ p^{ND_B} = \frac{\alpha(1-\gamma)}{4 + (2n-6)\gamma},\ P^{ND_B}_{cro} = P^{ND_B},$$

$$P^{ND_S} = \nabla_2[(n^5 + 3n^4 - 5n^3 - 13n^2 + 20n - 6)\gamma^2 + (5n^3 + 7n^2 - 24n + 10)\gamma + 6n - 4], \tag{7}$$

$$p^{ND_S} = \nabla_2[(n-1)(n^4 + \tfrac{5}{2}n^3 - 3n^2 - \tfrac{11}{2}n + 3)\gamma^2 + (4n^3 + \tfrac{1}{2}n^2 - \tfrac{21}{2}n + 5)\gamma - 2 + 3n], P^{ND_S}_{cro} = 2p^{ND_S},$$

$$\text{where } \nabla_2 \equiv \frac{\alpha(1-\gamma)}{2\,(2 + (n-3)\gamma)\,(1 + \gamma(n^2-1))[(n^3 + \frac{3}{2}n^2 - \frac{11}{2}n + 2)\gamma - 2 + 3n]}.$$

A.2 Proofs

Verification convention. Where closed forms are available, signs are verified symbolically on the admissible domain. Otherwise (discount regimes), inequalities are verified numerically along the anchored branch, with continuity ruling out sign reversals. Replication code available on request.

Proof of Proposition 1. Fix $r \in \{S, B\}$. The Stage-1 condition is $K_r(\delta;\gamma) = 0$, where K_r is a polynomial in δ with coefficients continuous in $\gamma \in [0,1)$. At $\gamma = 0$, $\delta = 1$ is the unique root of $K_r(\cdot;0)$ in $[1,\infty)$ and it is simple, i.e., $\partial_\delta K_r(1;0) \neq 0$. Hence, by the Implicit Function Theorem (IFT), there exists a unique local C^1 anchored branch $\delta^*_r(\gamma)$ with $\delta^*_r(0) = 1$. For all $\gamma \in [0,1)$, $K_r(1;\gamma) = -\gamma L_r(\gamma)$ with $L_r(\gamma) > 0$, so $K_r(1;\gamma) < 0$ for every $\gamma > 0$. The leading coefficient of K_r is positive, so $K_r(\delta;\gamma) \to +\infty$ as $\delta \to \infty$, and the Intermediate Value Theorem implies for each $\gamma \in (0,1)$ there exists a root $\hat{\delta}(\gamma) > 1$. By anchored root continuity, $\delta^*_r(\gamma)$ cannot satisfy $\delta^*_r(\gamma) \leq 1$ at any $\gamma > 0$ without crossing $\delta = 1$ at some positive parameter value; but $K_r(1;\gamma) = 0$ only at $\gamma = 0$. Therefore $\delta^*_r(\gamma) \geq 1$ for all $\gamma \in [0,1)$, with $\delta^*_r(\gamma) > 1$ for $\gamma > 0$. □

Proof of Lemma 1. By Proposition 1, $K_r(1;0) = 0$ and $\partial_\delta K_r(1;0) \neq 0$, so the IFT yields a unique local C^1 equilibrium branch $\delta_r^*(\gamma)$ with $\delta_r^*(0) = 1$. Let $\Delta_r(\gamma)$ denote the discriminant of $K_r(\cdot;\gamma)$ with respect to δ. Since the root at $\gamma = 0$ is simple, $\Delta_r(0) \neq 0$. Because Δ_r is a polynomial in γ, it can vanish only at finitely many points in $[0, 1)$; let S_r denote this set. For $\gamma \notin S_r$, the anchored root is simple and the IFT implies that $\delta_r^*(\gamma)$ is C^1. At points $\gamma \in S_r$, polynomial roots depend continuously on coefficients, so the anchored root varies continuously with γ. Hence $\delta_r^*(\gamma)$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus S_r$. □

