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Abstract

This paper analyzes the monetary policy trade-off between defending purchasing power of consumers and keeping moderate debt cost for borrowers, in the framework of a heterogeneous agent New Keynesian open economy hit by a foreign energy price shock. Raising the interest rate indeed combats the loss in purchasing power due to the energy shock through a real exchange rate appreciation: however, this comes at the expense of higher interest payments for debtors. The trade-off can be resolved by adopting a milder interest rate policy during the crisis in exchange for a prolonged contraction beyond the energy shock time span. This interest rate smoothing approach allows to still experience a real appreciation today, while spreading the impact on debt costs more evenly over time. This policy counterfactual is analyzed in a quantitative model of the UK economy under the 2022-2023 energy price hike, where the loss of consumers' purchasing power and the vulnerability of mortgage costs to higher policy rates have been elements of paramount empirical relevance.

JEL Codes: D14, D31, E52, G21, G51, Q43

Keywords: monetary policy, energy, heterogeneity, inequality, household debt

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1 Introduction

The years 2022 and 2023 witnessed a substantial rise in energy prices in advanced economies, exacerbating the inflationary pressures that had been steadily building since 2021; in response to the inflation surge, central banks reacted by raising the policy rates, aiming to curb inflationary pressures and to safeguard the real income of consumers. Generally, these interventions led to increased interest rates on variable-rate mortgages and fixed-rate mortgages due for renewal during the period of rate hikes. The case of the UK economy is particularly illustrative of this phenomenon: housing mortgages' cost are typically renegotiated every 5 years or less, making their interest rate particularly sensible to the movements in the policy rate set by the Bank of England (BoE).

The Central Banks' trade-off between shielding real income of consumers and maintaining moderate mortgage interest rates poses challenges for the formulation of a monetary policy reaction to an energy price shock. A contractionary interest rate policy effectively safeguards households' wages purchasing power by fostering a real exchange rate appreciation (by uncovered interest rate parity); on the other side, it increases the cost of mortgages.

The main theoretical result of the paper is that the trade-off between the protection of households' real income and preventing high interest rates for borrowers can be resolved once we account for monetary policy manipulating *the whole path of future interest rates.* If the central bank indeed commits to monetary tightening in the future, this implies a current real appreciation of domestic goods - through uncovered interest rate parity holding across the whole yield curve - that protects real wages' purchasing power ; therefore there is room to adopt a milder monetary policy at the onset of the shock, in order not to increase too much the financial burden on borrowers. The result of the paper echoes Silvana Tenreyro's argument in her final speech as Monetary Policy Committee member at the Bank of England, which stated that the monetary authority should commit in advance to a determined path of future interest rates, in order to partially offset the need to raise current rates in reaction to the surge in energy prices.

This paper analyses this trade-off in a small open economy new keynesian setting where agents are heterogeneous because of uninsurable idiosyncratic income risk. Agents trade in a liquid assets and are endowed with perpetual liabilities (mortgages) whose interest rate is in part fixed and in part variable, i.e. directly connected with the monetary policy rate. The presence of mortgages creates a quantitatively relevant adverse effect of contractionary monetary policy on the budget constraints of households. Agents' heterogeneity is a key assumption to make both the components of the trade-off (increases in temporary mortgage costs and falls in the real wage) quantitatively relevant from a welfare perspective:

households indeed are unable to fully absorb income and mortgage cost shocks due to a precautionary saving motive, which especially holds true for the ones closer to the borrowing limit. Moreover, a full heterogeneous agents environment allows to have both a real wage fall and mortgage cost increases to be quantitatively relevant in affecting consumption over the whole crossection of agents (differently from a two-agents models, where these effects would only be numerically important for the borrowing constrained agents).

Once obtained the theoretical results in terms of benefit of interest rate smoothing, I proceed to a quantitative assessment of the implications of the model in the UK economy. The model is fed with the actual current and expected interest rate hike implemented by the BoE, as well as by the actual energy price data. The model is constructed and calibrated to match data both in an "aggregate" dimension (CPI inflation, real exchange rate, real wage, aggregate mortgage cost) and to align with the incidence of mortgages on the cross-sectional households' consumption patterns. The reference panel data for this analysis, "Understanding Society", reports nearly exclusively food expenditure among various expenditure items: therefore, I focus on comparing the model's outcomes to the data in terms of the effects of mortgage cost increases on food consumption.

The quantitative results of the paper point out that a *smoothed* interest rate policy - characterized by the interest rate peaking at 1 percentage point less than in BoE implemented policy, and requiring an additional three years to land on the new long-term level - is able to attain the same real exchange appreciation over the energy crisis, while reducing the food consumption difference between mortgagors and non-mortgagors by 4% over 2022, thanks to the reduced interest rate surge.

Contribution to the literature The model builds on the framework by Auclert, Rognlie, Souchier, and Straub (2023b), which study fiscal and monetary response to energy shocks in a HANK-type small open economy. Bartocci, Cantelmo, Cova, Notarpietro, and Pisani (2024) studies a desirable fiscal and monetary mix in a two-agent and two-country model. Other recent literature studying the behavior of heterogeneous agents open economy in face of foreign shocks are Auclert, Rognlie, Souchier, and Straub (2023a) and Fukui, Nakamura, and Steinsson (2023) - for the case of depreciation shocks, and de Ferra, Mitman, and Romei (2020) - for sudden stops in capital inflows. This paper complements this strand of literature by analysing the trade-off - faced by a monetary policy reacting to the energy price shock - between fighting real wages deterioration and keeping moderate welfare costs for borrowers. Pieroni (2023) studies the inflation - output gap trade-off faced by monetary policy during an energy supply shock in a closed economy HANK environment. Also in his framework the government's choice is characterized by a tension between raising interest rates to fight inflation, and the aim of not penalizing too much borrowers though the cost of debt channel. However, it restricts monetary policy to a Taylor-rule without room for monetary smoothing. The 2022-2023 energy crisis gives rise to other sources of welfare loss, which have been analyzed by recent literature: Olivi, Sterk, and Xhani (2023) study optimal monetary policy when consumption baskets vary across households: their model does not display neither an open economy dimension (so an appreciation channel of monetary policy) nor a debt cost channel of interest rate policy, which are the key factors of the trade-off examined in my work. Gnocato (2025) analyses optimal monetary policy when energy shocks have asymmetric effects on employed and unemployed households.

My paper, while assessing the trade-off between purchasing power defense and mortgage cost moderation, explicitly takes into account distributional effects of interest rate hikes, effects which are investigated empirically and theoretically in Del Negro, Dogra, Gundam, Lee, and Pacula (2024). Factoring inequality outcomes in the assessment of monetary policy performance is a robust implication of optimal policy analysis in heterogeneous agents' models such as in Bhandari, Evans, Golosov, and Sargent (2021), Wolf (2023), Ragot (2017), Acharya, Challe, and Dogra (2021), Dávila and Schaab (2023) and Smirnov (2023). My paper naturally relates to this branch of literature by accounting for the asymmetric effect on monetary policy across the households' crossection in formulating an alternative monetary policy with respect to the benchmark one followed by the BoE over the energy crisis. In accordance with the findings from optimal policy literature, the proposed alternative suggests a "milder" contraction during the most severe stages of the economic cycle, to avoid excessively burdening borrowers. Chan, Diz, and Kanngiesser (2023) reach a similar conclusion, showing that their two-agent models—featuring a hand-to-mouth household—experience adverse effects from a contractionary interest rate hike in the context of an energy shock, even when the household is not directly engaged in borrowing.

The modelization of the heterogeneous agents' setting follow closely Nuño and Thomas (2022) and Achdou, Han, Lasry, Lions, and Moll (2021).

The paper is organized as follows: section 2 presents the model; section 3 analyzes the real appreciation - mortgage cost trade-off of the central bank, and provides the analytical result behind the interest rate smoothing policy prescription. Section 4 lays the ground for the quantitative application: it first presents the macro trends of the UK economy over the energy crisis and computes the empirical effect of mortgages on food consumption of households over the crossection; then proceeds to calibration and validation of the model. Section 5 explores the quantitative results of the model by comparing the benchmark BoE policy with a smoothed policy alternative. Section 6 concludes.

2 Model

The following general open economy framework builds on Auclert et al. (2023a) and Auclert et al. (2023b), while introducing two novel elements: long term bonds and mortgages (the latter modeled as perpetual debt, as in Burya and Davitaya (2022)), and food and non-food consumption (in order to construct a model-counterpart of food consumption variations analyzed in section 4).

2.1 Domestic households

A small open economy (the "domestic" economy) is populated by a unit mass of households, heterogeneous with respect to their wealth and their labor productivity. The discounted utility of a generic household i in economy j reads:

$$E_0 \int_0^\infty e^{\rho t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} \right] dt \tag{2.1.1}$$

where ρ is a subjective discount rate, σ is the coefficient of risk aversion, c_t is a Dixit-Stigliz consumption aggregator of a food c_t^f and non-food good c_t^{nf}, with elasticity ν and time-varying relative weight φ_t:

$$c_t = [\varphi_t^{\frac{1}{\nu}} c_{ft}{}^{\frac{\nu-1}{\nu}} + (1-\varphi_t)^{\frac{1}{\nu}} c_{nt}{}^{\frac{\nu-1}{\nu}}]^{\frac{\nu}{1-\nu}} \tag{2.1.2}$$

The Dixit-Stigliz formulation gives rise to the standard characterization of the price level as a harmonic average of the food and non-food goods:

$$p_t = [\varphi_t p_{ft}^{1-\nu} + (1-\varphi_t) p_{nt}^{1-\nu}]^{\frac{1}{1-\nu}} \tag{2.1.3}$$

Labor supply n_t is a bundle of a unit mass of labor varieties k supplied by the household:

$$n_t = \int_0^1 n_{kt} dk \tag{2.1.4}$$

where each variety's supply n_{kt} - equal across all household - is determined by a union, whose optimization problem will be discussed later.

Similarly to Nuño and Thomas (2022) I assuming that households trade in a nominal risk-free long-term bond a_t among themselves and witht he foreign economy. A bond issued a time t promises a stream of nominal payments $\{\delta e^{-\delta(s-t)}\}_{s\in(t,\infty)}$ summing up to one unit of domestic currency over the infinite lifetime of the bond. A fraction ω of households is also endowed with *mortgage stock*, equal across all of them, that enter the budget constraint under the form of an nominal perpetual debt paid at interest rate i_t^d, and whose proceeds are rebated equally to each domestic household. Therefore the remaining fraction $1 - \omega$ of households which are non-mortgagors (or "outright owners") still enjoy the stream of proceeds of mortgage revenues. I assume that the stock of mortgages is indexed to steady state inflation, while is inflated/deflated away by the effects of any extra-inflation above or below steady state. Formally, the real levels of mortgage stocks $D_t^r \equiv D_t/p_t$ follows a law of motion which takes into account the effect of inflation $\pi_t \equiv \dot{p}_t/p_t$ on its denominator:

$$\dot{D}_t^r = -D_t^r(\pi_t - \pi) \tag{2.1.5}$$

where π is steady state inflation. Indexing of this kind allows to obtain an overall steady state of the economy with $D^r \neq 0$ even when steady state inflation is different from zero.
The drift in the asset's dynamics is determined by the saving of the household, converted in asset units by division by the price X_t of the currently traded bond, net of the real reduction of asset amount by the amortization rate δ and inflation π_t:

$$\dot{a}_t = \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t)a_t \tag{2.1.6}$$

where $w_t \equiv W_t/p_t$ is the real wage, z_t is an idiosyncratic productivity shock that follows a diffusion process with parameters $\mu(z), \varsigma^2$; i_t^D is a household-specific interest rate on mortgages, and d_t and Π_t are dividends rebated to the household, generated respectively by the profits of firms and by the pooled economy-wide revenues from mortgages.
Each household's mortgage debt stock D is made up by a variable rate amount D^v and a fixed rate amount D^f, such that $D = D^v + D^f$. Both D^v and D^f have real value determined with the same process of (2.1.5): so the ratios D^v/D and D^f/D are constant over time. The variable rate mortgage yields interest rate i_t, anchored to the one provided by a security issued by the central bank (see section 2.5). The fixed rate mortgage consists instead in the sum of a continuum of mortgages of the same size D^f/S, indexed with subscript s and ranging from 0 to S:

$$D^f = \int_0^S D^f(s)ds \tag{2.1.7}$$

Each $D^f(s)$ entails a household-specific interest rate $i_t^f(s)$: this implies $i_t^f = \frac{1}{S}\int_0^S i_t^f(s)ds$.
At each period t, only the mortgage $s(t)$ gets its interest rate updated, where $s(t)$ is the remainder of the division of t/S: this introduces a S-interval periodicity in the update

of each mortgage s. When a mortgage $s(t)$ is renewed, it is paired with an interest rate $i^f_t(s) = i^f_{\tau\in[t,t+S)}(s)$, constant until next time of renewal $t+S$. I assume that this interest rate is set to the level that would guarantee to the foreign household the same total payment amount of domestic currency over the next S time interval that would be accrued if $D^f(s)$ were behaving as a variable rate mortgage (given the information set of the economy at time t). In other terms, the fixed interest rate is equal to the average of the variable rates over the time until the next mortgage rate renewal:

$$i^f_t(s) = i^f_{\tau\in[t,t+S)}(s) = \frac{1}{S}\int\limits_{[t,t+S)} i_\tau d\tau \tag{2.1.8}$$

It is here worth to highlight that the updating mechanism for i^d_t (2.1.8) is arbitrarily assumed in a stylized way to capture the forward-looking nature of the fixed rate of mortgage, and it will prove to be suitable to let the aggregate mortgage rate i^d_t track its empirical counterpart in section 4.4. Given the exogenous and non-tradable nature of the mortgage perpetuity D, the interest rate update rule for both fixed and variable mortgages is indeed detached from any market force in the model [1]. Let us define the aggregate interest rate on mortgages i^d_t as the weighted average of the fixed and variable rate:

$$i^d_t = \frac{D^f}{D} i^f_t + \frac{D^v}{D} i_t \tag{2.1.9}$$

It is worth highlighting here that the modelization of mortgages as non-tradable perpetual debt, distinct from the liquid long-term asset, is essential to accurately reflect the illiquid nature of this liability and its implications for household behavior, and allows us to capture the sizable impact of mortgage cost increases on household budget constraints—an effect documented in Section 3—that cannot be replicated when liabilities are assumed to be liquid, as in standard HANK models liquid debt instruments are typically subject to tight borrowing limits, which substantially restrict their capacity to generate large shifts in disposable income.

Households aim at maximizing lifetime utility (2.1.1) by choosing consumption, asset hold-

[1] The non-tradability of the perpetuity could be relaxed by assuming that the latter was sold only once in the life of the economy, by a private perfectly competitive intermediary with property rights equally split across all households, to a subset of agents (since then called "mortgagors") hit by a preference shock to current consumption such to drive them to wish to relax their current borrowing limit at the expense of future perpetual payments), while no unexpected shock had yet hit the economy. At the trade time, the perpetuity D would be expected to yield the same interest rate $\bar{i}$ as the long-term debt a_t, for the whole infinite horizon on the economy. After that moment, mortgagors would be locked-in with their mortgage position D and converge to a steady state distributions of assets and states - that one that will be treated in section 2.8.

ing under constraints (2.1.6) and the borrowing limit. The intertemporal problem of the household can be formulated recursively under the form of a Hamiltonian-Bellman-Jacobi equation for household with productivity realization z, asset holding a and mortgagor status $l = m, n$ (where m stands for mortgagor and n for non-mortgagor):

$$\rho V_t^l(a,z) = \max_{a_t, c_t^l} \left[\frac{c_t^{l\,1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} + s_t^l(a,z) \frac{\partial V_t}{\partial a} \right] + \mu(z) \frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2} \frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t} \quad (2.1.10)$$

where

$$s_t^l(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t^l - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if } l = m \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t^l + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if } l = n \end{cases} \quad (2.1.11)$$

We can define the joint density of wealth and productivity $f_t(a,z) = \omega f_t^m(a,z) + (1 - \omega) f_t^n(a,z)$, where $f_t^m(a,z)$ and $f_t^n(a,z)$ are the distributions of mortgagors and non-mortgagors (each one normalized to mass 1) , respectively. Their dynamics over time are governed by Kolmogorov-forward equations:

$$\frac{\partial f_t^l(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t^l(a,z) f_t^l(a,z)] - \mu(z) \frac{\partial V_t^l}{\partial z} + \frac{\varsigma^2}{2} \frac{\partial^2 V_t^l}{\partial z^2} \quad l = m, n \quad (2.1.12)$$

I will assume that the process for z is normalized such that the idiosyncratic productivity realizations aggregate to one:

$$\omega \int_z z f_t^m(a,z) dz + (1-\omega) \int_z z f_t^n(a,z) dz = 1 \quad (2.1.13)$$

Lastly, let us define C_t as aggregate consumption in the domestic economy - the integral of $c_t^l(a,z)$ over types m, n and all states a, z.

