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Analytics of the government expenditure multiplier with QE*

Vito Polito[1], Paulo Santos Monteiro[2], and Mike Wickens[3]

[1]University of Sheffield, CESifo

[2]University of York

[3]Cardiff University, University of York, CEPR, CESifo

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Abstract

This paper analyses the government expenditure multiplier at the zero lower bound (ZLB) in the presence of quantitative easing (QE), using a tractable New Keynesian model with financial frictions. We show that sufficiently large exogenous QE can lift the economy off the ZLB, thus yielding multipliers below one even for a small fiscal stimulus. Pre-commitment to a gradual QE unwinding further reduces the fiscal stimulus needed to keep multipliers below unity. When QE instead follows an instrument rule that responds to conventional monetary shortfalls, the multiplier can remain below one even at the ZLB. This result also holds under an optimally designed, welfare-based QE policy. Our analysis provides theoretical support for the growing empirical evidence that government spending multipliers can remain below unity at the ZLB.

JEL codes: E52, E58, E62.

Keywords: Government expenditure multiplier; Zero lower bound; Quantitative easing.

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1 Introduction

Macroeconomic theory based on dynamic general equilibrium modeling has long established that the output multiplier of government expenditure varies significantly depending on the size of the fiscal stimulus and the extent of monetary policy accommodation (Woodford, 2011). When conventional monetary policy responds to a fiscal expansion by raising real interest rates, theoretical models suggest that the fiscal multiplier is below one. By contrast, when monetary policy is constrained, such as when the zero lower bound (ZLB) on the short-term nominal interest rate is binding, the multiplier is often found to exceed one and can potentially be quite large, as long as the temporary increase in government spending is sufficiently small so that the ZLB remains binding. This distinction in the size of the government expenditure multiplier across the two regimes (away from and at the ZLB) has become widely accepted and has arguably shaped the policy framework for evaluating the effects of fiscal stimuli in periods of low interest rates when many central banks maintained policy rates near the ZLB for extended periods (Eggertsson, 2008, 2011; Erceg and Lindé, 2014).

However, this conventional view overlooks that, in practice, central banks can utilise a broad array of tools to stimulate inflation and output at the ZLB. Among the most prominent of these are large-scale asset purchases, often referred to as quantitative easing (QE), which became a prominent tool of monetary policy following the 2007/08 global financial crisis and again during the Covid-19 pandemic, when policy rates remained near the ZLB for extended periods. This paper reexamines the government expenditure multiplier at the ZLB in the presence of QE. Our methodological contribution lies in employing an analytically tractable New Keynesian framework, which allows us to derive closed-form solutions for the multiplier and to provide a transparent illustration of the effects of QE.

This is an important inquiry because the growing reliance on QE since the late 2000s has led central bank asset holdings in advanced economies to reach levels unseen since World War II (Bhattarai and Neely, 2022). At the same time, three strands of the empirical evidence suggest that QE may have a significant impact on the output multiplier of government expenditure. First, a growing body of empirical research, both for the United States and internationally, shows that QE is a close substitute for conventional monetary policy at the ZLB, stimulating output and prices similarly to interest rate cuts (Gambacorta et al., 2014; Swanson and Williams, 2014; Weale and Wieladek, 2016; Dahlhaus et al., 2018; Dell'Ariccia et al., 2018; Swanson, 2018, 2021). This evidence casts doubt on whether the ZLB is, in practice, a binding constraint for a central bank. Second, evidence from both reduced-form and structural macroeconomic models suggests that QE may offset the constraints on conventional policy. Debortoli et al. (2020) estimate a VAR with time-varying coefficients and find that macroeconomic responses in the United States changed little between the pre-2007/08 crisis period and the ZLB era, coining the ZLB irrelevance hypothesis. Using a medium-scale dynamic stochastic general equilibrium (DSGE) model, Sims and Wu (2021) find that output responses to government expenditure shocks are nearly identical on and off the ZLB, provided QE follows an instrument rule with feedback on output and inflation. Third, the econometric evidence on the magnitude of the government expenditure multiplier at the ZLB remains, at best, unsettled. Early studies suggest multipliers above one, but only on impact or in the very short-term (Ramey and Zubairy, 2018; Miyamoto et al., 2018). More recent work, finds the government expenditure multiplier to be consistently below one during ZLB periods after accounting for the asymmetric effects of positive and negative government spending shocks (Barnichon et al., 2022), or when more flexible econometric methodologies are employed (Inoue et al., 2024).

This paper aims to reconcile theory with empirical evidence. We build on the New Keynesian framework of Sims et al. (2023), in which financial frictions restrict some households to trading only in long-term bonds, and financial intermediaries face a binding leverage constraint on their holdings of long-term bonds. As in Gertler and Karadi (2013), QE affects real quantities and prices because the central bank purchases long-term bonds from intermediaries, relaxing their constraint and expanding credit to constrained households. We introduce exogenous government purchases, enabling the analysis of the multiplier in a way analogous to Woodford (2011), thus preserving analytical tractability and ensuring closed-form solutions for the government expenditure multiplier and transparency in the effects of QE. Crucially, our framework nests Woodford (2011) as a special case, allowing direct comparison of fiscal multipliers with and without QE in the New Keynesian setting.

We first establish how the financial frictions added to the New Keynesian model to make QE effective influence fiscal transmission away from the ZLB, providing a benchmark useful to evaluate the multiplier at the ZLB. To do so, we compare the multiplier in our New Keynesian model with two reference frameworks considered in Woodford (2011): a flexible-price equilibrium and a sticky-price model with a Taylor rule. Under flexible prices, the classical result that government spending crowds out private consumption holds, but financial frictions in our model attenuate this effect, yielding a multiplier lower than one, but higher than in the New Keynesian model with flexible prices. In the sticky-price model, fiscal stimulus raises output and inflation, prompting an interest rate response through the Taylor rule that dampens aggregate demand. This crowding out of demand is reduced by financial frictions, producing a multiplier larger than in the canonical New Keynesian model but still below one.

The ZLB is introduced through a perturbation of the zero-inflation perfect-foresight equilibrium in which the natural real rate turns negative absent policy intervention (following Eggertsson, 2011; Woodford, 2011). We first analyse an exogenous QE intervention, meaning it is independent of the size of the fiscal stimulus. As in Woodford (2011), the government spending multiplier at the ZLB is state-dependent: when fiscal stimulus is small, the multiplier exceeds one because constrained monetary policy lowers real interest rates in response to fiscal expansion; if fiscal stimulus is large enough to lift the economy off the ZLB, the multiplier falls below one as conventional policy raises the real rate. In our model, however, the fiscal stimulus required to exit the ZLB declines with the size of QE. Although QE does not affect the multiplier within each regime, it influences the multiplier through an extensive margin effect by shifting the threshold for regime switching. By boosting output and prices, an exogenous QE injection enables exit from the ZLB at lower levels of fiscal stimulus. If the exogenous QE stimulus is sufficiently large, so that conventional policy is restored, then the output multiplier is below one irrespective of the magnitude of the fiscal stimulus. Because the size of the central bank's steady state balance sheet amplifies the macroeconomic effects of exogenous QE, output multipliers below one become even more likely as the central bank's equilibrium asset holdings grow.

We next ask how the path of QE normalisation, as the natural rate returns to its long-run level, shapes fiscal multipliers at the ZLB. We model normalisation as a credible commitment, made at the onset of the ZLB episode, to maintain elevated central bank asset holdings with gradual unwinding occurring probabilistically once the natural rate recovers. In each period after the natural rate returns to its long-run level, asset holdings remain elevated with some probability. We show that this commitment raises expectations of future economic expansion, enhancing the effectiveness of QE at the ZLB. As a result, the fiscal stimulus required to exit the ZLB declines further. This mirrors the expectation channel of forward guidance on the nominal interest rate, highlighted by Eggertsson and Woodford (2003) and Eggertsson et al. (2004). Werning (2011) considers how optimal monetary policy at the ZLB, including forward guidance, affects the design of fiscal policy. In our paper, we show that commitment to gradual QE unwinding amplifies the extensive margin effect of QE, implying that persistent expansions of the central bank's balance sheet are associated with government spending multipliers below unity.

Next, we study the output multiplier at the ZLB when QE is determined according to an implementable instrument rule that conditions asset purchases on the shortfall in conventional monetary policy at the ZLB, defined as the gap between the desired nominal interest rate, which is negative in the face of a sufficiently adverse natural rate shock, and the ZLB. QE becomes inactive once the economy exits the ZLB, but substitutes conventional monetary policy while the ZLB remains binding, tightening in response to fiscal expansions.[1] Under such a rule, the extensive margin effect vanishes, but QE affects the government expenditure multiplier through an intensive margin channel, influencing its magnitude while the ZLB persists. If the rule-based response of QE to the fiscal stimulus is sufficiently aggressive, the multiplier can fall below unity even while the ZLB continues to bind. A larger central bank's steady state balance sheet amplifies the impact of QE: thus any increase in inflation and output triggered by the fiscal stimulus prompts a more aggressive QE tightening at the ZLB, leading to a lower government spending multiplier. This result helps reconcile the theory with empirical findings that government spending multipliers at the ZLB are often estimated to be below one (Barnichon et al., 2022; Inoue et al., 2024). The response of QE thus plays a crucial role in shaping the macroeconomic effects of fiscal policy.

Lastly, we analyse the effects of a central bank implementing QE at the ZLB through an optimal policy rule that minimises a micro-founded welfare function, subject to the Phillips and IS curve constraints. The central bank's loss function reflects not only traditional stabilisation objectives (minimising the volatility of inflation, the output gap, and government expenditure), but also consumption distribution between

[1] Such a rule is similar to that proposed, for example, in Gertler and Karadi (2011), although in their work the instrument rule targets credit spreads.

borrowers and savers, consistent with prior work emphasising the role of consumption inequality in optimal policy (e.g., Bilbiie, 2024; Bonciani and Oh, 2025; Wu and Xie, 2025).[2] We show that, during a ZLB episode, the optimal QE policy takes the form of a feedback rule and QE continues to operate through an intensive margin channel, influencing the magnitude of the multiplier at the ZLB. Optimal QE is shown to decline with the size of the fiscal stimulus. The optimal QE stimulus also declines the larger is the size of the central bank balance sheet in normal times. We characterise conditions under which optimal QE causes the government expenditure multiplier to fall below one even at the ZLB. We argue that these conditions are satisfied under conventional model calibrations.

Related Literature. The paper contributes to several strands of the literature on stabilisation policy at the ZLB. First, it speaks to the extensive body of work analyzing government spending multipliers in New Keynesian models (see, e.g., Christiano et al., 2011; Eggertsson, 2011; Woodford, 2011; Erceg and Lindé, 2014). This literature has long established two main propositions. First, a sufficiently aggressive, temporary expansion of fiscal policy can facilitate exit from the ZLB, when implemented alongside accommodative monetary policy. Second, the output multiplier of government expenditure exceeds unity when nominal interest rates are constrained at the ZLB. Our paper revisits these results within a setting where unconventional monetary policy, in the form of QE, plays a central role. We show that whether QE operates through direct stimulus or expectation channels, the conventional view overstates the size of the fiscal stimulus required to exit the ZLB. Moreover, if QE responds to the conventional monetary policy shortfall or depends on a micro-founded optimal rule, the output multiplier of government expenditure can be lower than unity even while the economy remains at the ZLB.

Second, our work contributes to the literature that uses New Keynesian models with constrained financial intermediaries to study the role of QE as a monetary policy instrument, (see, e.g., Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013; Del Negro et al., 2017; Sims and Wu, 2021). We build on the analytically tractable framework of Sims et al. (2023), which embeds financial frictions and QE in a

[2]Working with the same model we do, Wu and Xie (2025) also show that the quadratic approximation to the welfare-theoretical planner's loss function depends not only on the volatility of the output gap and inflation, but also the cross-sectional consumption dispersion between Savers and Borrowers. However, contrary to us, their analysis focuses on stabilisation outside of the ZLB, and they can show that, together, conventional monetary policy and QE eliminate the trade-off between aggregate stabilisation and redistribution. We show that at the ZLB, optimal QE alone cannot achieve both objectives.

four-equation New Keynesian model. While prior work has focused on the stabilising effects of QE on macroeconomic and financial shocks, we shift attention to its interaction with fiscal policy: studying how the government expenditure multiplier at the ZLB depends on the central bank's conduct of QE, including exogenous and rules-based asset purchases, as well as credible commitments on the path of balance sheet normalization.

Third, our paper relates to recent work on the design of optimal central bank balance sheet policies and their interaction with fiscal policy and other policy instruments. Corbellini (2024) and Wolf (2025) study the substitutability between interest rate policy and government transfers. More closely related to our work, Wu and Xie (2025) study fiscal multipliers in a New Keynesian model with QE, similar to Sims and Wu (2021) and Sims et al. (2023), but they abstract from the ZLB. Their focus is on how the method of fiscal financing affects multipliers in a setting without Ricardian equivalence. Bonciani and Oh (2025) also study optimal QE and forward guidance at the ZLB, using a micro-founded objective function of the central bank derived from the Sims et al. (2023) model. Like us, they emphasise the trade-off between stabilisation and redistribution, with the latter dampening optimal QE. In contrast, our paper considers how optimal QE, as well as announcements about the balance sheet normalization, affects the government expenditure multiplier. Thus, our paper also relates to a recent literature advocating for the gradual reduction of central bank balance sheets following large-scale asset purchases (Karadi and Nakov, 2021; Cantore and Meichtry, 2024; Harrison, 2024). Our focus is on how credible announcements about the path of balance sheet normalisation affect the output multiplier of government spending during ZLB episodes.

The paper is structured as follows. Section 2 outlines the economic environment and derives the multiplier under flexible and sticky prices away from the ZLB. Section 3 analyses the government expenditure multiplier with exogenous QE. Section 4 considers balance sheet normalisation. Sections 5 and 6 study the multiplier when QE follows, in turn, an instrument rule and an optimal policy rule. Section 7 summarizes the results and concludes. Additional analytical details are provided in the Appendices.

2 Model

We work with the four-equation New Keynesian model of Sims et al. (2023). This allows us to analyse the size of the government spending multiplier analytically, as in Woodford (2011), but in a setting where unconventional monetary policy can help overcome the ZLB. The model features two types of households, savers and borrowers, who differ in their degree of patience and access to financial markets, producers who set prices in a staggered fashion, financial intermediaries, the central bank, and the government.[3]

Production is organised across three stages: wholesalers hire labour and produce an undifferentiated wholesale good using a technology linear in employment; monopolistic retailers, $\nu \in [0, 1]$, purchase the wholesale good to produce a differentiated intermediate good, $y(\nu)$, and set prices subject to nominal frictions as in Calvo (1983), with $\theta \in (0, 1)$, the share of firms unable to reset price each period; lastly, a final good producer aggregates the differentiated intermediate goods to produce the final output, a Dixit and Stiglitz (1977) composite good, $Y = \left(\int_0^1 y(\nu)^{1/\epsilon} d\nu\right)^\epsilon$, with $\epsilon > 1$, the elasticity of substitution across differentiated intermediate varieties.

Patient households (the savers) consume, supply labour, and own equity shares in retail firms and financial intermediaries. They receive wage income and dividends from both firms and financial intermediaries, make equity transfers to financial intermediaries, pay taxes in the form of lump sums, and can only save through short-term, one-period deposits in the financial intermediaries. Savers have separable flow utility functions over their consumption, C, and employment, N, given by $u(C) - v(N) = C^{1-1/\sigma}/(1 - 1/\sigma) - N^{1+\eta}/(1 + \eta)$, with $\sigma > 0$, the elasticity of intertemporal substitution, and $\eta > 0$, the inverse Frisch elasticity of labour supply. Their discount factor is $\beta \in (0, 1)$. Impatient households (the borrowers) have a lower discount factor than savers, do not earn labour income, and finance consumption through borrowing and lump sum transfers received from the government. Their flow utility is given by $u\left(C^b\right) = \left(C^b\right)^{1-1/\sigma}/(1 - 1/\sigma)$, with C^b their consumption. Financial markets are segmented and, in particular, borrowers can only issue long-term nominal bonds.

Financial intermediaries have a lifetime span of one period, with a new intermediary entering the model

[3] Appendix A provides a more detailed description of the economic environment and the model's equilibrium conditions.

each period. The liability side of their balance sheet includes short-term deposits received from savers and equity injections. The latter includes two components, new equities from savers and equity transfers from the preceding financial intermediary, corresponding to the stock of outstanding long-term bonds held previously. On the asset side, there are reserves held with the central bank and long-term bonds. Financial intermediaries' holdings of long-term bonds are subject to a binding leverage constraint such that the value of long bonds held cannot exceed a multiple of the nominal net worth. This is imposed as an exogenous constraint. Similar to New Keynesian models that include financial intermediation in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), the binding leverage constraint could be micro-founded assuming that financial intermediaries may misappropriate funds.

There are two types of monetary policy. Conventional monetary policy changes the short-term nominal interest rate according to some rule. Unconventional monetary policy, namely QE, changes the size of the central bank's balance sheet. This includes long-term bonds on the asset side, interest-bearing reserves on the liabilities side. QE consists of creating interest-bearing reserves to finance purchases of long-term bonds issued by the borrowers that sit in the balance sheet of financial intermediaries. The transmission mechanism of QE to output and inflation includes two channels. When the central bank purchases long-term bonds, it eases the leverage constraint faced by financial intermediaries, leading to an expansion in the supply of credit to borrowers, which in turn stimulates aggregate demand.[4] Thus expansionary QE has effects on output and prices equivalent to those of a positive demand shock. At the same time, QE shifts resources from savers, who supply labour elastically, to borrowers, whose labour supply is inelastic. Consequently, QE generates a negative wealth effect for savers, reducing wages and real marginal costs in a manner similar to a favourable credit shock. For this reason QE is less inflationary than conventional monetary policy, a prediction generally supported by the empirical evidence indicating that, since the 2007/08 financial crisis, the expansion of central bank balance sheets in major advanced economies has had a weaker and less persistent effect on the price level compared to conventional monetary policy, see Gambacorta et al. (2014); Swanson (2023).[5] Further, in the log-linearised equilibrium, the size of

[4]The binding leverage constraint induces a positive interest rate spread between long- and short-term bonds. Thus expansionary QE reduces the (term) spread. The transmission mechanism of QE through spreads has long been formalised in theoretical models, see (see, e.g., Gertler and Karadi, 2011, 2013; Curdia and Woodford, 2011; Chen et al., 2012; Del Negro et al., 2017).

[5]Against this, see Aruoba et al. (2022).

the steady state central bank's balance sheet amplifies the macroeconomic effects of QE on output and inflation.

The government finances an exogenous stream of purchases, G, and transfers to borrowers by levying lump-sum taxes on savers. As in Woodford (2011), a change in the path of total government expenditure is assumed to imply a change in the path of lump-sum tax collections to maintain intertemporal government solvency.[6] Final output is purchased for consumption by households and by the government subject to the aggregate resources constraint, $Y = C + C^b + G$.

2.1 The aggregate economy

The equilibrium conditions can be approximated around the zero-inflation steady state to derive the Phillips and IS curves that determine the aggregate dynamics of inflation, π_t, and of the output deviation from its steady state, $\widehat{Y}_t \equiv \left(Y_t - \overline{Y}\right)/\overline{Y}$. These curves capture the transmission mechanisms of conventional monetary policy, which controls the nominal interest rate, i_t; unconventional monetary policy, measured as the deviation of QE from its steady state, $\widehat{\texttt{QE}}_t \equiv \left(\texttt{QE}_t - \overline{\texttt{QE}}\right)/\overline{\texttt{QE}}$; and fiscal policy, captured by the deviation of government consumption from its long-run steady state, expressed as a proportion of the steady-state output, $\widetilde{G}_t \equiv \left(G_t - \overline{G}\right)/\overline{Y}$. In particular, the equations describing the Phillips and IS curves are given by[7]

$$\pi_t = \kappa\left(\widehat{Y}_t - \Gamma\widetilde{G}_t\right) + \beta E_t\pi_{t+1} - \varsigma\widehat{\texttt{QE}}_t, \tag{1}$$

$$\widehat{Y}_t - \widetilde{G}_t - \xi\widehat{\texttt{QE}}_t = E_t\left(\widehat{Y}_{t+1} - \widetilde{G}_{t+1} - \xi\widehat{\texttt{QE}}_{t+1}\right) - \varphi\left(i_t - E_t\pi_{t+1} - r^n\right). \tag{2}$$

The parameter $\kappa \equiv \lambda\left(\frac{1}{\varphi} + \eta\right) > 0$ stands for the elasticity of inflation to changes in the output gap, where $\lambda \equiv (1-\theta)(1-\theta\beta)/\theta$ is the elasticity of inflation with respect to marginal cost, and $\varphi \equiv (1-g)(1-z)\sigma > 0$ is the intertemporal elasticity of substitution of total household expenditure, with $g \equiv \overline{G}/\overline{Y}$, the government consumption share of output, and $z \equiv \overline{C^b}/\left(\overline{C} + \overline{C^b}\right)$, the steady-state share of

[6]We abstract from inclusion of government bonds for analytical tractability. This is without loss of generality, because the qualitative effects of QE through government bond purchases mirror those for private bonds, differing only quantitatively by a constant fraction, as shown in Sims and Wu (2021).