Proof of Proposition 2. Fix $r \in \{S, B\}$ and let $G_r(\delta, \gamma) \equiv \partial_\delta \Pi_r(\delta, \gamma)$. By Lemma 1, the anchored solution $\delta_r^*(\gamma)$ is continuous on $[0, 1)$ and C^1 on $[0, 1) \setminus S_r$, with S_r finite. On the admissible set, Π_r is strictly concave in δ, so $\partial^2_{\delta\delta}\Pi_r < 0$ and the interior maximiser is locally unique. For $\gamma \in (0, 1) \setminus S_r$, differentiating $G_r(\delta_r^*(\gamma), \gamma) = 0$ yields $\delta_r^{*\prime}(\gamma) = -\frac{\partial_{\delta\gamma}\Pi_r(\delta_r^*(\gamma),\gamma)}{\partial^2_{\delta\delta}\Pi_r(\delta_r^*(\gamma),\gamma)}$. Along the anchored branch, $\partial_{\delta\gamma}\Pi_r(\delta_r^*(\gamma), \gamma) \geq 0$, while $\partial^2_{\delta\delta}\Pi_r < 0$, hence $\delta_r^{*\prime}(\gamma) \geq 0$ wherever it exists. Continuity extends the non-decreasing property across S_r. □

Proof of Lemma 2. At a symmetric equilibrium,

$$\frac{\mathrm{d}p_i}{\mathrm{d}p_j} = -\frac{\Delta_{ij}}{\partial^2 \pi_i / \partial p_i^2}, \qquad \Delta_{ij} \equiv \frac{\partial^2 \pi_i}{\partial p_i\, \partial p_j}. \tag{8}$$

Since $\partial^2 \pi_i / \partial p_i^2 < 0$ under all regimes, the sign of the best-response slope equals the sign of Δ_{ij}. Under FM and ND_S, $\Delta_{ij} < 0$ for $\gamma \in (0, 1)$, so $\mathrm{d}p_i/\mathrm{d}p_j < 0$: strategic substitutes. Under ND_B, $\Delta_{ij} > 0$ on $(0, 1)$, so $\mathrm{d}p_i/\mathrm{d}p_j > 0$: strategic complements. □

Proof of Proposition 3. For $n = 2$, equilibrium outcomes under FM, ND_S, and ND_B admit closed-form expressions. For each $H \in \{W, Q_{\mathrm{tot}}\}$, the differences $H^{ND_B} - H^{ND_S}$ and $H^{ND_S} - H^{FM}$ are rational functions of (γ, μ) on $(0, 1) \times [1/2, 1]$ with no singularities on the admissible set. Their signs are verified as described in the verification convention, implying $H^{ND_B} > H^{ND_S} > H^{FM}$ whenever the equilibrium is interior. □

Proof of Proposition 4. Fix $H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}$ and $R \in \{ND_S, D_B, D_S\}$. By Lemma 1, equilibrium outcomes are continuous in γ on $(0, 1)$, so $\Delta_{H,R}(\gamma) \equiv H^{ND_B}(\gamma) - H^R(\gamma)$ is continuous. For $R \in \{FM, ND_S\}$, $\Delta_{H,R}$ is rational in γ and its sign follows from symbolic verification. For

$R \in \{D_B, D_S\}$, outcomes are evaluated along the anchored equilibrium branch and the sign follows from numerical verification and continuity. Thus $H^{ND_B}(\gamma) > H^R(\gamma) > H^{FM}(\gamma)$, $\forall\gamma \in (0,1)$. □

Proof of Proposition 5. By Lemma 1, equilibrium prices are continuous in γ on $(0,1)$. In FM, ND$_S$, and ND$_B$, standalone and cross-network prices admit closed-form expressions, so the differences $p^{ND_S} - p^{ND_B}$, $p^{FM} - p^{ND_S}$ and their cross-network analogues are rational functions of γ with no singularities on $(0,1)$. Symbolic verification yields $p^{ND_B} < p^{ND_S} < p^{FM}$ and $P_{cro}^{ND_B} < P_{cro}^{ND_S} < P_{cro}^{FM}$. The absence of a uniform price minimiser follows from numerical evaluation over the admissible parameter range. □

