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Business cycle models with labour market frictions: the role of the matching function*

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Abstract

Standard business cycle models with search and matching frictions in the labour market increasingly rely on the assumption that firms face hiring, as opposed to, search costs in recruiting workers. We show that although this modification improves the model's empirical performance, it causes the matching function to play no role in macroeconomic dynamics. Assuming both costs can overcome this shortcoming but for reasonable parameter values it implies that matching efficiency shocks have no effects.

JEL classifications: E32, C52, J64

Keywords: DSGE models; Labour market; search and matching; unemployment; hiring costs.

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models with search and matching frictions in the labour market have become a major workhorse in applied monetary economics. In addition to providing an explanation for the existence of equilibrium unemployment, such models provide an additional source of endogenous persistence and are consistent with the fact that most labour adjustments occur on the employment or extensive margin (Andolfatto, 1996).

The dominant approach to modelling unemployment and labour market fluctuations is the search and matching Diamond-Mortensen-Pissarides (DMP) model (Mortensen and Pissarides, 1994) and first incorporated into a New Keynesian (NK) model in Walsh (2003). A key aspect of search models is the matching function, where unemployed workers searching for a job and firms posting vacancies in order to recruit workers interact to generate new matches. This innovation results in the model containing a new variable, vacancies, so that a theory of how jobs are created is required. Most of the literature assumes that firms face a cost per vacancy posted and that they will post vacancies until their expected returns net of the costs are driven to zero due to free entry.

Increasingly, NK models with labour market search have replaced the vacancy-posting costs with hiring costs. In other words, instead of facing a cost in searching for a worker, firms face costs post-match from changing the hiring rate and these can be thought of as costs involved in training a worker. Examples of such models include Gertler et al. (2008), Gertler and Trigari (2009), Hertweck (2013), Di Pace and Hertweck (2019) and Gertler et al. (2020). An appealing aspect of this feature is that it partly offsets the Shimer puzzle within a monetary economics context whereby the output response to interest rate shocks is much weaker than that observed in the data. Crucially, Christiano et al. (2016) use Bayesian methods to estimate a medium-scale DSGE model with labour market frictions and embodying both costs formulations, finding that hiring costs form by far the largest component.

However, we show that in a standard DMP New Keynesian model where vacancy-posting costs are replaced with hiring costs, the matching function plays no role: its specification, parameter values and shocks to matching efficiency only affect the number of vacancies posted by firms as well as the job filling probability but it cannot influence the behaviour of the model's remaining variables, including output and unemployment. Thus, reliance on hiring costs leads to the NK-DMP framework not truly being a search and matching model as the concept is generally understood.

2 Search and matching frictions in a one-period model

The irrelevance of the matching function for macroeconomic dynamics in the DMP model with hiring costs is best understood with a simple one-period model. We begin by solving the model with the usual DMP set-up where firms have to incur a cost per vacancy that is posted and then contrast the results by considering the effects of hiring costs.

The representative household contains many members who are either employed (n) or unemployed (u). Abstracting from any labour force participation decision and normalising the labour force to one we then have

$$u = 1 - n$$

Current employment equals the given initial value n_0 plus the new hires or matches

$$n = n_0 + m(u, v)$$

where $m(u, v)$ is the matching function, which combines the unemployed agents u and vacancies v to match workers to firms. We shall assume the following functional form

$$m = \bar{\mu} v^{\eta} u^{1-\eta}$$

so that η represents elasticity of matches with respect to vacancies and the constant $\bar{\mu}$ can be interpreted as a matching efficiency parameter. It will prove convenient to re-write the matching function in terms of labour market tightness, θ, defined as the ratio of vacancies to unemployment. We therefore have

$$\theta = \frac{v}{u} = \frac{v}{1-n}$$

and

$$m = \bar{\mu}\theta^{\eta-1}v$$

2.1 Firms

In order to recruit a worker the firm must post a vacancy, whose probability of being filled is given by

$$\mu(\theta) = \frac{m}{v} = \bar{\mu}\theta^{\eta-1}$$

and is taken as exogenous by the firm. Moreover, posting vacancies is costly. We assume that posting a vacancy involves a cost $A\kappa$, where A is the aggregate level of technology.[1]

The problem for the firm is to maximise its profit, Π, given by

$$\Pi = \max_{v} An - wn - A\kappa v$$

subject to

$$n = \mu(\theta)v + n_0$$

and where w denotes the real wage.