2.2 Final good producers

A mass of perfectly competitive firms produce either the food or non-food good, according to a CES production function in energy input y_{Et} (supplied by the foreign economy) and non-energy domestic input y_{Dt} (supplied by domestic producers):

$$y_{jt} = [(1-\alpha_E)^{\frac{1}{\epsilon}} y_{Dt}^{\frac{\epsilon-1}{\epsilon}} + \alpha_E^{\frac{1}{\epsilon}} y_{Et}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{1-\epsilon}} \quad j = f, n \quad (2.2.1)$$

where ϵ is the elasticity of substitution between energy and non-energy goods. Notice that, being the production function for the food and non-food good exactly equal, the marginal cost mc_t^f, mc_t^n for both goods is the same, and it immediately follows that $mc_t^f = mc_t^n =$

$p_{ft} = p_{nt} = p_t$ by perfect competition. The CES production function gives rise to the following formulation for the latter nominal marginal cost (equal to the final consumer's price p_t):

$$mc_t^f = mc_t^{nf} = p_{ft} = p_{nt} = p_t = [(1-\alpha_E)p_{Dt}^{1-\epsilon} + \alpha_E p_{Et}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{2.2.2}$$

where p_{Dt} and p_{Et} are respectively the prices of the non-energy and energy inputs.
The non-energy input y_{Dt} is in turn itself a CES aggregator of a home-produced good y_{Ht} and foreign-produced good y_{Ft}:

$$y_{Dt} = [(1-\alpha)^{\frac{1}{\eta}} y_{Ht}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} y_{Ft}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.2.3}$$

Where η is the elasticity of substitution between the domestic and foreign good. The price of the non-energy good can be derived as:

$$p_{Dt} = [(1-\alpha)p_{Ht}^{1-\eta} + \alpha p_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{2.2.4}$$

Final producers and producers of the non-energy good solve an optimal variety expenditure problem, which delivers a standard Dixit-Stigliz demand formulation for energy, domestic and foreign goods:

$$y_{Et} = \alpha_E \left(\frac{p_{Et}}{p_t}\right)^{-\epsilon} (y_{ft} + y_n t) \tag{2.2.5}$$

$$y_{Ht} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} (1-\alpha) \left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta} (y_{ft} + y_n t) \tag{2.2.6}$$

$$y_{Ft} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} \alpha \left(\frac{p_{Ft}}{p_{Dt}}\right)^{-\eta} (y_{ft} + y_n t) \tag{2.2.7}$$

It is worth highlighting that the economy-wide impact of the energy price P_{Et} is transmitted through the production sector, rather than appearing directly as a consumption good in households' baskets. This modeling choice reflects the case of the UK economy, where the direct effect of the energy shock on household consumption was mitigated by fiscal intervention, most notably through the Energy Price Guarantee (EPG).

2.3 Intermediate good producers

The intermediate domestic good y_{Ht} is produced by a competitive mass of firms which operate under a technology linear in aggregate labor N_t and aggregate productivity A:

$$Y_{Ht} = AN_t \tag{2.3.1}$$

This implies that dividends are zero ($d_t = 0$). Aggregate labor N_t is a Dixit-Stigliz aggregator of labor varieties:

$$N_t = \left(\int N_{kt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3.2}$$

where N_{kt} is the aggregate labor demand for variety k. The zero profit condition equates the real wage per unit of output to the price of the domestic good:

$$w_t \frac{1}{A} = \frac{p_{Ht}}{p_t} \tag{2.3.3}$$

Firms also face an optimal choice of the labor variety mix, leading to the standard optimal labor variety demand:

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t \tag{2.3.4}$$

where W_{kt} is the nominal wage in labor market k.

2.4 Unions

Each union k determines the labor supply of variety k, i.e. n_{kt} - equal across all households - standing ready to satisfy labor demand:

$$n_{kt} = N_{kt} \tag{2.4.1}$$

Following Wolf (2021), the union chooses the nominal wage W_{kt} at which it supplies labor in order to maximize the utility of the *average* agent; this utility is considered net of a nominal adjustment cost with respect to steady state wage inflation, and a real wage stabilization motive (the latter being introduced as in Auclert et al. (2023b):

$$\max \int_{\tau \geq 0} \exp \left[-\rho\tau \left(u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right) + N_{t+\tau}\lambda_{t+\tau} - \frac{\psi}{2}(\pi_{k,t}^{W} - \tilde{\pi}_{k}^{W})^2 N_{t+\tau} \right) \right] \tag{2.4.2}$$

where $\pi_{k,t}^{W}$ is wage inflation for labor variety k, and $\bar{\pi}_{k}^{W}$ is its steady state level, that works as a wage indexing benchmark, with wage increments higher or lower than this value yielding wage adjustment costs; λ_t is a stochastic wedge to marginal disutility of average agent.
As shown in the appendix, I solve the maximization problem subject to constraint (2.3.4) and the real labor earnings specification derived from the household block, obtaining the New Keynesian Phillips curve for inflation in the labor market:

$$\pi_t^{W} - \bar{\pi}_k^{W} = \frac{1}{(\rho + \vartheta) - \dot{N}_t/N_t} \left[\kappa \left(\chi N_t^{\phi} - \frac{\varepsilon - 1}{\varepsilon} w_t C_t^{-\sigma} + \lambda_t \right) + \dot{\pi}_t^{W} \right] \tag{2.4.3}$$

where π_W is aggregate wage inflation, and the slope κ is given by $\frac{\varepsilon}{\psi}$; the quantity λ_t acts an exogenous shock to the Phillips curve (analogously to a cost-push shock in a firms' sticky prices environment).

2.5 Central bank

The central bank trades a short term (instantaneous) risk-free asset with the foreign households. and sets its nominal return i_t according to a Taylor rule that responds to inflation deviations with respect the target $\bar{\pi}$, with time-varying intensity:

$$i_t = i_t^{ex} + \phi_{\pi,t}(\pi_t - \bar{\pi}) \tag{2.5.1}$$

where i_t^{ex} is an arbitrary component of interest rate setting. While a Taylor rule with a constant coefficient ϕ_π>1 guarantees a unique equilibrium, in the quantitative section my objective is to closely replicate the Bank of England's benchmark policy as observed in the data and to study alternative interest-rate trajectories that differ in their degree of smoothness over time. To allow for this flexibility, I let the policy parameter $\phi_{\pi,t}$ vary over time, subject to the requirement that it converges to a constant value $\phi_\pi > 1$ for all $t \geq T_i$. This terminal regime ensures determinacy from time T_i onward, and thus uniquely pins down the equilibrium path on the entire interval prior to T_i by backward induction.
In particular, setting the following policy:

$$i_t^{ex} = \bar{i} \text{ for } t \leq 0 \text{ and } t \geq T_i \tag{2.5.2}$$

$$\phi_{\pi,t} = \begin{cases} \phi_\pi \text{ for } t \leq 0 \text{ and } t \geq T_i \\ 0 \text{ for } t \in [0, T_i) \end{cases} \tag{2.5.3}$$

allows to i_t to be fully pinned down by the arbitrary component i_t^{ex} until time T_i, which in turn can be set to match closely the actual BoE policy, as well as can be make "smoother" to analyze policy couterfactuals.

2.6 Foreign economy

The rest of the world displays a representative household with constant consumption C^* of a non-energy good ($C^* = y^*$). The good is produced by a foreign representative firm, with technology symmetric to the final producers in the domestic economy ((2.2.3)):

$$y^* = [\alpha^{\frac{1}{\eta}} {y_{Ht}^*}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} {y_{Ft}^*}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.6.1}$$

where y^*_{Ht} and y^*_{Ft} are respectively the quantities of domestic and foreign input used by the foreign representative firm; note that the coefficient $(1-\alpha)$ is paired with y^*_{Ft}, due to home bias, mirroring expression (2.2.3).

Exported domestic goods are priced in foreign currency. Therefore, the foreign firms features the following Dixit-Stigliz demand for the domestic good:

$$y^*_{Ht} = \alpha \left(\frac{p^*_{Ht}}{p^*_t} \right)^{-\eta} y^* \tag{2.6.2}$$

Where p^*_{Ht} and p^*_t are the home good price and the foreign price level in foreign currency, respectively. The foreign price index p^*_t is given by the standard CES formulation, symmetric to (2.2.2):

$$p^*_t = [(1-\alpha) p^{*1-\eta}_{Ft} + \alpha p^{*1-\eta}_{Ht}]^{\frac{1}{1-\eta}} \tag{2.6.3}$$

with p^*_{Ft} being the price of the foreign good in foreign currency; I assume p^*_{Ft} to be itself a Dixit-Stigliz aggregator of a mass of varieties N^*, i.e. $p^*_{Ft} = \left(\int_0^{N^*} \tilde{p}^{*1-\eta}_{Ft}(n) dn \right)^{\frac{1}{1-\eta}}$. For $N^* \to \infty$, imposing symmetry across the foreign varieties' prices $\tilde{p}^*_{Ft}(n)$ implies $p^*_{Ft} \to p^*_t$ - namely, the foreign economy is "big" with respect to the domestic one, so its price index is not affected by domestic economy's price fluctuations.

Monetary policy in the foreign economy ensures full price stability:

$$p^*_t = p^*_{Ft} = 1 \tag{2.6.4}$$

where I normalize p^* to 1. I assume the law of one price to hold, hence I obtain:

$$p^*_{Ht} = p_{Ht} \mathcal{S}_t \tag{2.6.5}$$

$$p_{Ft} = p^*_{Ft} / \mathcal{S}_t = 1/\mathcal{S}_t \tag{2.6.6}$$

where $\mathcal{S}_t$ is the nominal exchange rate. Defining the real exchange rate as $Q_t = \mathcal{S}_t \frac{p_t}{p^*_t} = p_t \mathcal{S}_t$, and substituting y^* by C^* by foreign economy's good market clearing, we can rewrite foreign demand (2.6.2) as:

$$y^*_{Ht} = \alpha \left(\frac{p_{Ht}}{p_t} Q_t \right)^{-\eta} C^* \tag{2.6.7}$$

From the equation above, it can be noticed how a real appreciation (i.e. an increase in Q_t), leads foreign consumers to express a lower demand for the domestic good, which becomes relatively less convenient.

In the light of the foreign price stability and law of one price assumptions, and using the definition $Q_t = p_t \mathcal{S}_t$ and the price indexes (2.2.2) and (2.2.4), we obtain the real price of energy and the domestic and foreign goods as a functions of real exchange rate Q_t and energy price in foreign currency p^*_{Et}, that I assume to be exogenous :

$$\frac{p_{Et}}{p_t} = p^*_{Et}/Q_t \equiv p_E(Q_t, p^*_{Et}) \tag{2.6.8}$$

$$\frac{p_{Dt}}{p_t} = \left(\frac{1-\alpha_E p_E(Q_t, p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{\frac{1}{1-\epsilon}} \equiv p_D(Q_t, p^*_{Et}) \tag{2.6.9}$$

$$\frac{p_{Ht}}{p_t} = \left[\frac{1}{1-\alpha}\left(\frac{1-\alpha_E p_E(Q_t, p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{\frac{1-\eta}{1-\epsilon}} - \frac{\alpha}{1-\alpha} p_F(Q_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv p_H(Q_t, p^*_{Et}) \tag{2.6.10}$$

$$\frac{p_{Ft}}{p_t} = 1/Q_t \equiv p_F(Q_t) \tag{2.6.11}$$

The real price of energy p_{Et}/p_t depends positively on the foreign nominal price of energy p^*_{Et}, and negatively on the real exchange rate Q_t: domestic goods' appreciation indeed makes imported energy relatively cheaper. Conversely, the price of the non-energy good p_{Dt} is negatively related to the price of energy, so it is decreasing in p^*_{Et} and increasing in Q_t. p_{Ft}/p_t depends negatively on the real exchange rate: real appreciations indeed reduce the price of the foreign good relatively to the domestic one. The real price of the domestic good, p_{Ht}/p_t, depends negatively on both the real price of energy and the real price of foreign goods: therefore, a real appreciation (i.e. and increase in Q_t) boosts the real price of domestic goods by making energy and foreign goods relatively cheaper. An increase in energy price p^*_{Et} instead lowers p_{Ht}/p_t by reducing the relative price of domestic goods with respect to energy.

I assume that the foreign household can invest in the domestic households' long-term bond, in a short term foreign asset yielding nominal return $i^* + \xi_t$ (with ξ_t being a time varying component), and in the central bank's asset at rate i_t, as discussed in section **??**. To rule out arbitrage opportunities between between the latter two options, their return needs to be equal (uncovered interest parity, "UIP"):

$$i_t = i^* - \frac{\dot{\mathcal{S}}_t}{\mathcal{S}_t} + \xi_t \tag{2.6.12}$$

The condition can also be expressed in real terms:

$$i_t - \pi_t = i^* - \pi^* - \frac{\dot{Q}_t}{Q_t} + \xi_t \tag{2.6.13}$$

where $\pi^* = 0$ due price stability in the foreign economy.

The foreign households also need to face no arbitrage between investing in the domestic central bank's asset and in the domestic households' long term bond; therefore the two returns need to be equalized:

$$i_t - \pi_t = \frac{\dot{X}_t}{X_t} + \frac{\delta}{X_t} - (\delta + \pi_t) \tag{2.6.14}$$

Iterating forward the equation above allows to pin down the price of bonds at time t:

$$X_t = \int_t^\infty \delta e^{-\left[\int_t^s (i_\tau + \delta(\tau - t) d\tau\right]} ds \tag{2.6.15}$$

2.7 Equilibrium

Given a path for the exogenous component of interest rate i_t^{ex} and energy prices p_{Et}^*, an initial distribution of wealth and productivity $f_0(a, z)$, foreign consumption C^*, and targeted inflation $\bar{\pi}$, a competitive equilibrium is defined as a path for households' choices $(a_t, c_{ft}, c_{nt}, c_t)$, firms' choices $(N_t, y_{ft}, y_{nt}, y_{Ht}, y_{Et})$, unions' choices (n_t, π_t^W), prices $(p_H(Q_t, p_{Et}^*)$ $, p_E(Q_t, p_{Et}^*), p_F(Q_t), w_t, Q_t, X_t)$, aggregate quantities $(Y_{ft}, Y_{nt}, Y_{Ht}, C_t)$ and distributions $(f_t(a, z))$, consistent with the Kolmogorov forward dynamics (2.1.12)) such that households and firms optimize, and the following market clearing conditions in the goods and labor market are satisfied, as well as the uniform rebating rule for mortgage payment revenues:

$$\begin{aligned}
Y_{Ht} =& (1-\alpha_E)\left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon}(1-\alpha)\left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta}(Y_{ft}+Y_{nt}) + \alpha\left(\frac{p_{Ht}^*}{p_t^*}\right)^{-\eta} C^* = \\
=& (1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t, p_{Et}^*)^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha(p_F(Q_t)/p_D(Q_t, p_{Et}^*))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft}+Y_{nt}) \\
&+ \alpha\left(p_H(Q_t, p_{Et}^*)Q_t\right)^{-\eta} C^* && (2.7.1) \\
C_t =& Y_{ft} + Y_{nt} && (2.7.2) \\
Y_{Ht} =& AN_t && (2.7.3) \\
N_t =& n_t && (2.7.4) \\
\Pi_t =& \omega D_t^r i_t^d && (2.7.5)
\end{aligned}$$

where (2.7.1) is market clearing in the domestic good's market[2], (2.7.2) is market clearing in the final goods' market[3], (2.7.3) is market clearing the labor market, and (2.7.4) stands

[2]Condition (2.7.1) is retrieved by substituting for p_{Dt}/p_t and p_{Ht}/p_{Dt} by using the price indexes (2.2.2) and (2.2.4) and results (2.6.8)-(2.6.11).

[3]Since $p_{ft} = p_{nt} = p_t$, the aggregate demands for the food and non-food goods write $C_{ft} = \varphi_t C_t$ and $C_{nt} = (1 - \varphi_t)C_t$. By market clearing in the two markets, we have $Y_{ft} = C_{ft}$ and $Y_{nt} = C_{nt}$, hence, since

for the assumptions of households complying with the unions' choices in setting their labor supply (by symmetry among unions, $\int_0^1 n_{kt} dk \equiv n_t \ \forall k$). The goods market clearing condition (2.7.1) in particular is given by the sum of domestic demand (the first term on the right hand side) and foreign demand (the second term on the right hand side).

2.8 Steady state

In order to obtain a stationary value for D_t^r, I need calibrate the model to obtain constant inflation at target ($\pi_t = \bar{\pi}$): this is achieved by imposing ${}_t i^{ex}$ equal to the stationary interest rate $\bar{i}$ in the steady state version of UIP ((2.6.13)), with ξ being a stationary value for ξ_t. The model exhibits an infinite number of steady states, each one indexed by a value for the stationary real stock of mortgage $\bar{D}^r$. This is due to the fact that any nonzero inflation path $\pi_t \in [0, \infty)$, for given initial stock D, determines a different limit value of D_t^r (for $t \to \infty$) - determined by the extent to which the inflation path reduces the real mortgage stock over time, according to (2.1.5) . The following discussion will characterize a steady state for *given* $\bar{D}^r$.

The real domestic price $p_H(Q)$ is determined uniquely by the steady state $\bar{Q}$, and so is $\bar{w}$, by (2.3.3). Therefore, by (2.1.6), each household's consumption in home and foreign good $c(a, z)$ is determined uniquely by $\bar{Q}$, the steady state interest rate $\bar{i}$ (which also pins down the mortgage rate $i^d = i$), labor $\bar{N}$ and the states a, z (provided that I already substitute for the mortgage proceeds' rebating rule (2.7.5) and the labor supply compliance (2.7.4)). This implies that the drift function $s(a, z)$ depends only on $\bar{i}$, $\bar{Q}$ and $\bar{N}$ and the states a, z. Then, by setting to 0 the left hand side of (2.1.12), we can obtain the whole steady state distribution $f^l(a, z)$ - and then the overall distribution $f(a, z)$- as a function of $\bar{i}$, $\bar{Q}$ and $\bar{N}$. Aggregate consumption $\bar{C}$ is defined as the integral over steady state consumption for each combination of states, given the stationary distribution $\bar{f}(a, z)$; since both the idiosyncratic consumption levels $\bar{c}(a, z)$ and the distribution $\bar{f}(a, z)$ are determined by $\bar{i}$, $\bar{Q}$ and $\bar{N}$, we can then retrieve the following parsimonious functional formulation for $\bar{C}$:

$$\bar{C} = C(\bar{i}, \bar{Q}, \bar{N}) \tag{2.8.1}$$

Given $\bar{D}^r$, $\bar{i} = i^*$ and the stationary price of energy $\bar{p}_E^*$, equations (2.4.3),(2.7.1),(2.7.2),(2.7.3), (2.8.1),(2.6.13) define a system of six equations in six variables: $\bar{\pi}, \bar{Y}_H, \bar{N}, \bar{C}, (\bar{Y}_{ft} + \bar{Y}_{nt}), \bar{Q}$. If mortgages are 0 ($\bar{D}^r = 0$), the model shocks are small enough in size to guaranteee that

$\varphi_t \in (0, 1)$, we obtain $C_t = Y_{ft} + Y_{nt}$

the dynamics revert to the *initial* steady state: heterogeneous agents small open economy models can indeed feature stable steady states thanks to the convergence property of the asset distribution (beyond Auclert et al. (2023a), see also Nuño and Thomas (2022) de Ferra et al. (2020)[4]).
However, allowing for $\bar{D}^r > 0$ leads to convergence to a different final steady state from the initial one, due to the different final mortgage stock D^r, if inflation π_t is different from π at any point in time.
Notice that the discussion so far relies on the assumption of no structural parametric changes over the dynamics of the model, which would mechanically lead to a different final steady state. This however will be the case for the quantitative analysis of section 4 and 5, which will postulate a different final stationary interest rate both in the domestic and foreign economy, $\tilde{i} = i^* + \tilde{\xi} > \bar{i} = \bar{i}^* + \bar{\xi}$ (with $\bar{\xi}$ and $\tilde{\xi}$ being respectively the initial and final stationary value for ξ_t) providing an additional reason behind the attainment of a different final steady state, in addition to the inflation-driven adjustment of the mortgage stock.