Proof of Proposition 6. Fix $R \in \{ND_B, ND_S, D_B, D_S\}$ and define $\Delta_R(\gamma) \equiv \pi^R(\gamma) - \pi^{FM}(\gamma)$. By Lemma 1, Δ_R is continuous on $(0,1)$. In the no-discount regimes, Δ_R is rational in γ and symbolic verification shows it has a unique zero $\tilde{\gamma}_R \in (0,1)$, with Δ_R strictly positive for $\gamma < \tilde{\gamma}_R$ and strictly negative for $\gamma > \tilde{\gamma}_R$. In the discount regimes, profits are evaluated along the anchored equilibrium branch. Numerical root search identifies a unique zero on $(0,1)$ and continuity rules out additional crossings, establishing the stated threshold property. □

Proof of Proposition 7. Fix $n = 3$ and $H \in \{W, Q_{\text{tot}}\}$. Ratios H^{ND_B}/H^{FM} and H^{ND_B}/H^{ND_S} are rational functions of (γ, μ) without singularities on $(0,1)\times[1/2,1]$. Symbolic verification shows both ratios exceed one throughout the admissible set, implying $H^{ND_B} > H^{FM}$ and $H^{ND_B} > H^{ND_S}$. □

Proof of Proposition 8. Fix $H \in \{W, \text{CS}, Q_{\text{tot}}\}$ and $R \in \{FM, ND_S, D_S, D_B\}$. Let $\Delta_R^H(\gamma) \equiv H^{ND_B}(\gamma) - H^R(\gamma)$. By Lemma 1, Δ_R^H is continuous on $(0,1)$. For $R \in \{FM, ND_S\}$, Δ_R^H is rational in γ and its sign follows from symbolic verification. For $R \in \{D_S, D_B\}$, outcomes are evaluated along the anchored equilibrium branch and the sign follows from numerical verification and continuity. Thus $\Delta_R^H(\gamma) > 0$ for all $\gamma \in (0,1)$. □

Proof of Proposition 9. Fix an integer $n \geq 4$ and $H \in \{W, Q_{\text{tot}}\}$. For each fixed n, the ratio $R_H(\gamma, \mu, n) \equiv H^{ND_B}(\gamma, \mu, n)/H^{ND_S}(\gamma, \mu, n)$ is rational in (γ, μ) with no singularities on $(0,1) \times [1/2, 1]$. Symbolic verification shows $R_H(\gamma, \mu, n) > 1$ throughout the admissible set, implying $H^{ND_B}(\gamma, \mu, n) > H^{ND_S}(\gamma, \mu, n)$. □

Proof of Lemma 3. From (8), strategic interaction is governed by the sign of Δ_{ij}. Under ND_B, Δ_{ij} admits a closed-form expression satisfying $\Delta_{ij} > 0$ for all $\gamma \in (0,1)$ and all $n \geq 2$, implying strategic complements. Under FM and ND_S, Δ_{ij} is rational in (γ, n) and takes both positive and negative values on the admissible set, so the sign can vary with n. □

Proof of Proposition 10. Fix $H \in \{W, \mathrm{CS}, Q_{\mathrm{tot}}\}$. For each fixed integer $n \geq 2$, the difference $\Delta_H(n,\gamma) \equiv H^{ND_B}(n,\gamma) - H^{ND_S}(n,\gamma)$ is rational in γ with no singularities on $(0,1)$. Symbolic verification shows $\Delta_H(n,\gamma) > 0$ for all $\gamma \in (0,1)$ and all $n \geq 2$, completing the proof. □

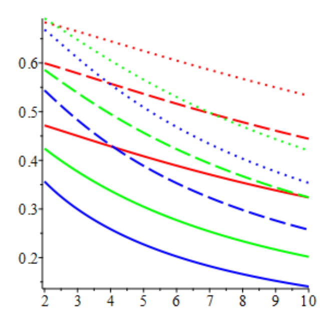
Proof of Proposition 11. Fix a calibrated elasticity η and let $\gamma_\eta(n)$ denote the associated parameter path. For each outcome H, define $\Delta_H(n) \equiv H^{ND_B}(\gamma_\eta(n), n) - H^{FM}(\gamma_\eta(n), n)$. Along the calibrated path, write $n = 1/m$ and express the ratio $R_H(n) = H^{ND_B}/H^{FM}$ as a Taylor expansion in m: $R_H(n) = c_0(s) + a_1(s)m + a_2(s)m^2 + a_3(s)m^3 + O(m^4)$, where $s = n^2\gamma_\eta(n)$ lies in a compact interval.