Combining the first order conditions for vacancies and employment we obtain

[1] As in Blanchard and Galí (2010), we make the vacancy-posting cost proportional to the aggregate level of technology as it simplifies the results without altering the conclusions.

$$A - w = \frac{A\kappa}{\mu(\theta)} \tag{1}$$

which represents the free entry condition. Given that an unfilled vacancy results in no output being produced, the firm's surplus from a match is given by

$$\mathcal{S}_F = A - w \tag{2}$$

2.2 Households

We assume that there is a representative household containing many members, with a fraction n of these being employed while the remainder are unemployed, so that

$$u = 1 - n$$

Household members do not choose the amount of work that they supply, n, but they will accept a match as long as they obtain a utility gain from doing so. The utility of the household is given by

$$\mathcal{W}(n) = U(c) - V(n)$$

while their budget constraint is

$$c = wn + \Pi$$

Household members will then accept a job offer whenever $\mathcal{W}'(n) \geq 0$, which occurs when

$$w \geq \frac{V'(n)}{U'(c)}$$

Moreover, for the household the surplus from the match is given by

$$\mathcal{S}_H = w - \frac{V'(n)}{U'(c)} \tag{3}$$

2.3 Wage determination

Assuming that wages are set according to Nash bargaining, firms and workers maximise the aggregate surplus from the match, which is given by

$$\mathcal{S} = \mathcal{S}_H^{\phi} \mathcal{S}_F^{1-\phi} \tag{4}$$

where ϕ represents the worker's share. The wage that maximises the surplus is then given by

$$w = \phi A + (1-\phi)\frac{V'(n)}{U'(c)} \tag{5}$$

2.4 Aggregation and equilibrium

Substituting out vacancies by using $v = (1-n)\theta$, the equilibrium equations are given by

$$\begin{aligned} c + A\kappa(1-n)\theta &= An \\ A - w &= \frac{A\kappa}{\mu(\theta)} \\ w &= \phi A + (1-\phi)\frac{V'(n)}{U'(c)} \\ n &= \mu(\theta)(1-n)\theta + n_0 \end{aligned} \tag{6}$$

The first three equations represent the aggregate resource constraint; the free entry condition and the equilibrium real wage. The last equation is simply the law of motion for employment. In this model the structure of the matching function (as well as matching efficiency shocks via $\bar{\mu}$) is contained in $\mu(\theta)$.

In order to obtain simple analytical expressions we assume that $U(c) = \ln c$ and $V(n) = \gamma n$ so that $V'(n)/U'(c) = \gamma c$. For comparison purposes, it should be noted that the equations can be simplified to a system containing n and θ only:

$$\frac{\kappa}{\mu(\theta)} = (1-\phi)\left[1-\gamma\left(n-\kappa(1-n)\theta\right)\right]$$
$$n = \mu(\theta)(1-n)\theta + n_0$$

Therefore, employment and the remaining variables in the model depend on the parameters in the matching function $\bar{\mu}$ and η.

2.5 The DMP model with hiring costs

Define the hiring rate x as

$$x = \frac{m}{n_0}$$

We now assume that firm's objective is[2]

$$\Pi = \max_v An - wn - A\kappa_h x n_0$$

In other words, as in Gertler et al. (2008), the firm incurs a cost not in searching for a worker by posting vacancies, but in training the employee once the match has been made.

The law of motion of employment can be written as

$$n = n_0 + xn_0$$

Combining the first order conditions for employment and the hiring rate the free entry condition now becomes

$$A - w = A\kappa_h \tag{7}$$

[2]In principle the parameter κ is not the same as in the previous model but we are 'recycling' notation for simplicity.

which replaces (1).

The model is now given by the following equations

$$\begin{aligned} c + A\kappa_h x n_0 &= An \\ A - w &= A\kappa_h \\ w &= \phi A + (1-\phi)\frac{V'(n)}{U'(c)} \\ n &= n_0 + x n_0 \end{aligned} \tag{8}$$

Crucially, the model no longer contains $\mu(\theta)$: employment (and hence unemployment) as well as the remaining three variables above (consumption, wages and the hiring rate) are invariant to the formulation of the matching function or to matching efficiency shocks. In other words, the search and matching component of the model is superfluous.