[7]Appendix A presents the model's log-linear equilibrium conditions and describes the derivation of the Phillips and IS curves, equations (1) and (2).

borrower consumption. The parameter $\Gamma \in (0,1)$, as shown below, measures the government expenditure multiplier with flexible prices. Thus, $\widehat{Y}_t - \Gamma \widetilde{G}_t$ corresponds to the output gap defined as the the number of percentage points by which aggregate output exceeds the flexible-price equilibrium level. The parameter $\varsigma \equiv (\lambda/\varphi)\,\xi$ stands for the elasticity of inflation to changes in QE, with $\xi \equiv (1-g)\,z\left(\bar{\mathtt{b}}^{\mathrm{cb}}/\bar{\mathtt{b}}\right) \in (0,1)$, the elasticity of private expenditure to changes in QE, which is positively related to the size of the central bank's balance sheet in steady state, $\bar{\mathtt{b}}^{\mathrm{cb}}$, as a proportion of the total the steady-state real value of long-term bonds, $\bar{\mathtt{b}}$ in the economy. Thus, increase of the relative size of the central bank balance sheet amplifies the impact of QE on output and inflation. The parameter ς enters the Phillips curve with a negative sign because of the wealth effect exerted by QE on savers described above. The parameter $r^n \equiv -\log\beta$ stands for the short-term real interest rate consistent with a stationary equilibrium, price stability and the absence of any unconventional monetary policy (so that, $\widehat{Y}_t = 0$, $\pi_t = 0$ and $\widehat{\mathtt{QE}}_t = 0$, for all t), which we refer to as the long-run natural rate of interest. Crucially, all elasticity parameters that determine aggregate output and inflation, namely κ and ς in the Phillips curve (1) and ξ and φ in the IS equation (2), are a function of z, the degree of financial frictions in the economy. Simply setting $z = 0$ recovers the canonical New Keynesian model with government expenditure, used in Woodford (2011) to study the analytics of the government expenditure multiplier. This is important because it means that Woodford (2011)'s model is nested in equations (1) and (2) as a special case, thereby providing a useful benchmark against which to relate our results on the government expenditure multiplier.

Figure 1 highlights our key observation, that motivates this paper: the substitutability between conventional monetary policy and QE. It is based on a standard calibration of the model in which the ZLB does not bind and represented by equations (1), (2), a Taylor (1993)-type rule for the nominal rate of interest and an autoregressive process of order one for QE. We illustrate the response of inflation and output to two alternative policy shocks each calibrated to raise the output gap by one percent. The first is a conventional expansionary monetary policy shock (a surprise reduction in the nominal interest rate). The second is an unanticipated increase in the central bank's asset purchases (a QE shock). Both output and inflation rise in response to expansionary shocks from conventional monetary policy and QE.[8] These results suggest

[8]The lower inflation response to the QE shock, compared to the conventional monetary policy shock for the same output gap increase, indicates that QE is less inflationary.

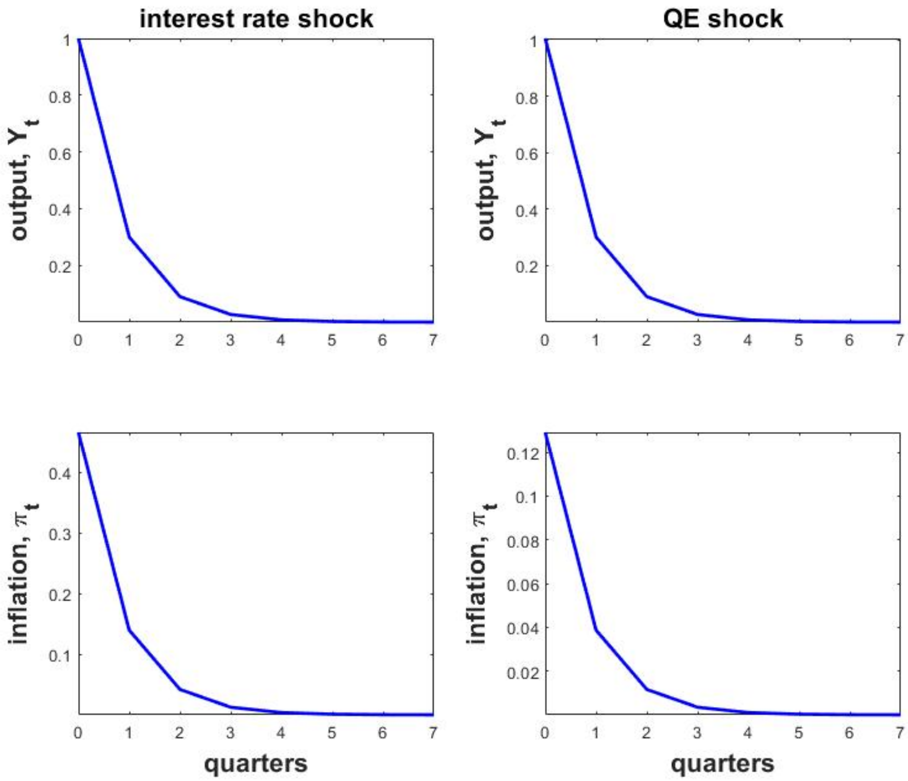


Figure 1: Percentage change of output and inflation to an expansionary shock to the interest rate (left panel) and QE (right panel). Both shocks are calibrated to deliver a one percent increase in the output gap. The model consists of the Phillips curve (1); the IS curve (2); the interest rate rule $i_t = r^n + 1.5\pi_t + 0.5\widehat{Y}_t + \epsilon_{i,t}$, with $\epsilon_{i,t} = \rho_{\epsilon_i}\epsilon_{i,t-1} + u_{i,t}$; and the QE rule $\widehat{\mathtt{QE}}_t = \rho_{qe}\widehat{\mathtt{QE}}_{t-1} + u_{qe,t}$. Both $u_{qe,t}$ and $u_{i,t}$ are independent and identically distributed errors with zero mean and unit variance. The model parameters are calibrated as follows: $\theta = 3/4$, $\beta = 0.99$, $\eta = 2$, $\sigma = 1$, $g = 0.2$, $z = 0.33$, $\overline{\mathtt{b}}^{\text{cb}}/\overline{\mathtt{b}} = 0.3$.

that, even when the nominal interest rate is constrained by the ZLB, monetary policy remains effective. It therefore raises questions about the government expenditure multiplier's divergence between ZLB-binding and non-binding periods, particularly when QE operates systematically like conventional monetary policy. The remainder of the paper delves into this issue in detail.

2.2 Multiplier with flexible prices

We first characterise the government expenditure multiplier that is obtained under flexible prices.[9] To do this, we simply make use of the property in the New Keynesian model with Calvo (1983) prices, that if $\pi_t = 0$ for all t (strict inflation targeting), then the resulting equilibrium allocations correspond

[9] Appendix B gives additional details on all analytical results included in this section.

to the flexible price allocations, as there are no relative price distortions in the absence of inflation. The consequence of imposing $\pi_t = 0$ for all t is that, in the absence of QE (with $\widehat{\mathtt{QE}}_t = 0$), the flexible-price equilibrium level of output is determined from the Phillips curve in equation (1) as

$$\widehat{Y}_t = \Gamma \widetilde{G}_t.$$

Recall that $\widetilde{G}_t \equiv \left(G_t - \overline{G}\right)/\overline{Y}$, the change in government expenditure from its long-run level relative to steady-state output. Thus, under flexible prices, an increase in government expenditure totaling one percent of GDP raises output by Γ percentage points, the government expenditure multiplier.

To uncover the determinants of Γ, we combine the equilibrium labour supply and market clearing conditions for the full model. In particular, making use of the labour market equilibrium conditions, we obtain

$$\left(Y - C^b - G\right)^{-1/\sigma} = \left(\frac{\epsilon}{\epsilon - 1}\right) Y^{\eta}.$$

Implicit differentiation in the neighborhood of the steady state yields

$$\Gamma \equiv \frac{d\widehat{Y}}{d\widetilde{G}} = \frac{dY}{dG} = \frac{1/\varphi}{1/\varphi + \eta} < 1. \tag{3}$$

This establishes that, with flexible prices, the government expenditure multiplier is less than unity. One interesting feature of the result in equation (3) is that the limit of Γ is unity as the share of total consumption going to borrowers goes to one, $z \to 1$. This is because an increase in aggregate demand caused by higher government expenditure only generates crowding out through the effect it exerts on employment. But as $z \to 1$, we have $\varphi \to 0$, meaning that the intertemporal elasticity of substitution of private expenditure goes to zero. This happens because only savers undertake intertemporal labour supply substitution, and aggregate private sector expenditure is no longer crowded out by an increase in government consumption if their consumption share becomes negligibly small.

Conversely, as $z \to 0$, financial frictions cease to have any bearing on the determination of the equilibrium. The model then reduces to a standard neoclassical general equilibrium framework with flexible prices and perfect competition. As a result, the government expenditure multiplier near the steady state becomes

$\Gamma_{\texttt{neoclassical}} \equiv 1/\left(1+\eta(1-g)\sigma\right)$. This is the same government expenditure multiplier determined in Woodford (2011), and represents a lower bound for Γ in (3).

2.3 Multiplier with sticky prices and a Taylor rule

The above result determines the government expenditure multiplier under flexible prices or under a strict inflation target, such that $\pi_t = 0$ for all t, thus avoiding any relative price dispersion even with sticky prices. Next, we study how the distortions caused by staggered-price stickiness influence the size of the government expenditure multiplier when monetary policy is set following an implementable Taylor-type rule of the form

$$i_t = r^n + \phi_\pi \pi_t + \phi_y \left(\widehat{Y}_t - \Gamma \widetilde{G}_t\right), \tag{4}$$

with $\phi_\pi > 1$ and $\phi_y > 0$ as in Taylor (1993), and $\widehat{Y}_t - \Gamma \widetilde{G}_t$ denoting the welfare relevant output gap.[10]

For the present, we still assume that the economy is never at the ZLB and there is no QE, so that $\widehat{\texttt{QE}}_t = 0$ for all t. To derive the multiplier, we follow the same steps as in Woodford (2011). Thus, we conjecture a linear solution for the endogenous variables that takes the form

$$\begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ i_t - r^n \end{bmatrix} = \begin{bmatrix} \gamma_y \\ \gamma_\pi \\ \gamma_i \end{bmatrix} \widetilde{G}_t,$$

and we let government expenditure be serially correlated according to $E_t \widetilde{G}_{t+1} = \rho \widetilde{G}_t$, with $\rho \in (0,1)$. We then substitute the conjectured solution in (1), (2) and (4) and employ the method of undetermined coefficients to obtain γ_y, γ_π, and γ_u. This yields the government expenditure multiplier under the Taylor rule, given by

$$\Gamma_{\texttt{Taylor rule}} = \gamma_y = \frac{1-\rho+\psi\Gamma}{1-\rho+\psi} < 1, \tag{5}$$

with $\psi \equiv \varphi\left(\phi_y + \kappa\left(\phi_\pi - \rho\right)\right)/\left(1-\rho\beta\right) > 0$. Consequently, $\Gamma < \Gamma_{\texttt{Taylor rule}} < 1$. Thus, the government expenditure multiplier when monetary policy is set according to the Taylor rule in (4) is larger than

[10] Appendix B provides additional analytical details on the multiplier derived in this section.

that obtained under the strict inflation targeting regime, but is still smaller than one.

This multiplier is analogous to that obtained in Woodford (2011) using a Taylor rule as in (4). It is higher than under flexible prices, because the Taylor rule allows inflation to increase in response to the fiscal stimulus; but it is lower than one because the real interest rate rises in response to the increases in inflation and the output gap. The extent to which the rise of the real interest rate dampens the government expenditure multiplier depends on the intertemporal elasticity of substitution of private expenditure, φ. In our model, however, this elasticity depends on the share of consumption of borrowers, z. The larger this share, the smaller is the intertemporal elasticity of substitution, and the smaller is the coefficient ψ. For this reason, the multiplier is higher than that predicted by the canonical New Keynesian model in a similar setting with sticky prices and a Taylor rule.[11]

3 Fiscal policy at the ZLB with exogenous QE

We now describe the conditions under which the economy may encounter the ZLB. Following Eggertsson (2011) and Woodford (2011), we assume a two-state Markov equilibrium comprised of an initial crisis state triggered by a large negative shock to the natural interest rate. This requires extending the model to include a natural real rate shock, δ_t, so that instead of (2) we now have the following IS equation

$$\widehat{Y}_t - \widetilde{G}_t - \xi\widehat{\mathtt{QE}}_t = E_t\left(\widehat{Y}_{t+1} - \widetilde{G}_{t+1} - \xi\widehat{\mathtt{QE}}_{t+1}\right) - \varphi\left(i_t - E_t\pi_{t+1} - r^n + \delta_t\right), \tag{6}$$

with $r^n_t = r^n - \delta_t$, the natural real rate at date t.[12]

In period t there is a sufficiently large shock, $\delta_t = \delta_L$, such that the natural rate becomes negative, $r^n_t = r^n - \delta_L < 0$, and the ZLB constraint is binding in the absence of any fiscal stimulus or QE. Each period after t there is a probability $\mu \in (0, 1)$ that the shock persists and the natural rate remains negative,

[11] As $z \to 1$, the multiplier $\Gamma_{\mathtt{Taylor\ rule}}$ becomes larger and closer to one. Similarly, the government expenditure multiplier is larger with a flat Philips curve (small κ).

[12] The nature of the shock that brings the economy to the ZLB affects the size of the government expenditure multiplier. Mertens and Ravn (2014) consider sunspot shocks, such as a drop in household confidence, where higher government expenditure deepens households' pessimism, thereby dampening the fiscal stimulus effects and yielding multipliers below one at the ZLB. To align with the canonical New Keynesian literature described above, we instead work with fundamental shocks, like a rise in households' preference for future consumption.

whereas with probability $1-\mu$ the natural real rate returns to its normal value, $r_t^n = r^n > 0$. We denote the date at which the natural real rate returns to its normal value as $T > t$, with T a random variable. Once this occurs, both monetary and fiscal policies are conducted so that the economy returns immediately to the perfect foresight steady-state equilibrium, $\widetilde{G}_{T+s} = \widehat{Y}_{T+s} = \pi_{T+s} = 0$, for all $s \geq 0$. The Taylor rule describing conventional monetary policy is adjusted to explicitly reflect that the short-term interest rate is constrained by the ZLB with the result that

$$i_t = \max\left\{0, r^n + \phi_\pi \pi_t + \phi_y \left(\widehat{Y}_t - \Gamma \widetilde{G}_t\right)\right\}. \tag{7}$$

The model is entirely forward-looking and can yield a unique bounded solution for the endogenous variables, $\left(\widehat{Y}_t, \pi_t, i_t\right)$ during the crisis state, such that $\left(\widehat{Y}_t, \pi_t, i_t\right) = \left(\overline{Y}_L, \overline{\pi}_L, i_L\right)$ for all $t < T$. Of course, the existence of such an equilibrium hinges on whether solving forward the equilibrium conditions (1), (6) and (7) yields a unique bounded stationary solution. This is the case as long as the model parameters satisfy the restriction

$$\mathcal{R} \equiv (1-\mu)(1-\beta\mu) - \kappa\varphi\mu > 0, \tag{8}$$

which places an upper limit on the private sector's expectation about the duration, $1/(1-\mu)$, of the negative interest rate shock.[13]

The characterisation of the two-state Markov equilibrium is completed by specifying the paths of government expenditure and QE, given by

$$\begin{bmatrix} \widetilde{G}_t \\ \widehat{\mathtt{QE}}_t \end{bmatrix} = \begin{cases} \left(\overline{G}_L \geq 0, \overline{\mathtt{QE}}_L \geq 0\right)' & \text{for } t = 0, \dots, T-1, \\ (0,0)' & \text{for } t \geq T, \end{cases} \tag{9}$$

implying that government expenditure and QE are allowed to rise above their perfect-foresight steady-state equilibrium values while the economy is in the crisis state. In particular, the path of QE in (9) means that the central bank balance sheet expands only during the crisis state, thereby mirroring central bank behaviour in major advanced economies following the 2007/08 financial crisis and the Covid-19 pandemic.

[13] Appendix C provides additional details on all analytical results presented in this section.

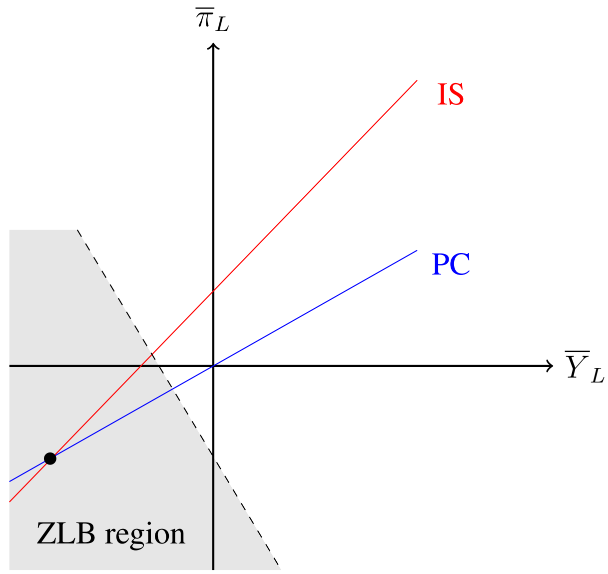


Figure 2: Equilibrium at the ZLB in the absence of fiscal stimulus and QE.

We start by considering the simplest hypothesis for the design of QE in (9). Namely, that the size of the QE stimulus, $\overline{\mathtt{QE}}_L > 0$, is exogenous, meaning that the amount of QE is independent, in equilibrium, of the amount of fiscal stimulus.[14]

Given the equilibrium allocations in the perfect foresight state, we can solve the model backward and determine the equilibrium allocations while $t < T$ by replacing the paths of government expenditure and QE in (9) into equations (1) and (6), obtaining the Phillips and IS curves during the crisis state as

$$\overline{\pi}_L = \kappa\left(\overline{Y}_L - \Gamma\overline{G}_L\right) + \beta\mu\overline{\pi}_L - \varsigma\overline{\mathtt{QE}}_L, \tag{10}$$

$$\overline{Y}_L = \mu\overline{Y}_L + (1-\mu)\,\overline{G}_L + (1-\mu)\,\xi\overline{\mathtt{QE}}_L - \varphi\left(i_L - \mu\overline{\pi}_L - \overline{r}_L\right), \tag{11}$$

where $\overline{r}_L = r^n - \delta_L < 0$. As in Woodford (2011), the equilibrium values of output and inflation during the crisis depend on the extent of any fiscal stimulus undertaken. In our framework, however, these values also depend on the type of QE policy implemented while the economy is in the crisis state.

Figure 2 illustrates graphically the equilibrium solution to the system of equations (10) and (11)when the ZLB constraint is binding. According to the the Taylor rule (7), $i_L = 0$. Thus, the IS curve is positively

[14] Notice that such an exogenous QE rule obtains if, for example, QE is set contingent on the exogenous natural rate shock, δ_L. Thus, this is analogous to the assumption in Del Negro et al. (2017) where QE is represented as a function of an exogenous liquidity shock, and also that in Sims et al. (2023), where QE is represented as a function of an exogenous credit shock.

sloped in the output-inflation space. Condition (8) ensures that the two lines intersect in the southwest quadrant of the figure, implying that the economy experiences deflation and output contraction. The shaded area corresponds to the region in which the ZLB is binding. For a sufficiently large shock, δ_L, to the natural real rate, the Phillips and IS curves meet in this region. Using equations (10) and (11), and imposing $i_L = 0$, we obtain the following equilibrium values for inflation and the output at the ZLB

$$\overline{\pi}_L = \frac{1}{1-\beta\mu}\Big[\kappa\varkappa_r\overline{r}_L + \kappa\varkappa_g\overline{G}_L + (\kappa\varkappa_{qe} - \varsigma)\,\overline{\mathtt{QE}}_L\Big], \tag{12}$$

$$\overline{Y}_L - \Gamma\overline{G}_L = \varkappa_r\overline{r}_L + \varkappa_g\overline{G}_L + \varkappa_{qe}\overline{\mathtt{QE}}_L, \tag{13}$$

with $\varkappa_r \equiv (1-\beta\mu)\,\varphi/\mathcal{R}$, $\varkappa_g \equiv (1-\mu)\,(1-\beta\mu)\,(1-\Gamma)\,/\mathcal{R}$ and $\varkappa_{qe} \equiv \xi\,((1-\mu)\,(1-\beta\mu) - \lambda\mu)\,/\mathcal{R}$. Given condition (8) and the fact that $\Gamma < 1$ from (3), it is straightforward to veryfy that $\varkappa_r > 0$, $\varkappa_g > 1-\Gamma > 0$, $\varkappa_{qe} > \xi > 0$ and $\kappa\varkappa_{qe} - \varsigma > 0$. Thus, at the ZLB an increase in either government expenditure or QE raises both inflation and output. The coefficient $\varkappa_g$ is independent of $\overline{\mathtt{b}}^{\mathrm{cb}}$, whereas $\varkappa_{qe}$ depend positively on it: the size of the central bank's steady state balance sheet does not affect the equilibrium response of output and inflation to government expenditure but enhances the response to QE.