For each criterion, $c_0(s)$ is uniformly bounded away from one on this interval (except for the welfare case at $\eta = -0.3$, where it remains arbitrarily close to one). The correction terms admit uniform bounds, so there exists N_η such that $R_H(n) > 1$ for all $n \geq N_\eta$. For the remaining finite set $2 \leq n < N_\eta$, direct evaluation confirms the stated inequalities. □

A.3 Market-wide fare elasticity

Let P_{av} the quantity-weighted average fare at the symmetric free-market equilibrium, taken as the empirical pricing benchmark. The industry elasticity is computed under a small uniform fare scaling, $(P, P_{cro}, p) \mapsto (1+\varepsilon)(P, P_{cro}, p)$: $\eta = \frac{\mathrm{d}\ln Q_{\mathrm{tot}}}{\mathrm{d}\ln P_{\mathrm{av}}} = \frac{1}{Q_{\mathrm{tot}}}\left(\frac{\partial Q_{\mathrm{tot}}}{\partial P}P + \frac{\partial Q_{\mathrm{tot}}}{\partial P_{cro}}P_{cro} + \frac{\partial Q_{\mathrm{tot}}}{\partial p}p\right)$. Under symmetry, this yields a closed-form expression as a function of (γ, n) which can be inverted numerically to map empirical elasticities η into the corresponding substitutability parameter γ, yielding the cal-

Figure 9: Free-market share of single-leg components, $s = 2 - S_{\mathrm{PT}}$.



Note: Solid/dashed/dotted: $\mu = 0.5/0.75/1$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

ibration used in the main text:

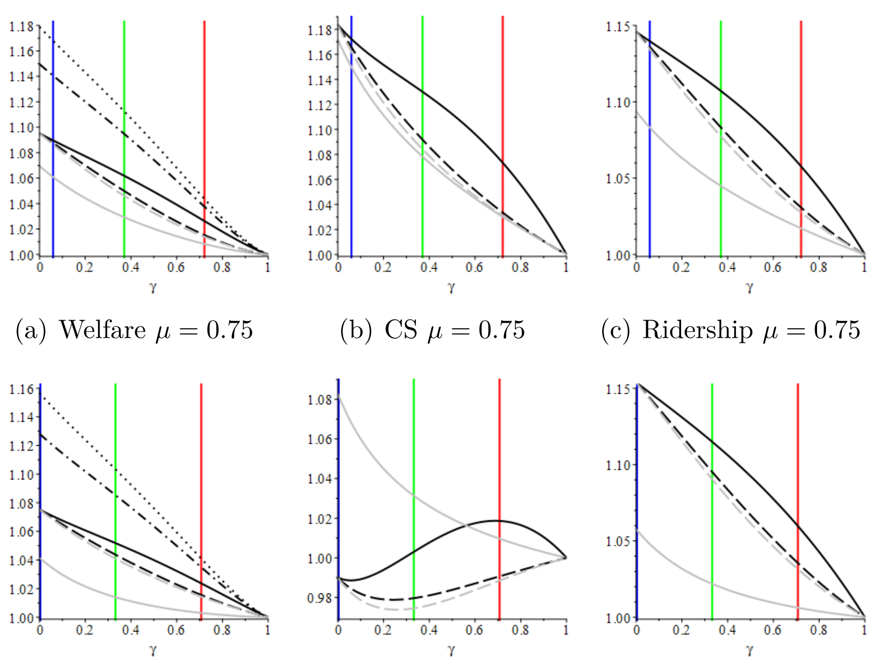
$$\eta(\gamma, n) = \frac{(-7n^3+2n^2+15)\gamma^3+(-13n^3+3n^2+21n+35)\gamma^2+(-6n^3+n^2+25n+25)\gamma+6n+5}{(3n-9)\gamma^3+(-n-30)\gamma^2+(-8n-24)\gamma-4n-5}. \tag{9}$$

A.4 Demand Composition and Robustness in μ

FM observed transfer intensity $S_{\mathrm{PT}} \approx 1.5$-$1.7$ (American Public Transportation Association, 2007) implies a single-leg share $s = 2 - S_{\mathrm{PT}} \approx 0.3$-$0.5$, and a corresponding μ. Figure 9 shows that the benchmark $\mu = 0.5$ (solid lines) aligns with this range across calibrated elasticities at small n, supporting its use in the main analysis.