We next consider a dynamic New Keynesian model to show that the finding obtained above still holds.

3 The model

The main elements of the model follow Ravenna and Walsh (2008) closely so the description will be kept brief.[3] Households are identical and they are uniformly distributed on the unit interval. The representative household has a unit measure of workers, who supply labour inelastically to wholesale firms so that the proportion of the household in employment is denoted by N_t. Consumption risks are fully pooled within the household so that all of its members will enjoy the same level of consumption.

Households maximise utility, which depends on consumption of the final good only. We abstract from the intensive margin and assume that household members are either employed or searching for work. Households supply labour to wholesale firms that produce

[3] Unlike Ravenna and Walsh (2008), we assume that the unemployed do not receive an income in order to maintain comparability with the one-period model presented above. This modification does not alter the conclusions of the paper.

a homogeneous good, which they then sell to retailers, who turn these into differentiated goods and operate under monopolistic competition.

The household's problem is then given by

$$W_t(N_t, B_t) = max \{U(C_t) + \beta E_t W_{t+1}(N_{t+1}, B_{t+1})\} \tag{9}$$

subject to

$$P_t C_t + B_{t+1} = P_t w_t N_t + R_{t-1} B_t + P_t \Pi_t^r$$

where C_t denotes consumption, with price P_t and B_{t+1} represents one-period risk-free bonds purchased in period t with gross return R_t. Employed members receive a real wage w_t and Π_t^r represents the profits of the retail firms.

The household purchases C_t consumption goods from the retailers, which is a Dixit-Stiglitz aggregate of the individual goods i, defined as

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

so that the price level is given by

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

Household optimisation implies

$$U'(C_t) = \beta E_t \left[U'(C_{t+1}) \frac{R_t}{\Pi_{t+1}} \right]$$

where $\Pi_t = P_t / P_{t-1}$ denotes the gross inflation rate.

Each period, a fraction ρ of the matched jobs is exogenously terminated and searching workers (s_t) consist of those previously unemployed as well as those whose match has ended. Searching workers have a probability p_t of finding a match within the period so that the law of motion of aggregate employment is given by

$$N_t = (1 - \rho)N_{t-1} + p_t s_t \tag{10}$$

and the proportion of searching workers follows

$$s_t = 1 - (1 - \rho)N_{t-1} \tag{11}$$

3.1 Wholesale firms

Wholesale firm i produces a homogeneous output Y_{it}^w using only labour and subject to an aggregate technology shock Z_t

$$Y_{it}^w = Z_t N_{it}$$

In order to obtain new employees, the firm must post vacancies v_{it}.

3.2 Benchmark case: vacancy-posting costs

In the benchmark case, each posted vacancy involves a cost κ_v in final good units so that v_t is a Dixit-Stiglitz aggregate of the same form as consumption. Therefore, total private demand for final goods is

$$E_t = P_t \left(C_t + \kappa_v v_t \right)$$

The objective of wholesale firm i is to maximise

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\lambda_{t+j}}{\lambda_t} \right) \Pi_{i,t+j}$$

with

$$\Pi_{i,t+j} = \left(\frac{P^w_{t+j}}{P_{t+j}}\right) Y^w_{i,t+j} - \kappa_v v_{i,t+j} - w_{t+j} N_{i,t+j} \tag{12}$$

where λ_t is the marginal utility of consumption for the household, P^w_t is the price of the wholesale good with v_{it} denoting the number of vacancies posted by the firm. The maximisation is subject to the production function and the law of motion of employment, given by

$$N_{it} = (1-\rho)N_{i,t-1} + v_{it}q(\theta_t)$$

with the number of matches in period t being equal to the number of vacancies posted times the probability that a vacancy will be filled, $q(\theta_t)$, where $\theta_t \equiv v_t/s_t$ denotes labour market tightness.

Combining the first order conditions we obtain

$$\frac{\kappa_v}{q(\theta_t)} = \frac{P^w_t}{P_t} Z_t - w_t + \beta(1-\rho)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa_v}{q(\theta_{t+1})}\right] \tag{13}$$

This equation states that the cost of posting a vacancy equals the expected present value of a new match. In addition, we also have the free-entry condition

$$V^J_t = \frac{\kappa_v}{q(\theta_t)} \tag{14}$$

where V^J_t is the value of a match to the firm so that the cost of posting a vacancy equals its expected value.