To characterise the conditions under which the economy enters the ZLB, and how a fiscal or QE policy stimulus may affect them, it is useful to introduce the shadow nominal interest rate, i^s_L. This is defined as the value to which the nominal interest rate would be set if the ZLB constraint did not exist and negative interest rates were possible, and is given by [15]

$$i^s_L \equiv r^n + \phi_\pi\overline{\pi}_L + \phi_y\left(\overline{Y}_L - \Gamma\overline{G}_L\right). \tag{14}$$

The advantage of defining the shadow interest rate in equation (14) is that we can now determine the size of the natural interest rate shock required for the ZLB to be binding. After replacing into (14) the equilibrium solutions for inflation and output in (12) and (13), and solving for $i^s_L = 0$, we find that the

[15]The term "shadow interest rate" is typically used in the literature to capture the effective stance of monetary policy when the interest rate is at the ZLB, but other instruments, such as QE, are used to provide monetary policy stimulus (Krippner, 2013; Wu and Xia, 2016). Our use of the term "shadow interest rate" is different, describing the interest rate that would be prescribed by the monetary policy rule if negative interest rates were possible.

ZLB constraint binds in the neighborhood of $\overline{G}_L = \overline{\mathtt{QE}}_L = 0$ if and only if

$$\delta_L > r^n + \left[\left(\frac{\phi_\pi \kappa}{1-\beta\mu} + \phi_y\right)\varkappa_r\right]^{-1} r^n, \tag{15}$$

or, equivalently, if there is a sufficiently large natural rate shock δ_L, such that the natural real rate, $\overline{r}_L = r^n - \delta_L$, is negative in the absence of fiscal and QE stimuli. From the linearity of the system, it follows that for small policy perturbations ($\overline{G}_L \simeq 0$ and $\overline{\mathtt{QE}}_L \simeq 0$), the ZLB is still be binding, and the shadow rate (14) is negative. We also know, from (12) and (13), that both government expenditure and QE raise inflation and the output gap, therefore the shadow rate is increasing in both policy instruments

$$\frac{di_L^s}{d\overline{G}_L} > 0, \quad \text{and} \quad \frac{di_L^s}{d\overline{\mathtt{QE}}_L} > 0. \tag{16}$$

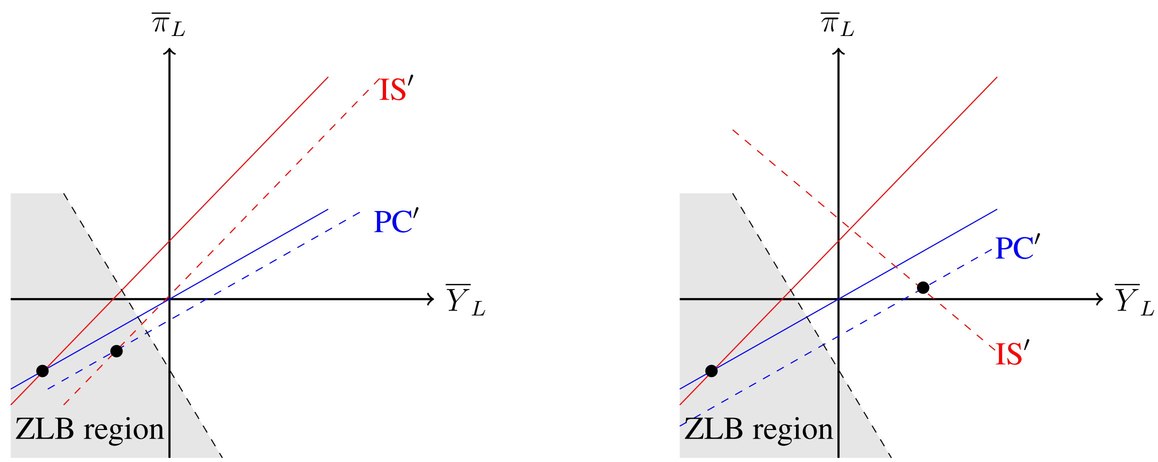
In other words, a sufficiently large fiscal or monetary stimulus can help the economy exit the ZLB. Next, we consider the size of the government expenditure multiplier when the central bank enacts a QE programme during the crisis period, $t < T$, under two possible scenarios: a small stimulus that leaves the economy at the ZLB; and a large stimulus that brings the economy outside the ZLB.

3.1 Exogenous QE and small stimulus

We first restrict attention to a scenario where the fiscal and QE stimuli are small, $\overline{G}_L \simeq 0$ and $\overline{\mathtt{QE}}_L \simeq 0$, implying that despite the stimulus the ZLB constraint remains binding. From (13) we see that for a small fiscal stimulus that leaves the economy at the ZLB during the crisis state the government expenditure multiplier is given by

$$\Gamma_{\text{ZLB benchmark}} = \varkappa_g + \Gamma = \frac{1-\mu-\overline{\psi}\Gamma}{1-\mu-\overline{\psi}} > 1, \tag{17}$$

with $\overline{\psi} = \varphi\mu\kappa/(1-\beta\mu)$. The decomposition $\varkappa_g + \Gamma$ shows that the magnitude of the multiplier while the economy remains inside the ZLB depends neither on the extent of the exogenous QE injection nor on the size of the central bank's steady state balance sheet, being determined entirely by the elasticity of output to government expenditure and the flexible price multiplier. The next decomposition helps establish the sign of the multiplier: given condition (8) and the fact that $\Gamma \in (0, 1)$ from (3), this ZLB multiplier must



(a) Equilibrium with small stimulus

(b) Equilibrium with large stimulus

Figure 3: Equilibrium at the ZLB under small and large fiscal and monetary stimuli

be greater than unity, consistently with the conventional view from New Keynesian theory. For this reason, we take this as a benchmark for the value of the multiplier at the ZLB.

Intuition for why $\Gamma_{\text{ZLB benchmark}}$ is less than one, can be obtained directly from the equilibrium solution for inflation in equation (12), deriving the change in inflation following the fiscal stimulus which is given by

$$\frac{d\overline{\pi}_L}{d\overline{G}_L} = \frac{\kappa}{1-\beta\mu}\Big(\Gamma_{\text{ZLB benchmark}} - \Gamma\Big) > 0.$$

Thus, as long as the economy remains at the ZLB following the increase in government expenditure, the real interest rate falls producing a government expenditure multiplier above unity. The effects of a small government expenditure shock are represented in Figure 3a. When the fiscal stimulus is small, the crisis state equilibrium is inside the ZLB region, where the IS and Phillips curves are both positively sloped. The fiscal stimulus increases inflation and, as the economy is in the ZLB region, this causes the real interest rate to become even more negative, further increasing output.

The size of the benchmark multiplier at the ZLB depends on z, the consumption share of the borrowers. When this share is large, the multiplier is smaller and approaches unity, with

$$\lim_{z\to 1} \Gamma_{\text{ZLB benchmark}} = 1.$$

The reason for this is that as z goes to unity, intertemporal substitution becomes less powerful and, thus, the path of the real interest rate is no longer as important in determining the size of the multiplier. As z may be interpreted as a measure of liquidity-constrained households, we conclude that at the ZLB the fiscal multiplier is larger when fewer consumers are liquidity-constrained. This is in contrast to what happens when the economy is not at the ZLB.

3.2 Exogenous QE and large stimulus

A sufficiently large fiscal stimulus or QE raises the shadow rate above zero, allowing the economy to exit the ZLB. In this case, the interest rate and the shadow rate coincide for all $t < T$, and the relevant IS curve corresponds to that in equation (11) with $i_L = i_L^s > 0$. Making use of equations (10) and (14) to substitute, in turn, for $\overline{\pi}_L$ and i_L^s in equation (11), and totally differentiating yields the government expenditure multiplier

$$\Gamma_{\text{ZLB exit}} = \frac{1 - \mu + \widetilde{\psi}\Gamma}{1 - \mu + \widetilde{\psi}} < 1, \tag{18}$$

with $\widetilde{\psi} \equiv \varphi \left(\phi_y + \kappa \left(\phi_\pi - \mu\right) / \left(1 - \beta\mu\right)\right) > 0$.

Two results about the government expenditure multiplier in equation (18) stand out. First, the size of the multiplier $\Gamma_{\text{ZLB exit}}$ is independent of both the extent of the exogenous QE injection that moves the economy out of the ZLB and the size of the central bank's steady state balance sheet. Second, the Taylor rule becomes active again because the economy exits the ZLB. For this reason, expression (18) corresponds exactly to that of the multiplier $\Gamma_{\texttt{Taylor rule}}$ in (5), with the probability μ replacing the coefficient that measures the serial correlation of government expenditure, ρ. Thus, when the fiscal stimulus and QE are sufficiently large to exit the ZLB, the government expenditure multiplier returns to be less than one. This is illustrated in Figure 3b. With a sufficiently large stimulus, the new equilibrium is achieved outside the ZLB region, and the IS equation yields the conventional negative relationship between inflation and output. An increase in inflation causes the central bank to raise the nominal interest rate enough for the real interest rate to increase and lower output.

Like the benchmark multiplier, $\Gamma_{\text{ZLB exit}}$ also depends on the degree of intertemporal substitution in the household sector. This time, however, the government expenditure multiplier increases, approaching unity as the consumption share of the borrowers becomes larger so that

$$\lim_{z \to 1} \Gamma_{\text{ZLB exit}} = 1.$$

As z goes to unity, the dampening impact of the rise in the real interest rate on the multiplier due to intertemporal substitution becomes less powerful, so that the multiplier goes to unity.

3.3 State-dependent government expenditure multiplier

We have two possible regimes which are endogenously determined and, in particular, are contingent on the size of the fiscal and QE stimuli. We are now able to identify which of the two multipliers, $\Gamma_{\text{ZLB benchmark}} > 1$ and $\Gamma_{\text{ZLB exit}} < 1$, prevails when the economy is in the crisis state and, thus, obtain a state-dependent characterisation of the government expenditure multiplier.

From (16) we know that a sufficiently large fiscal stimulus, $\overline{G}_L$, causes the economy to exit the ZLB regime. Consequently, we can obtain the threshold level, $\overline{G}^{\text{TH}}$, required to exit the ZLB. To do this we use equations (12) and (13) to substitute for $\overline{\pi}_L$ and $\overline{Y}_L$ in (14), and solve for the level of $\overline{G}_L$ required to obtain $i_L^s = 0$, which yields the government expenditure threshold

$$\overline{G}^{\text{TH}} = -\frac{\varkappa_r}{\varkappa_g}\overline{r}_L - \left[\frac{(1-\beta\mu)/\varkappa_g}{\phi_\pi\kappa + \phi_y(1-\beta\mu)}\right]\left[r^n + \left(\phi_\pi\frac{\kappa\varkappa_{qe} - \varsigma}{1-\beta\mu} + \phi_y\kappa\right)\overline{\mathtt{QE}}_L\right]. \tag{19}$$

Thus, as long as the size of the fiscal stimulus is above the threshold $\overline{G}^{\text{TH}}$, the economy is outside the ZLB region and the multiplier in the crisis state is $\Gamma_{\text{ZLB exit}} < 1$.

This result has similarities to that of Woodford (2011), in that at the ZLB the government expenditure multiplier is greater than one only for a sufficiently small fiscal stimulus. However, the threshold $\overline{G}^{\text{TH}}$ in (19) also depends negatively on both the magnitude of the exogenous QE injection, $\overline{\mathtt{QE}}_L$, and the size of the central bank's steady state balance sheet, $\overline{\mathtt{b}}^{\text{cb}}$. In other words, the conventional view, which disregards QE, overstates the size of the fiscal stimulus required to exit the ZLB.

It is even possible to determine a boundary value for the exogenous QE programme required to ensure that the government expenditure multiplier in the crisis state is always less than one. Setting $\overline{G}^{\text{TH}} = 0$ and solving equation (19) for $\overline{\texttt{QE}}_L$, this threshold level is given by

$$\overline{\texttt{QE}}^{\text{TH}} = -\left(\phi_\pi \frac{\kappa\varkappa_{qe} - \varsigma}{1-\beta\mu} + \phi_y\kappa\right)^{-1}\left[\left(\phi_y + \frac{\phi_\pi\kappa}{1-\beta\mu}\right)\varkappa_r\overline{r}_L + r^n\right] > 0. \tag{20}$$

The sign follows from the term in square brackets being negative due to condition (15). The restriction required to ensure the ZLB constraint binds in the neighborhood of $\overline{G}_L = \overline{\texttt{QE}}_L = 0$. Hence, if the exogenous QE stimulus is "sufficiently large", meaning that $\overline{\texttt{QE}}_L \geq \overline{\texttt{QE}}^{\text{TH}}$, then the government expenditure multiplier in the crisis state is always less than one, irrespective of the size of the fiscal stimulus. The threshold $\overline{\texttt{QE}}^{\text{TH}}$ is negatively related to the equilibrium elasticity of inflation to QE, captured by $\kappa\varkappa_{qe} - \varsigma$. This implies that central banks with relatively large balance sheets require a smaller exogenous QE injection to exit the ZLB.

To summarize, when QE is set exogenously and the economy is at the ZLB the (local) fiscal multiplier in the neighborhood of $\overline{G}_L = 0$ is contingent on the amount of QE, as follows

$$\Gamma_{\text{QE EXO}}\left(\overline{\texttt{QE}}_L\right) \equiv \texttt{I}\left(\overline{\texttt{QE}}_L < \overline{\texttt{QE}}^{\text{TH}}\right)\Gamma_{\text{ZLB benchmark}} + \left[1 - \texttt{I}\left(\overline{\texttt{QE}}_L > \overline{\texttt{QE}}^{\text{TH}}\right)\right]\Gamma_{\text{ZLB exit}}, \tag{21}$$

where $\texttt{I}(\bullet)$ denotes an indicator function that determines whether the economy is in the ZLB or not. According to (21), exogenous QE does not affect the value of the government expenditure multiplier within each regime (at or away from the ZLB). It shifts the threshold at which the economy transitions between regimes. In other words, exogenous QE influences the multiplier through an extensive margin effect, by altering the fiscal stance required to induce regime switching. Likewise, the size of the central bank's steady state balance sheet has only impact on the extensive margin (reducing the size of the region corresponding to a multiplier larger than one), but does not affect the intensive margin (the value of the multiplier within each regime).

4 Balance sheet normalisation and the multiplier at the ZLB

In the previous section, we studied the government expenditure multiplier when QE immediately returns to its pre-crisis level once the shock to the natural interest rate subsides, as specified in (9). We now turn to the case where the central bank unwinds QE gradually after the natural interest rate returns to its positive long-run equilibrium, a scenario motivated by the tendency of central banks to maintain QE beyond the restoration of output and price stability. For example, the Fed ended the three rounds of asset purchases known as QE1, QE2, and QE3 by late 2014, and began gradually shrinking its balance sheet as assets matured starting in October 2017 (see Kuttner, 2018). Similar gradual normalisation programmes are pursued by most central banks of advanced economies (see Bhattarai and Neely, 2022).

To analyse the impact on the multiplier of the gradual unwinding of QE we assume that, once the economy enters the ZLB, the central bank makes a credible promise that it will maintain a nonzero level of QE for some time after the natural rate of interest returns to its long-run value. The duration of this extended phase of QE is treated as a random variable: once $r_t^n = r^n > 0$ there is a probability $\mu_\tau \in (0,1)$ that $\widehat{\mathtt{QE}}_t = \upsilon\overline{\mathtt{QE}}_L$, $\upsilon > 0$, while QE might return to zero, $\overline{\mathtt{QE}}_t = 0$, with probability $1-\mu_\tau$. The parameter υ measures QE in the transition path after the natural rate returns to its long-run level: the central bank can either increase QE, $\upsilon > 1$, extend it, $\upsilon = 1$, or gradually unwind it, $\upsilon \in (0,1)$.[16]

The model equilibrium now comprises three Markov states: the crisis state (when the natural rate is negative), followed by a transition state, of expected duration $1/(1-\mu_\tau)$, during which QE is still positive but the economy's natural rate is positive and the conventional Taylor rule is active. This is followed by the end of the QE programme, when the economy rests in the perfect foresight long-run equilibrium. To determine the new equilibrium, let the triplet $\left(\overline{Y}_\tau, \overline{\pi}_\tau, i_\tau\right)$ denote the output gap, inflation rate and nominal rate of interest during the transitional state. The equilibrium is solved backward in two steps.[17]

In the first step, output and inflation during the transition state are determined. Since there is no fiscal stimulus and the Taylor rule is active, the only state variable in the transition state is QE. Therefore,

[16] This strategy to study balance sheet normalisation echoes the analysis in Werning (2011), who studies announcements about the path of interest rates under commitment (forward guidance), also distinguishing across three phases, the liquidity trap phase, a second phase immediately out of the liquidity trap and, finally, the long-run equilibrium.

[17] See Appendix D for more details on this solution.

inflation and output are given by $\overline{\pi}_\tau = \tau_\pi \overline{\mathtt{QE}}_\tau$ and $\overline{Y}_\tau = \tau_y \overline{\mathtt{QE}}_\tau$, respectively, where $\tau_\pi \equiv \frac{\kappa\tau_y - \varsigma}{1-\beta\mu_\tau}$ and $\tau_y \equiv \frac{\xi(1-\mu_\tau)(1-\beta\mu_\tau)+\varphi(\phi_\pi-\mu_\tau)\varsigma}{(1-\beta\mu_\tau)(1-\mu_\tau+\psi_\tau)} > 0$, with $\psi_\tau \equiv \varphi\left[\phi_y + \frac{\kappa(\phi_\pi-\mu_\tau)}{(1-\beta\mu_\tau)}\right] > 0$. The coefficient τ_y is positive because $\phi_\pi - \mu_\tau > 0$, whilst $\tau_\pi > 0$, as long as $\mu_\tau < 1 - \xi\phi_y/\eta$. Thus, a gradual unwinding of QE increases output and prices during the transition state.

Output expands during the transition phase, $\tau_y > 0$, because QE stimulates borrowers' consumption, raising aggregate demand, as captured by the term $\xi(1-\mu_\tau)(1-\beta\mu_\tau)$ in the numerator of τ_y. Conventional monetary policy offsets this by raising the nominal interest rate, as captured by the term ψ_τ, but less aggressively than usual, since the negative wealth effect of QE on savers dampens the inflationary pressures by expanding their labour supply, as captured by $\varphi\left(\phi_\pi - \mu_\tau\right)\varsigma$. Prices rise during the transition, $\tau_\pi > 0$, if the inflationary impact of stronger demand outweighs the deflationary wealth effects of QE, a condition that depends on the expected duration of the transition phase.

Given the solution for the transition state, the equilibrium values of inflation and output in the crisis state are determined as

$$\overline{\pi}_L = \frac{\kappa}{1-\beta\mu}\left(\overline{Y}_L - \Gamma\overline{G}_L\right) - \frac{\varsigma^A}{1-\beta\mu}\overline{\mathtt{QE}}_L, \tag{22}$$

$$\overline{Y}_L - \Gamma\overline{G}_L = \varkappa_g\overline{G}_L + \varkappa_{qe}^A\overline{\mathtt{QE}}_L + \varkappa_r\overline{r}_L, \tag{23}$$

where $\varsigma^A \equiv \varsigma - (1-\mu)\,\mu_\tau\beta\upsilon\tau_\pi$ and $\varkappa_{qe}^A \equiv \varkappa_{qe} + (1-\mu)\,\mu_\tau\varkappa_r\upsilon\left[\frac{\tau_y}{\varphi} + \frac{\tau_\pi}{(1-\beta\mu)} - \frac{\xi}{\varphi}\right]$. The solutions for inflation and output, in turn, equations (22) and (23), mirror those under immediate QE unwinding given by equations (10) and (13), with ς^A and $\varkappa_{qe}^A$ replacing ς and $\varkappa_{qe}$. The difference is that gradual QE unwinding amplifies inflation and output dynamics in the crisis state. Inflation rises during the transition, meaning $\tau_\pi > 0$, generating expectations of higher future inflation, which offsets QE's deflationary wealth effect and raises crisis-state inflation, with $\varsigma^A < \varsigma$. The elasticity of output gap to QE, $\varkappa_{qe}^A$, is shaped by two channels: anticipated future expansion stimulates demand, as captured by the term $\frac{\tau_y}{\varphi} + \frac{\tau_\pi}{(1-\beta\mu)}$, while expected conventional monetary policy tightening partially offsets it, through $\frac{\xi}{\varphi}$. The net effect is positive if $\tau_y > \underline{\tau}_y > 0$ (see Appendix D). Consequently, $\varkappa_{qe}^A > \varkappa_{qe}$. The amplification of demand is stronger when: (i) the natural rate shock is less persistent (corresponding to a lower μ); or when (ii) the QE unwinding is slower (corresponding to a higher μ_τ); or, finally, (iii) if the QE during the transition period is larger (meaning a higher υ).