3 Trading off appreciations with mortgage costs

In this section I analyse the impact of an energy price shock on crossectional household income, and later will introduce the trade-off faced by monetary policy in its reaction. Starting from a steady state configuration for the domestic economy, I will take into account an unexpected and temporary rise in the price of energy p^*_{Et} ($P^*_{Et} > \bar{P}^*_E$ and $P^*_{Es} = \bar{P}^*_E$ for $s \in (t, \infty)$). Given the results obtained in model outline, we can express the real income of a mortgagor household with states a, z (i.e. $\delta a_t + z_t w_t n_t - D^r_t i^d_t + \Pi_t$), net of the coupon payment δa_t, as follows:

$$z p_H(Q_t, p^*_{Et}) Y_{Ht} - (1-\omega) D^r_t i^d_t (i_{s\in[t,t+S)})) \tag{3.0.1}$$

Where labor income is a function of domestic output Y_{Ht} and and the real price of domestic good $p_H(Q_t, p^*_{Et})$, while the mortgage rate i^d_t is expressed as a function of all the future short-term interest rates until $t+S$ (i.e. $i^d_t(i_{s\in[t,t+S)})))$. The equilibrium expression $(1-\omega)D^r_t i^d_t$ stands for mortgage payment net of revenues Π_t. Notice that I decide not to include coupon payments δa_t in the income specification (3.0.1), as they do not depend directly on the energy price variation, nor on the interest rate policy (they depend instead indirectly on these shocks through the endogenous response of the household in adjusting its asset stock a_t).
A jump in p^*_{Et} makes domestic goods relatively more attractive than energy, increasing overall

[4]Then it is not needed to resort to debt-elastic interest rates as commonly done in representative agent models without international risk sharing.

world demand for domestic goods relatively to demand for the foreign ones. This effect is captured in equation (2.7.1) , and has a positive impact on domestic output Y_{Ht} (expenditure switching channel, ES). On the other side, an increase in p^*_{Et} lowers the firms' revenue per unit of output, and then wages, i.e. the term $p_H(Q_t)$ in equation (3.0.1) (terms of trade channel, TT). This last effect is produced by the higher price of energy relative to domestic goods, which passes through on domestic real wages.

If ES is stronger, households will enjoy a higher current wage income, while if TT dominates they will suffer from a current wage income loss. By looking at equation (2.7.1), with elasticities ϵ and η low enough the effect of energy price on demand for domestic good is muted: therefore the expenditure switching channel is dominated by the terms of trade channel. This is the case I will focus from now onwards, as it allows the energy price shock to induce a real income loss (as in Auclert et al. (2023b)).

Let us now assume rigid prices and zero wedges in the UIP[5]: $\pi_s = \xi_s = 0 \ \forall s > t$. Let us define the final steady state real exchange level $\tilde{Q}$ [6]. If the central bank reacts to the shock by producing an increase in the interest rate i_t by a contractionary monetary policy, that implies $dQ_t < 0$ by the UIP condition (2.6.13). In order to have this movement being consistent with a reversion to the initial steady state, Q_t needs to jump at the onset of the shock: the economy experiences a real appreciation. Intuitively, the domestic currency temporarily soars before depreciating over time back to its steady state level: this reduces the incentive to invest in domestic assets and restores indifference between the two countries' investment opportunities. The real exchange rate appreciation in turn passes through the real domestic wages by the firms' pricing condition (2.3.3), restoring some purchasing power for the household: analytically, in equation (3.0.1),the real wage term $p_H(Q_t, p^*_{Et})$ is negatively affected by the shock to p^*_{Et} but positively affected by the increase in Q_t. The interest rate hike fights the fall in real income by a domestic real appreciation.

However, the rise in the interest rate i_t affects real income (3.0.1) also through a higher outflows in terms of mortgage payment, as the aggregate mortgage rate i^d_t rises due to the increase in the short-term interest rate (by the mechanisms unraveled in equations (2.1.8) and (2.1.9) (debt-cost channel of an interest rate increase). The effects of an interest rate hike on consumption of mortgagors poses a trade-off to central bank's policy: on one side, the whole households' crossection suffers a weaker real income loss, on the other, mortgagors incur into a higher cost of debt.

The key aspect, however, is that whether the interest rate hike is frontloaded or smoothed

[5]This is obtained by assuming fully rigid nominal wages, so non-energy good prices P_{Dt}, together with the fact that $P^*_{Es} = \bar{P}^*_E$ for $s > t$.

[6]Following the discussion of Section 2.8, the initial shock to prices lowers the final real mortgage stock and implies a different final steady state real exchange rate from the initial one.

can make a significant difference to mitigate this trade off. Indeed you can achieve an appreciation of the current exchange rate even if the interest rate hike is smoothed over time. Let us consider the policy maker willing to attain the level $Q_t = Q^* > \bar{Q}$. The forward iteration of the UIP condition (2.6.13) up to infinity (bearing in mind that we are in the case $\pi_s = \xi_s = 0 \ \forall s > t$) yields:

$$\ln Q^* - \ln \tilde{Q} = \int_t^\infty (i_\tau - i^*) d\tau \tag{3.0.2}$$

So the current real exchange rate depends on the whole sum of future interest rates.
The question to be posed here is whether the trade-off between current appreciation and mortgage cost increase can be relaxed by distributing the latter over a protracted time span, leveraging the forward looking nature of Q_t. This can be engineered by an increase in the *future* short term interest rates $\int_t^\infty (i_\tau - i^*) d\tau$ (*interest rate smoothing*); notice that this would nevertheless come at the expense of Q_t and i_t being persistently above steady state beyond t, when it would be not anymore needed.
What does this interest rate smoothing strategy implies for the current variation in the mortgage cost, $i_t^d - i^*$? I will answer to this question by considering first two simple extreme cases ($S \to 0$ and $S = \infty$), and then I will analyze the general case for any fixed term horizon.

1. Case $S \to 0$ (short maturity mortgages). The fixed rate behaves as a variable rate ($i_t^f = i_t$) (we can see that by plugging the limit $S \to 0$ inside (2.1.8)). So, by equation (2.1.9), the variation in mortgage cost ($i_t^d - i^*$) boils down to $i_t - i^*$. A smoothed pattern for the policy rate i_t over time maps exactly into the same pattern for i_t^d, so interest rate smoothing is extremely effective in shifting the mortgage cost burden of an appreciation forward in time.

2. Case $S \to \infty$ (long maturity mortgages). The aggregate fixed mortgage interest rate at t is the average of the previously renewed mortgage rates down to time $t - S$ (set at i^* since the economy was in steady state before t) and the current renewed rate at the forward looking value $\frac{1}{S} \int_{[t,t+S)} i_\tau d\tau$ (see equation (2.1.8)). Therefore, the variation in i_t^f (i.e. $\dot{(i)}_t^f$ is given by:

$$\dot{(i)}_t^f = \frac{1}{S} \left(\frac{1}{S} \int_{[t,t+S)} i_\tau d\tau - i^* \right) \tag{3.0.3}$$

where for $S \to \infty$, the expression above equals zero. The intuition behind this result is that the size of the sub-mortgages getting their interest rate updated in the interval dt, i.e. (dt/S), goes to 0. Hence, by (2.1.9), the overall variation in mortgage cost is $(\dot{i})^d_t = \frac{D^v}{D}(\dot{i})_t$. The total mortgage rate deviation is pinned down only by variable rate mortgages variations. Therefore, the rationale to implement interest rate smoothing is more limited and given exclusively by the aim to smooth out variable rate mortgage cost increases over time.

The two simple cases above represent two extreme cases with respect to the extent to which interest rate smoothing shifts ahead the mortgage cost burden: significantly in the case $S \to 0$ and minimally in the case $S \to \infty$. Hence it is reasonable to expect that this policy would be more desirable the lower is the mortgage horizon S, as showed below. Consider the variation at t of mortgage cost (according to equation (2.1.9)):

$$(\dot{i})^d_t = \left(1 - \frac{D_v}{D}\right)(\dot{i})^f_t + \frac{D^v}{D}(\dot{i})_t \tag{3.0.4}$$

We can then substitute for (3.0.3):

$$(\dot{i})^d_t = \left(1 - \frac{D_v}{D}\right)\frac{1}{S^2}\int\limits_{[t,t+S)}(i_\tau - i^*)d\tau + \frac{D^v}{D}(\dot{i})_t \tag{3.0.5}$$

Substituting for (3.0.2) we obtain:

$$(\dot{i})^d_t = \left(1 - \frac{D_v}{D}\right)\frac{1}{S^2}\left(\ln Q^* - \ln\tilde{Q} - \int\limits_{[t+S,\infty)}(i_\tau - i^*)d\tau\right) + \frac{D^v}{D}\left(\ln Q^* - \ln\tilde{Q} - \int\limits_{(t,\infty)}(i_\tau - i^*)d\tau\right) \tag{3.0.6}$$

And finally rearranging, we obtain:

$$(\dot{i})^d_t = \left[\left(1 - \frac{D_v}{D}\right)\frac{1}{S^2} + \frac{D^v}{D}\right](\ln Q^* - \ln\tilde{Q}) - \underbrace{\left[\left(1 - \frac{D_v}{D}\right)\frac{1}{S^2}\int\limits_{[t+S,\infty)}(i_\tau - i^*)d\tau + \frac{D^v}{D}\int\limits_{(t,\infty)}(i_\tau - i^*)d\tau\right]}_{\text{smoothing effect}} \tag{3.0.7}$$

Equation (3.0.7) provide the key analytical result to understand why monetary policy smoothing can relax the trade-off between appreciation of Q_t and increase in i^d_t. Adopting a smoothed policy allows to achieve the target Q^* at the expense of a lower mortgage rate variation $(\dot{i})_t$ - effect captured in the term $\int_{t+S}^{\infty}(i_\tau - i^*)d\tau$ (the raise in interest rates beyond the fixed mortgage term $t + S$ entails indeed no effect on the currently updating fixed rate

i_t^f) and in the term $\int\limits_{(t,\infty)} (i_\tau - i^*)d\tau$ (the raise in interest rates beyond t has not effect on the current variable rate i_t).

For a higher mortgage term S (higher maturity), interest rate smoothing is less effective in mitigating the increase in mortgage costs (the impact becoming minimal for $S \to \infty$, as discussed previously). This is due to:

1. the impact of future monetary contraction on today's rate i_t^d is active for a longer time span $[t, t+S]$ (analytically, the"innocuous" forward guidance term $\int\limits_{t+S}^{\infty} (i_\tau - i^*)d\tau$ shrinks).

2. a smaller fraction of mortgages are updated at t, so shifting the debt cost burden ahead in time is quantitatively less important in the determination of $(\dot{i})_t$ (analytically, this is given by the smaller term $\frac{1}{S}$).

The results indicate that smoothing the interest rate path during an energy shock is beneficial from a welfare perspective, as it allows for real exchange rate appreciation while reducing the immediate pressure on mortgage costs. By avoiding sharp rate hikes, policymakers can mitigate the financial burden on households during the shock period.

However, this approach has a long-term cost. Prolonged monetary accommodation leads to higher future mortgage rates due to delayed monetary tightening, extending beyond the energy shock. This creates a forward guidance challenge, where the policymaker must assess whether short-term relief outweighs the future burden. A detailed quantitative analysis, as outlined in the next sections, is necessary to determine the overall welfare impact of this trade-off.

4 A quantitative application to the UK economy

4.1 The UK case in data

The surge in energy prices starting from 2021 had significant consequences for the UK economy. As depicted in Figure 4.1, the real industrial energy price index for electricity, gas, and other fuels surged by approximately 150% from 2021 to 2023. This surge in energy prices translated into a surge in CPI inflation, which peaked at 11% in 2023. Real wages, as illustrated in Figure 4.1, experienced a fall from the second half of 2021 onwards, resulting in a decline in the purchasing power of workers and households. In response to the inflation surge brought on by the increase in energy prices, the Bank of England responded decisively. Between 2021 and 2024, the bank significantly raised nominal interest rates, climbing

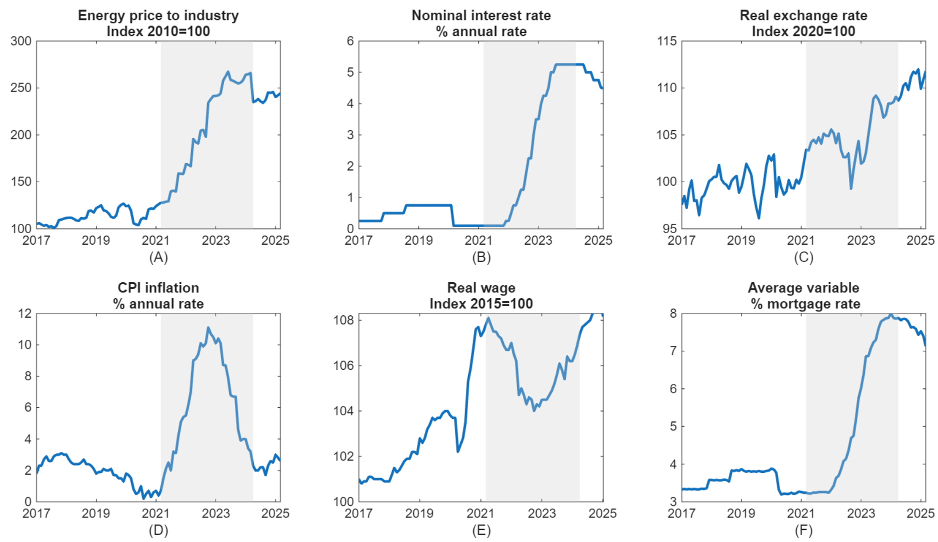


Figure 4.1: Energy prices to industry (quarterly data), policy rate, real exchange rate, CPI inflation rate, real wage and average variable mortgage rate. Source: Office for National Statistics, BoE and FRED database)

from 0.25% to approximately 5%. This shift in nominal interest rates held implications for mortgages' cost dynamics. Notably, at the beginning of 2021, 28% of mortgages were characterized by a 2-years fixed rate duration, while another 21% were featuring a variable rate[7]. These features of the mortgage market determined a discernible increase in the aggregate economy-wide mortgage rate, which climbed from 2% to almost 3.5% between 2021 and 2024. The facts presented above demonstrate the challenging trade-off faced by the Bank of England. Striking a balance between restoring real wage values and keeping borrowing rates moderate for mortgages was a complex task: while the former objective required a tight monetary policy to contain inflation, the latter was calling for a loose interest rate setting.
In what follows, I will further dig into the relevance of the increase in mortgage rates in affecting crossectional consumption. Leveraging data from the "Understanding Society" survey, a longitudinal panel that tracks information across various households in the UK over time, I explore the dynamics within two interview waves: 2020-2021 and 2021-2022.
In particular, I restrict the the analysis to households interviewed both in 2021 and in 2022, in order to track their consumption behavior over time. I include in the sample only households categorized as either housing mortgagors or outright homeowners. Households with tenure status changing between the two interview waves are also excluded, leading to a final sample of 2,477 households. The survey associates to each household its food consumption consumed at home, in addition to income, demographic and geographical characteristics.

[7]Source: Office of National Statistics

Table 1: Descriptive Statistics (monthly), households in 2021 interview wave

Per capita food consumption (£)			Income (£)		% mortgagors
Decile	Mean	Std	Mean	Std	
Bottom 20%	89.3	21.1	4'099	2'242	63%
Bottom 40%	112.8	28.9	4'084	2'348	61%
Bottom 60%	134.0	39.0	4'143	2'649	59%
Bottom 80%	157.6	53.8	4'137	2.742	56%
100%	201.6	120.5	4.128	2.779	54%

Due to the importance of distributional outcomes of a mortgage cost surge in the current framework, it is convenient to express descriptive statistics of the sample with respect to different subsamples of the distribution of food consumption in the pre-energy shock interview wave (i.e. 2021). The total sample of households is indeed split into 5 subsamples according to the position held by each household in the 2021 consumption distribution, namely the bottom 20%,40%,60%,80%,100% of the distribution. I restrict my analysis to the variation in annual food consumption, due to the limited range of expenditure items captured in the survey. For each household, I compute the percentage variation in per-member household food consumption (given by the ratio between household food consumption and household size $C_f(i,t) = food\_consumption(i,t)/size(i,t)$) between the 2021 and 2022 response:

$$\Delta_{c,f}(i, 2022) = \left[\frac{C_f(i, 2022)}{CPI\_food(2022)} - \frac{C_f(i, 2021)}{CPI\_food(2022)}\right] \Big/ \frac{C_f(i, 2021)}{CPI\_food(2022)} \quad (4.1.1)$$

where $C_f(i, 2022)$ is the food consumption value for household i reported in 2022, and $C_f(i, 2021)$ is the value stated by the same respondent in the previous 2021 interview. The variations is adjusted for changes in the food price index, in order to track only movements in real expenditure for food.