Wages are determined via the Nash bargaining solution in which workers receive a share equal to b. Denoting V^S_t the net marginal value of employment to the representative household, the function to be maximised is[4]

[4] As the model assumes that wages are flexible, the solution is the same whether the bargaining occurs

$$\max_{w_t} \left(V_t^S\right)^b \left(V_t^J\right)^{1-b}$$

This gives rise to the sharing rule

$$(1-b)V_t^S = bV_t^J$$

While the value of firm's surplus is given by (14), in deriving the worker's surplus we note that a worker who is employed receives w_t. Moreover, the match has a continuation value of V_{t+1}^S, with a discount factor that takes into account that the match may not continue into the next period and adjusted by the probability of remaining unemployed in $t+1$. The worker's surplus is therefore

$$V_t^S = w_t + \beta(1-\rho)E_t \frac{\lambda_{t+1}}{\lambda_t} \left(1 - \theta_{t+1} q_{t+1}\right) V_{t+1}^S \tag{15}$$

3.3 Alternative case: hiring costs

Following Gertler and Trigari (2009), Gertler et al. (2008) and Di Pace and Hertweck (2019), among others, rather than positing that posting vacancies is costly, we assume that firms face a cost when hiring labour. That is, the cost arises after the match. The profit function (12) is therefore now replaced with

$$\Pi_{i,t+j} = \left(\frac{P_{t+j}^w}{P_{t+j}}\right) Y_{i,t+j}^w - \frac{1}{2}\kappa_h x_{it}^2 N_{i,t-1} - w_{t+j} N_{i,t+j} \tag{16}$$

where

$$x_{it} = \frac{q(\theta_t) v_{it}}{N_{i,t-1}}$$

represents the firm's hiring rate. In this case, the job creation condition (13) and the value of the firm (14) are replaced by

over real or nominal wages.

$$\kappa_h x_t = V_t^J \tag{17}$$

and

$$V_t^J = \frac{P_t^w}{P_t} Z_t - w_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\rho) V_{t+1}^J + \frac{1}{2} \kappa_h x_{t+1}^2 \right] \tag{18}$$

respectively. It should be noted that the assumption of quadratic, as opposed to linear, hiring costs does not affect the key results of this paper, nor the assumption that the costs pertain to the firm's hiring rate rather than the aggregate hiring rate, as in Hertweck (2013) and Di Pace and Hertweck (2019).

3.4 Retail firms

There is a continuum of monopolistically competitive retail firms, indexed by $j \in [0, 1]$. Retailers purchase the homogeneous goods from wholesalers, differentiate them and then sell the resulting final output to households (for consumption) and wholesale firms (for vacancies). Retailers face a common real marginal cost given by

$$mc_t = \frac{P_t^w}{P_t}$$

Retailers are subject to Calvo (1983) pricing with price-stickiness parameter ω so that the objective of the firm is to set $P_t(j)$ in order to maximise

$$E_t \sum_{i=0}^{\infty} (\beta\omega)^i \frac{\lambda_{t+i}}{\lambda_t} \left[\left(\frac{P_t(j) - P_{t+i}^w}{P_{t+i}} \right) Y_{t+i}(j) \right]$$

Expenditure minimisation implies that the demand for the final good produced by firm j is given by

$$Y_{t+i}(j) = \left(\frac{P_t(i)}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i}$$

with aggregate demand for the final good being equal to E_t/P_t. Defining the optimal re-set price P_t^* and noting that the aggregate price index implies that

$$\Pi_t^{1-\varepsilon} = \omega + (1-\omega)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

we can obtain the New Keynesian Phillips curve.

3.5 Market clearing

In equilibrium,

$$Y_t(i) = C_t(i) + v_t(i)$$

for all $i \in [0,1]$ and all t. Defining aggregate output as $Y_t \equiv \left(\int_0^1 Y_t(j)^{1-1/\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$

it follows that under vacancy-posting costs the aggregate resource constraint equals

$$Y_t = C_t + \kappa_v v_t \tag{19}$$

whereas under hiring costs it becomes

$$Y_t = C_t + \frac{1}{2}\kappa_h x_t^2 N_{t-1} \tag{20}$$

with

$$Y_t^w = Y_t \Delta_t \tag{21}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$ is a measure of price dispersion, which equals zero in a first order approximation to a zero inflation steady state.