The equilibrium solutions for inflation and the output gap in equations (22) and (23) imply that gradual QE unwinding once the natural rate of interest becomes positive does not alter the impact of a fiscal stimulus on output during the crisis state. The government expenditure multiplier can take two possible values: either $\Gamma_{\text{ZLB benchmark}} > 1$, as in equation (17) if QE and the fiscal stimulus are small enough to leave the economy at the ZLB, or $\Gamma_{\text{ZLB exit}} < 1$, as in equation (18), if QE and the fiscal stimulus are large enough so that the economy exists the ZLB. However, the circumstances under which the multiplier may or may not exceed unity are distinct. From the solutions in equations (22) and (23), and using the same reasoning as above, it follows that the government expenditure multiplier in the crisis state is always less than unity, irrespective of the size of the fiscal stimulus, if the QE injection at the ZLB exceeds the threshold

$$\overline{\mathtt{QE}}^{\text{TH,A}} = -\frac{1}{\chi_{qe}^{A}}\left[\left(\phi_y + \frac{\phi_\pi \kappa}{1-\beta\mu}\right)\varkappa_r \overline{r}_L + r^n\right] > 0, \tag{24}$$

where $\chi_{qe}^{A} \equiv \left[\left(\frac{\phi_\pi \kappa}{1-\beta\mu} + \phi_y\right)\varkappa_{qe}^{A} - \frac{\phi_\pi \varsigma^A}{(1-\beta\mu)}\right] > \chi_{qe}$. The QE threshold in (24) is analogous to that derived in equation (20). The magnitude of χ_{qe}^{A} relative to χ_{qe} is determined on the basis that $\varkappa_{qe}^{A} > \varkappa_{qe}$ and $\varsigma^A < \varsigma$. Crucially, this implies that $\overline{\mathtt{QE}}^{\text{TH,A}} < \overline{\mathtt{QE}}^{\text{TH}}$, meaning that commitment to gradually unwind QE once the natural rate of interest turns positive reinforces QE's impact at the extensive margin. Thus, the multiplier retains the state-dependent structure of equation (21), but with a larger region corresponding to the regime where the multiplier is less than one. This result highlights how the conventional view can further overstate the fiscal stimulus required to exit the ZLB by overlooking the expectation channels associated with gradual QE unwinding. As previously highlighted in relation to the threshold in equation (21), the region corresponding to the multiplier being less than one increases with the size of the central bank's steady state balance sheet, which amplifies the effectiveness of the exogenous QE injection in moving the economy out of the ZLB.

5 Multiplier under an instrument rule for QE

Next, we examine the analytics of the government expenditure multiplier at the ZLB when the central bank conducts QE using a simple, implementable rule.[18] The rule exploits the substitutability between QE and conventional monetary policy by setting asset purchases as a function of the monetary policy shortfall at the ZLB, as follows

$$\begin{aligned} \overline{\mathtt{QE}}_L &\equiv -\vartheta \left(i_L^s - i_L \right), \\ &= -\vartheta \min \left\{ 0, i_L^s \right\}, \end{aligned} \tag{25}$$

where i_L^s is the shadow rate defined in equation (14) and the parameter $\vartheta > 0$ governs how aggressive the QE rule is. The conventional monetary policy shortfall at the ZLB is given by the difference between the shadow and the actual nominal interest rate. By linking asset purchases to the shortfall in conventional monetary policy, equation (25) ensures that QE is undertaken only when the ZLB becomes a binding constraint. When $i_L^s - i_L < 0$, the ZLB constraint is binding and there is a conventional monetary policy shortfall. If instead $i_L^s - i_L = 0$, the interest rate prescribed by the Taylor rule is positive and, thus, the ZLB constraint is not binding.

Naturally, the instrument rule (25) alters the dynamics of inflation and the output gap at the ZLB, and a unique bounded solution at the ZLB exists if and only if the parameters satisfy

$$\mathcal{R} + \mathcal{R}\varkappa_{qe}\vartheta\phi_y + \xi\lambda\eta \left(1 - \mu\right)\vartheta\phi_\pi > 0, \tag{26}$$

where $\vartheta\phi_\pi > 0$ and $\vartheta\phi_y > 0$ measure QE's response to, in turn, inflation and the output gap. Like condition (8), the parameter restrictions in (26) can also be interpreted as placing an upper limit on how long the private sector expects the ZLB to persist, this time, however, conditional on the QE rule in (25). Provided that (8) holds, and since $\vartheta > 0$, condition (26) is satisfied, enabling us to proceed with the analysis of the multiplier using the same two-state Markov equilibrium framework as before.

The equilibrium with a binding ZLB and the QE instrument rule (25) is illustrated in Figure 4. A large natural rate shock, that ensures condition (15) is satisfied, causes the ZLB constraint to bind in the

[18] Additional details for all results in this section are in Appendix E.

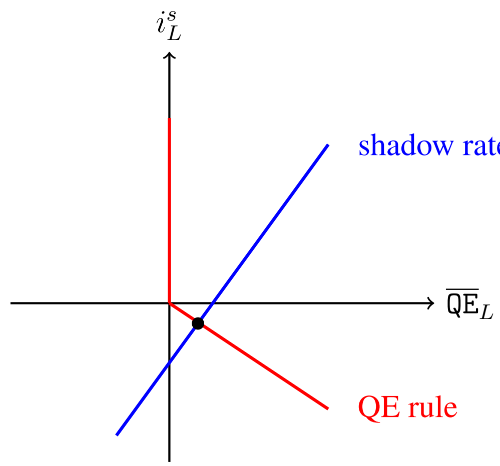


Figure 4: An instrument rule for QE at the ZLB

neighborhood of $\overline{G}_L = \overline{\mathtt{QE}}_L = 0$. This leads to a negative shadow rate and positive QE, as required by (25). The red line represents the QE rule. When the shadow rate is positive, QE is zero; but when the shadow rate is negative (that is, when the ZLB binds) QE becomes positive and it increases as the shadow rate falls further. The blue line is the shadow rate. As in (16), more QE raises the output gap and inflation, thereby increasing the shadow rate. The equilibrium is given by the intersection of the two curves.

The upshot of having an instrument rule for QE, is that the equilibrium shadow rate can be obtained as a function of only government expenditure and the natural rate shock. To see this, consider the shadow rate equation (14), and make use of equations (12) and (13) to substitute for, in turn, inflation and the output gap. This yields

$$i_L^s = r^n + \varrho_r \overline{r}_L + \varrho_g \overline{G}_L + \varrho_{qe} \overline{\mathtt{QE}}_L, \tag{27}$$

with $\varrho_r \equiv \left[\frac{\kappa\phi_\pi}{1-\beta\mu} + \phi_y\right] \varkappa_r > 0$, $\varrho_g \equiv \left[\frac{\kappa\phi_\pi}{1-\beta\mu} + \phi_y\right] \varkappa_g > 0$ and $\varrho_{qe} \equiv \left[\frac{(\kappa-\varsigma/\varkappa_{qe})\phi_\pi}{1-\beta\mu} + \phi_y\right] \varkappa_{qe} > 0$. The equilibrium solution for the shadow rate in (27) can be replaced in (25) to determine the corresponding equilibrium solution for QE as

$$\overline{\mathtt{QE}}_L = \begin{cases} -\left(1+\vartheta\varrho_{qe}\right)^{-1} \vartheta \left(r^n + \varrho_r \overline{r}_L + \varrho_g \overline{G}_L\right) & \text{if } i_L^s \leq 0, \\ 0 & \text{if } i_L^s > 0. \end{cases} \tag{28}$$

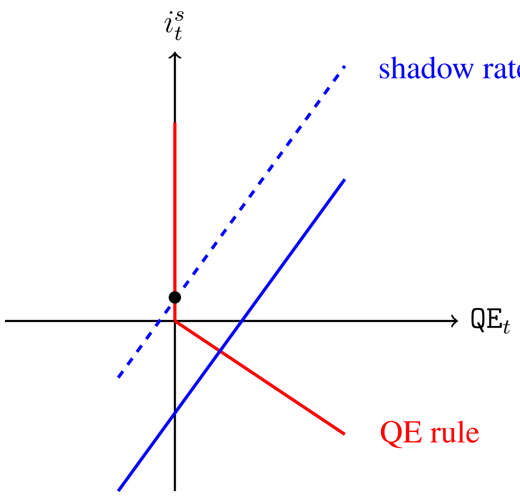
Equation (28) shows that at the ZLB, i.e. when there is a conventional monetary policy shortfall, $i_L^s \leq 0$, QE is entirely determined by the natural rate shock δ_L and government expenditure $\overline{G}_L$. According to (28), the equilibrium response of QE to a fiscal stimulus is dampened by the size of the central bank's steady state balance sheet. As captured by ϱ_{qe} in equation (27), the central bank balance sheet amplifies the sensitivity of the shadow rate to QE. Thus, a larger balance sheet allows the central bank to achieve the same macroeconomic effects on inflation and output through a smaller QE adjustment, thereby reducing the need for a more aggressive policy response.

From equation (28) it follows that the ZLB binds if and only if government expenditure is below the threshold

$$\overline{G}^{\text{TH,IR}} = -\frac{\varkappa_r}{\varkappa_g}\overline{r}_L - \left[\frac{(1-\beta\mu)/\varkappa_g}{\kappa\phi_\pi + (1-\beta\mu)\,\phi_y}\right] r^n > 0, \tag{29}$$

where the positive sign is established making use (15). Compared with that in equation (19) under exogenous QE, the government expenditure threshold derived under the QE instrument rule in (25) does not depend on QE. For the purpose of the government expenditure multiplier, this means that the extensive margin effect of QE vanishes when the central bank determines asset purchases according to an implementable instrument rule that conditions QE on the magnitude of the shortfall in conventional monetary policy at the ZLB. Nevertheless, the government expenditure threshold in equation (29) still implies that, as in the case of exogenous QE, there are two possible scenarios, illustrated in Figure 5. When the fiscal stimulus is large, such that government expenditure is set above the threshold level, $\overline{G}_L \geq \overline{G}^{\text{TH,IR}}$, the ZLB is no longer binding and the fiscal multiplier is given by $\Gamma_{\text{ZLB exit}} < 1$, defined in equation (18). This corresponds to the scenario illustrated graphically in panel (a) of Figure 5, with the equilibrium obtained when the shadow rate is positive and, thus, there is zero QE.

When, instead, $\overline{G}_L < \overline{G}^{\text{TH,IR}}$, the fiscal stimulus is small and the economy stays at the ZLB. This is illustrated in panel (b) of Figure 5. In this scenario, an increase in government expenditure has two effects on the multiplier. On the one hand, as the economy remains at the ZLB following the increase in government expenditure, the real interest rate falls resulting in a multiplier above unity, equivalent to the benchmark multiplier at the ZLB in (17) for exogenous QE. On the other hand, through the instrument rule in (25), QE dampens the output expansion at the ZLB caused by the increase in government expenditure, thereby reducing the multiplier. Consequently, the government expenditure multiplier under a QE



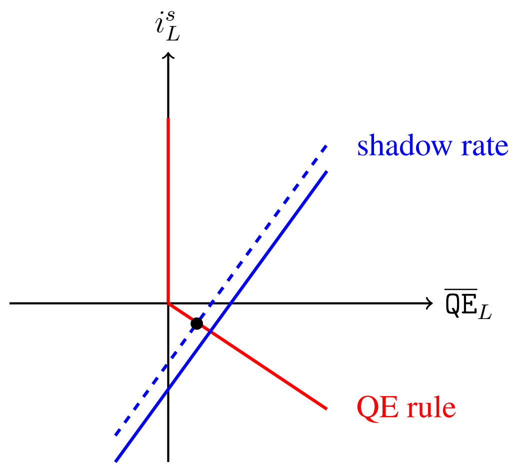


Figure 5: Fiscal stimulus under an instrument rule for QE at the ZLB

instrument rule, $\Gamma_{\text{QE IR}}$, can be decomposed as

$$\Gamma_{\text{QE IR}} = \Gamma_{\text{ZLB benchmark}} + \varkappa_{qe} \frac{d\overline{\mathtt{QE}}_L}{d\bar{G}_L}. \tag{30}$$

The result in equation (30) shows that when QE is conducted according to an instrument rule as in (25), the government expenditure multiplier while the economy remains inside the ZLB region is affected by QE. In other words, QE influences the government expenditure multiplier through an intensive margin channel. The strength of this intensive margin effect is measured by the product of the elasticity of output to changes in QE, $\varkappa_{qe} > 0$, and the elasticity of QE to changes in government expenditure, $\frac{d\overline{\mathtt{QE}}_L}{d\bar{G}_L} = -\varrho_g / \left(1/\vartheta + \varrho_{qe}\right) < 0$. From (30) it is evident that a larger size of the central bank's steady state balance sheet corresponds to a smaller multiplier under the instrument rule.[19] This has a clear economic interpretation: because the central bank balance sheet amplifies the impact of QE, any increase in output and inflation induced by the fiscal stimulus is subject to a more aggressive QE tightening at the ZLB, resulting in a lower multiplier.

Crucially, for a small fiscal stimulus that keeps the economy at the ZLB, the multiplier in (30) can be less than unity if the instrument rule is sufficiently active, i.e. for a large value for ϑ. To see this, note first that

[19]The first term on the right side of (30) does not depend on $\bar{\mathtt{b}}^{\text{cb}}$, $\frac{d\varkappa_{qe}}{d\bar{\mathtt{b}}^{\text{cb}}} = \frac{\varkappa_{qe}}{\bar{\mathtt{b}}^{\text{cb}}} > 0$, and $\frac{d\varrho_{qe}}{d\bar{\mathtt{b}}^{\text{cb}}} = \left[\frac{\kappa\phi_\pi}{1-\beta\mu} + \phi_y\right] \frac{\varkappa_{qe}}{\bar{\mathtt{b}}^{\text{cb}}} > 0$. Thus, it must be that $\frac{d\frac{d\overline{\mathtt{QE}}_L}{d\bar{G}_L}}{d\bar{\mathtt{b}}^{\text{cb}}} = -\varrho_g / \left(1/\vartheta + \varrho_{qe}\right)^2 \frac{d\varrho_{qe}}{d\bar{\mathtt{b}}^{\text{cb}}} < 0$.

the multiplier in equation (30) can be equivalently written as:

$$\Gamma_{\text{QE IR}} = \frac{1 - \mu + \widehat{\psi}\Gamma}{1 - \mu + \widehat{\psi}}, \tag{31}$$

with $\widehat{\psi} = [\mathcal{R}\varkappa_{qe}\vartheta\phi_y + (1-\mu)\,\xi\kappa\vartheta\phi_\pi - \kappa\varphi\mu] \,/\, (1 - \beta\mu - \varsigma\vartheta\phi_\pi)$. From equation (31), it follows that the government expenditure multiplier is less than one if and only if

$$\vartheta > \overline{\vartheta} \equiv \frac{\kappa\varphi\mu}{\mathcal{R}\varkappa_{qe}\phi_y + (1-\mu)\,\xi\kappa\phi_\pi} > 0. \tag{32}$$

Thus, equation (32) identifies when QE's response to the conventional monetary policy shortfall is sufficiently aggressive to ensure that the government expenditure multiplier at the ZLB is less than unity.[20] The empirical evidence discussed in the Introduction, which finds multipliers below one during recent ZLB episodes, could, therefore, suggest the implementation of a sufficiently aggressive QE rule. At the same time, the result in (32) shed light on why medium-scale DSGE models return output responses to government expenditure shocks at the ZLB that resemble those outside the ZLB, when using a QE rule with direct feedback on inflation and output (Sims and Wu, 2021) .

It is natural at this stage to ask whether plausible or existing calibrations of the parameter ϑ, which captures QE's response to the conventional monetary policy shortfall, satisfy the condition for this response to be sufficiently aggressive such that the government expenditure multiplier falls below unity even when the ZLB remains binding in equation (32). We believe that the answer is likely to be affirmative. According to the calibration described for Figure 1, the threshold value required to ensure $\Gamma_{\text{QE IR}} < 1$ is $\overline{\vartheta} > 1.91$. Estimates in the empirical literature typically suggest that a QE injection of \$600 billion leads to a roughly 15 basis point decline in long-term yields (see, e.g., Swanson, 2021). Assuming a linear relationship and a steady state value of Treasury securities held by the Fed of 0.795 trillions, this implies $\vartheta = 5.67$.[21] Sims and Wu (2021) use instead $\vartheta = 7$, for a QE rule with direct feedback on inflation and output. Both these calibrations in our model imply a government expenditure multiplier necessarily lower than one at the

[20] As the rule (25) sets QE contingent on the shadow rate, this implies that for a given ϑ, the elasticity of QE to changes in government expenditure is also higher, the larger are ϕ_π and ϕ_y. In that sense, an aggressive conventional monetary policy rule also causes the QE rule to be more aggressive and, thus, lowers the government expenditure multiplier.

[21] The steady state value is calculated, using data from FRED, as the average of Treasury securities held by the Federal Reserve (WSHOTSL) on 18 December 2008 (\$629,397 million) and 5 December 2020 (\$779,715 million).

ZLB.

As with the other multipliers derived so far, intertemporal substitution remains a key force determining the size of the multiplier under the instrument rule (25): when the share of total consumption going to borrowers approaches unity, $z \to 1$, the government expenditure multiplier, $\Gamma_{\text{QE IR}}$, goes to one, a result that follows from the differentiation of $\Gamma_{\text{QE IR}}$ with respect to z through L'Hôpital's rule (see Appendix E).

6 Multiplier under an optimal QE rule

We now examine the government expenditure multiplier when the central bank implements an optimal QE policy rule, based on a quadratic approximation to the weighted sum of the savers and borrowers' utility.[22] Deriving such an approximation is not straightforward, because savers and borrowers have different discount factors. This heterogeneity complicates the formulation of an utilitarian welfare function, (see, e.g., Feldstein, 1964; Gollier and Zeckhauser, 2005). Specifically, if the central bank maximises the average discounted sum of the flow utility for each group while adhering to each household's discount factor, this would result in a non-stationary solution where the consumption of the most impatient individuals (borrowers) diminishes over time. To avoid this issue, we assume that the central bank cares equally about both households and uses a common factor, β, to discount the sum of their flow utilities. Additionally, we follow the approach in Woodford (2011) and assume that households (presumably, both borrowers and savers, although this is inconsequential) derive utility flows directly from government expenditure.

The central bank wishes to maximise the total expected discounted utility of the two agents, as follows

$$E_t \sum_{j=0}^{\infty} \beta^j \left[u\left(C_{t+j}\right) - v\left(N_{t+j}\right) + u\left(C^b_{t+j}\right) + \mathcal{G}\left(G_{t+j}\right) \right], \tag{33}$$

where $u\left(C\right)$, $u\left(C^b\right)$ and $v\left(N\right)$ represent, respectively, the per-period utility from consumption for savers, the utility from consumption for borrowers, and the disutility from labour for the savers, as previously described, whereas $\mathcal{G}\left(\bullet\right)$ is an increasing and concave function reflecting the benefits to

[22] Additional details for all results in this section are in Appendix F.

households of public spending. We evaluate the optimal QE policy, assuming, as before, a two-state Markov environment where the economy starts at the ZLB with the natural rate of interest $r_t^n = \overline{r}_L < 0$, and returns to $r_t^n = r^n > 0$, with probability $1-\mu$ each period. Given this equilibrium structure, a quadratic approximation to the central bank's loss function (33), valid for small disturbances around the efficient (first-best) steady state, at the ZLB yields

$$\mathtt{L} = \frac{1}{2}\left[\overline{\pi}_L^2 + \frac{\kappa}{\epsilon}\left(\overline{Y}_L - \Gamma\overline{G}_L\right)^2 + \frac{\kappa}{\epsilon}\left(\frac{\eta_g}{1/\varphi} + 1 - \Gamma\right)\Gamma\overline{G}_L^2 + \varsigma\mathtt{I}_L\right]. \tag{34}$$

The first three terms in the loss function (34) are exactly the same as those obtained in the analysis of Woodford (2011) of optimal policy with a representative agent. They capture the *stabilisation* motive of the central bank, as measured by welfare losses associated with, in turn, the volatilities of inflation, the output gap and government expenditure. The weights attached to each of these three objectives depend on the private's sector parameters, except for $\eta_g = -\left(\mathcal{G}''/\mathcal{G}'\right)\overline{Y}$, which depends on the concavity of the $\mathcal{G}(\bullet)$ function and, thus, the diminishing marginal utility of government expenditure. The last term $\mathtt{I}_L$ of the loss function (34) is, of course, different from what is obtained with the representative agent model, and is given by

$$\begin{aligned}\mathtt{I}_L &= -\frac{1}{2}\left(\frac{1-g}{b^{cb}/b}\right)\overline{C}_L\overline{C}_L^b \\ &= -\left(\overline{Y}_L - \Gamma\overline{G}_L\right)\overline{\mathtt{QE}}_L + (1-\Gamma)\,\overline{G}_L\overline{\mathtt{QE}}_L + \xi\overline{\mathtt{QE}}_L^2.\end{aligned} \tag{35}$$

The first representation of $\mathtt{I}_L$ captures a *redistributive* motive of the central bank, as measured by welfare losses from consumption inequality between borrowers and savers: for a given average consumption, $\left(\overline{C}_L + \overline{C}_L^b\right)/2$, the product $\overline{C}_L\overline{C}_L^b$ is minimised the more equal are $\overline{C}_L$ and $\overline{C}_L^b$.[23] The redistribution motive emerges in two-agent New Keynesian models, as consumption inequality enters directly into the planner's objective (see, e.g., Bilbiie, 2024; Bonciani and Oh, 2025; Wu and Xie, 2025).