In order to capture distributional effects of mortgage cost increases along the households' crossection, I regress the consumption variation $\Delta^j_{c,f}(i, 2022)$ on a dummy $I_M(i)$, which assumes value 1 if the household owns its house through a mortgage and 0 if it is an owner outright; in the regression I control for the total net household real income variation between the two interview waves, $\Delta income(i, 2022) = \frac{income(i,2022)}{CPI(2022)} / \frac{income(i,2021)}{CPI(2021)}$. An additional vector X of regressors include government office regions as a geographical controls, and both number of children and number of people in working age as demographical controls. The model specification, for each quintile $\mathcal{Q}^j$ of the consumption distribution for $C(i, 2021)$, writes:

$$\Delta^j_{c,f}(i, 2022) = \beta^j_0 + \beta^j_1 * I_M(i) + \beta^j_2 * \Delta income(i, 2022) + \beta^j_3 X_t(i) + \varepsilon(i) \tag{4.1.2}$$
$$\forall\ i \text{ s.t. } C_f(i, 2021) \leq \mathcal{Q}^j(C_f(i, 2021))$$

The results up are summed up in Table 2.

Table 2: Regression results for consumption variation $\Delta^j_c(i, 2022)$

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.2997***	-0.1565***	-0.0828	-0.0651	-0.0690
	(0.1033)	(0.0588)	(0.0985)	(0.0743)	(0.0595)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.2907***	-0.1520***	-0.0850	-0.0633	-0.0715
	(0.1031)	(0.0589)	(0.0980)	(0.0739)	(0.0593)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	No	No	No	No	No
Mortgagor	-0.4197***	-0.2742***	-0.1085	-0.0573	-0.0228
	(0.0891)	(0.0508)	(0.0843)	(0.0634)	(0.0511)
Demographic controls	No	No	No	No	No
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.4143***	-0.2723***	-0.1092	-0.0569	-0.0248
	(0.0889)	(0.0508)	(0.0839)	(0.0631)	(0.0510)
Demographic controls	No	No	No	No	No
Regional controls	No	No	No	No	No
Δ% income control	Yes	Yes	Yes	Yes	Yes
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Observations	495	991	1486	1982	2477

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

Consistently with the prediction of heterogeneous agents literature, households which are able to afford lower consumption levels have also a low capacity to financially absorb income shocks (like a mortgage cost increase). We can indeed notice how the coefficients of the "Mortgagor" dummy increase in size and significance as we consider subsamples closer to the bottom of the consumption distribution. In particular, controlling for locations and demographic characteristics, the the bottom 20% and 40% of the distribution displayed respectively a 30% and 16% consumption loss of mortgagors with respect to outright owners - with a statistical significance of 1%; the other samples (bottom 60%, 80% and 100%) feature instead lower and not significant consumption effect from mortgage holding. Overall,

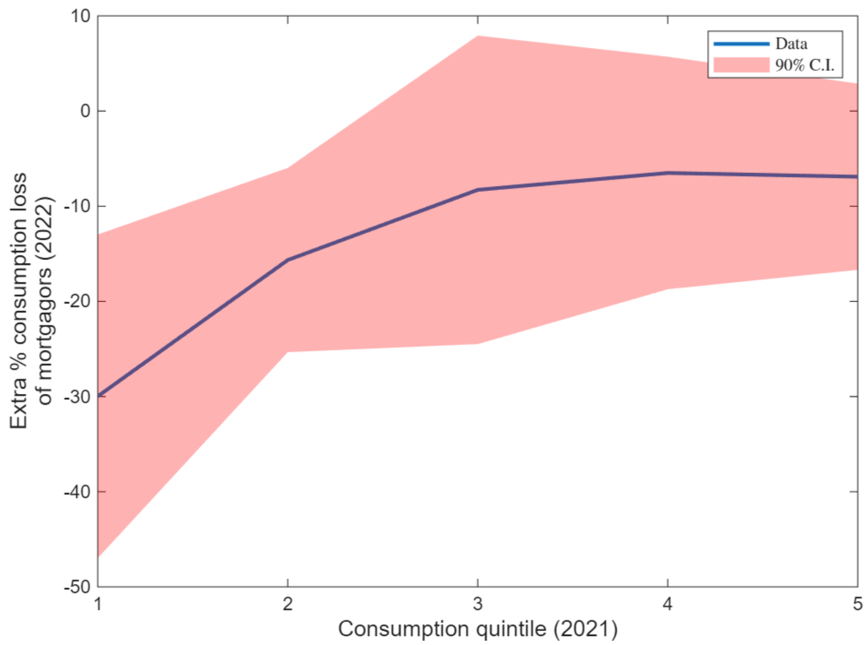


Figure 4.2: Coefficient β^j of mortgagor dummy in the regression for consumption variation $\Delta_c^j(i, 2022)$, with all controls, for households lying below each 2021 wealth quintile D_j. Shaded area: 90% confidence bandwidth

controlling for geographical locations does not change significantly the estimated impact of mortgage holding, while controlling for demographics drops this impact from 42% to 30%, suggesting that household's composition is a determinant of the mortgagor/owner outright status of the household, as well as of its consumption variation over the 2021-2022 time span. In what follows, I will tailor the calibration of the model to capture the empirical estimates in the case with all controls (first line of Table 2), reported graphically in Figure 4.2.

4.2 Sticky expectation adjustment

The model is initialized at a steady state with target inflation $\bar{\pi}$ and exogenous interest rate $i_t^{ex} = \bar{i}$, and the Taylor rule coefficient at $\phi_\pi > 1$ (as discussed in section 2.5). At time 0, the economy is subject to an energy price shock and an interest rate shock (the latter of the type detailed in section 2.5) - both of which are assumed not to be expected until time 0 = January 2021, that can be approximately identified as the onset of the energy price crisis and Bank of England's response (see Figure 4.1). Iterating forward (2.4.3) and (2.6.13) pins down π_0 and Q_0 as:

$$\pi_0^W = \tilde{\pi}^W + \kappa \int_t^\infty \frac{N_\tau}{N_t} e^{-\rho(\tau - t)} \left(\chi N_t^\phi - \frac{\varepsilon - 1}{\varepsilon} w_t C_t^{-\sigma} + \lambda_t \right) \tag{4.2.1}$$

$$Q_0 = \tilde{Q} + \int_t^\infty (i_\tau - \pi_\tau - (i^* - \pi_\tau^* - \xi_\tau)) d\tau \tag{4.2.2}$$

where $\tilde{\pi}^W$ and $\tilde{Q}$ are respectively the new steady states of wage inflation and real exchange rate. At 0 suddenly the expected paths $\lambda_{[t,\infty)}$, $N_{[t,\infty)}$, $w_{[t,\infty)}$, $C_{[t,\infty)}$ and $i_{[t,\infty)}$ change all at once, implying potentially sizable jumps in Q_0 and π_0^W that we do not observe in the data. In order to allow instead for a smooth reaction to the shock, following Gabaix (2020), I assume that unions' and foreign investors' expectations - respectively with respect to inflation and real exchange rate - are behaviorally biased towards expectational anchors given by their initial steady state values.

In the current setting it implies that the prospective inflation increment at any time t, $\mathbb{E}_t \dot{\pi}_t^W$, is dampened with respect to perfect foresight by a component $\vartheta_t(\pi_{k,t}^W - \tilde{\pi}_k^W)$, i.e. $\tilde{\mathbb{E}}_t \dot{\pi}_t = \dot{\pi}_t - \vartheta_t(\pi_{k,t}^W - \tilde{\pi}_k^W)$, where $\mathbb{E}_t$ is the biased expectation operator. This builds on the idea that the agent (in this case, the union), due to a behavioral bias, allocates always some probability $\int_t^{t+\Delta} \vartheta_u du$ on a hypothetical outcome in which the prospective inflation $\pi_{k,t+\Delta}^W$ is equal to an expectational anchor (in this case, $\tilde{\pi}^W$) instead of the rationally expected value under perfect foresight $\pi_{k,t+\Delta}^W$. Analogously to unions, foreign investors are have and expectational anchor for the prospective $Q_{t+\Delta}$ given by the initial steady state value $\bar{Q}$, implying $\tilde{\mathbb{E}}_t \frac{\dot{Q}_t}{Q_t} = \mathbb{E}_t \frac{\dot{Q}_t}{Q_t} - \vartheta_t \frac{Q_t - \bar{Q}}{Q_t}$.

My assumption distances from Gabaix (2020) in assuming that θ_t is time varying and equal to 0 in steady state: this reflect the fact that, in presence of an update of their information set, unions and foreign investors transition gradually away from their given expectational anchor, converging to perfect foresight. In other terms, ϑ_t exhibits a jump from zero to an initial value $\theta_0 > 0$, and then decreases gradually back to 0.

Equations (4.2.1) and (4.2.4) now become:

$$\pi_0 = \tilde{\pi} + \kappa \int_0^\infty \frac{N_\tau}{N_t} e^{-\rho(\tau - 0) - \int_0^\tau \vartheta_1(u) du} \left(\chi N_\tau^\phi - \frac{\varepsilon - 1}{\varepsilon} w_\tau C_\tau^{-\sigma} + \lambda_\tau \right) d\tau \tag{4.2.3}$$

$$Q_0 = \tilde{Q} + \int_0^\infty e^{-\int_0^\tau \vartheta_1(u) du} (i_\tau - \pi_\tau - (i^* - \pi_\tau^* - \xi_\tau)) d\tau \tag{4.2.4}$$

which, for a decreasing path of ϑ_t, now feature a more modest variation with respect to their initial steady state level. I choose to adopt a convenient functional form for $\vartheta_1()$ such that the behavioral discounting it implies between any time t and $t + \Delta$, i.e. (where Δ is the monthly time step used for the model simulation as discussed later) is equal to a constant coefficient Θ elevated at the power of $1 + \frac{t}{\Delta}$.

$$\int_t^{t+\Delta} \vartheta_1(u)du = \Theta^{1+\frac{t}{\Delta}} \tag{4.2.5}$$

that is given for functional form $\vartheta_1(u) = \frac{\ln \Theta/\Delta}{\Theta - 1}\Theta^{u/\Delta}$. For $\Delta = 1/12$ (monthly time step), this formulation states simply that, starting from period 0, and considering only the discounting effect of the time varying coefficient $\theta_1()$, the expectational anchors $\tilde{\pi}^W$ and $\tilde{Q}$ will be weighted Θ the first month, by Θ^2 the second month and so forth.

4.3 Calibration

The main forces at play in the model are the "open economy" dimension, that generates the adverse effects of the price of energy on domestic real wages through a terms-of-trade effect, and the "mortgage" dimension, which mediates the transmission of contractionary interest rate policy on crossectional consumption through the mortgage cost variation faced by households. Therefore my calibration strategy aims at matching salient features of the UK economy along both these dimensions.

Parameters. Following the calibration of Chan et al. (2023), tailored to the energy importer economies, I set the energy share in production α_e to 0.05 and the elasticity between labor and energy ϵ to 0.15 (built on UK estimates), the price elasticity of world demand for domestic exports η to 0.35, and the export share α to 0.25. The time step Δ is 1/12 (monthly unit periods). The slope of the Phillips curve is set to 0.0238, consistent with the UK wage stickiness parameter reported in Albuquerque et al. (2025) [8]. The sticky expectation coefficient Θ is set at 0.8 - implying that the expectational anchors $\bar{Q}$ and $\tilde{\pi}$ are weighted 80% at period 0 *January 2021) by unions and foreign investors respectively: this guarantees a no-jump reaction of inflation and the real exchange rate at the onset of the shock under all the policy alternatives analyzed in the next section. The process assumed in equation (4.2.5) guarantees that the role of expectational anchor fades away quickly by the end 2021, it shrinks already to a 5% relative weight in expectation formation).
With regards to the household crossectional dimension, I set $\sigma = 1$, $\phi = 2$ and $\rho = 0.05$ aligned with common parametrization choices; the borrowing limit is close to 0 ($\bar{a} = -0.2$)

[8]In a Calvo wage stickiness setup, κ would be equal to $\frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$, that equals 0.0238 for $\theta_w = 0.8637$ (the value in Albuquerque et al. (2025))

Parameter	Definition	Value	Source/Target
		Households	
ρ	Household discount factor	0.05	Literature
σ	Household risk aversion	1	Literature
ϕ	Inverse Frisch elasticity	2	Literature
$\mu(z)$	Mean of the diffusion process	$(-0.383\log(z)+\varsigma^2/2)z$	Steady state variance of log income in UK
ς	St. dev, of the diffusion process	0.6189	UK data (D'Amico and Fazio (2025))
δ	Amortization rate, LT bonds	0.021	Nuño and Thomas (2022)
$\bar{a}$	Borrowing limit	-0.02	Literature
		Mortgages	
D	Mortgage stock	8	Magnitude of crossectional Δ consumption (section 4.5)
S	Fixed rate mortgage duration	28	ONS UK (2021)
D_v/D	% variable rate mortgages	21%	ONS UK (2021)
ω	% mortgagors	54%	Understanding society survey (2021)
		Labour Unions	
κ	Slope of Phillips curve	0.0238	UK wage stickiness from Albuquerque et al. (2025)
		Firms and international trade	
α_e	Energy share in production	0.05	Chan et al. (2023) - Energy importer countries
ϵ	CES degree energy-labour in production	0.15	Chan et al. (2023) - UK estimates
η	Elasticity of world demand for domestic goods	0.35	Chan et al. (2023) - Energy importer countries
α	Foreign preference for domestic exports	0.25	Chan et al. (2023) - Export share ≈ 0.25
		Monetary Policy	
$\bar{i}$	Initial steady state interest rate (with $\bar{\xi}=0$)	0.5% yearly	Pre-energy crisis path
$\tilde{i}$	Final steady state interest rate	3% yearly	Post-energy crisis path (BoE projections)
ϕ_π	Taylor rule coefficient	1.5	Albuquerque et al. (2025) - UK estimate
		Expectation Adjustment	
Θ	Weight to expectational anchor at month 0	0.8	No jump in π_0 and Q_0 under all considered policies

Table 3: Calibrated parameters

according to standard assumptions in incomplete markets environments. As far as the households' idiosyncratic income process is concerned, I set ς^2 to match the variance of the UK log-income idiosyncratic process as computed in D'Amico and Fazio (2025)[9]. I calibrate the variance of the initial steady state distribution of log-income to 0.5, following the same "Understanding Society" sample used in section 4.1, that, given ς^2, yields a drift $\mu(z)=\left(-0.383\log(z)+\frac{\varsigma^2}{2}\right)z$.

The long-term bonds amortization rate δ is set to 0.22, consistent with a bond duration of 4.5 years (see Nuño and Thomas (2022)). The fraction of mortgagors replicates the data for the full "Understanding Society" survey sample (54%, see Table 1). The average fixed rate mortgage duration S is set to 28 months, matching the average duration of fixed rate mortgages at the beginning of 2021[10]. The mortgage stock is calibrated at $D=8$, to match the magnitude of the consumption effect of the 2022 mortgage cost increase as in Figure 4.2 (as carried out in section 4.5).

[9]The calibrated value of ς^2 is given by the total variance of the persistent and transitory idiosyncratic income shock as from the canonical "persistent plus transitory specification" model estimated in their paper

[10]In the first quarter of 2021, the Office of National statistics reports 27.8% of the total outstanding mortgages being 2-years fixed rate, 6.7% with 3 and 4 years fixed, 42% with 5-years fixed, 2.1% with a rate fixed for more than 5 years, and a 21.4% of variable rate.

Shocks. The model is subject to an energy price shock and an interest rate policy - both of which agents are assumed not to expect until time 0 = January 2021, and then to perfectly foresee from 0 onwards, consistently with an "MIT" shock assumption. The shocks follow a lognormal time profile starting in January 2021 and is set to match the observed data pattern shown in Figure 4.3. A key modeling choice concerns the time horizon of the data used to fit the shock. On the one hand, extending the horizon improves the fit over the full observed series; on the other hand, it requires assuming that agents correctly anticipate developments further into the future. To balance these two needs, I restrict the fitting horizon to data up to December 2023, which captures the core of the energy price crisis and constitutes the main focus of this quantitative analysis.
As far as the interest rate is concerned, monetary policy is modeled as from section 2.5, with the Taylor rule coefficient is switched to 0 from period $t = 0$, leaving room for the arbitrary component i_t^{ex} to track the BoE rates time series. The right tail of the lognormal model input for i_t (i,.e. at the right of the *argmax* of the curve) is truncated when the implied value for i_t would fall below the final steady state $\tilde{i}$, i.e. in July 2027: from then onwards, i_t^{ex} is set at the new level $\tilde{i}$, and monetary policy reverts to its stationary Taylor rule configuration with $\phi_\pi > 1$ (so July 2027 is the terminal value T_i for arbitrary interest rate setting as from section 2.5). The lognormal time profile of i_t^{ex} and P_{Et}^* is described in the following equations:

$$P_{Et}^* = \bar{P}_E^* + K_e Lognormal_{\mu_e,\sigma_e^2}(t) \ \forall t \geq 01/2021 \tag{4.3.1}$$

$$i_t = \begin{cases} \bar{i} \ \ \forall t \in [01/2021, T_{const}] \\ \bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t) \ \forall t \in [11/2021, \text{argmax}(i_t)] \\ \max\{\bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t) \ , \ \tilde{i}\} \ \forall t > \text{argmax}(i_t) \end{cases} \tag{4.3.2}$$

where $K_e = 47.38, K_i = 3.85, \mu_e = 3.7, \mu_i = 4, \sigma_e = 0.55, \sigma_i = 0.85$ are parameters set to match closely the data counterpart. The first 10 months of constant interest rates (i.e. from $t = 01/2021$ to $T_{const} = 10/2021$) are set to replicate the lagged response of the BoE with respect to the beginning of the inflation surge (see Figure 4.3).
Energy price data are given by the quarterly overall energy cost to industry index (detrended with respect to a linear projection based on post-Brexit time frame, i.e. 2017-2021) - recalling that the energy enters the model as a domestic firms' input.
As far as policy rate data are concerned, they are retrieved from the BoE dataset. Due to the role of the current and future interest rate hikes in shaping macroeconomic outcomes

-as from the theoretical result of section 3 - I consider a data sample for interest rate that extends further in the horizon with respect to the energy price data, until August 2024 and further encompassing the available official BoE projections up to that date. The path points out to a gradual interest rate cut - already initiated in April 2024 - to be implemented at a progressively slower pace. Consistently with this assumption, I assume a final "landing" stationary value for the BoE rate i_t of 3% (annualized). In line with the data, I assume the initial $\bar{i}$ equal to 0.5% and zero inflation (see Figure 4.3). Moreover I choose the final real interest rate $\tilde{i} - \tilde{\pi}$ to be equal to the initial one $\bar{i} - \tilde{\pi}$, which implies a final inflation rate $\tilde{\pi}$ equal to 2.5%, in line with the BoE target. Due to the key role of the real interest rate in influencing steady state aggregate demand (2.8.1)[11], equality between its initial and final level imply numerically similar values between the steady state levels of the endogenous steady state variables, especially $\bar{Q}$ ad $\tilde{Q}$, and $\bar{w}$ and $\tilde{w}$ respectively.