Figures 10 and 12 report welfare, consumer surplus, and aggregate ridership relative to the free market for $\mu \in 0.75, 1$, respectively for $n = 2$ and $n \in [2, 50]$. Figure 11 reports outcomes for $n = 3$ relative to ND_B.

Figure 10: Robustness: welfare, consumer surplus, and ridership relative to the free market for $\mu \in \{0.75, 1\}$ under Duopoly.

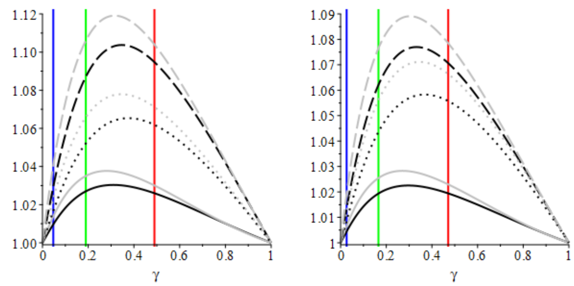


(a) Welfare $\mu = 0.75$ (b) CS $\mu = 0.75$ (c) Ridership $\mu = 0.75$

(d) Welfare $\mu = 1$ (e) CS $\mu = 1$ (f) Ridership $\mu = 1$

Note: Black/grey: bundled/standalone benchmark; solid/dashed: no-discount/discount. Panel (a): welfare (dash-dot/dot = low/high augmentation). Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

Figure 11: Robustness: welfare, consumer surplus, and ridership of D_B and D_S relative to ND_B for $\mu \in \{0.75, 1\}$ under Triopoly.



(a) $\mu = 0.75$ (b) $\mu = 1$

Note: Black/grey: bundled/standalone benchmark; solid/dashed/dot: $W/CS/Q_{tot}$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