The second formulation of (35), by contrast, recasts $\mathtt{I}_L$ as a function of QE, the output gap, $\left(\overline{Y}_L - \Gamma\overline{G}_L\right)$, and the fiscal stimulus, $\overline{G}_L$. This formulation reveals the trade-off faced by the planner in implementing

[23] Because the social planner values savers and borrowers equally and discounts their utilities using the same factor β, the first-best allocation requires equal consumption shares, implying $z = 1/2$. Alternatively, if the first-best steady state is unattainable, QE would appear in the welfare function to account for the impact of stabilisation policy on consumption dispersion. This aligns with Benigno and Woodford (2005)'s analysis of optimal monetary policy in the New Keynesian model with a distorted steady state.

QE at the ZLB. Since, the output gap is negative at the ZLB and we are considering a small stimulus that leaves the economy at the ZLB, we see from (35) that $\mathtt{I}_L$, is increasing in the level of QE. Although stabilisation at the ZLB calls for QE interventions, aversion to inequality limits its optimal level, as QE amplifies the consumption gap between savers and borrowers, leading to convex welfare losses.

From equation (35) we also have that

$$\frac{d^2\mathtt{I}_L}{d\overline{\mathtt{QE}}_L d\overline{G}_L} = \left(1 - \frac{d\overline{Y}_L}{d\overline{G}_L}\right)\overline{\mathtt{QE}}_L, \tag{36}$$

with the upshot that, if the government spending multiplier is less than one, then an increase in government spending raises welfare losses. The intuition is simple: with a fiscal multiplier less than one, an increase in government spending crowds out the consumption of savers. Thus QE, by raising the consumption of borrowers, further widens the gap with savers, resulting in greater welfare losses from inequality.

6.1 Optimal QE rule

With a small fiscal stimulus such that the economy remains at the ZLB, i.e. $i_L = 0$, maximisation of the loss function (34) subject to the private sector constraints given by the Phillips and IS curves in equations (10) and (11) yields the optimal QE rule

$$\overline{\mathtt{QE}}^{\star}_L = -\vartheta^{\star}\left[\phi^{\star}_{\pi}\overline{\pi}_L + \phi^{\star}_y\left(\overline{Y}_L - \Gamma\overline{G}_L\right) - \phi^{\star}_{\mathtt{I}}\left(\overline{Y}_L - \overline{G}_L\right)\right], \tag{37}$$

with $\vartheta^{\star} \equiv 1/\varsigma$, $\phi^{\star}_{\pi} \equiv 2(1-\mu)(\varkappa_{qe} - \xi)/[\varphi\mu(2\xi - \varkappa_{qe})] > 0$, $\phi^{\star}_y \equiv 2\kappa\varkappa_{qe}/[\epsilon(2\xi - \varkappa_{qe})] > 0$ and $\phi^{\star}_{\mathtt{I}} \equiv \varsigma/(2\xi - \varkappa_{qe}) > 0$. The signs of the coefficients $\phi^{\star}_y$, $\phi^{\star}_{\pi}$, and $\phi^{\star}_{\mathtt{I}}$ follow from the conditions required to ensure that the optimal QE rule in equation (37) yields a unique bounded equilibrium at the ZLB. The first two terms on the right side of (37) are isomorphic to the corresponding terms for inflation and output gap in the instrument rule in equation (25). The last term, involving the deviation of private expenditure from the steady state, $\overline{Y}_L - \overline{G}_L$ (which is negative at the ZLB), follows from the redistribution motive discussed above: QE reallocates resources away from savers, thereby widening the gap between their consumption and that of borrowers.

Two features of the optimal QE rule in equation (37) are worth observing. The first concerns the interaction between QE and the fiscal stimulus. Upon differentiating $\overline{\mathtt{QE}}_L^\star$ with respect to $\overline{G}_L$, and making use of the equilibrium solutions for inflation and the output at the ZLB in (12) and (13), it follows that $\frac{\partial \overline{\mathtt{QE}}_L^\star}{\partial \overline{G}_L} < 0$. Thus, according to the optimal rule in equation (37), QE necessarily influences the government expenditure multiplier, reducing its size while the economy remains in the ZLB, the intensive margin channel. The second observation concerns the impact of the size of the central bank's steady state balance sheet. In (37), the coefficients $\phi_y^\star$, $\phi_\pi^\star$ and $\phi_{\mathtt{I}}^\star$ are independent from $\overline{\mathtt{b}}^{\text{cb}}$, whereas the coefficient $\vartheta^\star$ is decreasing in $\overline{\mathtt{b}}^{\text{cb}}$. Thus, under the optimal rule, the response of QE becomes less aggressive as the size of the central bank's steady state balance sheet increases, through the coefficient $\vartheta^\star$. This has a clear economic interpretation: a larger balance sheet amplifies the effects of QE proportionally. As a result, the optimal QE policy scales down with the size of the central bank's balance sheet.

6.2 Optimal QE and the fiscal multiplier

In order to study the analytics of the government expenditure multiplier we replace the equation for QE in (37) in the Phillips and IS equations (10) and (11), respectively. From the equilibrium solution for output, it follows that the government expenditure multiplier at the ZLB when the central bank sets QE according to the optimal rule in equation (37) is given by

$$\Gamma_{\text{QE Opt}} \equiv \frac{1-\mu-\psi_{\mathtt{I}}\phi_{\mathtt{I}}^\star + \psi^\star \Gamma}{1-\mu-\psi_{\mathtt{I}}\phi_{\mathtt{I}}^\star + \psi^\star}, \tag{38}$$

where $\psi^\star \equiv \frac{\mathcal{R}\varkappa_{qe}\vartheta^\star\phi_y^\star + \xi(1-\mu)\kappa\vartheta^\star\phi_\pi^\star - \varphi\mu\kappa}{1-\beta\mu-\phi_\pi^\star}$ and $\psi_{\mathtt{I}} \equiv \frac{\mathcal{R}\varkappa_{qe}\vartheta^\star}{1-\beta\mu-\phi_\pi^\star}$. The size of the government expenditure multiplier $\Gamma_{\text{QE Opt}}$ in (38) depends on the magnitude of the response coefficients $\phi_y^\star$, $\phi_\pi^\star$ and $\phi_{\mathtt{I}}^\star$ of the optimal QE rule. These coefficients are themselves determined by the parameters of: the welfare loss function, the Phillips and IS curves that constrain policy actions, and the expected duration of the ZLB.

Two results about the government expenditure multiplier $\Gamma_{\text{QE Opt}}$ in (38) are worth highlighting. First, the multiplier $\Gamma_{\text{QE Opt}}$ is entirely independent from the size of the central bank's steady state balance sheet, as seen from the coefficients $\psi^\star$ and $\psi_{\mathtt{I}}$ that do not depend on $\overline{\mathtt{b}}^{\text{cb}}$. As noted above, the effects of QE are proportional to the size of the central bank's steady state balance sheet, but for that same reason, optimal QE is inversely proportional to the central bank's steady state balance sheet. Accordingly, the output multiplier of government spending, driven by the QE response under the optimal rule, is likewise unaffected by the size of central bank's steady state balance sheet.

Second, the multiplier $\Gamma_{\text{QE Opt}}$ is less than one for any calibration of the model's parameters that satisfy

$$\phi_{\pi}^{\star} > \overline{\phi}_{\pi} \equiv \frac{\varsigma\varphi\mu}{(1-\mu)\,\xi} > 0 \tag{39}$$

$$\phi_{y}^{\star} > \overline{\phi}_{y} \equiv -\frac{(1-\mu-\psi_{\mathrm{I}}\phi_{\mathrm{I}}^{\star})\,(1-\beta\mu-\phi_{\pi}^{\star}) + [\xi\,(1-\mu)\,\vartheta^{\star}\phi_{\pi}^{\star} - \varphi\mu]\,\kappa}{\mathcal{R}\varkappa_{qe}\vartheta^{\star}}. \tag{40}$$

The coefficients $\overline{\phi}_{\pi}$ and $\overline{\phi}_{y}$, therefore, identify when the optimal QE rule is sufficiently aggressive to ensure that the government expenditure multiplier at the ZLB is lower than unity. While the sign of $\overline{\phi}_{\pi}$ is unambiguous, the sign of $\overline{\phi}_{y}$ depends on model parameters. Neither $\overline{\phi}_{\pi}$ nor $\overline{\phi}_{y}$ in (39) and (40) depends on the size of the central bank's steady state balance sheet.

Figure 6 illustrates our results about the optimal QE rule and the government expenditure multiplier in this section. Panel (a) shows that, at the ZLB, the optimal QE injection is negatively related to the size of the fiscal stimulus, meaning that QE impacts on the government expenditure multiplier while the economy remains in the ZLB, the intensive margin channel. Panel (b) shows that the optimal QE stimulus, while the economy is at the ZLB, decreases as the the size of the central bank's steady state balance sheet increases. Panel (c) shows instead that under the optimal QE rule, the government expenditure multiplier is invariant to the size of the central bank's steady state balance sheet. Figure 6 also shows that if the expected duration of the ZLB is lower, corresponding to a smaller μ, then the fiscal multiplier associated with a given increase in government spending is larger and, therefore, the optimal QE is lower.

7 Conclusion

This paper revisits the analytics of the government expenditure multiplier within a New Keynesian framework that incorporates financial frictions and QE, aiming to reconcile theoretical predictions with empirical findings under conditions where the ZLB on the nominal interest rate binds. The New Keynesian literature has long emphasised two main results concerning fiscal policy at the ZLB. First, a sufficiently

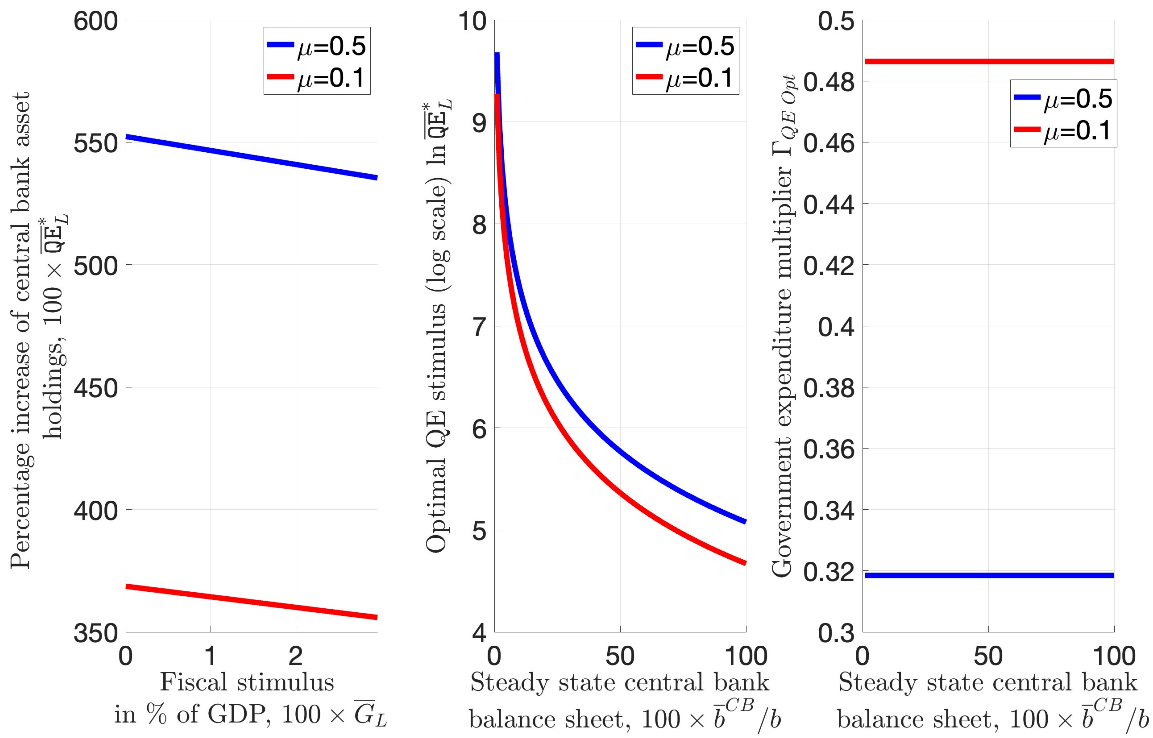


Figure 6: Response at the ZLB of: Optimal QE to a government expenditure stimulus, in % of GDP (Panel a); Optimal QE (Panel b) and government expenditure multiplier (Panel c) to change in the size of the central bank's steady state balance sheet. The model consists of the Phillips curve (10); the IS curve (11); the interest rate $i_t = 0$; and the optimal QE rule (37). Parameters are calibrated as in Figure 1, with $z = 0.5$ and $\epsilon = 1.5$.

aggressive temporary increase of government expenditure can help the economy exit the ZLB. Second, the output multiplier of government expenditure is greater than one at the ZLB due to constrained monetary policy. We show that accounting for QE challenges these results. By suitably adapting the analytically tractable four-equation model of Sims et al. (2023), we provide closed-form insights into how QE alters fiscal transmission and its impact on the output multiplier of government expenditure.

Our analysis yields several key results. Away from the ZLB, the presence of financial frictions attenuates the crowding-out effects typically associated with fiscal expansions, raising the multiplier compared to the frictionless benchmark but keeping it below unity. When the ZLB binds, QE influences the multiplier through two distinct channels. If asset purchases by the central bank are determined irrespective of the size of the fiscal stimulus, QE's impacts the government expenditure multiplier through an extensive margin effect, lowering the fiscal stimulus needed to exit the ZLB and thereby reducing the window over which the multiplier exceeds one. Commitment to a gradual normalisation path for QE once the economic shocks that moved the economy into the ZLB subside further reinforces this extensive margin effect, implying that persistent expansions of the central bank's balance sheet are associated with government spending multipliers below unity. In contrast, when QE follows an instrument rule or is set according to an optimal policy derived from a micro-founded welfare loss function, it operates on the intensive margin, directly moderating the multiplier even while the economy remains at the ZLB.

Our findings suggest that QE may effectively neutralise the expansionary effects of fiscal stimulus at the ZLB by restoring monetary policy's ability to tighten in response to rising demand. This calls into question the robustness of policy prescriptions that advocate aggressive fiscal stimulus during liquidity traps without accounting for the role of QE. More broadly, the results highlight the necessity of incorporating unconventional monetary tools into macroeconomic analyses of fiscal policy effectiveness.

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Online Appendix

A Non-linear model and log-linearisation

This appendix has three parts. First, we present in more detail the economic environment underlying the model of the economy from which the key equations (1) and (2) are derived. The model is based on the New Keynesian framework of Sims et al. (2023) to which we make two modifications. First, we render the binding leverage constraint faced by financial intermediary deterministic, thereby removing the "credit shocks" that affect the Phillips and IS curves in the original model. Second, we include exogenous government purchases. These modifications allow us, as shown in the main text, to derive a New Keynesian with QE that nests the model of Woodford (2011) as a special case. The remaining two parts of the appendix, state the log-linear equilibrium conditions and use them to derive the Phillips and IS equations in (1) and (2) in the main text.

Non-linear model. There are two types of households, savers and borrowers. Although there is an equal mass of both household types (measure one), the steady-state consumption of borrowers as a share of total consumption is $z \in (0, 1)$. The final aggregate output produced in this economy is either purchased by households or by the government. Thus the resource constraint is

$$Y_t = C_t + C_t^b + G_t, \tag{A.1}$$

where C_t denotes the consumption of savers, C_t^b is the consumption of borrowers, and G_t is government consumption. In addition to the two household types and the government, the economy also features financial intermediaries and a central bank. The production sector includes competitive final good producers, monopolistic retailers who set prices subject to the Calvo (1983) friction, and wholesale producers who hire labour services from savers. Next, we describe the problem solved by each agent type. *Savers* have preferences over their individual consumption and labour supply, N, represented by the following utility function

$$E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-1/\sigma} - 1}{1 - 1/\sigma} + \frac{N_{t+j}^{1+\eta}}{1+\eta}\right)\right], \tag{A.2}$$

with $\beta \in (0,1)$, the discount factor, $\sigma > 0$, the elasticity of intertemporal substitution, and $\eta > 0$, the inverse Frisch elasticity of labour supply. Savers earn income from employment, and their holdings of financial assets and, thus, face the following flow budget constraint

$$P_t C_t + S_t = W_t N_t + R^s_{t-1} S_{t-1} + P_t \left(D^p_t + D^{\mathtt{fi}}_t + T_t - X^{\mathtt{fi}}_t \right), \tag{A.3}$$

where P_t is the aggregate price level of the final good; S_t are one-period deposits held by savers at date t; W_t is the wage rate; R^s_{t-1} is the nominal gross return on the bonds purchased at date $t-1$; D^p_t and $D^{\mathtt{fi}}_t$ are the dividends received from the household's ownership of, in turn, intermediate producer firms and financial intermediaries; T_t are net transfers received by savers from the government; and $X^{\mathtt{fi}}_t$ are exogenously determined injections of equity to the financial intermediaries. Savers choose sequences for their consumption, employment, and savings to maximise (A.2) subject to the sequence of budget constraints (A.3). The necessary conditions to solve this problem are

$$\frac{W_t}{P_t} = C_t^{1/\sigma} N_t^{\eta}, \tag{A.4}$$

$$1 = \beta R^s_t E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{1/\sigma} \left(\frac{1}{1+\pi_{t+1}} \right) \right], \tag{A.5}$$

where $\pi_{t+1} = (P_{t+1}/P_t) - 1$, is the inflation rate between dates t and $t+1$. *Borrowers* do not supply labour, have preferences over consumption represented by the utility function

$$E_t \left[\sum_{j=0}^{\infty} \beta_b^j \left(\frac{{C^b_{t+j}}^{1-1/\sigma} - 1}{1 - 1/\sigma} \right) \right], \tag{A.6}$$

and are more impatient than savers, since $\beta_b^t \in (0, \beta)$. The only sources of income for borrowers are transfers received from the government and revenue raised from selling long-term bonds (of which they are the only originators) to financial intermediaries. These long-term bonds are modeled as perpetuities with decaying coupon payments at rate $\gamma \in (0,1)$. Hence, borrowers face the flow budget constraint

$$P_t C^b_t + B_{t-1} = \mathcal{Q}_t \mathtt{NB}_t + P_t T^b_t, \tag{A.7}$$

where T_t^b are the net transfers received from the government (and taken as given by the private sector), and $\mathtt{NB}_t = B_t - \gamma B_{t-1}$ corresponds to the issuing of new long-term bonds, sold at price $\mathcal{Q}_t$, with B_t the coupon liability due in $t+1$ from the outstanding long-term bonds, given by

$$B_t = \sum_{s=0}^{t} \gamma^s \mathtt{NB}_{t-s}, \tag{A.8}$$

following the decaying structure of coupon payments. We assume transfers from the government to borrowers to be given by $T_t^b = (1+\gamma)(B_{t-1}/P_t)$, which substituting in the budget constraint (A.7) yields

$$P_t C_t^b = \mathcal{Q}_t B_t. \tag{A.9}$$

As in Sims et al. (2023), this bail-out assumption allows the elimination of a state variable, therefore obtaining the aggregate equations for inflation and output in the model used in the main text. However, borrowers are not hand-to-mouth consumers, since their consumption must still obey the following Euler equation

$$1 = \beta_b E_t \left[\left(\frac{C_t^b}{C_{t+1}^b} \right)^{1/\sigma} \left(\frac{R_{t+1}^b}{1+\pi_{t+1}} \right) \right], \tag{A.10}$$

with

$$R_{t+1}^b = \frac{1 + \gamma \mathcal{Q}_{t+1}}{\mathcal{Q}_t}. \tag{A.11}$$

Financial intermediaries (banks) are modeled assuming that in each period there is a new competitive stand-in bank that replaces the incumbent and receives an equity injection from savers, given by

$$P_t X_t^{\mathtt{fi}} = P_t \overline{X}^{\mathtt{fi}} + \gamma \mathcal{Q}_t B_{t-1}^{\mathtt{fi}}. \tag{A.12}$$

This equity injection is made out new fresh equity, $P_t \overline{X}^{\mathtt{fi}}$, and the outstanding long-term bonds held by previous intermediaries, $\gamma \mathcal{Q}_t B_{t-1}^{\mathtt{fi}}$. Besides the equity injection, the stand-in bank also attracts deposits from savers, S_t. Together, these internal and external funds are used to invest in two kinds of assets, reserves held with the central bank $\mathtt{RE}_t^{\mathtt{fi}}$ and long-term bonds $B_t^{\mathtt{fi}}$ purchased from borrowers. Thus, the balance sheet constraint of the financial intermediary reads as

$$S_t + P_t\overline{X}^{\mathtt{fi}} + \gamma \mathcal{Q}_t B^{\mathtt{fi}}_{t-1} = \mathtt{RE}^{\mathtt{fi}}_t + \mathcal{Q}_t B^{\mathtt{fi}}_t. \tag{A.13}$$