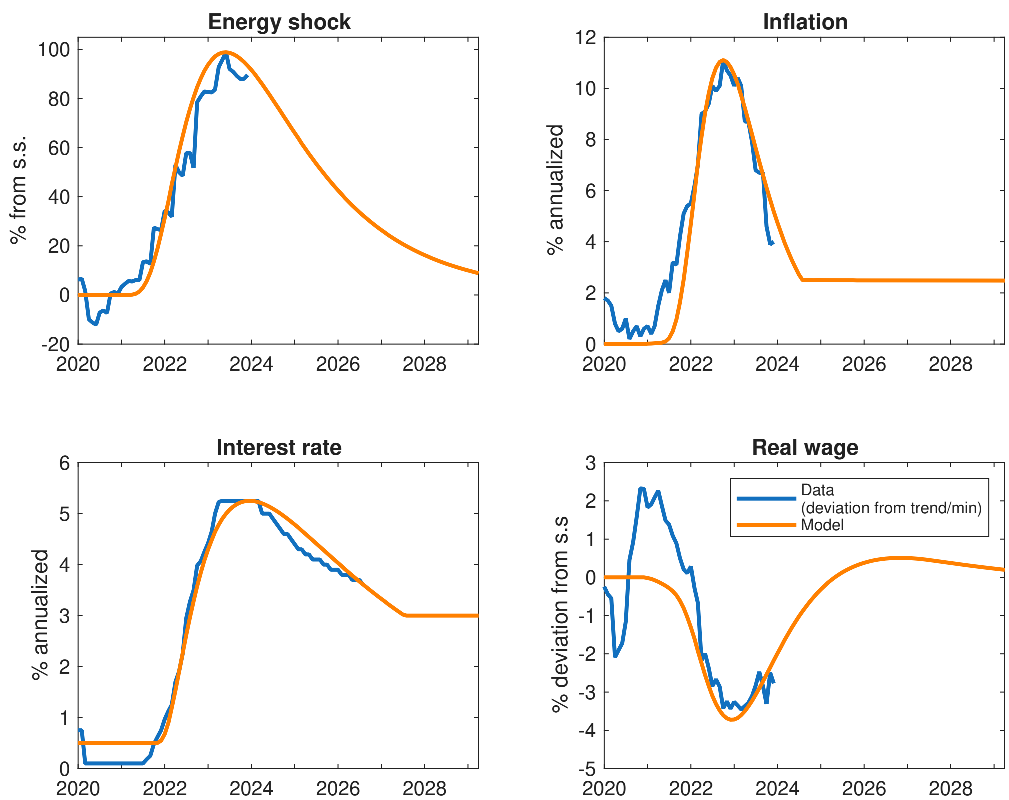


Figure 4.3: Energy price to industry, CPI inflation, interest rate, real wage, vs. data (Re-elaboration from series by Office for National Statistics and BoE). Energy price and real wage removed trends are computed on the 2017-2021 time sample, while interest rate and inflation are presented in raw data.

Once having established a path for the exogenous shocks i_t^{ex} and P_{Et}^{*}, I still need to specify

[11]Consider that the return on households' domestic bonds need to be equal to the real interest rate, by equation (2.6.14)

a pattern for the UIP and Phillips curve wedges ξ_t and λ_t, subject to the final steady states $\tilde{\xi} = \tilde{i} - \tilde{\pi} - i^*$ and $\tilde{\lambda} = 0$ (the latter set consistently with inflation at the target $\tilde{\pi}$). The two wedges are set to let the paths of w_t and π_t track the data counterparts, given by real regular pay in the UK and by the CPI inflation in annualized values. Similarly to the strategy adopted before, the paths of the wage w_t (in percentage deviations from the final steady state $\tilde{w}$) and π_t are assumed as well to follow a lognormal profile starting at time 0 = January 2021, whose paths are reported in Figure 4.3, and compared with the data benchmark. The latter are given respectively by the detrended real regular pay in the UK as from ONS data, and by the annualized CPI inflation. The magnitude and hump-shape (resp. u-shape) of CPI inflation (resp. real wage) is successfully replicated by the model output, with inflation peaking at 11% level, and real wages falling beyond 3% below trend.

4.4 Model validation at the aggregate level

The model is solved, by looping over the final steady state real mortgage stock $\bar{D}^r$, and aggregate consumption, building on the solution method by Achdou et al. (2021). Conveniently, the Phillips curve (2.4.3), UIP (2.6.13), pricing condition of home good firm (2.3.3), as well as the market clearing conditions (2.7.1)-(2.7.3) can be solved separately, for given aggregate consumption path, by using Dynare "perfect foresight" routine - details are reported in the appendix. The impulse responses for the real exchange rate (in percentage deviation from final steady state $\tilde{Q}$) and the variable rate on mortgages (in absolute deviations from the initial stady state $\bar{i}$) are reported in Figure 4.4 and compared with the data. The latter are given by - respectively - 1) the Real Broad Effective Exchange Rate for UK given by the FRED database, detrended with respect to a linear projection based on the same time frame used to compute the linear trend of the energy shock, i.e. 2017 - 2020; 2) the average variable rate paid by UK households as from BoE data, in absolute deviations from the plateau reached in 2021 (see Figure 4.1).
The impulse response function for the Q_t deviation successfully replicates the magnitude and persistence of the sustained appreciation realized from the beginning on the BoE contractionary policy. On the other side, the simple assumption linking the variable rate to the central bank's policy rate i_t proves effective in tracking the actual increasing path path of the average variable rate payment as from the data series.

4.5 Model validation in the crossectional consumption response

So far calibration choices were not discussed in detail with respect to the per-household mortgage stock D . The goal of this section is to present a calibration choice of this parameter,

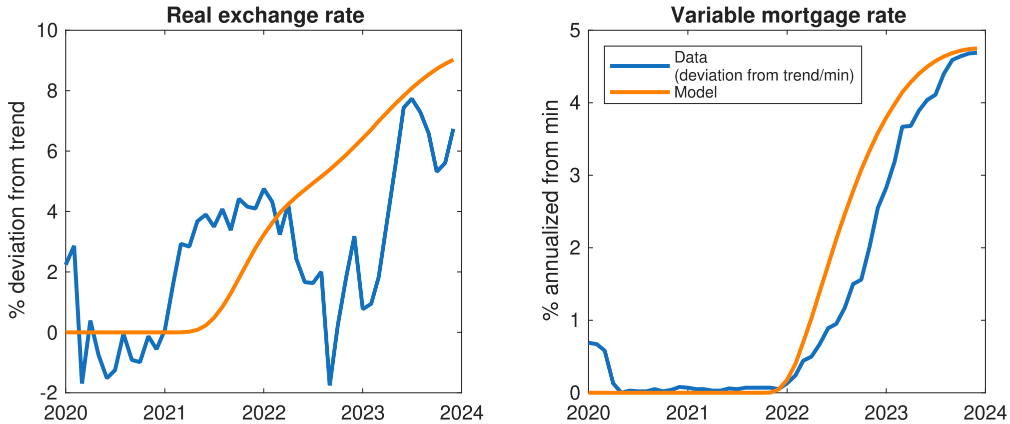


Figure 4.4: Output of the model: Real exchange rate and aggregate mortgage rate i_t^d (respectively in relative and absolute difference with respect to their initial steady state values), vs. data (Re-elaboration from series by Office for National Statistics and BoE). Real exchange rate is presented in deviation from the linear trend, computed with respect to the 2017-2020 time sample. Aggregate mortgage rates is showed in absolute differences from the 2021 plateau.

suitable to let the models replicate the difference in the 2021-2022 percentage consumption variation between the mortgagors and non-mortgagors (Figure 4.2).
The main challenge that needs to be addressed by the validation method consists in producing a discrete sample of mortgage and non-mortgage households with food consumption and income variations between 2021 and 2022, in order to implement a regression of the type (4.1.2) on the simulation output.

4.5.1 A model-generated crossectional effect of mortgages

In order to compare the effect of the mortgage cost increase on crossectional household consumption with the data output in Table 2, I need to perform a regression of the same type on the data delivered by the model: that requires, for each household starting at node a, z in period 01/2021, to identify the model implied variation of food consumption between 2021 and 2022 $\Delta^j_{c,f}(a,z)$ and variation of income $\Delta income(a,z)$, which I formulate as the ratio of the average expected consumption and income in 2022, on their initial values (as of January 2021), given the initial state node a, z:

$$\Delta^j_{c,f}(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[c_{ft}|c_{f,01/2021} = c_f(a,z)\right]}{c_{f,01/2021}} - 1 \qquad (4.5.1)$$

$$\Delta^j_{y}(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[y_{t}|y_{01/2021} = y(a,z)\right]}{y_{01/2021}} - 1 \qquad (4.5.2)$$

where income y_t is defined as the resources flow accrued to the household:

$$y_t = \delta a_t + z_t w_t n_t + \Pi_t - D_t^r i_t^d \tag{4.5.3}$$

The asset a and shock z are discretized along grids with dimension I and J respectively, which deliver discretized vectors $\{g_t\}_t, \{C_{ft}\}_t, \{y_t\}_t$ with size $I * J \times 1$, which comprise respectively the density, food consumption and income for each state node a, z. Following Achdou et al. (2021), we can also derive for each period a transition matrix $\mathcal{G}_t^{t+1}$ such that $g_{t+1} = \mathcal{G}_t^{t+1} g_t$; therefore, by multiplying the transition matrices from $t = 01/2021$ to subsequent $t < 12/2022$, we obtain the transition matrix that map $g_{01/2021}$ to g_t:

$$g_t = \mathcal{G}_{01/2021}^t g_{01/2021} \tag{4.5.4}$$

Each column of the matrix $\mathcal{G}_{01/2021}^t$ (hence, each row of the transpose $(\mathcal{G}_{01/2021}^t)^T$) represents the distribution of outcomes in t conditional on state a, z in steady state. Then I can recover the expected consumption (resp., income) in 2022, conditional on the household being characterized by states a, z at 01/2021, and hence the variations introduced in equations (4.5.1)-(4.5.2):

$$\Delta_{c,f}^j(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * C_{ft}}{c_{f,01/2021}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * C_{ft}\right] - \ln c_{f,01/2021} \tag{4.5.5}$$

$$\Delta_{y}^j(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * y_t}{y_{01/2021}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * y_t\right] - \ln y_{01/2021} \tag{4.5.6}$$

At this stage, discretizing $g_{01/2021}$ into a frequency allows to obtain a countable number of households - indexed by i - each one with consumption $c_{01/2021}(i)$. I can then rank the resulting sample of model household according to $c_{01/2021}(i)$, to obtain the initial discretized distribution of consumption. Notice that, alongside the derivation carried out in this section, I can also formulate an expression for the variation in total consumption basket c_t, analogous to (4.5.5)

$$\Delta_{c}^j(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * C_{t}}{c_{01/2021}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^t)^T * C_{t}\right] - \ln c_{01/2021} \tag{4.5.7}$$

4.5.2 From total nondurable to food consumption

The total nondurable consumption values for each model household ($c_t(i)$) are necessary inputs to generate the model-implied idiosyncratic food consumption levels and hence to draw a comparison with the empirical section results', as previously discussed. Given the consumption aggregator (2.1.2) and the result $p_{ft} = p_t$, food consumption is given by:

$$c_{ft}(i) = \varphi_t c_t(i) \tag{4.5.8}$$

Similarly to Aguiar and Bils (2015)[12], I take the logarithm of equation above, that boils down to:

$$\ln c_{ft}(i) = \Phi_t + \ln c_t(i) \tag{4.5.9}$$

where $\Phi_t = \ln \phi_t$ is a time varying coefficient. Let us assume that φ_t (and then Φ_t) varies on a yearly basis; taking time difference between the expected value of 2022 consumption and 01/2021, we get:

$$\Delta_{c,f} = \Delta\Phi_{2022} + \Delta_c(i) \tag{4.5.10}$$

where the quantities $\Delta_{c,f}(i)$ and $\Delta_c(i)$ are defined respectively by (4.5.5) and (4.5.7), and $\Delta\Phi_{2022}$ is given by $\Delta\Phi_{2022} = \Phi_{2022} - \Phi_{2021}$.

Let us now run a regression on the model output, mirroring the empirical counterpart (4.1.2), with the exception of being performed on total nondurable consumption instead of exclusively food:

$$\begin{aligned} &\Delta_c^j(i) = \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i) + \varepsilon(i) \\ &\forall\ i \text{ s.t. } c_{01/2021}(i) \leq \mathcal{Q}^j(c_{01/2021}) \end{aligned} \tag{4.5.11}$$

where $\Delta y(i)$ is the percentage income variation of household i between 01/2021 and year 2022 (given by expression (4.5.6), $I_M(i)$ is the previously defined indicator function for mortage holders, and $\mathcal{Q}^j(c_{01/2021})$ is the $j-th$ quintile of the steady state model consumption distribution. Results are summarized in Table 4.

Once having estimated the coefficients γ_0^j, γ_1^j, γ_2^j, we substitute for the linear prediction (4.5.11) inside (4.5.10):

$$\Delta_{c,f}(i) = \Delta\Phi_{2022} + \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i) + \varepsilon(i) \tag{4.5.12}$$

The coefficient γ_1^j provides the impact of mortgage holding on 2021 -2022 food consumption

[12] The authors perform instead a linear approximation around the crossectional average of $c_t(i)$.

Table 4: Regression results for consumption variation $\Delta_c^j(i, 2022)$ (model output)

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.2615	-0.1828	-0.1288	-0.0798	-0.0595
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Δ% income control	Yes	Yes	Yes	Yes	Yes

Note: All values are significant as the regression is performed on the whole model population

variation, a model counterpart of the empirical estimate of β_1^j retrieved in section 4.1 and plotted in Figure 4.2 for each sample of the model consumption distribution in 12/2021. Note that, while the model accounts percentage variations in food consumption deviating from the ones in total consumption by the factor Φ_{2022}, the average *difference* between percentage consumption variations of mortgagors and non-mortgagors is the same both with respect to food and total consumption, and measured by the factor γ_1^j.

Table 5: Consumption variation $\Delta_{c,f}^j(i, 2022)$. Model vs. Data.

Variable		**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	Data	-0.2997***	-0.1565***	-0.0828	-0.0651	-0.0690
		(0.1033)	(0.0588)	(0.0985)	(0.0743)	(0.0595)
	Model (γ_1^j)	-0.2615	-0.1828	-0.1288	-0.0798	-0.0595
Bottom % of $C(i, 2021)$		20%	40%	60%	80%	100%

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

Figure 4.5 compares the crossectional effects of the mortgage cost increase as from the simulation's outcome, to the empirical counterpart illustrated in Figure 4.2, and to the outcome which would arise in a setting with near-zero idiosyncratic shock ($\varsigma^2 = 0.0001$). The model replicates closely the negative relationship between the position held in the consumption distribution at the beginning of 2021 and the extra-consumption loss with respect to owners outright over the crossection of mortgagors, with households at the bottom of the distribution suffering most in food consumption terms. No confidence bandwidths arise in the model-based regression, as the latter is performed on the whole model population. In the near-zero idiosyncratic shock case, the heterogeneity dimension of the model is shut down, as all agents have nearly the same propensity to consume: consequently the impact on consumption of the mortgage cost increase is equal across all quintiles of the steady state distribution

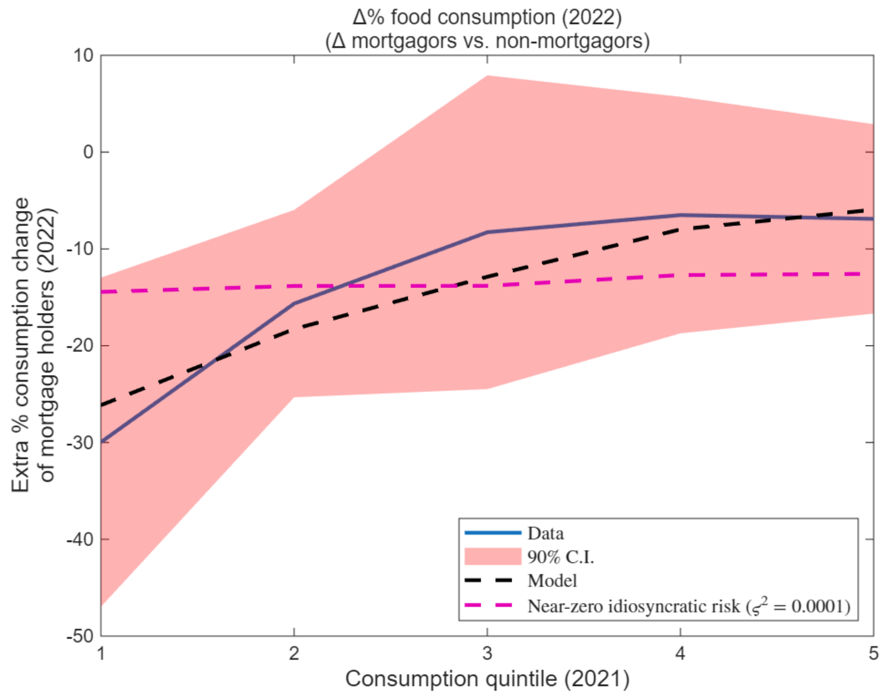


Figure 4.5: Coefficient β^j of mortgagor dummy in the regression for food consumption variation $\Delta^j_{c,f}(i)$, for households lying below 01/2021 consumption deciles $\mathcal{Q}^j$. Shaded area: 90% confidence bandwidth of the empirical results. Model vs Data.