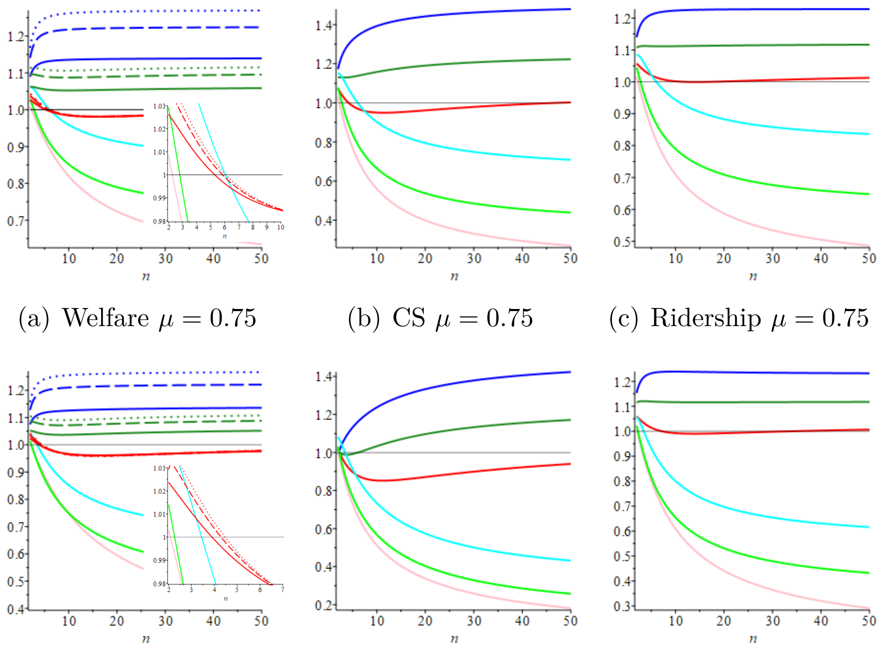
A.5 External-cost calibration

Converting induced ridership changes into avoided external costs via marginal external costs (MEC) from UK TAG (Department for Transport, 2023a), let d_c, d_t denote diverted *person* trips from car/taxi per induced bus boarding, with occupancies $\omega_c = \omega_t$.[23] with average trip lengths L_c, L_t. Under TAG diversion rates (A5.4.6), occupancies (A1.3.3), and NTS0303d trip lengths (Department for Transport, 2020), avoided vehicle-km per induced bus boarding are $\kappa = \frac{d_c}{\omega_c} L_c + \frac{d_t}{\omega_t} L_t = \frac{0.24}{1.57} 13.52 + \frac{0.12}{1.57} 8.05 \approx 2.68$ vkm. TAG reports marginal external costs (pence per vkm) by geography (A5.4.2), comprising congestion, safety, air quality, greenhouse gases, noise, and infrastructure.[24] For England (ex London), we use *Other Urban* and *Inner/Outer Conurbations* values, yielding $MEC \in [33.5, 51.1]$ p/vkm and avoided external cost per induced bus boarding $b_{\text{ext}} = \kappa \cdot \frac{\text{MEC}}{100} \in [£0.90, £1.37]$. Total 2019 England (ex London) bus passenger journeys were $J_0 = 2.113$ billion, with revenue £3.506 billion (BUS01;BUS04 Department for Transport, 2025), implying an average fare per boarding of $f_{\text{board}} \approx £1.66$.

Since observed boardings and model quantities scale proportionally, $\%\Delta Q^R_{\text{tot}}$ also equals the percentage change in boardings. The associated externality benefit is therefore $b_{\text{ext}} \times \%\Delta Q^R_{\text{tot}} \times J_0$. Avoided vkm and CO_2 emissions are reported as $\Delta \text{VKM}^R = \%\Delta Q^R_{\text{tot}} \times J_0 \times \kappa$, $CO_{2,R} = \frac{(e_c - e_b)\, \Delta \text{VKM}^R}{10^6}$, using a 100 g/km per-passenger car-bus emissions differential.[25] In the main text we

[23] Taxi occupancies are typically higher; using car occupancy is conservative.
[24] TAG marginal external costs exclude fiscal transfers.
[25] Official UK conversion factors imply average car emissions of about 115 gCO_2e per passenger-km after

Figure 12: Robustness: welfare, consumer surplus, and ridership relative to the free market for $\mu \in \{0.75, 1\}$ and $n \in [2, 50]$.



(a) Welfare $\mu = 0.75$ (b) CS $\mu = 0.75$ (c) Ridership $\mu = 0.75$

(d) Welfare $\mu = 1$ (e) CS $\mu = 1$ (f) Ridership $\mu = 1$

Note: Blue/DarkGreen/Red: H^{ND_B}/H^{FM}; Cyan/LightGreen/Pink: H^{ND_S}/H^{FM}; $H \in \{W, CS, Q_{tot}\}$. Calibrations: $\eta = -0.3$, $\eta = -0.7$, $\eta = -1.1$.

report *augmented welfare*, applying the proportional uplift Eq. (5), where $\beta = \frac{b_{\text{ext}}}{f_{\text{board}}} \in [0.54, 0.83]$. This standardises the external benefit per induced boarding in fare-equivalent terms while preserving welfare rankings.[26]

A.6 Network-size choice: model and results

We extend the baseline model to allow firms to choose network size. Each firm i selects the number of complementary component pairs it operates, denoted n_i, so that total network size is $n = n_1 + n_2$. Each additional (x, y) pair adds a symmetrically differentiated option to firm i's portfolio. Preferences and demand follow Eq. (1), with n determined endogenously.

Under symmetry, all within-firm bundles share price P_{ii} and all standalone components share

adjusting for occupancy (Department for Business, Energy & Industrial Strategy, 2019). Marginal bus emissions are smaller when additional passengers are absorbed via higher load factors.