The financial intermediary's profits are then distributed in the form of dividends to savers, and are given by

$$P_t D^{\mathtt{fi}} = \left(R^b_{t+1} - R^s_t\right)\mathcal{Q}_t B^{\mathtt{fi}}_t + \left(R^{re}_{t+1} - R^s_t\right)\mathtt{RE}^{\mathtt{fi}}_t + R^s_t\left(P_t\overline{X}^{\mathtt{fi}} + \gamma\mathcal{Q}_t B^{\mathtt{fi}}_{t-1}\right), \tag{A.14}$$

where R^b_t and R^{re}_t are, in turn, the return on long-term bonds and the return on central bank reserves. Financial intermediaries face a leverage constraint relative to their holdings of long-term bonds, such that

$$\mathcal{Q}_t \mathtt{b}^{\mathtt{fi}}_t \leq \Theta\overline{X}^{\mathtt{fi}}, \tag{A.15}$$

with $\Theta > 0$, the leverage ratio, and where $\mathtt{b}^{\mathtt{fi}}_t = B^{\mathtt{fi}}_t/P_t$, is the real value of long-term bond holdings by the financial intermediary. In what follows, we assume the leverage constraint (A.15) is always binding. Sims et al. (2023) treat the leverage ratio Θ as a stochastic parameter, which they then interpret as a source of credit shock. This is not necessary for the purpose of our analysis, hence we treat Θ as a constant parameter. Finally, as in Sims et al. (2023), the financial intermediary prices cash flows using the stochastic discount factor of savers, but to do so in a myopic way, in the sense that the incumbent bank does not price the capital that is distributed after it is replaced by a new bank. This assumption is made for tractability, as it makes the problem of the bank static. The optimality conditions governing the behaviour of the stand-in bank are

$$\Omega_t = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\left(\frac{R^b_{t+1} - R^s_t}{1+\pi_{t+1}}\right)\right], \tag{A.16}$$

$$0 = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\left(\frac{R^{re}_{t+1} - R^s_t}{1+\pi_{t+1}}\right)\right], \tag{A.17}$$

where $\Omega_t > 0$ stands for the Lagrange multiplier associated with the leverage constraint (A.15), being positive because the constraint is assumed to be binding in every period t. **Production** is organised across three stages: wholesale production, retail production, and final good production. The wholesale production is carried out by a competitive stand-in firm that produces an undifferentiated wholesale good using the linear technology in labour at a constant marginal cost

$$\mathtt{mc}_t = W_t/P_t, \tag{A.18}$$

corresponding to the real wage. The retail producers (owned by savers) are monopolistic competitive firms, $\nu \in [0,1]$, who purchase the wholesale good to produce a differentiated intermediate good, $y(\nu)$, and set prices subject to nominal frictions as in Calvo (1983), with $\theta \in (0,1)$ the share of firms unable to reset price each period; lastly, the representative final good producer aggregates the differentiated intermediate goods, to produce the final good, so that

$$Y_t = \left(\int_0^1 y_t(\nu)^{1-1/\epsilon} \, d\nu \right)^{1/(1-1/\epsilon)}, \tag{A.19}$$

with $\epsilon > 1$, the elasticity of substitution across differentiated intermediate varieties. Thus, each intermediate retail is confronted with the constant elasticity demand function

$$y_t(\nu) = \left[\frac{p_t(\nu)}{P_t} \right]^{-\epsilon} Y_t, \tag{A.20}$$

with, in turn, $p_t(\nu)$, the price set by retailer $\nu \in [0,1]$, and $P_t = \left(\int_0^1 p_t(\nu)^{1-\epsilon} \, d\nu \right)^{1/(1-\epsilon)}$, the aggregate price index. Given the demand from the final output producers, the optimal reset price chosen by the retail firms allowed to change their price, $P_t^\star$, satisfies the condition

$$0 = E_t \left[\sum_{s=0}^{\infty} (\theta\beta)^s \left(\frac{C_t}{C_{t+s}} \right)^{1/\sigma} \left(\frac{P_t^\star}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \left(\frac{P_t^\star}{P_{t+s}} - \frac{\epsilon}{\epsilon - 1} \mathtt{mc}_{t+s} \right) \right]. \tag{A.21}$$

Since the reset price is the same for each firm allowed to change price, and the probability of being allowed to reset price is the same across all retailers, the aggregate price index evolves as follows

$$P_t = (1-\theta) P_t^{\star 1-\epsilon} + \theta P_{t-1}^{1-\epsilon}. \tag{A.22}$$

Imposing market clearing conditions in both the intermediate goods and labour markets yields

$$N_t = \int_0^1 y(\nu)\, d\nu = v_t^p Y_t, \tag{A.23}$$

with

$$v_t^p = \int_0^1 \left(p_t(\nu)/P_t\right)^{-\epsilon} d\nu, \tag{A.24}$$

a measure of price dispersion in the intermediate goods sector. *The central bank and the government* are in charge of monetary and fiscal policy. Alongside controlling the nominal rate of interest i_t according to a Taylor (1993)'s rule when the ZLB constrain is not binding, the central bank can also create reserve deposits, which are used to purchase long-term bonds from borrowers. Thus, the central bank's balance sheet is simply given by

$$\mathtt{RE}_t = \mathcal{Q}_t B_t^{\mathtt{cb}}, \tag{A.25}$$

with $B_t^{\mathtt{cb}}$ the holdings of long-term bonds by the central bank. The real value of the long-term bonds held by the central bank corresponds to the amount of QE in the economy. Therefore we have that

$$\mathcal{Q}_t \mathtt{b}_t^{\mathtt{cb}} = \mathtt{QE}_t. \tag{A.26}$$

with $\mathtt{b}_t^{\mathtt{cb}} = B_t^{\mathtt{cb}}/P_t$, the real value of central bank long-term bond holdings. In equilibrium, the reserve deposits created by the central bank must be held by the financial intermediaries, and the total outstanding long-term bonds issued by borrowers must be held by the financial intermediaries and the central bank, so that, in turn

$$\begin{aligned} \mathtt{RE}_t &= \mathtt{RE}_t^{\mathtt{fi}}, \\ \mathtt{b}_t &= \mathtt{b}_t^{\mathtt{fi}} + \mathtt{b}_t^{\mathtt{cb}}, \end{aligned} \tag{A.27}$$

where $\mathtt{b}_t = B_t/P_t$, is the real value of outstanding bonds. Any revenue the central bank earns from its holdings of long-term debt is transferred to the government and, likewise, the central bank's interest payments on reserves are funded by the government. The government purchases final goods spending an amount $P_t G_t$. As explained above, it also transfers an amount $P_t T_t^b$ to borrowers. Any surplus (deficit) the government incurs is transferred to (taxed from) savers. All transfers (taxes) are lump sums. Thus, the government budget constraint is

$$R_t^b B_{t-1}^{\mathtt{cb}} + P_t G_t + P_t T_t^b = P_t T_t + R_t^{re} \mathtt{RE}_t. \tag{A.28}$$

Log-linear equilibrium conditions. The non-linear equilibrium conditions presented above are linearised around a non-stochastic steady state with zero trend inflation. A bar above a variable without subscript denotes the non-stochastic steady state of that variable. A hat above a variable denotes percentage deviation from steady state, e.g., $\widehat{Y}_t = (Y_t - \overline{Y})/\overline{Y}$. We consider all variables in percentage deviation from steady state, except for government expenditure that is expressed relative to steady-state output, $\widetilde{G}_t = (G_t - \overline{G})/\overline{Y}$, with $g = \overline{G}/\overline{Y}$. The resulting linear system of equilibrium conditions is as follows

$$\widehat{Y}_t = (1-g)(1-z)\,\widehat{C}_t + (1-g)\,z\widehat{C}_t^b + \widetilde{G}_t, \tag{A.29}$$

$$\widehat{W}_t - \widehat{P}_t = \frac{\widehat{C}_t}{\sigma} + \eta \widehat{N}_t, \tag{A.30}$$

$$0 = E_t\left(\widehat{C}_{t+1} - \widehat{C}_t\right) - \sigma\left(\widehat{R}_t^s - E_t\pi_{t+1}\right), \tag{A.31}$$

$$0 = E_t\left(\widehat{C}_{t+1}^b - \widehat{C}_t^b\right) - \sigma\left(E_t\widehat{R}_{t+1}^b - E_t\pi_{t+1}\right), \tag{A.32}$$

$$\widehat{R}_t^b = \gamma\beta_b\widehat{Q}_t - \widehat{Q}_{t-1}, \tag{A.33}$$

$$\sigma\widehat{\Omega}_t = \sigma(\overline{R}^b/\overline{\Omega})E_t\widehat{R}_{t+1}^b - \sigma(\overline{R}^s/\overline{\Omega})\widehat{R}_t^s - E_t\left(\widehat{C}_{t+1} - \widehat{C}_t\right) - \sigma E_t\pi_{t+1}, \tag{A.34}$$

$$\widehat{R}_t^s = \widehat{R}_t^{re} = i_t - r^n, \tag{A.35}$$

$$\widehat{Y}_t = \widehat{N}_t, \tag{A.36}$$

$$\widehat{v}_t^p = 0, \tag{A.37}$$

$$\widehat{\mathtt{mc}}_t = \widehat{W}_t - \widehat{P}_t, \tag{A.38}$$

$$\widehat{P}_t^\star - \widehat{P}_{t-1} = (1-\theta\beta)\,\widehat{\mathtt{mc}}_{t+s} + \theta\beta E_t\left(\widehat{P}_{t+1}^\star - \widehat{P}_t\right) + \pi_t, \tag{A.39}$$

$$\pi_t = (1-\theta)\left(\widehat{P}_t^\star - \widehat{P}_{t-1}\right), \tag{A.40}$$

$$\widehat{C}_t^b = \widehat{\mathcal{Q}}_t + \widehat{\mathtt{b}}_t, \tag{A.41}$$

$$\widehat{\mathtt{b}}_t = \left(1 - \frac{\overline{\mathtt{b}}^{\mathrm{cb}}}{\overline{\mathtt{b}}}\right)\widehat{b}_t^{\mathrm{fi}} + \left(\frac{\overline{\mathtt{b}}^{\mathrm{cb}}}{\overline{\mathtt{b}}}\right)\widehat{\mathtt{b}}_t^{\mathrm{cb}}, \tag{A.42}$$

$$\widehat{\mathcal{Q}}_t + \widehat{\mathtt{b}}_t^{\mathrm{cb}} = \widehat{\mathtt{QE}}_t, \tag{A.43}$$

$$\widehat{\mathcal{Q}}_t + \widehat{\mathtt{b}}_t^{\mathrm{fi}} = 0. \tag{A.44}$$

Equations (A.29) to (A.44) form a system of sixteen linear equilibrium conditions, which determines the endogenous variables: $\widehat{Y}_t$, $\widehat{C}_t$, $\widehat{C}_t^b$, $\widehat{N}_t$, $\left(\widehat{W}_t - \widehat{P}_t\right)$, $\left(\widehat{P}_t^{\star} - \widehat{P}_{t-1}\right)$, π_t, v^p, $\widehat{\mathtt{mc}}_t$, $\widehat{\mathcal{Q}}_t$, $\widehat{\Omega}$, $\widehat{R}_t^s$,$\widehat{R}_t^b$, $\widehat{\mathtt{b}}_t$, $\widehat{\mathtt{b}}_t^{\mathtt{fi}}$, $\widehat{\mathtt{b}}_t^{\mathtt{cb}}$.

Aggregate inflation and output. Combining equations (A.39), (A.40), yields

$$\pi_t = \lambda \widehat{\mathtt{mc}}_t + \beta E_t \pi_{t+1}, \tag{A.45}$$

with $\lambda = (1-\theta)(1-\theta\beta)/\theta$. In turn, combining (A.30), (A.36), and (A.38) gives

$$\widehat{\mathtt{mc}}_t = \eta \widehat{Y}_t + \frac{\widehat{C}_t}{\sigma}. \tag{A.46}$$

Making use of equation (A.29) to substitute in (A.46) we obtain

$$\widehat{\mathtt{mc}}_t = \left(\frac{1}{\varphi} + \eta\right)\widehat{Y}_t - \left[\frac{1}{\sigma}\frac{z}{(1-z)}\right]\widehat{C}_t^b - \frac{\widetilde{G}_t}{\varphi}, \tag{A.47}$$

with $\varphi = (1-g)(1-z)\sigma > 0$. Next, combining (A.41), (A.42), (A.43) and (A.44), yields

$$\widehat{C}_t^b = \left(\frac{\overline{\mathtt{b}}^{\mathrm{cb}}}{\overline{\mathtt{b}}}\right)\widehat{\mathtt{QE}}_t. \tag{A.48}$$

Using (A.48) to substitute in (A.47), we obtain

$$\widehat{\mathtt{mc}}_t = \left(\frac{1}{\varphi} + \eta\right)\widehat{Y}_t - \left[\frac{1}{\sigma}\frac{z}{(1-z)}\left(\frac{\overline{\mathtt{b}}^{\mathrm{cb}}}{\overline{\mathtt{b}}}\right)\right]\widehat{\mathtt{QE}}_t - \frac{\widetilde{G}_t}{\varphi}. \tag{A.49}$$

We define the natural level of output as the output obtained when $\widehat{\mathtt{mc}}_t = 0$ and $\widehat{\mathtt{QE}}_t = 0$, so that, from equation (A.49), we obtain

$$\widehat{Y}_t^n = \Gamma \widetilde{G}_t, \quad \text{with} \quad \Gamma = \frac{1/\varphi}{1/\varphi + \eta}. \tag{A.50}$$

Combining (A.49) and (A.50), it is possible to represent the aggregate marginal cost in terms of the welfare-relevant output gap, corresponding to $\left(\widehat{Y}_t - \widehat{Y}_t^n\right) = \left(\widehat{Y}_t - \Gamma\widetilde{G}_t\right)$, as follows

$$\widehat{\mathtt{mc}}_t = \left(\frac{1}{\varphi} + \eta\right)\left(\widehat{Y}_t - \Gamma\widetilde{G}_t\right) - \left[\frac{1}{\sigma}\frac{z}{(1-z)}\left(\frac{\bar{\mathtt{b}}^{\mathtt{cb}}}{\bar{\mathtt{b}}}\right)\right]\widehat{\mathtt{QE}}_t. \tag{A.51}$$

Making use of (A.51) to substitute for the marginal cost in (A.45) yields the Phillips curve equation (1) in the main text. Finally, to obtain the IS curve corresponding to equation (2) in the main text, combine (A.29), (A.31), (A.35) and (A.48).

B Multipliers with flexible and sticky prices

Flexible prices. With flexible prices, there is no price dispersion across retail producers and, therefore, we have that

$$v_t^p = \left(\int_0^1 \frac{p_t(\nu)}{P_t} d\nu\right)^{-\epsilon} = 1,$$

and, consequently, $Y_t = N_t$, from equation (A.23). Furthermore, with flexible prices, we have that

$$\frac{W_t}{P_t} = \mathtt{mc}_t = \left(\frac{\epsilon - 1}{\epsilon}\right). \tag{B.1}$$

Combining (A.4) and (B.1) yields

$$C_t^{-1/\sigma} = \left(\frac{\epsilon}{\epsilon - 1}\right) N_t^{\eta}, \tag{B.2}$$

Finally, making use of equation (A.1) and the aggregate production function to substitute for, in turn, C_t and N_t, yields

$$\left(Y_t - C_t^b - G_t\right)^{-1/\sigma} = \left(\frac{\epsilon}{\epsilon - 1}\right) Y_t^{\eta}. \tag{B.3}$$

Implicit differentiation of (B.3), in the neighborhood of the steady state, yields

$$\begin{aligned}\frac{dY}{dG} &= \frac{(1/\sigma)\left(\overline{Y}-\overline{C}^{b}-\overline{G}\right)^{-1/\sigma-1}}{(1/\sigma)\left(\overline{Y}-\overline{C}^{b}-\overline{G}\right)^{-1/\sigma-1}+\eta\left(\epsilon/\left(\epsilon-1\right)\right)\overline{Y}^{\eta-1}} \\ &= \frac{1/\sigma}{1/\sigma+\eta\left(1-g\right)\left(1-z\right)} = \frac{1/\varphi}{1/\varphi+\eta} = \frac{d\widehat{Y}}{d\widetilde{G}} \equiv \Gamma \in (0,1)\,,\end{aligned} \tag{B.4}$$

corresponding to the multiplier with flexible prices in equation (3).

Sticky prices. When prices are staggered à la Calvo (1983), the ZLB does not bind, monetary policy follows a Taylor (1993)'s type rule and there is absence of QE, the model of the economy includes equations (1) and (2), with the restriction that $\widehat{\mathtt{QE}}_t = 0$ for all $t \geq 0$, and equation (4). Making use of the conjectured solution for output, inflation and government expenditure reported in the main text, yields

$$\gamma_\pi \widetilde{G}_t = \kappa\left(\gamma_y-\Gamma\right)\widetilde{G}_t+\rho\beta\gamma_\pi\widetilde{G}_t, \tag{B.5}$$

$$\left(\gamma_y-1\right)\widetilde{G}_t = \rho\left(\gamma_y-1\right)\widetilde{G}_t-\varphi\left(i_t-\gamma_\pi\rho\widetilde{G}_t-r^\star\right), \tag{B.6}$$

$$i_t = r^n+\phi_\pi\gamma_\pi\widetilde{G}_t+\phi_y\left(\gamma_y-\Gamma\right)\widetilde{G}_t. \tag{B.7}$$

Combining (B.6) and (B.7), yields

$$\left(\gamma_y-1\right)\widetilde{G}_t = \rho\left(\gamma_y-1\right)\widetilde{G}_t-\varphi\phi_y\left(\gamma_y-\Gamma\right)\widetilde{G}_t-\varphi\left(\phi_\pi-\rho\right)\gamma_\pi\widetilde{G}_t. \tag{B.8}$$

Next, from (B.5) we obtain

$$\gamma_\pi = \frac{\kappa\left(\gamma_y-\Gamma\right)}{1-\rho\beta}, \tag{B.9}$$

and plugging into (B.8) yields

$$\left[1-\rho+\varphi\phi_y+\frac{\varphi\left(\phi_\pi-\rho\right)\kappa}{1-\rho\beta}\right]\gamma_y = 1-\rho+\varphi\phi_y\Gamma+\frac{\varphi\left(\phi_\pi-\rho\right)\Gamma}{1-\rho\beta}. \tag{B.10}$$

Rearranging terms, we obtain the multiplier $\Gamma_{\mathtt{Taylor\ rule}}$ in equation (5).

C Fiscal policy at the ZLB with exogenous QE

This appendix comprises five parts, each separately detailing how we determine the existence of the ZLB equilibrium in (8), the signs of the coefficients in the inflation and output gap equations (12) and (13), the required interest rate shock to trigger the ZLB in equation (15), the multiplier with a small fiscal stimulus in equation (17), and the multiplier with a large fiscal stimulus in (18).

Existence of a unique equilibrium. As discussed in the main text, a pre-requisite for the analysis of the government expenditure multiplier at the ZLB is that there exists a unique bounded solution for the endogenous variables, $\left(\widehat{Y}_t, \pi_t, i_t\right) = \left(\overline{Y}_L, \overline{\pi}_L, i_L\right)$, for all $t < T$. This solution exists as long as the condition in equation (8) is met. To verify this condition, consider the parameters $\varkappa_r$, $\varkappa_g$ and $\varkappa_{qe}$ of the equilibrium solution for inflation and output gap in equations (12) and (13). We see that if $(1-\mu)(1-\beta\mu) = \varphi\mu\kappa$, the parameters $\varkappa_r$, $\varkappa_g$, and $\varkappa_{qe}$ become infinitely large, and no equilibrium exists.[24] Instead, if $(1-\mu)(1-\beta\mu) < \varphi\mu\kappa$, then $\varkappa_r < 0$, and with $\bar{r}_L < 0$, $\overline{G}_L \simeq 0$ and $\overline{\mathtt{QE}}_L \simeq 0$, we obtain from the IS equation (13) that $\overline{Y}_L > 0$ and from the Phillips curve in equation (12) that $\overline{\pi}_L > 0$. But then, from equation (7), we have $i_L > 0$, which violates the conjectured solution, $i_L = 0$. Thus, for an equilibrium solution to exist, where the ZLB condition is binding, the restriction (8) must be satisfied.

Coefficients of equilibrium inflation and output. As described in the main text, the signs of the parameters $\varkappa_r$, $\varkappa_g$ and $\varkappa_{qe}$ in the equilibrium solutions for inflation and output gap in equations (12) and (13) follow from the restriction (8) and that $\Gamma < 1$ from (3). In particular, the numerator of $\varkappa_{qe}$ is positive because when condition (8) holds, then $(1-\beta\mu)(1-\mu) > \mu\kappa\varphi$. Because $\kappa = \lambda\left(\frac{1}{\varphi} + \eta\right)$, it follows that $\mu\varphi\kappa > \lambda\mu$.

Enter the ZLB. To determine the size of the shock to the natural interest rate required to reach the ZLB, we replace the solutions for inflation and output gap in equations (12) and (13) into the shadow rate equation in (14), and then solve for $i^s_L = 0$, to obtain:

$$r^n + \left(\frac{\phi_\pi\kappa}{1-\beta\mu} + \phi_y\right)\left(\varkappa_g\overline{G}_L + \varkappa_r\overline{r}_L\right) + \left[\left(\frac{\phi_\pi\kappa}{1-\beta\mu} + \phi_y\right)\varkappa_{qe}\overline{\mathtt{QE}}_L - \frac{\phi_\pi\varsigma}{1-\beta\mu}\right]\overline{\mathtt{QE}}_L < 0.$$

[24] Eggertsson (2011) calls this configuration the deflationary black hole.

In the absence of fiscal and QE stimulus, i.e when stimulus $\overline{G}_L = \overline{\mathtt{QE}}_L = 0$, this reduces to:

$$r^n + \left(\frac{\phi_\pi \kappa}{1-\beta\mu} + \phi_y\right) \varkappa_r \overline{r}_L < 0.$$

Using the definition of $\overline{r}_L = r^n - \delta_L$, yields condition (15).