(around 14% loss with respect to non-mortgagors). Therefore the heterogeneity dimension of the model is a key element to match the stronger impact of the shock throughout the consumption distribution.

5 Smoothing interest rate policy

5.1 The equilibrium effect of the benchmark BoE policy

The impulse response of the variables under the interest rate path set by the BoE (from onwards labelled as i_t^{bmk}, where "*bmk*" being short for "benchmark"), which were showcased in the previous section, underlie a real appreciation effect that fights the real income loss due to the energy price shock, along the lines discussed in section 3. Through the UIP condition, the stronger is the increase in the interest rate, the higher is the shift of the whole real exchange rate path. In order to show that, Figure 5.1 compares the equilibrium pattern for CPI inflation, nominal and real $(i_t - \pi_t)$ interest rate, real exchange rate, real wage and aggregate mortgage rate i_t^d to the one that would materialize with a milder interest rate policy ("moderate hike", in short *mh*) implemented. Such alternative policy is constructed

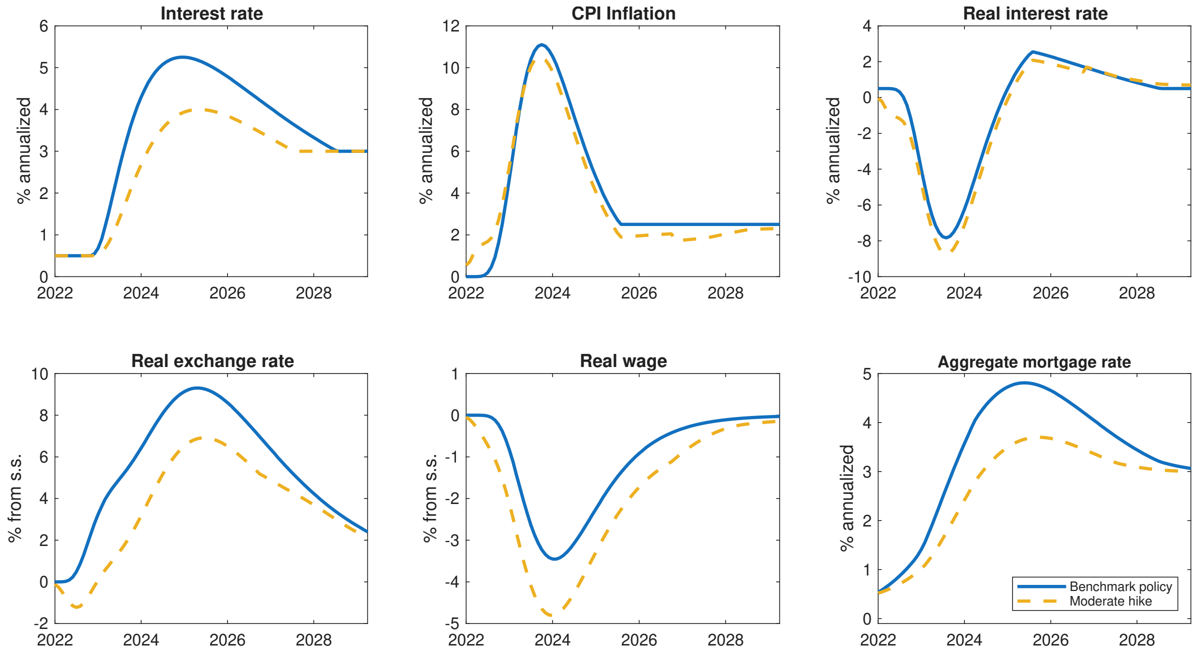


Figure 5.1: Impulse response functions to the energy shock. Benchmark policy vs. Moderate hike.

as imposing the parameters $\sigma_i^{mh} = 0.75$ and $K_i^{mh} = 2.71$ (lower than the $\sigma_i = 0.85$ and $K_i = 3.85$ of the benchmark). While lowering σ_i reduces the mass in the tails of the interest rate path, the decrease in K_i shrinks the whole path downwards - resulting in the interest rate path peaking at 4% points instead of 5.25%. The date of reversion of monetary policy to Taylor rule T_i is again identified by the period in which i_t^{ex} lands on the long run level $\tilde{i}$ (now T_i = July 2026 instead of July 2027).

Through UIP, the lower interest rate determines a lower exchange rate appreciation - the path of Q_t indeed is shifted down by by 2 percentage points with respect to the benchmark scenario. The lower appreciation proves less effective in defending real wages - w_t drops more severely, down to an extra 2% over 2023. On the other side, the milder increase in the nominal interest rate results in a lower path for the aggregate mortgage rate—by as much as 1 percentage point between 2024 and 2025.

Figure 5.2 showcases the average % variation in food consumption between 01/2021 and year 2022, isolating the effect of real wage fall alone, for the households lying below each j quintile of the 01/2021 consumption distribution. The overall variation in consumption ($\Delta_{c,f}^{total}$) is defined as the average of the total expected extra variation in consumption over the crossection with respect to a scenario without any aggregate shock. The effect of wages is isolated by subtracting from this variation the one that would be obtained by exogenously fixing the real wages to steady state in the partial equilibrium outcome of the households'

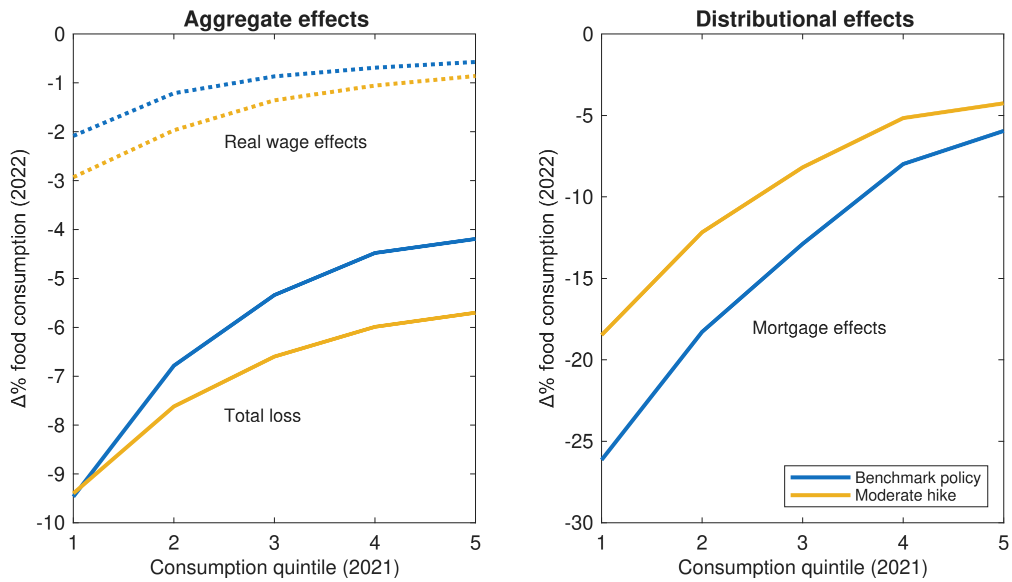


Figure 5.2: Left: food consumption % fall over 2022 for each consumption quintile of the 12/2021 consumption distribution (total $\Delta_{c,f}^{total}$ and decomposed by real wage effect $\Delta_{c,f}^{w}$). Right: 2022 Δ% consumption difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.5.10)). Benchmark policy vs. Moderate hike.

block. The total $\Delta_{c,f}^{total}$ and real wage-driven $\Delta_{c,f}^{w}$ variations can then be defined as follows:

$$\Delta_{c,f}^{j,total} = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^{t})^T * C_{ft}}{c_{f,01/2021}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^{t,no\ shock})^T * C_{ft}^{no\ shock}}{c_{f,01/2021}} \tag{5.1.1}$$

$$\Delta_{c,f}^{j,w=\bar{w}} = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^{t,\ w=\bar{w}})^T * C_{ft}^{w=\bar{w}}}{c_{f,01/2021}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{01/2021}^{t,no\ shock})^T * C_{ft}^{no\ shock}}{c_{f,01/2021}} \tag{5.1.2}$$

$$\Delta_{c,f}^{j,w} = \Delta_{c,f}^{j,total} - \Delta_{c,f}^{j,w=\bar{w}} \tag{5.1.3}$$

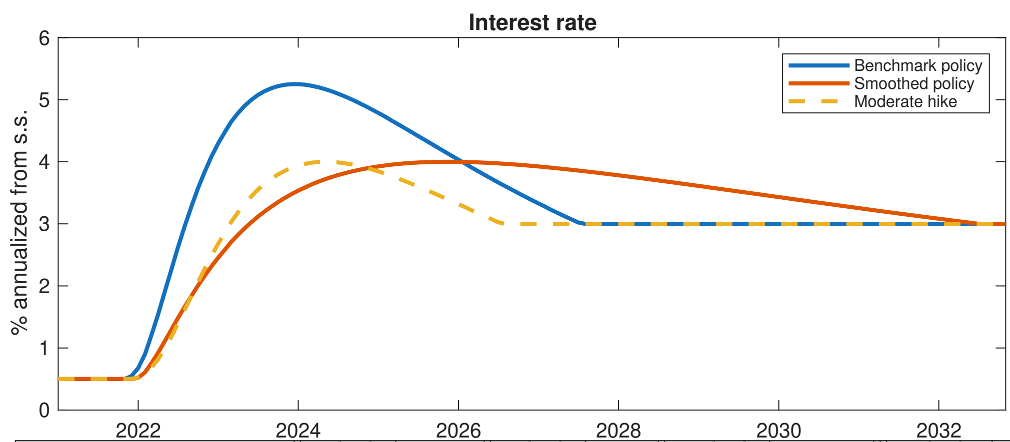
where $\mathcal{G}_{01/2021}^{t}$ and C_{ft} are respectively the transition matrix from 01/2021 to time t, and the food consumption level.

Figure 5.2 well captures the trade-off implied by the "moderate hike" policy between real exchange rate appreciation and mortgage costs. The alternative policy produces a worse impact of energy shock on consumption through a real wage fall - as real exchange rate appreciation is milder ; this consumption loss is more pronounced for households at the bottom of the distribution, whose consumption falls by about 2% rather than 3% due to the decline in wages. The loss becomes progressively smaller for higher-income quintiles, which are better able to absorb the shock to purchasing power thanks to their larger wealth buffers. However, the lower interest rate involves a better performance in terms of consumption of mortgagors, who can enjoy a weaker increase in the aggregate mortgage rate and see their consumption inequality gap with non-mortgagors being reduced by approximately 8 percentage points the bottom of the distribution: again, the beneficial effect of a lower mortgage payment becomes less sizable the more we move up the distribution.

5.2 A smoothed interest rate policy alternative

In what follows, I will introduce a new candidate policy ("smoothed policy", in short sm), which assumes a lognormal profile specified in the same way as in the benchmark policy (equation (4.3.2)); where the parameters K_i^{sm},μ_i^{sm}, σ_i and T_{const} are set to make this alternative policy peaking at the same 4% level as the moderate hike, while exhibiting a "smoother" profile, leading to a reversion to the long run interest rate four years later than in the benchmark. The graphical path of the new candidate policy is illustrated in Figure 5.3, where the it is compared to the benchmark and the moderate hike previously considered in Figure 5.1.

Consistently with the theoretical results of section 3, the stabilization power of the smoothed



Benchmark policy	σ_i^{bmk}	0.85	μ_i^{bmk}	4	K^{bmk}	3.85	T_{const}^{bmk}	10
Smoothed policy	σ_i^{sm}	1.2	μ_i^{sm}	5.3	K^{sm}	2.71	T_{const}^{sm}	12
Moderate hike	σ_i^{mh}	0.75	μ_i^{mh}	4	K^{mh}	10.26	T_{const}^{mh}	10

Figure 5.3: Benchmark policy vs. smoothed policy and moderate hike. Graphical path and functional form

policy alternative succeeds in achieving the same defense of consumers' purchasing power as in the benchmark policy. This can be seen in the bottom-left plot of Figure 5.4, where until 2024 the real exchange rate path under the smoothed policy is quantitatively similar to the benchmark, implying a similar patterns of real wages as well. This in turn implies that the effect of the energy shock on consumption through the real wage is equal between the benchmark and smoothed policy across all the household distribution quintiles, as shown in the left graph of Figure 5.5. On the other side, the mortgage rate i_t^d in the smoothed policy takes lower values until 2026 (see Figure 5.4), tracking the behavior of the same variable under the moderate hike alternative. As highlighted in the right plot of Figure 5.5, this translates into a significantly lower consumption drop for mortgagors with respect to non-mortgagors, that overlaps closely with the moderate hike outcome (consumption inequality between mortgagors and non-mortgagors shrinks by 8% for the household in the bottom quintile): smoothed policies are successful in partially closing the inequality gap between the two types of agents, without affecting the performance in terms of real exchange rate appreciation during the energy crisis.

While the equally milder initial rise in the interest rate implemented by both policies delivers an equal relief on mortgagors' consumption (right graph of Figure 5.5), only the smoothed policy is able to achieve that without affecting negatively the real exchange rate, since it sustains it at the same level implied by the benchmark policy through the 4-years further protracted interest rate hike. Therefore, the smoothed policy overperforms the moderate hike, by matching the benchmark policy's outcome in reducing the consumption loss due to real wage fall.

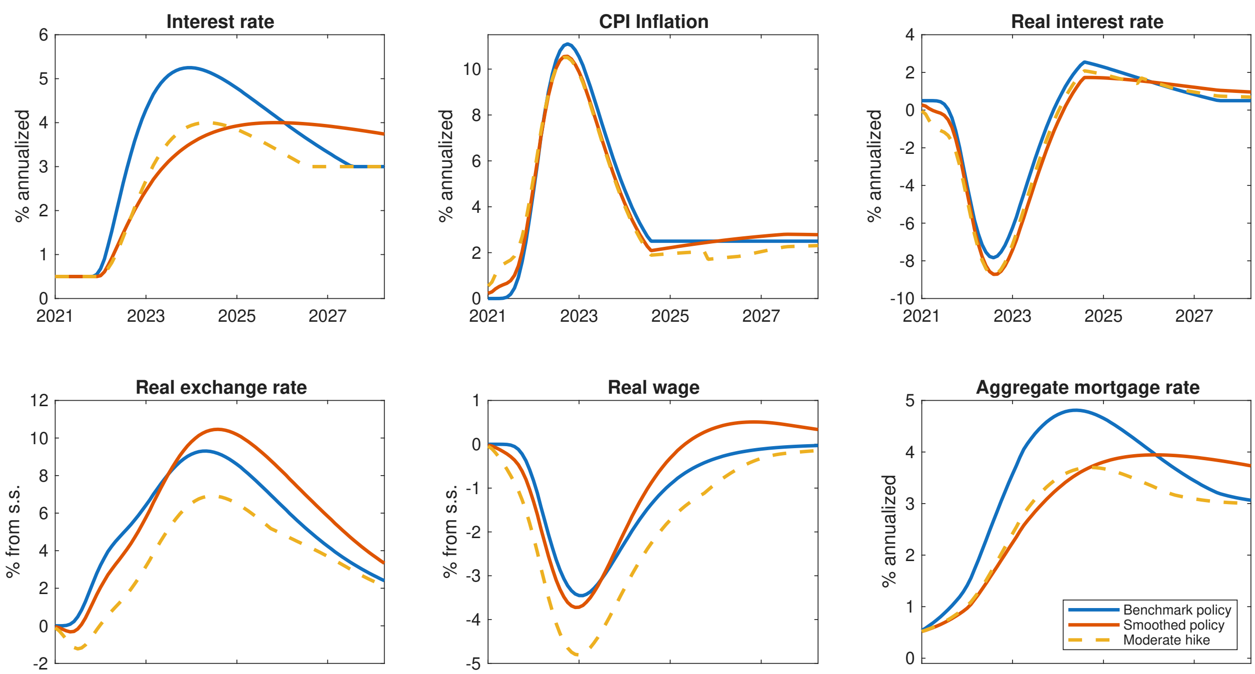


Figure 5.4: Impulse response functions to the energy shock. Benchmark policy vs. Smoothed policy and Moderate hike.

From the discussion above, we can see how the quantitative results confirm the theoretical prescriptions coming from the stylized model of section 3: a smoothing motive of the interest rate policy relaxes the trade-off between real appreciation and mortgage cost increase, which

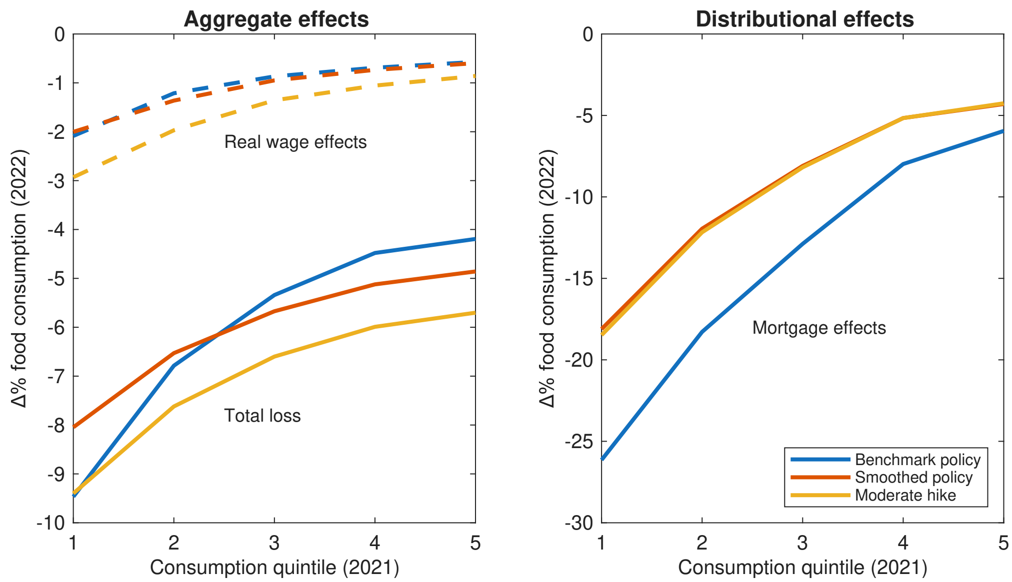


Figure 5.5: Left: food consumption % fall in 2022 for each consumption quintile of the 01/2021 consumption distribution (total $\Delta_{c,f}^{total}$ and decomposed by real wage effect $\Delta_{c,f}^{w}$). Right: 2022 % consumption fall difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.5.10)). Benchmark policy vs. Smoothed policy and Moderate hike.

instead was still relevant in the simple moderate hike case: the consumption loss due to real wage fall is indeed the approximately the same between the benchmark and the smoothed policy, while the latter can achieve more moderate mortgage rates and therefore a lower impact on consumption of mortgagors.