[26] We value avoided car and taxi external costs only; marginal bus external costs are not netted out, consistent with short-run demand absorption through higher load factors.

Table 1: External benefits of ND_B vs. FM by n and elasticity

		Low		High	
		$\eta = -0.7$	$\eta = -1.1$	$\eta = -0.7$	$\eta = -1.1$
$n = 2$	Welfare add-on (£m/yr) [%]*	186.1 [75.0]	237.3 [95.7]	283.8 [114.4]	362.0 [146.0]
	ΔVKM (m)	555.4	708.4	555.4	708.4
	CO_2 (t) [%]†	55,540 [1.9]	70,840 [2.4]	55,540 [1.9]	70,840 [2.4]
$n = 3$	Welfare add-on (£m/yr) [%]*	197.4 [79.6]	307.6 [124.0]	301.2 [121.4]	469.1 [189.2]
	ΔVKM (m)	589.4	918.1	589.4	918.1
	CO_2 (t) [%]†	58,940 [2.0]	91,810 [3.1]	58,940 [2.0]	91,810 [3.1]

Notes: Low/High correspond to limits MEC $\in [33.5, 51.1]$ p/vkm, implying $\beta \in [0.54, 0.83]$. England (ex-London) operating support £248m is from 2018/19 BUS statistics (Department for Transport, 2025). *Percentages in [.] on the welfare rows express the add-on as a share of operating support. †Percentages in [.] on the CO_2 rows express the saving as a share of UK buses tailpipe CO_2 in 2019 of 3.0 $MtCO_2e$ (ENV0201, Department for Transport, 2023b).

price p_i. Quantities scale with network size through the number of available bundles and components. Holding prices fixed, a larger network raises utility through a standard variety effect. Firms choose network size prior to price competition. At Stage 0, each firm incurs a fixed overhead F_0, ensuring interior participation. At Stage 1, firms simultaneously choose $n_i \in \{1, 2, 3\}$. At Stage 2, prices are set as in the base model under the relevant regime.

Table 2: Knife-edge equilibria: types, expected profits, welfare, and network size

(η, F)	Regime	Equilibrium type	$E(\pi_i)$	$E(W)$	$E(n)$
$(-0.7, 0.10)$	FM	mixed (support $\{1, 2\}$): $\Pr(1) = 0.767$; $\Pr(2) = 0.233$	0.1213	0.7086	2.5
	ND_B	PSNE $(1, 1)$	0.1309	0.7278	2.0
$(-1.1, 0.25)$	FM	mixed (support $\{2, 3\}$): $\Pr(2) = 0.680$; $\Pr(3) = 0.320$	0.1483	1.4914	4.6
	ND_B	PSNE $(2, 2)$	0.1744	1.5165	4.0
		PSNE $(1, 3)$ and $(3, 1)$	0.2255	1.4748	4.0

Operating costs, $C_i = (k + mZ_i)Z_i + n_i F$, combine fixed and density-dependent components, where $Z_i \equiv n_i^2 Q_{ii} + n_i n_j Q_{ij} + \frac{n_i}{2}(X_i + Y_i)$ and Z_i denotes total bundled-equivalent passenger volume internalised by firm i. The parameter $k > 0$ captures baseline marginal cost, $m < 0$ captures economies of density, and $F > 0$ is a per-pair fixed cost.

Restricting $n_i \in \{1, 2, 3\}$ keeps network-size tractable. Following McHardy (2024), we set $(k, m) = (0.1, -0.02)$ and choose $F \in \{0.10, 0.25\}$ (i) ensures best responses lie on this grid and (ii) generate knife-edge cases where FM can sustain larger networks than ND_B. Substitutability is set in line with medium- and long-run demand conditions, corresponding to $\eta \in \{-0.7, -1.1\}$. The resulting equilibria, reported in Tables 2 and 3, underpin the comparisons in Section 6.

Table 3: Nudging F upward ($\eta = -1.1,\ F = 0.28$)

Regime	Equilibrium type	$E(\pi_i)$	$E(W)$	$E(n)$
FM	PSNE $(2,2)$	0.1350	1.2624	4.0
ND_B	PSNE $(2,2)$	0.1144	1.3965	4.0