Multiplier small fiscal stimulus. The multiplier with a small fiscal and QE stimulus in equation (17) follows immediately by differentiating the equilibrium solution for output in equation (13) with respect to $\overline{G}_L$ and then combining terms using the definition of $\varkappa_g$.

Multiplier with large fiscal stimulus. With a large fiscal stimulus that makes the ZLB no longer binding even though the real rate is still negative, the equilibrium is fully caractherised by the triplet of constant values $(\overline{\pi}_L, \overline{Y}_L, i_L)$ that solves the Phillips and the IS curves in (10) and (11), with $i_L = i_L^s$ as in equation (14). The multiplier is now derived as follows. First replace the interest rate in equation (14) on the right side of the IS curve in (11) to derive output as

$$\overline{Y}_L = \frac{\varphi(\mu - \phi_\pi)}{1-\mu+\varphi\phi_y}\overline{\pi}_L + \frac{1-\mu+\varphi\phi_y\Gamma}{1-\mu+\varphi\phi_y}\overline{G}_L - \frac{\varphi}{1-\mu+\varphi\phi_y}\delta_L + \frac{(1-\mu)\xi}{1-\mu+\varphi\phi_y}\overline{\mathtt{QE}}_L.$$

Then replace in the above $\overline{\pi}_L$ from equation (10) to obtain the equilibrium solution for output:

$$\begin{aligned}\overline{Y}_L \quad = \quad & \frac{(1-\mu+\varphi\phi_y\Gamma)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa\Gamma}{(1-\mu+\varphi\phi_y)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa}\overline{G}_L \\ & - \frac{(1-\beta\mu)\varphi}{(1-\mu+\varphi\phi_y)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa}\delta_L - \frac{[(1-\mu)(1-\beta\mu) - (\mu-\phi_\pi)\lambda]\varphi\xi}{(1-\mu+\varphi\phi_y)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa\varphi}\overline{\mathtt{QE}}_L.\end{aligned}$$

Differentiating the above with respect to $\overline{G}_L$ yields:

$$d\overline{Y}_L/d\overline{G}_L = \frac{(1-\mu+\varphi\phi_y\Gamma)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa\Gamma}{(1-\mu+\varphi\phi_y)(1-\beta\mu) - \varphi(\mu-\phi_\pi)\kappa} = \frac{1-\mu+\varphi\phi_y\Gamma - \frac{\varphi(\mu-\phi_\pi)\kappa}{(1-\beta\mu)}\Gamma}{1-\mu+\varphi\phi_y - \frac{\varphi(\mu-\phi_\pi)}{(1-\beta\mu)}\kappa},$$

which corresponds to the multiplier $\Gamma_{\text{ZLB exit}}$ obtained in (18).

D Normalisation and the multiplier at the ZLB

Transitional state. As explained in the main text, the model of the economy now needs to be solved backward in two steps. In the first, given the equilibrium allocation for the perfect foresight state, the solution for output, inflation, and the nominal rate of interest is determined from the system

$$
\begin{aligned}
\overline{\pi}_\tau &= \kappa \overline{Y}_\tau + \beta \mu_\tau \overline{\pi}_\tau - \varsigma \overline{\mathtt{QE}}_\tau \\
\overline{Y}_\tau - \xi \overline{\mathtt{QE}}_\tau &= \mu_\tau \left(\overline{Y}_\tau - \xi \overline{\mathtt{QE}}_\tau \right) - \varphi \left(i_\tau - \mu_\tau \overline{\pi}_\tau - r^n \right) \\
i_\tau &= r^n + \phi_\pi \overline{\pi}_\tau + \phi_y \overline{Y}_\tau .
\end{aligned}
$$

After solving for the inflation rate in the Phillips curve and replacing the Taylor rule in the IS equation, the system reduces to

$$
\begin{aligned}
\overline{\pi}_\tau &= \frac{\kappa}{1 - \beta \mu_\tau} \overline{Y}_\tau - \frac{\varsigma}{1 - \beta \mu_\tau} \overline{\mathtt{QE}}_\tau , \\
\overline{Y}_\tau - \xi \overline{\mathtt{QE}}_\tau &= \mu_\tau \left(\overline{Y}_\tau - \xi \overline{\mathtt{QE}}_\tau \right) - \varphi \left(\phi_\pi \overline{\pi}_\tau + \phi_y \overline{Y}_\tau - \mu_\tau \overline{\pi}_\tau \right) .
\end{aligned}
$$

If we replace the inflation rate from the Phillips curve into the IS equation, the equilibrium level of output is given by

$$
\left[1 - \mu_\tau + \varphi \phi_y - \varphi \left(\mu_\tau - \phi_\pi \right) \frac{\kappa}{1 - \beta \mu_\tau} \right] \overline{Y}_\tau = \left[\xi \left(1 - \mu_\tau \right) - \frac{\varphi \left(\mu_\tau - \phi_\pi \right) \varsigma}{1 - \beta \mu_\tau} \right] \overline{\mathtt{QE}}_\tau ,
$$

which gives the solution $\overline{Y}_\tau = \tau_y \overline{\mathtt{QE}}_\tau$ reported in the main text. Replacing this solution into the Phillips curve yields the inflation rate during the transitional state $\overline{\pi}_\tau = \tau_\pi \overline{\mathtt{QE}}_\tau$, also reported in the main text.

The response coefficient of inflation to QE during the transitional state is positive as long as $\kappa \tau_y - \varsigma > 0$. After replacing the definition of τ_y and multiplying through, this becomes

$$
\frac{\kappa \xi \left(1 - \mu_\tau \right) \left(1 - \beta \mu_\tau \right) + \kappa \varphi \left(\phi_\pi - \mu_\tau \right) \varsigma - \varsigma \left(1 - \beta \mu_\tau \right) \left(1 - \mu_\tau + \psi_\tau \right)}{\left(1 - \beta \mu_\tau \right) \left(1 - \mu_\tau + \psi_\tau \right)} > 0,
$$

which is positive as long as the numerator is positive. This can be written as

$$(\kappa\xi - \varsigma)(1-\mu_\tau)(1-\beta\mu_\tau) + \kappa\varphi(\phi_\pi - \mu_\tau)\varsigma - \varsigma(1-\beta\mu_\tau)\psi_\tau > 0.$$

Using the definition of ψ_τ, cancelling terms and simplifying gives $(\kappa\xi - \varsigma)(1-\mu_\tau) - \varsigma\varphi\phi_y > 0$. Using the definitions of κ and ς, then gives as $\mu_\tau < 1 - \xi\phi_y/\eta$, as reported in the main text.

The crisis state. In the second stage, the equilibrium in the crisis state is determined conditional on the transitional state solution. At the ZLB the dynamics of inflation and output are now governed by

$$\overline{\pi}_L = \kappa\left(\overline{Y}_L - \Gamma\overline{G}_L\right) + \beta\mu\overline{\pi}_L + \beta(1-\mu)\mu_\tau\overline{\pi}_\tau - \varsigma\overline{\mathtt{QE}}_L,$$
$$\overline{Y}_L - \overline{G}_L - \xi\overline{\mathtt{QE}}_L = \mu\left(\overline{Y}_L - \overline{G}_L - \xi\overline{\mathtt{QE}}_L\right) + (1-\mu)\mu_\tau\left(\overline{Y}_\tau - \xi\overline{\mathtt{QE}}_\tau\right) + \varphi\left[\mu\overline{\pi}_L + (1-\mu)\mu_\tau\overline{\pi}_\tau + \overline{r}_L\right].$$

Thus inflation at the ZLB is

$$\overline{\pi}_L = \frac{\kappa}{1-\beta\mu}\left(\widehat{Y}_L - \Gamma\widetilde{G}_L\right) + \frac{\beta(1-\mu)\mu_\tau}{1-\beta\mu}\pi_\tau - \frac{\varsigma}{1-\beta\mu}\overline{\mathtt{QE}}_L.$$

After replacing this in the IS equation, the equilibrium level of output is determined as

$$\overline{Y}_L - \Gamma\overline{G}_L = \varkappa_r\overline{r}_L + \varkappa_g\overline{G}_L + \varkappa_{qe}\overline{\mathtt{QE}}_L + \mu_\tau\frac{(1-\mu)}{\mathcal{R}}\left[(1-\beta\mu)\left(\overline{Y}_\tau - \overline{\mathtt{QE}}_\tau\right) + \varphi\overline{\pi}_\tau\right].$$

This is then combined with the solutions for output and inflation during the transitional state, $\widehat{Y}_\tau$ and π_τ, to be rewritten in the more compact form as in equation (23) in the main text.

We need to evaluate when the coefficient $\varkappa_{qe}^A$ in equation (23) is positive. This is true as long as

$$\frac{\tau_y}{\varphi} + \frac{\tau_\pi}{(1-\beta\mu)} - \frac{\xi}{\varphi} > 0.$$

After replacing the definition of τ_π and multiplying through this inequality becomes

$$\frac{(1-\beta\mu)(1-\beta\mu_\tau)\tau_y + \varphi\kappa\tau_y - \varphi\varsigma - (1-\beta\mu)(1-\beta\mu_\tau)\xi}{\varphi(1-\beta\mu)(1-\beta\mu_\tau)} > 0,$$

which holds as long as the numerator is positive or, equivalently

$$\tau_y > \underline{\tau}_Y \equiv \frac{\varphi\varsigma + \xi\left(1-\beta\mu\right)\left(1-\beta\mu_\tau\right)}{\left(1-\beta\mu\right)\left(1-\beta\mu_\tau\right)+\varphi\kappa}.$$

E Multiplier under an instrument rule for QE

Existence of the unique equilibrium. First we need to determine a condition for the existence of a unique bounded solution when QE is set according to the instrument rule (25). Upon replacing the shadow rate in equation (14) into the instrument rule (25), and then substituting this on the right sides of the Phillips and IS curves in equations (10) and (11), respectively, the dynamics of inflation and the output gap at the ZLB are determined by the system

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t\widehat{Y}_{t+1} \end{bmatrix} = \mathbf{D}\begin{bmatrix} \pi_t \\ \widehat{Y}_t \end{bmatrix},$$

where

$$\mathbf{D} = \begin{bmatrix} \frac{1}{\beta\mu} - \frac{\varsigma}{\beta\mu}\vartheta\phi_\pi & -\left(\frac{\kappa}{\beta\mu} + \frac{\varsigma}{\beta\mu}\vartheta\phi_y\right) \\ -\frac{\varphi}{\mu\beta} + \frac{[\beta(1-\mu)\xi+\varsigma\varphi]}{\mu\beta}\vartheta\phi_\pi & \left(\frac{1}{\mu} + \frac{\kappa\varphi}{\mu\beta} + \frac{[\beta(1-\mu)\xi+\varsigma\varphi]}{\mu\beta}\vartheta\phi_y\right) \end{bmatrix}.$$

The existence of a unique bounded solution at the ZLB requires the two eigenvalues of the matrix $\mathbf{D}$, e_1 and e_2, to be both greater than one. Making use of the result for 2×2 matrices that the product of the eigenvalues is equal to the determinant of the matrix and the sum of the eigenvalues is equal to the trace of the matrix, it follows that

$$\begin{aligned} e_1 e_2 &= \det\left(\mathbf{D}\right) = \frac{\left(1-\mu\right)\xi}{\beta\mu^2}\vartheta\phi_y + \frac{\kappa\left(1-\mu\right)\xi}{\beta\mu^2}\vartheta\phi_\pi + \frac{1}{\beta\mu^2} - \frac{\varsigma}{\beta\mu^2}\vartheta\phi_\pi, \\ e_1 + e_2 &= tr\left(\mathbf{D}\right) = \frac{1}{\beta\mu} - \frac{\varsigma}{\beta\mu}\vartheta\phi_\pi + \frac{1}{\mu} + \frac{\kappa\varphi}{\mu\beta} + \frac{\left[\beta\left(1-\mu\right)\xi + \varsigma\varphi\right]}{\mu\beta}\vartheta\phi_y. \end{aligned}$$

The two eigenvalues of the matrix $\mathbf{D}$ are both greater than one if $\det\left(\mathbf{D}\right) - tr\left(\mathbf{D}\right) > -1$. After incorporating the above solutions and simplifying, it can be shown that when the economy is at the ZLB and QE follows the instrument rule in equation (25) a unique bounded solution exists if and only if

$$(1-\beta\mu)(1-\mu)-\mu\varphi\kappa+[\xi(1-\mu\beta)(1-\mu)-\mu\varsigma\varphi]\vartheta\phi_y+(\kappa\xi-\varsigma)(1-\mu)\vartheta\phi_\pi>0,$$

which can be equivalently expressed in the more compact form given by equation (26) using $\kappa\xi-\varsigma=\lambda\eta\xi$, condition (8), and the definition of $\varkappa_{qe}$. It is straightforward to verify that the inequality in equation (26) holds as long as the model parameters meet the condition in equation (8) and $\vartheta>0$. Note also that when $\vartheta=0$, which occurs if the central bank does not implement any QE at the ZLB or, equivalently, QE is exogenous, the requirement for the existence of a unique bounded solution in equation (26) reduces to $(1-\beta\mu)(1-\mu)-\mu\varphi\kappa>0$, which is condition (8).

Decomposition multiplier under instrument rule. The decomposition of the multiplier in equation (30) is determined as follows. Starting from the equilibrium solution for output in equation (13), we know that

$$\frac{\partial\overline{Y}_L}{\partial\overline{G}_L}=\Gamma+\varkappa_g+\varkappa_{qe}\frac{\partial\overline{\mathtt{QE}}_L}{\partial\overline{G}_L},$$

where $\varkappa_g+\Gamma=\Gamma_{\text{ZLB benchmark}}$ according to (17). To determine $\frac{\partial\overline{\mathtt{QE}}_L}{\partial\overline{G}_L}$, we differentiate the equilibrium solution for QE in equation (28) with respect to government expenditure to obtain at the ZLB

$$\frac{\partial\overline{\mathtt{QE}}_L}{\partial\overline{G}_L}=-\frac{\vartheta\varrho_g}{(1+\varrho_{qe}\vartheta)},$$

which then gives (30).

Derivation multiplier under instrument rule. To determine the multiplier in (31), we replace the ZLB branch of the QE rule in equation (25) on the right side of the Phillips and IS curves in equations (10) and (11), respectively to obtain:

$$\begin{aligned}\overline{\pi}_L &= \frac{(\kappa+\varsigma\vartheta\phi_y)}{(1-\beta\mu-\varsigma\vartheta\phi_\pi)}\left(\overline{Y}_L-\Gamma\overline{G}_L\right)+\frac{\varsigma\vartheta}{[(1-\beta\mu)-\varsigma\vartheta\phi_\pi]}r^n\\ \overline{Y}_L &= \overline{G}_L-\xi\vartheta\phi_y\left(\overline{Y}_L-\Gamma\overline{G}_L\right)+\frac{[\varphi\mu-(1-\mu)\xi\vartheta\phi_\pi]}{(1-\mu)}\overline{\pi}_L+\frac{\varphi}{(1-\mu)}\overline{r}_L-\xi\vartheta r^n.\end{aligned}$$

Combining, the equilibrium solution for output is found to be:

$$\left(1-\mu+\widehat{\psi}\right)\overline{Y}_L=\left(1-\mu+\widehat{\psi}\Gamma\right)\overline{G}_L+\frac{\varsigma\vartheta\left[\varphi\mu-(1-\mu)\,\xi\vartheta\phi_\pi\right]}{1-\beta\mu-\varsigma\vartheta\phi_\pi}r^n-(1-\mu)\,\xi\vartheta r^n+\varphi\overline{r}_L.$$

Differentiation with respect to $\overline{G}_L$ immediately returns the government expenditure multiplier in equation (31).

Analytics multiplier under instrument rule. To determine when the government expenditure multiplier in equation (31) is less than one, we set $\Gamma_{\text{QE IR}}<1$ and solve through the inequality as

$$\frac{\widehat{\psi}\,(\Gamma-1)}{1-\mu+\widehat{\psi}}<0.$$

Using the definition of $\widehat{\psi}$ and solving through, the above becomes

$$\frac{\{\xi\,(\mathcal{R}+\varphi\eta)\,\vartheta\phi_y+(1-\mu)\,\xi\kappa\vartheta\phi_\pi-\kappa\varphi\mu\}\,(\Gamma-1)}{\mathcal{R}+\xi\,(\mathcal{R}+\varphi\eta)\,\vartheta\phi_y+\xi\lambda\eta\,(1-\mu)\,\vartheta\phi_\pi}<0.$$

The denominator is positive given condition (26). Consequently, since $\Gamma-1<0$, the multiplier $\Gamma_{\text{QE IR}}$ is less than one if the expression in the curly bracket is positive, which is condition (32) in the main text.

Multiplier under instrument rule when $z\to 1$. Determining the value of the multiplier $\Gamma_{\text{ZLB IR}}$ as the share of borrowers in the economy tends one, i.e as $z\to 1$, it is not straightforward. Although we have previously shown that $\lim\limits_{z\to 1}\Gamma=1$, the coefficient $\widehat{\psi}$ in equation (31) also depends on z and $\lim\limits_{z\to 1}\widehat{\psi}=\frac{\pm\infty}{-\infty}$. We therefore apply L'Hôpital's rule and find that

$$\lim_{z\to 1}\widehat{\psi}=\lim_{z\to 1}-\frac{\mathcal{K}_1+(1-\mu)\left[\frac{\mathcal{K}_2}{(1-z)^2}\right]}{\left[\frac{\mathcal{K}_2}{(1-z)^2}\right]}$$

with $\mathcal{K}_1=(1-g)\frac{b^{cb}}{b}\left\{(\mathcal{R}\varkappa_{qe}/\xi)\,\upsilon\phi_y+\lambda\eta\left[(1-\mu)\,\upsilon\phi_\pi+\sigma\mu\frac{b}{b^{cb}}\right]\right\}$ and $\mathcal{K}_2=\lambda\upsilon\phi_\pi b^{cb}/\left(\sigma b\right)$, coefficients comprising of model parameters independent from z. From the above, it follows that

$$\lim_{z\to 1}\widehat{\psi}=-\left(1-\mu\right),$$

which is an asymptote for the multiplier $\Gamma_{\text{ZLB IR}}$. It is however easy to verify that as z grows to one $\widehat{\psi}$ tends to $-(1-\mu)$ from below, i.e. $\lim_{z\to 1^-} \widehat{\psi} = [-(1-\mu)]^-$, which ensures that the denominator of $\Gamma_{\text{ZLB IR}}$ in equation (31) is positive in the neighbourhood of one. This implies that $\lim_{z\to 1^-} \Gamma_{\text{ZLB IR}} = 1$.