5.3 Welfare implications

As a further step with respect to the policy experiment carried out so far, I proceed to investigate the welfare implications of adopting the smoothed interest rate policy. Given the perfect foresight nature of the model, the lifetime welfare of any household is embedded in its value function $V_0^m(a,z)$ or $V_0^{nm}(a,z)$, where 0 is the time index for the first period of the simulation, and m and nm are respectively indexes for mortgagor and an non-mortgagor household. The analysis of the previous section pointed out that interest rate smoothing, during the initial stages of the energy crisis, relieves the mortgage cost burden without giving up real wage defence; however, the interest rate remains higher for a longer time, making real exchange and wages' appreciation more persistent even beyond the energy price crisis - and less needed, as the energy price would have already decreased substantially (see Figure 4.3); moreover the interest rate smoothing involves an undesirable longer protraction of high mortgage rates, as can be observed in the bottom-right plot of Figure 5.4, where i_t^d under the smoothed policy overtakes the one produced by the benchmark policy from year 2026 onwards. The adverse effect of this kind of "forward guidance" intervention needs to be taken into account in order to quantitatively evaluate the welfare implications of the smoothing policy: such implications are nonetheless encoded in the initial level of the value functions $V_0^m(a,z)$ or $V_0^{nm}(a,z)$, which can be averaged across the initial idiosyncratic shocks to obtain average value functions per asset level $V_0^m(a)$ and $V_0^{nm}(a)$. Figure 5.6 reports on the left an "inequality" measure given by the difference between $V_0^m(a)$ and $V_0^{nm}(a)$: mortgagors are worse off than non-mortgagors in both policy scenarios, due to mortgage costs burdening both over the dynamics and in the final steady state; however, implementing the smoothed policy allows to reduce inequality between the two household class, thanks to its initial mitigation effect on mortgage rates during the energy price crisis. In the current scenario a policymaker caring about inequality would then consider the smoothed policy as a "less costly" measure, from a welfare perspective, to tackle the impact of the energy shocks on the economy. Total utilitarian welfare, defined as the average discretized value function at time 0, i.e. $\sum\limits_{a,z} g_t(a,z)v(a,z)$ increases from by 0.0072, pointing out that the reduction in inequality is not achieved at the expense of a lower economywide utility. In order to substantiate the welfare increase in terms of consumption unit, I compute in the benchmark scenario the consumption subsidy that would need to be accrued to every household over 2021, taking the equilibrium consumption and labor choices as given, in order to yield the same total welfare of the smoothed policy outcome. In other terms, I seek to compute the subsidy k^* such that:

$$\sum_{t=12/2021}^{T} \beta^t \sum_{a,z} g_t(a,z)u(c_t^{bmk}(1+k_t), n_t^{bmk}) = \sum_{t=12/2021}^{T} \beta^t \sum_{a,z} g_t(a,z)u(c_t^{sm}, n_t^{sm}) \tag{5.3.1}$$

$$\text{with } k_t = \begin{cases} k^* \text{ if } t \leq 12/2021 \\ 0 \text{ if } t > 12/2021 \end{cases}$$

with $\beta = \frac{1}{1+\rho\Delta}$ and T being the last simulation period. The resulting 2021 subsidy k^* is equal to 2.39%, implying that the consumption path of all the households (and consequently aggregate consumption) would need to be shifted upward over 2021 by this percentage amount in order to guarantee the achievement of the same total welfare as in the smoothed policy case (see right plot of Figure 5.6).

5.4 Testing the model implications: increasing fixed mortgage horizon

A corollary policy prescriptions coming from the discussion of section 3 is that a longer time horizon S for fixed rates' renewal leads to a weaker effect of interest rate smoothing in relax-

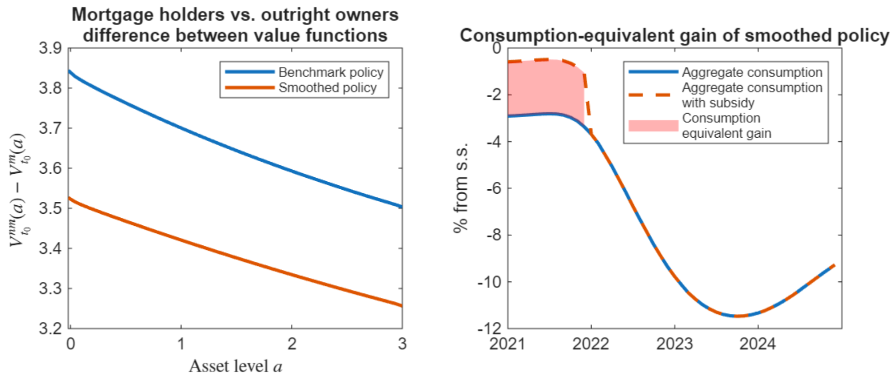


Figure 5.6: Value functions' difference at the first simulation period, for each asset level, Benchmark policy vs. Smoothed policy. (left plot). Consumption-equivalent gain of adopting the Smoothed policy (right plot)

ing the trade-off between exchange rate appreciation and mortgage costs, due to the lower sensibility of the current mortgage rate to future short term interest rate variations. I can test this implication in the current quantitative setting, by assessing the impulse responses under the same shock and three candidate policies of last section, with the exception of S being now set to five years instead of the 28 months years calibrated so far. Note that the pressure of contractionary policy on real exchange rate determination (through UIP) is the same as in the previous section, as the policy paths for i_t are the same as the ones considered before. On the other side, given the lower stickiness of fixed rate mortgages, the overall mortgage rate i^d_t displays for both the benahcmark and smoothed policy a stronger reaction in magnitude with respect to the baseline, with the benchmark policy's mortgage rate peak amounting to 4% (as opposed to the 5% peak of the baseline), as showed in Figure 5.7.
What is now the impact of the different policies on mortgage rates? As a result of the increased influence of the fixed-rate mortgage channel on the overall mortgage channel i^d_t, the impact of rising mortgage costs on mortgagors' consumption is dampened in both the policy options (see the bottom-right plot of Figure 5.7), with the consumption effects of mortgage cost increases now being different by 4 percentage points between the two policies a the bottom quintile of the household distribution (with respect to the 8% difference of the benchmark case). This confirms the analytical prediction outlined in section 3.

6 Conclusion

The trade-off between shielding the real wage of households and maintaining moderate costs for mortgagors in response to an energy price shock through an interest rate hike presents a complex challenge. While an increase in the interest rate can protect the purchasing power

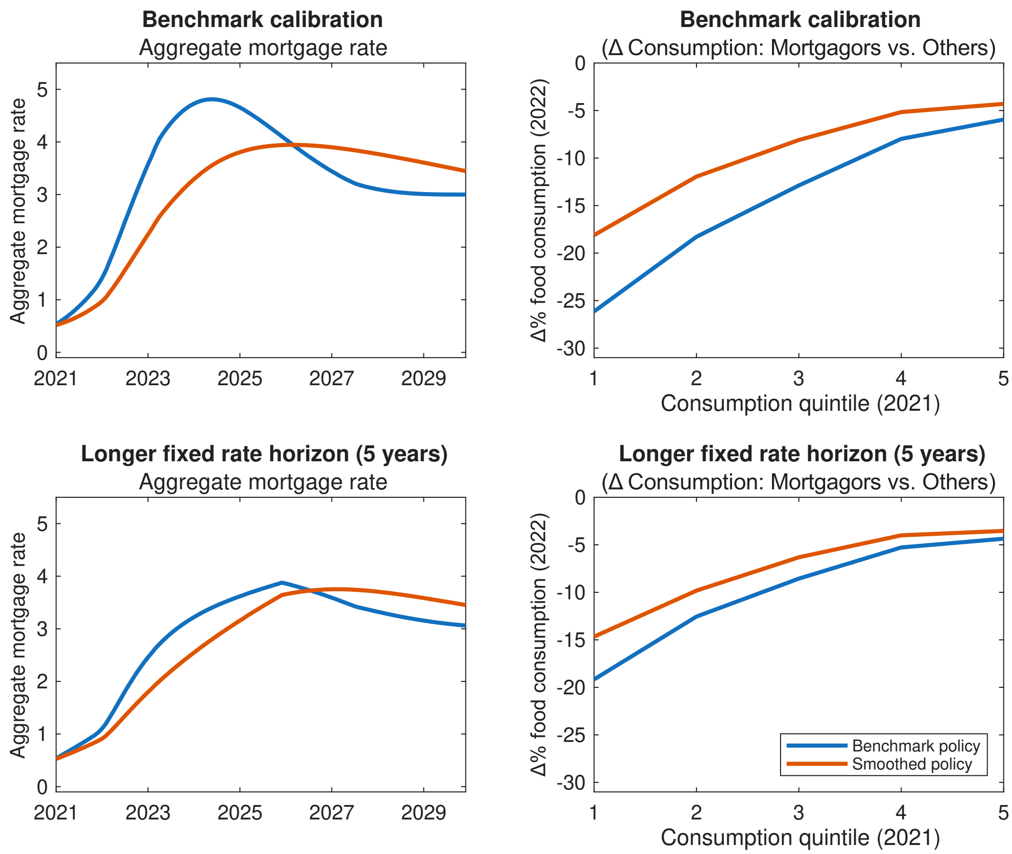


Figure 5.7: Policy's impact of mortgages for different fixed rate time horizons. Benchmark vs. Smoothed policy.

of households via real exchange rate appreciation, it also leads to higher mortgage rates. The benchmark contractionary policy implemented by the Bank of England (BoE) during 2022-2023 resulted in significant consumption losses for mortgagors, particularly those at the lower end of the consumption distribution. To address these challenges, this paper has explored an alternative strategy that employs milder and prolonged interest rate hikes. This approach delivers the same real exchange rate appreciation while spreading mortgage costs over a longer horizon, thereby reducing the immediate burden on mortgagors. The effect is decreasing in the length of fixed rate mortgage contracts.

This strategy presents a balanced approach to monetary policy, that would lead to more equitable welfare outcomes in the face of energy price shocks. A natural extension for this paper would therefore consist in a fully microfounded normative analysis, in the spirit of the literature about optimal policy in HANK.

References

Acharya, S., Challe, E., & Dogra, K. (2021, November). *Optimal Monetary Policy According to HANK* (Staff Working Papers No. 21-55). Bank of Canada.

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., & Moll, B. (2021, 04). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *The Review of Economic Studies*, *89*(1), 45-86.

Aguiar, M., & Bils, M. (2015, September). Has Consumption Inequality Mirrored Income Inequality? *American Economic Review*, *105*(9), 2725-2756.

Albuquerque, D., Chan, J., Kanngiesser, D., David Latto, S. L., Singh, S., & Žáček, J. (2025). *Decompositions, forecasts and scenarios from an estimated DSGE model for the UK economy* (Tech. Rep.).

Auclert, A., Rognlie, M., Souchier, M., & Straub, L. (2023a). *Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel.* (Download paperRevise and Resubmit at American Economic Review)

Auclert, A., Rognlie, M., Souchier, M., & Straub, L. (2023b). *Managing an energy shock: Fiscal and monetary policy.* (Download paperRevise and Resubmit at American Economic Review)

Bartocci, A., Cantelmo, A., Cova, P., Notarpietro, A., & Pisani, M. (2024). Monetary and fiscal policy responses to fossil fuel price shocks. *Energy Economics*, *136*, 107737. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0140988324004456` doi: https://doi.org/10.1016/j.eneco.2024.107737

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2021). Inequality, business cycles, and monetary-fiscal policy. *Econometrica*, *89*(6), 2559-2599.

Burya, A., & Davitaya, M. (2022). *Monetary Policy Implications of Heterogeneous Mortgage Refinancing* (Tech. Rep. No. 21-55).

Chan, J., Diz, S., & Kanngiesser, D. (2023, September). *Energy prices and household heterogeneity: monetary policy in a Gas-TANK* (Bank of England working papers No. 1041). Bank of England.

D'Amico, M., & Fazio, M. (2025). *Modelling income risk dynamics in the uk: a parametric approach* (Tech. Rep.).

de Ferra, S., Mitman, K., & Romei, F. (2020). Household heterogeneity and the transmission of foreign shocks. *Journal of International Economics*, *124*, 103303. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0022199620300222` (NBER International Seminar on Macroeconomics 2019) doi: https://doi.org/10.1016/j.jinteco.2020.103303

Del Negro, M., Dogra, K., Gundam, P., Lee, D., & Pacula, B. (2024, July). *On the Distributional Effects of Inflation and Inflation Stabilization* (Liberty Street Economics No. 20240702). Federal Reserve Bank of New York.

Dávila, E., & Schaab, A. (2023, February). *Optimal monetary policy with heterogeneous agents: Discretion, commitment, and timeless policy* (Working Paper No. 30961). National Bureau of Economic Research.

Fukui, M., Nakamura, E., & Steinsson, J. (2023, May). *The Macroeconomic Consequences of Exchange Rate Depreciations* (NBER Working Papers No. 31279). National Bureau of Economic Research, Inc.

Gabaix, X. (2020, August). A behavioral new keynesian model. *American Economic Review*, *110*(8), 2271–2327. Retrieved from https://www.aeaweb.org/articles?id=10.1257/aer.20162005 doi: 10.1257/aer.20162005

Gnocato, N. (2025). Energy price shocks, unemployment, and monetary policy. *Journal of Monetary Economics*, *151*, 103734. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304393225000054 doi: https://doi.org/10.1016/j.jmoneco.2025.103734

Nuño, G., & Thomas, C. (2022). Optimal redistributive inflation. *Annals of Economics and Statistics*(146), 3–64.

Olivi, A., Sterk, V. S., & Xhani, D. (2023). *Optimal Policy Rules in HANK* (Tech. Rep. No. 21-55).

Pieroni, V. (2023). Energy shortages and aggregate demand: Output loss and unequal burden from hank. *European Economic Review*, *154*, 104428.

Ragot, X. (2017). *Optimal Monetary Policy in a Liquidity Trap with Heterogeneous Agents* (Working Papers No. hal-03459028). HAL.

Smirnov, D. (2023). *Optimal Monetary Policy in HANK* (Tech. Rep.).

Wolf, C. (2021, August). *Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result* (NBER Working Papers). National Bureau of Economic Research, Inc.

Wolf, C. (2023). *Optimal Policy Rules in HANK* (Tech. Rep.).

Appendix

A Derivation of the New Keynesian Phillips curve

Following Wolf (2023), I assume that ech union k seeks to maximize the *utility of the average household* [13], i.e, a fictitious agent consuming the average amount over the household's crossection, and subject to the same supply schedule across labor varieties - set by the unions. The utility is evaluated net of an inflation and real wage stabilization cost Ψ_t. The maximization problem writes:

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau\left(\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\} - \Psi_t\right)\right] = \tag{A.0.1}$$

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau\left(\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\} - \frac{\psi}{2}\pi_{k,t}^{W2} N_{t+\tau}\right)\right] \tag{A.0.2}$$

where $\pi_{k,t}^W$ is wage inflation for labor variety k, and the parameter ψ scales the inflation cost. The problem is subject to: 1) the average real labor earning at time $t+\tau$ being given by:

$$Z_{t+\tau} = \frac{1}{P_{t+\tau}} \int_0^1 W_{kt+\tau}\left(\frac{W_{kt+\tau}}{W_{t+\tau}}\right)^{-\varepsilon} N_{t+\tau} dk \tag{A.0.3}$$

2) the envelope condition:

$$\frac{\partial C_{t+\tau}}{\partial W_{kt+\tau}} = \frac{\partial Z_{t+\tau}}{\partial W_{kt+\tau}} = \frac{1}{P_{t+\tau}} \int_0^1 (1-\varepsilon)\left(\frac{W_{kt+\tau}}{W_{t+\tau}}\right)^{-\varepsilon} N_{t+\tau} dk \tag{A.0.4}$$

and 3) the effect of the kth-variety nominal wage W_{kt} on labor supply, that , due to the N_{kt} determination $N_{kt} \equiv \int_0^1 \left(\frac{W_{kt}}{W_t}\right)^{-\varepsilon} N_t dk$,, writes:

$$\frac{\partial N_{kt}}{\partial W_{kt}} = -\varepsilon \frac{N_{kt}}{W_{kt}} \tag{A.0.5}$$

The problem can be formulated as a Hamilton-Bellman-Jacobi equation:

$$\rho J(W,t) = \max_{\pi^w}\left[\{u\left(C_t\right) - v\left(N_t\right)\} - \frac{\psi}{2}\pi_k^{W2} N_t\right] + J_W(W,t) W \pi_k^W + J_t(W,t) \tag{A.0.6}$$

where $J(W,t)$ is the real value of a union with wage W. Taking the envelope and first order

[13] This is a convenient assumption to model the way union aggregates preferences, because it allows to abstract inflation dynamics from distributional outcomes; an alternative is to assume maximization of the average utility of households for some arbitrary weights

conditions and imposing symmetry across all k, we get:

$$J_W(W,t)W = \psi \pi^W N \tag{A.0.7}$$

$$\left(\rho - \pi^W\right) J_W(W,t) = \frac{\varepsilon}{W}\left[\tilde{N} v'\left(\tilde{N}\right) - \frac{\varepsilon - 1}{\varepsilon}\tilde{N} w u'\left(C\right)\right] + J_{WW}(W,t) W \pi^W + J_{Wt}(W,t) \tag{A.0.8}$$

Differentiating (A.0.7) with respect to time gives

$$J_{WW}(W,t)\dot{W} + J_{Wt}(W,t) = \frac{\psi \tilde{N} \dot{\pi}^W}{W} + \frac{\psi \dot{N} \pi^W}{W} - \frac{\psi \pi^W \tilde{N}}{W}\frac{\dot{W}}{W} \tag{A.0.9}$$

Substituting the above expression and (A.0.7) inside (A.0.8) we obtain the Phillips curve as presented in section 2.4 (equation (2.4.3)), with $\kappa^W = \frac{\varepsilon}{\psi}$.