F Multiplier under an optimal QE rule

Welfare criterion function. As explained in the main text, our starting point is that the central bank wishes to maximise the following objective function in equation (33), which can be written as:

$$\mathbf{W} = E_t \sum_{j=0}^{\infty} \beta^j \mathbf{w}_{t+j}, \tag{F.1}$$

where $\mathbf{w}_t = u\left(Y_t - C_t^b - G_t\right) - v\left(N_t\right) + u\left(C_t^b\right) + \mathcal{G}\left(G_t\right)$. The central bank is assumed to have enough instruments to implement an efficient (first-best optimal) steady-state equilibrium, satisfying the following conditions: $u'\left(\overline{C}\right) = u'\left(\overline{C}^b\right)$ (which implies $\overline{C} = \overline{C}^b$ and thus $z = 1/2$), $v'\left(\overline{N}\right) = u'\left(\overline{C}\right)$, $\mathcal{G}'\left(\overline{G}\right) = u'\left(\overline{C}\right)$, and $\overline{Y} = \overline{N} = \overline{C} + \overline{C}^b + \overline{G}$. Considering small disturbances around the efficient steady state, the loss function of the central bank can be expressed in consumption equivalent, as follows

$$\mathbf{L} = E_t \left[\sum_{j=0}^{\infty} \beta^j \mathtt{L}_{t+j}\right], \tag{F.2}$$

with $\mathtt{L}_t = -\left(\mathbf{w}_t - \overline{\mathbf{w}}\right) / \left(u'\left(\overline{C}\right)\overline{Y}\right)$, the flow loss function of the central bank. The second-order Taylor expansion of $\mathbf{w}_t$ around steady state is given by

$$\begin{aligned}
\mathbf{w}_t - \overline{\mathbf{w}} = {} & u'\left(Y_t - \overline{Y}\right) - u'\left(G_t - \overline{G}\right) \\
& + \frac{u''}{2}\left[\left(Y_t - \overline{Y}\right)^2 + \left(G_t - \overline{G}\right)^2 + 2\left(C_t^b - \overline{C}^b\right)^2\right] \\
& - u''\left[\left(Y_t - \overline{Y}\right)\left(G_t - \overline{G}\right) + \left(Y_t - G_t - \overline{Y} + \overline{G}\right)\left(C_t^b - \overline{C}^b\right)\right] \\
& - v'\left(N_t - \overline{N}\right) - \frac{v''}{2}\left(N_t - \overline{N}\right)^2 + \mathcal{G}'\left(G_t - \overline{G}\right) + \frac{\mathcal{G}''}{2}\left(G_t - \overline{G}\right)^2,
\end{aligned} \tag{F.3}$$

where all the derivatives are evaluated at the steady-state equilibrium, we use the fact that $\overline{C} = \overline{C}^b$, and we denote $u'\left(\overline{C}\right) = u'$, $u''\left(\overline{C}\right) = u''$, $v'\left(\overline{N}\right) = v'$, $v''\left(\overline{N}\right) = v''$, $\mathcal{G}'\left(\overline{G}\right) = \mathcal{G}'$, $\mathcal{G}''\left(\overline{G}\right) = \mathcal{G}''$. Making use of the resource constraint, $Y_t - G_t = C_t + C_t^b$, and imposing $\mathcal{G}'\left(\overline{G}\right) = u'\left(\overline{C}\right)$, in the efficient steady-state, yields

$$\begin{aligned}
\mathbf{w}_t - \overline{\mathbf{w}} = {} & u'\left(Y_t - \overline{Y}\right) + \frac{u''}{2}\left[\left(Y_t - \overline{Y}\right)^2 + \left(G_t - \overline{G}\right)^2 + 2\left(C_t^b - \overline{C}^b\right)^2\right] \\
& - u''\left[\left(Y_t - \overline{Y}\right)\left(G_t - \overline{G}\right) + \left(C_t + C_t^b - \overline{C} - \overline{C}^b\right)\left(C_t^b - \overline{C}^b\right)\right] \\
& - v'\left(N_t - \overline{N}\right) - \frac{v''}{2}\left(N_t - \overline{N}\right)^2 + \frac{\mathcal{G}''}{2}\left(G_t - \overline{G}\right)^2, \\
= {} & u'\left(Y_t - \overline{Y}\right) + \frac{u''}{2}\left[\left(Y_t - \overline{Y}\right)^2 + \left(G_t - \overline{G}\right)^2\right] \\
& - u''\left[\left(Y_t - \overline{Y}\right)\left(G_t - \overline{G}\right) + \left(C_t - \overline{C}\right)\left(C_t^b - \overline{C}^b\right)\right] \\
& - v'\left(N_t - \overline{N}\right) - \frac{v''}{2}\left(N_t - \overline{N}\right)^2 + \frac{\mathcal{G}''}{2}\left(G_t - \overline{G}\right)^2.
\end{aligned} \tag{F.4}$$

Next, applying the approximation $\left(\frac{X_t - \overline{X}}{\overline{X}}\right) \simeq \widehat{X}_t + \frac{\widehat{X}_t^2}{2}$ to each term in (F.4), yields

$$\begin{aligned}
\frac{\mathbf{w}_t - \overline{\mathbf{w}}}{u'\,\overline{Y}} = {} & \widehat{Y}_t + \frac{1}{2}\left(\widehat{Y}_t^2 - \frac{\widehat{Y}_t^2 + \widetilde{G}_t^2 - 2\widehat{Y}\widetilde{G}_t}{\varphi} + \frac{1-g}{\sigma}\widehat{C}_t\widehat{C}_t^b\right) \\
& - \widehat{N}_t - \frac{(1+\eta)\,\widehat{N}_t^2 + \eta_g\widetilde{G}_t^2}{2} + \mathcal{O}(3),
\end{aligned} \tag{F.5}$$

with $\mathcal{O}(3)$ collecting all the terms of third order or higher, and where we use the fact that $\overline{Y} = 2\overline{C}^b/(1-g)$, and with $1/\varphi = -\left(u''/u'\right)\overline{Y} = 2/\left(\sigma(1-g)\right)$ and $\eta_g = -\left(\mathcal{G}''/\mathcal{G}'\right)\overline{Y}$.

Next, consider the equilibrium condition

$$\widehat{N}_t = \widehat{v}_t^p + \widehat{Y}_t, \tag{F.6}$$

where, up to a second-order approximation, we have

$$\widehat{v}_t^p = \ln\int_0^1 \left(\frac{P_t(\nu)}{P_t}\right)^{-\epsilon} d\nu \simeq \left[\frac{\epsilon \text{var}\left(P_t(\nu)\right)}{2}\right], \tag{F.7}$$

with var $(\bullet)$ that denotes the cross-sectional variance of prices. Substituting (F.6) and (F.7) in (F.5), yields

$$\begin{aligned}\frac{\mathbf{w}_t-\overline{\mathbf{w}}}{u'\,\overline{Y}} &= -\frac{1}{2}\left(\frac{1}{\varphi}+\eta\right)\left[\left(\widehat{Y}_t-\Gamma\widetilde{G}_t\right)^2+(1-\Gamma)\,\Gamma\widetilde{G}_t^2\right]+\left(\frac{\xi/\sigma}{\overline{\mathtt{b}}^{\mathrm{cb}}/\overline{\mathtt{b}}}\right)\widehat{C}_t\widehat{C}_t^b \\ &\quad -\frac{\eta_g\widetilde{G}_t^2}{2}-\frac{\epsilon\mathrm{var}\left(P_t\left(j\right)\right)}{2}+\mathcal{O}\left(3\right),\end{aligned} \tag{F.8}$$

where, recall, $\xi=(1-g)\,z\left(\overline{\mathtt{b}}^{\mathrm{cb}}/\overline{\mathtt{b}}\right)$, with $z=1/2$ in the efficient steady state.

Finally, the quadratic approximation to the central bank's welfare function is obtained by making use of the result in Woodford (2003), that

$$\sum_{t=0}^{\infty}\beta^t\mathrm{var}\left(P_t\left(j\right)\right)\simeq\frac{\theta}{(1-\beta\theta)\,(1-\theta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2. \tag{F.9}$$

Combining (F.8) and (F.9), yields

$$\sum_{t=0}^{\infty}\beta^t\mathtt{L}_t=\sum_{t=0}^{\infty}\;\frac{\beta^t}{2}\left[\pi_t^2+\frac{\kappa}{\epsilon}\left(\widehat{Y}_t-\Gamma\widetilde{G}_t\right)^2+\frac{\kappa}{\epsilon}\left(\frac{\eta_g}{1/\varphi}+1-\Gamma\right)\Gamma\widetilde{G}_t^2+\left(\frac{\xi/\sigma}{\overline{\mathtt{b}}^{\mathrm{cb}}/\overline{\mathtt{b}}}\right)\widehat{C}_t\widehat{C}_t^b\right], \tag{F.10}$$

Finally, assuming small disturbances around the steady state and the two-state Markov equilibrium structure described in the main text, we obtain the loss function at the ZLB in equation (34).

Derivation optimal QE rule. To derive the optimal QE policy, we make use of the economy's resource constraint (in log-linear form) in equations (A.29), the borrowers' budget constraint in equation (A.48), alongside the definitions of the parameters φ and ξ, to rewrite the inequality term (35) as

$$\mathtt{I}_L=-\left(\overline{Y}_L-\xi\overline{\mathtt{QE}}_L-\overline{G}_L\right)\overline{\mathtt{QE}}_L. \tag{F.11}$$

As a result the loss function in equation (34) can be written as

$$\mathtt{L}=\frac{1}{2}\left[\overline{\pi}_L^2+\frac{\kappa}{\epsilon}\left(\overline{Y}_L-\Gamma\overline{G}_L\right)^2+\frac{\kappa}{\epsilon}\left(\frac{\eta_g}{1/\varphi}+1-\Gamma\right)\Gamma\overline{G}_L^2-\varsigma\left(\overline{Y}_L-\xi\overline{\mathtt{QE}}_L-\overline{G}_L\right)\overline{\mathtt{QE}}_L\right] \tag{F.12}$$

and the Lagrangian function of the central bank can then be formulated as

$$\mathscr{L} = \mathtt{L} + \lambda_{\text{pc}} \left(\overline{\pi}_L - \kappa \overline{Y}_L + \kappa \Gamma \overline{G}_L - \beta \mu \overline{\pi}_L + \varsigma \overline{\mathtt{QE}}_L \right) + \lambda_{\text{is}} \left[\overline{Y}_L - \overline{G}_L - \xi \overline{\mathtt{QE}}_L - \frac{\varphi \left(\mu \overline{\pi}_L + \overline{r}_L \right)}{(1-\mu)} \right].$$

The system of first-order conditions is

$$\overline{\pi}_L : \ \overline{\pi}_L + \lambda_{\text{pc}} \left(1 - \beta \mu\right) - \frac{\lambda_{\text{is}} \varphi \mu}{1-\mu} = 0, \tag{F.13}$$

$$\overline{Y}_L : \ \omega_y \left(\overline{Y}_L - \Gamma \overline{G}_L \right) - \frac{\varsigma}{2} \overline{\mathtt{QE}}_L - \lambda_{\text{pc}} \kappa + \lambda_{\text{is}} = 0, \tag{F.14}$$

$$\overline{\mathtt{QE}}_L : \ -\varsigma \left(\overline{Y}_L - \overline{G}_L \right) + 2 \varsigma \xi \overline{\mathtt{QE}}_L + 2 \left(\lambda_{\text{pc}} \varsigma - \lambda_{\text{is}} \xi \right) = 0, \tag{F.15}$$

$$\lambda_{\text{pc}} : \ \overline{\pi}_L = \kappa \overline{Y}_L - \kappa \Gamma \overline{G}_L + \beta \mu \overline{\pi}_L - \varsigma \overline{\mathtt{QE}}_L, \tag{F.16}$$

$$\lambda_{\text{is}} : \ \widehat{Y}_L = \overline{G}_L + \xi \overline{\mathtt{QE}}_L + \frac{\varphi \mu}{(1-\mu)} \overline{\pi}_L + \frac{\varphi}{(1-\mu)} \overline{r}_L. \tag{F.17}$$

After combining the above first-order conditions, and using the definitions of $\mathcal{R}$ and $\varkappa_{qe}$, the optimal QE rule is derived as:

$$\overline{\mathtt{QE}}_L = \frac{2 \left(1-\mu\right) \left(\varkappa_{qe} - \xi \right)}{\varsigma \varphi \mu \left(\varkappa_{qe} - 2\xi \right)} \overline{\pi}_L - \frac{\left[\varsigma - 2 \left(\kappa / \epsilon \right) \varkappa_{qe} \right]}{\varsigma \left(\varkappa_{qe} - 2\xi \right)} \overline{Y}_L + \frac{\left[\varsigma - 2 \left(\kappa / \epsilon \right) \varkappa_{qe} \Gamma \right]}{\varsigma \left(\varkappa_{qe} - 2\xi \right)} \overline{G}_L,$$

which is then equivalently written in the more compact form of equation (37), using the definitions of the coefficients $\phi_y^\star$, $\phi_\pi^\star$ and $\phi_{\mathtt{I}}^\star$ in the main text.

Sign of optimal QE rule coefficients. The sign of the the coefficients $\phi_y^\star$, $\phi_\pi^\star$ and $\phi_{\mathtt{I}}^\star$ is determined as follows. First note that the existence of a unique bounded solution at the ZLB under the optimal QE rule in equation (37) requires that

$$\mathcal{R} + \mathcal{R} \varkappa_{qe} \vartheta^\star \left(\phi_y^\star - \phi_{\mathtt{I}}^\star \right) + \xi \lambda \eta \left(1-\mu\right) \vartheta^\star \phi_\pi^\star > 0,$$

which is the same stability requirement as with the instrument rule in equation (26), but $\vartheta^\star \left(\phi_y^\star - \phi_{\mathtt{I}}^\star \right)$ and $\vartheta^\star \phi_\pi^\star$ replace $\vartheta \phi_\pi$ and $\vartheta \phi_y$, respectively. Consequently, since $\vartheta^\star > 0$, sufficient conditions for the existence of a unique equilibrium at the ZLB when the central bank follows the optimal QE rule in equation (37) are that $\phi_y^\star > \phi_{\mathtt{I}}^\star$ and $\phi_\pi^\star > 0$. As noted in the main text, $\varkappa_{qe} > \xi > 0$. This implies that the numerator of $\phi_\pi^\star$ is positive. The numerators of both $\phi_y^\star$ and $\phi_{\mathtt{I}}^\star$ are positive too. Thus, existence of a unique bounded equilibrium under the optimal QE rule requires the denominators of all three coefficients to be positive, or equivalently that

$$2\xi - \varkappa_{qe} > 0. \tag{F.18}$$

As a consequence it necessarily follows that $\phi_{\mathtt{I}}^\star > 0$, which then implies that $\phi_y^\star > \phi_{\mathtt{I}}^\star > 0$.

Response of QE to fiscal stimulus. To evaluate whether and how QE responds to a fiscal stimulus under the optimal rule, consider differentiating the QE rule in equation (37) with respect to $\overline{G}_L$:

$$\frac{\partial \overline{\mathtt{QE}}_L^\star}{\partial \overline{G}_L} = -\vartheta^\star \phi_\pi^\star \frac{\partial \overline{\pi}_L}{\partial \overline{G}_L} - \vartheta^\star \left(\phi_y^\star - \phi_{\mathtt{I}}^\star\right) \frac{\partial \overline{Y}_L}{\partial \overline{G}_L} + \vartheta^\star \left(\phi_y^\star \Gamma - \phi_{\mathtt{I}}^\star\right). \tag{F.19}$$

From the equilibrium solutions of inflation and the output at the ZLB in equations (12) and (13), we know that:

$$\begin{aligned}
\frac{\partial \overline{\pi}_L}{\partial \overline{G}_L} &= \frac{1}{1-\beta\mu}\left[\kappa\varkappa_g + \left(\kappa\varkappa_{qe} - \varsigma\right)\frac{\partial \overline{\mathtt{QE}}_L}{\partial \overline{G}_L}\right], \\
\frac{\partial \overline{Y}_L}{\partial \overline{G}_L} - \Gamma &= \varkappa_g + \varkappa_{qe}\frac{\partial \overline{\mathtt{QE}}_L}{\partial \overline{G}_L}.
\end{aligned}$$

Replacing into (F.19) and rearranging yields:

$$\frac{\partial \overline{\mathtt{QE}}_L^\star}{\partial \overline{G}_L} = \frac{-\frac{\vartheta^\star \phi_\pi^\star \kappa \varkappa_g}{1-\beta\mu} - \vartheta^\star \left(\phi_y^\star - \phi_{\mathtt{I}}^\star\right)\left(\Gamma + \varkappa_g\right) + \vartheta^\star\left(\phi_y^\star\Gamma - \phi_{\mathtt{I}}^\star\right)}{\left[1 + \frac{\vartheta^\star \phi_\pi^\star (\kappa\varkappa_{qe} - \varsigma)}{1-\beta\mu} + \vartheta^\star\left(\phi_y^\star - \phi_{\mathtt{I}}^\star\right)\varkappa_{qe}\right]}.$$

The denominator is always positive, because $\kappa\varkappa_{qe} - \varsigma > 0$, as noted in the main text, and $\phi_y^\star > \phi_{\mathtt{I}}^\star$, as determined above from the sufficient conditions for the existence of a unique equilibrium at the ZLB when the central bank follows the optimal QE rule in equation (37). Thus the derivative exists. Further, the derivative is negative so long as the numerator is negative or:

$$-\phi_\pi^\star \kappa \varkappa_g - \left(\phi_y^\star - \phi_{\mathtt{I}}^\star\right)\left(\Gamma + \varkappa_g\right)\left(1 - \beta\mu\right) + \left(\phi_y^\star \Gamma - \phi_{\mathtt{I}}^\star\right)\left(1-\beta\mu\right) < 0.$$

This inequality can be equivalently expressed as a boundary value on μ, that is:

$$\mu < \frac{1}{\beta}\left[1+\frac{\phi_{\pi}^{\star}\kappa\varkappa_{g}}{\left(\phi_{y}^{\star}-\phi_{\mathtt{I}}^{\star}\right)\varkappa_{g}+\phi_{\mathtt{I}}^{\star}\left(1-\Gamma\right)}\right].$$

The right side is always a number greater than one, thus the inequality holds for sure since $\mu < 1$ by definition. Thus, it must be that $\frac{\partial \overline{\mathtt{QE}}_{L}^{\star}}{\partial G_{L}} < 0$.

Derivation multiplier under optimal QE rule. The optimal QE rule in equation (37) can then be replaced on the right sides the Phillips and IS curves in equations (10) and (11), respectively. From the resulting equilibrium solution for output it follows that the government expenditure multiplier is then given by (38), where the coefficients $\psi^{\star}$ and $\psi_{\mathtt{I}}$ are defined in the main text.

Analytics of multiplier under optimal QE rule. The size of the government expenditure multiplier $\Gamma_{\text{QE Opt}}$ in equation (38) is established as follows. As a preliminary step it is worth observing the multiplier $\Gamma_{\text{QE Opt}}$, like all other multipliers derived in the paper, takes the form of a ratio $(\mathcal{A}+\mathcal{B}\Gamma)\,/\,(\mathcal{A}+\mathcal{B})$, with coefficients $(\mathcal{A},\mathcal{B}) \lesseqgtr 0$ and $\Gamma \in (0,1)$. Such a ratio is definitely less than one in either of these two cases: $\mathcal{B} < 0$ and $\mathcal{A}+\mathcal{B} < 0$ (**Case 1**), or $\mathcal{B} > 0$ and $\mathcal{A}+\mathcal{B} > 0$, (**Case 2**). [25] Bearing this result in mind, we can verify that as long as QE is sufficiently aggressive under the optimal rule, i.e. the coefficients of the optimal QE rule meet the restrictions $\phi_{\pi}^{\star} > \overline{\phi}_{\pi}$ and $\phi_{y}^{\star} > \overline{\phi}_{y}$, then the multiplier $\Gamma_{\text{QE Opt}}$ in equation (38) is necessarily less than one. To see this, first note that $\psi^{\star}$ and $\psi_{\mathtt{I}}$ share the same denominator. This denominator is positive as long as $\phi_{\pi}^{\star} < 1-\beta\mu \equiv \phi_{\pi,D}$. Further, even when $\phi_{y}^{\star} \simeq 0$, the numerator of $\psi^{\star}$ is positive as long as $\phi_{\pi}^{\star} > \frac{\varsigma\varphi\mu}{(1-\mu)\xi} \equiv \phi_{\pi,N}$. From (8) it follows that $\phi_{\pi,D} > \phi_{\pi,N}$. Consider then two separate cases. **Case 1**. Suppose $\phi_{\pi}^{\star} > \phi_{\pi,D}$, then: 1) $\psi_{\mathtt{I}} < 0$. 2) (Den $\psi^{\star}$) < 0, (Num $\psi^{\star}$) > 0, $\Rightarrow \psi^{\star} < 0$, and becomes even more negative if $\phi_{y}^{\star}$ is large. 3) $1-\mu-\psi_{\mathtt{I}}\phi_{\mathtt{I}}^{\star} > 0$, because $1-\mu > 0$ and $-\psi_{\mathtt{I}}\phi_{\mathtt{I}}^{\star} > 0$. 4) (Den $\Gamma_{\text{QE Opt}}$) $= \underbrace{1-\mu-\psi_{\mathtt{I}}\alpha_{\mathtt{I}}^{\star}}_{>0}+\psi^{\star}$ is less than 0 if $\phi_{y}^{\star}$ large. Equivalently, $1-\mu-\psi_{\mathtt{I}}\phi_{\mathtt{I}}^{\star}+\psi^{\star} < 0$ if $\phi_{y}^{\star} > \overline{\phi}_{y}$, where the threshold $\overline{\phi}_{y}$ is defined in equation (40) after solving this inequality. 5) Consequently, the multiplier $\Gamma_{\text{QE Opt}}$ is necessarily less than one. **Case 2**. Suppose

[25] It is straightforward to verify that both $\Gamma_{\mathtt{Taylor\ rule}} < 1$ in equation (5) and $\Gamma_{\text{ZLB exit}} < 1$ in equation (18) meet the requirements of Case 2. Similarly, the inequality condition in (32) ensures that the multiplier $\Gamma_{\text{QE IR}} < 1$ in equation (31) can only fall in either Case 1 or Case 2, depending on the magnitude of ϑ.

$\phi_{\pi,N} < \phi_\pi^\star < \phi_{\pi,D}$, then: 1) $\psi_{\mathtt{I}} > 0$. 2) (Den $\psi^\star$) > 0, (Num $\psi^\star$) > 0, $\Rightarrow \psi^\star > 0$ and the larger is $\phi_y^\star$ the larger is $\psi^\star$. 3) $1 - \mu - \psi_{\mathtt{I}}\phi_{\mathtt{I}}^\star$ could be either positive, in which case $1 - \mu - \psi_{\mathtt{I}}\phi_{\mathtt{I}}^\star + \psi^\star > 0$ and the multiplier is less than one. 4) or $1 - \mu - \psi_{\mathtt{I}}\alpha_{\mathtt{I}}$ could be negative, however still $1 - \mu - \psi_{\mathtt{I}}\phi_{\mathtt{I}}^\star + \psi^\star > 0$ if $\phi_y^\star$ is large, which again requires $\phi_y^\star > \overline{\phi}_y$, as defined in (40). 5) Consequently, the multiplier $\Gamma_{\text{QE Opt}}$ is necessarily less than one. In summary, cases 1 and 2 imply that the government expenditure multiplier $\Gamma_{\text{QE Opt}}$ in equation (38) is necessarily less than one as long as the QE is sufficiently aggressive under the optimal policy, where sufficiently aggressive for the optimal QE rule in equation (37) requires $\phi_\pi^\star > \overline{\phi}_\pi$ and $\phi_y^\star > \overline{\phi}_y$.