B Equilibrium conditions

The model equilibrium is described by the following set of conditions:

$$\rho V_t^l(a,z) = \max_{a_t, c_t^l}\left[\frac{c_t^{l\,1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} + s_t^l(a,z)\frac{\partial V_t}{\partial a}\right] + \mu(z)\frac{\partial V_t^l}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t^l}{\partial z^2} + \frac{\partial V_t^l(a,z)}{\partial t} \quad l = m, n$$

$$c_t^l(a,z)^{-\sigma} = \frac{\partial V_t^l(a,z)}{\partial a}$$

$$s_t(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t^l - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if } l = m \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t^l + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if } l = n \end{cases}$$

$$\frac{\partial f_t^l(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t^l(a,z) f_t^l(a,z)] - \mu(z)\frac{\partial V_t^l}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t^l}{\partial z^2} \quad l = m, n$$

$$C_t = \omega \int_0^1 c_t^m(a,z) f_t^m(a,z) da dz + (1-\omega)\int_0^1 c_t^n(a,z) f_t^n(a,z) da dz$$

$$w_t \frac{1}{A} = p_H(Q_t, P_{Et}^*)$$

$$\dot{D}_t^r = -D_t^r \pi_t$$

$$i_t^d = \frac{D^f}{D} i_t^f + \frac{D^v}{D} i_t$$

$$i_t^f = \frac{1}{S}\int_0^S i_t^f(s) ds$$

$$i_t^f(s) = i^f_{\tau \in [t, t+S)}(s) = \frac{1}{S}\int_{[t,t+S)} i_\tau d\tau$$

$$\pi_t^W = \frac{1}{\rho - \dot{N}_t / N_t}\left[\kappa\left(\chi N_t^\phi - \frac{\varepsilon - 1}{\varepsilon} w_t C_t^{-\sigma}\right) + \dot{\pi}_t^W\right]$$

$$X_t = \int_t^\infty \delta e^{-\left[\int_t^s i_s + \delta(s-t)\right]} ds$$

$$i_t - \pi_t = i^* - \pi^* - \frac{dQ_t}{Q_t} + \xi_t$$

$$Y_{Ht} = (1-\alpha_E)\left(\frac{1 - \alpha_E p_E(Q_t, p_{Et}^*)^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1 - \alpha(p_F(Q_t)/p_D(Q_t, p_{Et}^*))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft} + Y_{nt}) +$$
$$\alpha\left(p_H(Q_t, p_{Et}^*) Q_t\right)^{-\eta} C^*$$

$$Y_{ft} + Y_{nt} = C_t$$

$$Y_{Ht} = A N_t$$

$N_t = n_t$
$\Pi_t = \omega D_t^r i_t^d$
$d_t = Y_{Ht} - AN_t$
$w_t = W_t/p_t$
$\pi_t = \dot{p}_t/p_t$
$\pi_t^W = \dot{W}_t/W_t$
$i_t = i_t^{ex} + \phi_{\pi,t}(\pi_t - \bar{\pi})$

Which in order, are: the Hamiltonian-Bellman-Jacobi equation, the optimality condition of the household, the drift function, the Kolmogorov-Forward equation, the definition of aggregate consumption, the domestic good producers' pricing, the evolution of real mortgage stock, the definition of mortgage rate, fixed mortgage rate, and fixed rate of submortgage s. Then we have the Phillips curve, the pricing of long term bonds, the UIP condition, the market clearing conditions and the mortgage revenues rebating rule. Finally we have the definition of dividends, real wage, price inflation, wage inflation and interest rate setting equation

C Solution algorithm

C.1 Steady state

Under the benchmark policy, the model is solved numerically in an analogous way to the method presented in Achdou et al. (2021), by iteration over the aggregate consumption path. Prior to considering the solution over the dynamics it is necessary to solve for the initial steady state of the model through the following steps:

1. Use the calibrated value for $\bar{Q}$ to obtain the wage

$$\bar{w} = p_H(\bar{Q}, p_E^*)/A \tag{C.1.1}$$

2. Since the real wage is constant, $\bar{\pi}^W = \bar{\pi}^W$.

3. Substituting for constant inflation at steady state in the (2.4.3), we get

$$N = \left(\frac{\varepsilon - 1}{\varepsilon}\tilde{w}\tilde{C}^{-\sigma}\frac{1}{\chi}\right)^{\frac{1}{\phi}} \tag{C.1.2}$$

4. Guess aggregate consumption $\bar{C}$

5. Solve the household problem by iteration on the HJB equation (see Nuño and Thomas (2022) for solving a household block presence of idiosyncratic risk and investment in long-term bonds).

6. Compute the resulting aggregate consumption $\bar{C} = \omega \int_0^1 \bar{c}^m(a,z) f^m(a,z) dadz + (1 - \omega) \int_0^1 \bar{c}^n(a,z) f^n(a,z) dadz$ and repeat points 4-6 until convergence.

7. From the equilibrium conditions (2.7.1)-(2.7.2), we obtain:

$$A\bar{N} = (1-\alpha_E) \left(\frac{1 - \alpha_E p_E(\bar{Q}, p_E^*)^{1-\epsilon}}{1-\alpha_E} \right)^{-\frac{\epsilon}{1-\epsilon}} (1-\alpha) \left(\frac{1 - \alpha (p_F(\bar{Q})/p_D(\bar{Q}, p_E^*))^{1-\eta}}{1-\alpha} \right)^{-\frac{\eta}{1-\eta}} \tilde{C} + \alpha \left(p_H(Q, p_E^*) \bar{Q} \right)^{-\eta} C^* \tag{C.1.3}$$

   From which we can retrieve the value for foreign consumption C^* consistent with the stationary equilibrium

Once computed the initial steady state, I already exploited the degree of freedom provided by C^*, so , in order to compute the final steady state, characterized by a different D^r, I need to solve the system of equations (C.1.1),(C.1.2),(C.1.3) (now with $\tilde{C}$, $\tilde{w}$ and $\tilde{N}$, $\tilde{Q}$ showing up instead of $\bar{C}$, $\bar{w}$, $\bar{N}$ and $\bar{Q}$) together with aggregate demand (equation (2.8.1)):

$$\tilde{C} = C(\tilde{i}, \tilde{Q}) \tag{C.1.4}$$

that is a system of four variables ($\tilde{w}$,$\tilde{Q}$,$\tilde{N}$,$\tilde{C}$) in four equations. Since (C.1.4) has to be solved numerically as in points 5-6, I proceed as follows:

1. Guess $\tilde{C}$

2. Use (C.1.1),(C.1.2),(C.1.3) to get $\tilde{Q}$, $\tilde{w}$,$\tilde{N}$

3. Use $\tilde{w}$ and to solve for the households' optimization and aggregate into an updated guess $\bar{C}'$ ((as in point 5 and 6 of the initial steady state computation))

4. Update the guess:

$$\bar{C} = \bar{C} + \theta(\bar{C}' - \bar{C}) \tag{C.1.5}$$

   until convergence of the quantity $|\bar{C} - \bar{C}'|$ to a threshold small enough. The sign and magnitude of the coefficient θ depends on the parameters of the model. For my parametrization and initial guess for $\bar{C}$, imposing a positive θ leads to an explosive feedback-loop between $\bar{C}$ and $\bar{w}$, while a negative θ^S (=0.5) allows to reach convergence.

C.2 Dynamics

Let us now turn the attention to the solution over the dynamics following an unexpected shock to p^*_{Et}, under perfect foresight. Let us use the notation $\{\}$ for sequences. The algorithm unfolds as follows:

- Assume a long time horizon T for the discretized variables' path
- Start with the inputs for $\{i_t^{ex}\}$, $\{\pi_t\}$, $\{w_t\}$, $\{p^*_{Et}\}$. Notice that, to the extent that the inflation sequence allows for a time T^π such that 1) $\pi_{t \geq T^\pi} = \tilde{\pi}$ and 2) the date T_i of reversal of interest rate to Taylor rule is greater then T^π; then $\{i_t\} = \{i_t^{ex}\}$. The interest rate is indeed pinned down entirely by i_t^{ex} before T_i by assumption, while after T_i it is still equal to i_t^{ex} because there is no Taylor rule response (inflation is at steady state).
- Compute $\{Q_t\}$ by using $w_t = \{p_H(Q_t, p^*_{Et})/A\}$

Then go through the following loop:

1. Guess a value for the final steady state real mortgage stock $\tilde{D}^r$ and compute the final steady state through the same steps showcased in the second part of section C.1
2. Guess a value for $\{C_t\}$
3. Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (2.7.1)-(2.7.2))
4. Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π_t^W backward by the New Keynesian Phillips curve, starting from $\pi_T^W = 0$
5. Use the identity $\pi_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi_t^W}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$), to compute $\{\pi_t^W\}$
6. Use the UIP condition (2.6.13) to back out the path of wedges $\{\xi_t\}$ such that the values of Q_t obtained initially are consistent with the inflation path $\{\pi_t\}$
7. Starting from $D^r_{-1} = D$, use $\{\pi_t\}$ to compute the path for the real mortgage stock $\{D_t^r\}$ up to time T (leading to a final value D_T^r not necessarily equal to the guess $\tilde{D}^r$
8. At each t, compute $i_t^d = \frac{D^f}{D} i_t^f + \frac{D^v}{D} i_t$. Following the assumptions of section 2.1, we can express $i_t^f = \frac{1}{S}((S-1)i_t^f + \frac{1}{S}\sum_{\tau=0}^{S} i_{t+\tau})$ (S needs to be $\in \mathbb{N}$). If $t + \tau > T$, $i_{t+\tau}$ si set to $\tilde{i}$ (final steady state).

9. Solve the household problem with long term bonds holding (see Nuño and Thomas (2022) backward, starting from the value functions of the final steady state computed previously.

10. Compute the new path for aggregate consumption $\{C'_t\}$ by aggregating up idiosyncratic consumption levels

11. Update C_t as $C_t = (1-\upsilon)C_t + \upsilon C'_t$ for an arbitrary coefficient $\upsilon \in (0,1)$

12. Iterate until convergence of $\max |\{C_t\} - \{C'_t\}|$ to some low threshold value.

13. Update $\tilde{D}_r$ as $\tilde{D}_r = (1-\upsilon^D)\tilde{D}_r + \upsilon^D D^r_T$ for an arbitrary coefficient $\upsilon^D \in (0,1)$

14. Iterate until convergence of $\max |\tilde{D}^r - D^r_T|$ to some low threshold value.

C.3 Alternative policies

In order to solve the model for the alternative policies, we consider $\{\pi_t\}$ and $\{w_t\}$ endogenous, while $\{\xi_t\}$ and $\{\lambda_t\}$ are exogenous (they are obtained by simulating the dynamics under the benchmark case). The algorithm unoflds as follows:

- Assume a long time horizon T for the discretized variables' path.
- Consider the exogenous inputs $\{\vartheta_t\}$, $\{\xi_t\}$, $\{\lambda_t\}$, $\{i^{ex}_t\}$, $\{p^*_{Et}\}$, $\{\phi_{\pi,t}\}$, π^{target}, where π^{target} is the process for target inflation showing up in the Phillip curve (equal to $\bar{\pi}$ in the initial steady state, and by $\tilde{\pi}$ from $t = 0$ onwards). The policy time-varying coefficient $\phi_{\pi,t}$ is set to 0 until T_i and to ϕ_π from T_i onwards, as discussed in section 2.5.

1. Guess a value for the final steady state real mortgage stock $\tilde{D}^r$ and compute the final steady state through the same steps showcased at the beginning of the current section.

2. Guess a value for $\{C_t\}$

3. Use the "perfect foresight" Dynare routine to compute the paths for $\{Q_t\}$, $\{\pi^W_t\}$, P_{Dt}/P_t, w_t, N_t, π_t, i_t, given the dicretized version of equations (2.6.13), (2.4.3), (2.6.9), (2.6.10) (where I substitute away P_{Ht}/P_t by (2.3.3)), (2.7.1) (where I substitute away Y_{Ht} by (2.7.3) and $Y_{ft}+Y_{nt}$ by (2.7.2)), (2.5.1) and $\pi_t = \pi^W_t - \dot{w}_t$ (that derives from the definition of the real wage $w_t = W_t/P_t$). Teh exogenous shock to feed Dynare with are the ones detailed previously - $\{\vartheta_t\}$, $\{\xi_t\}$, $\{\lambda_t\}$, $\{i^{ex}_t\}$, $\{p^*_{Et}\}$, $\{\phi_{\pi,t}\}$, π^{target} - in addition to the guessed path for $\{C_t\}$. Initial and final model configurations are given by the initial and final steady state of the model (the latter computed in step 1).

4. Go through the point 7-14 as for the benchmark policy algorithm, and iterate until convergence of $\max|\tilde{D}^r - D^r_T|$ to some low threshold value.

D Sensitivity Analysis for a lower mortgage stock

The following robustness checks aims at generating the impulse responses to the same energy shock analyzed in the body of the text, under different parametrizations of the mortgage stock amount. I compare the effects of mortgage cost increase on consumption under the benchmark scenario ($D = 8$), with the case of a calibration $D = 5$ (Figure D.1). The paths for Q_t and w_t are unaltered, implying that the degree of redistribution of resources between mortgagors and non-mortgagors due to a mortgage rate shock does not have sizable aggregate implications. At the cross-sectional level, the reduction in consumption inequality between mortgagors and non-mortgagors is considerably less sizable (with the adoption of the smoothed policy, the consumption difference between the two groups shrinks by 5 percentage points instead of the 8 points of Figure 5.5). Overall, the same qualitative mechanisms produced in the balseline calibration carry over to this case.

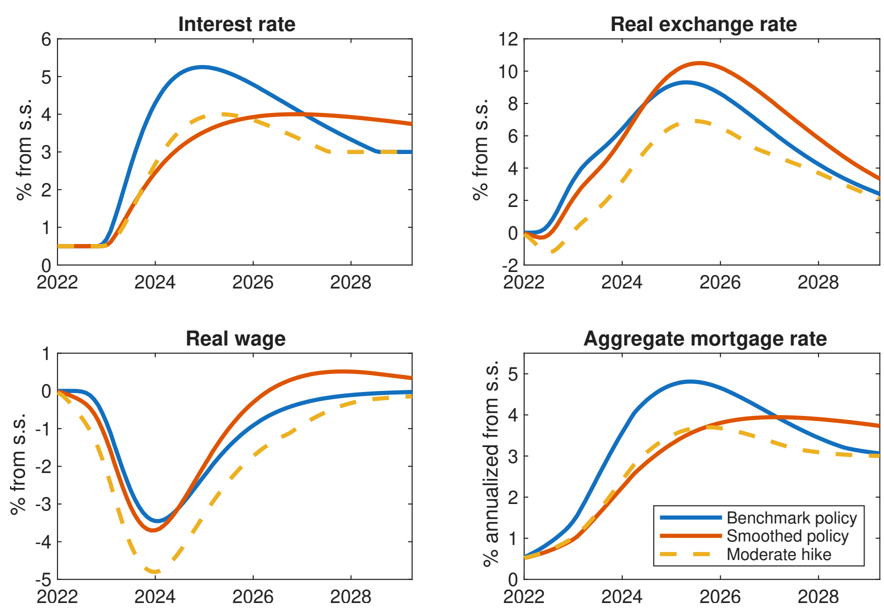


Figure D.1: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Case $D = 5$

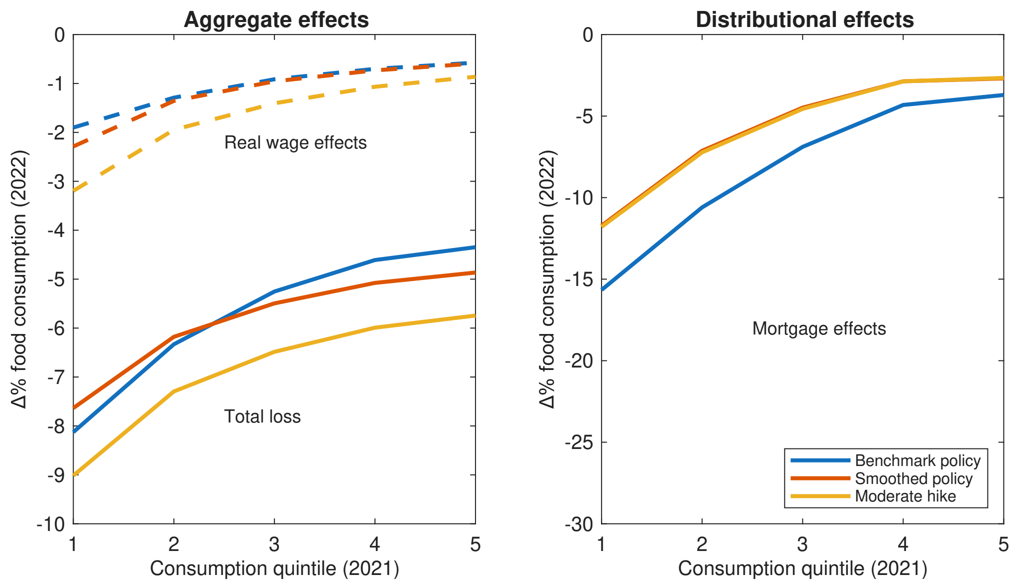


Figure D.2: Left: food consumption % fall in 2022 for each consumption quintile of the 01/2021 consumption distribution (total $\Delta_{c,f}^{total}$ and decomposed by real wage effect $\Delta_{c,f}^{w}$). Right: 2022 % consumption fall difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.5.10)). Benchmark policy vs. Smoothed policy and Moderate hike. Case $D = 5$.