3.6 Monetary policy

We close the model with a simple rule for the nominal interest rate

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} e^{\varepsilon_{ut}} \tag{22}$$

Table 1: Equilibrium conditions

Description	Model equation	
Consumption Euler equation	$\lambda_t = \beta E_t\left[\frac{\lambda_{t+1}R_t}{\Pi_{t+1}}\right]$	[i]
Marginal utility of consumption	$\lambda_t = C_t^{-\sigma}$	[ii]
Price index	$\Pi_t^{1-\varepsilon} = \omega + (1-\omega)\left(\frac{\tilde{G}_t}{\tilde{H}_t}\Pi_t\right)^{1-\varepsilon}$	[iii]
Auxliary equation	$\tilde{G}_t = \bar{\mu}\lambda_t\frac{Y_t}{\mu_t} + \beta\omega\tilde{G}_{t+1}\Pi_{t+1}^{\varepsilon}$	[iv]
Auxiliary equation	$\tilde{H}_t = \lambda_t Y_t + \beta\omega\tilde{H}_{t+1}\Pi_{t+1}^{\varepsilon-1}$	[v]
Interest rate rule	$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} e^{\varepsilon_{ut}}$	[vi]
Law of motion for labour	$N_t = (1-\rho)N_{t-1} + M_t$	[vii]
Value of match to worker	$V_t^S = w_t + \beta(1-\rho)E_t\frac{\lambda_{t+1}}{\lambda_t}(1-\theta_{t+1}q_{t+1})V_{t+1}^S$	[viii]
Wage sharing rule	$(1-b)V_t^S = bV_t^J$	[ix]
Labour market tightness	$\theta_t = \frac{v_t}{s_t}$	[x]
Probability of searcher obtaining match	$p_t = \frac{M_t}{s_t}$	[xi]
Vacancy-filling rate	$q_t = \frac{M_t}{v_t}$	[xii]
Fraction of searching workers	$s_t = 1-(1-\rho)N_{t-1}$	[xiii]
Unemployment	$u_t = 1 - N_t$	[xiv]
Matching function	$M_t = e^{\varepsilon_{mt}}\eta v_t^{\xi} s_t^{1-\xi}$	[xv]
Domestic output	$Y_t = \frac{Z_t N_t}{\Delta_t}$	[xvi]
Hiring rate	$x_t = \frac{q_t v_t}{N_{t-1}}$	[xvii]
Technology shock	$log(Z_t) = \rho_z \log(Z_{t-1}) + \varepsilon_{at}$	[xviii]
Under vacancy-posting costs		
Free entry condition	$\kappa_v = q_t V_t^J$	[ixx_a]
Job creation condition	$\frac{\kappa_v}{q_t} = \frac{Z_t P_t^w}{P_t} - w_t + \beta(1-\rho)E_t\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\kappa_v}{q_{t+1}}\right)$	[xx_a]
Aggregate resource constraint	$Y_t = C_t + \kappa_v v_t$	[xxi_a]
Under hiring costs		
Free entry condtion	$\kappa_h x_t = V_t^J$	[ixx_b]
Job creation condition	$\kappa_h x_t = \frac{Z_t P_t^w}{P_t} - w_t + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left((1-\rho)\,\kappa_h x_{t+1} + \frac{1}{2}\kappa_h x_{t+1}^2\right)$	[xx_b]
Aggregate resource constraint	$Y_t = C_t + \frac{1}{2}\kappa_h x_t^2 N_{t-1}$	[xxi_b]

where bars denote steady state values and ε_{ut} represents a shock to the interest rate rule, assumed to be white noise.

4 Steady state and parameterisation

The full model is shown in Table 1. To analyse the dynamic properties of the model, we solve a log-linear version that is approximated around its deterministic steady state, whose equations are presented in (23)-(25). We approximate the model around a zero-inflation steady state ($\bar{\Pi} = 1$) and we set a steady state price mark-up – $\bar{\mu} \equiv \bar{P}^w/\bar{P}$ – to one, reflecting the implicit assumption that a subsidy offsets the distortion caused by monopolistic competition. The steady state rate of employment, $\bar{N}$, and the vacancy-filling rate, $\bar{q}$, are set to 0.9 and 0.7, respectively.[5]

Given the above and with a steady state value of technology $\bar{Z} = 1$, we can solve for output and the remaining labour market variables. The remaining variables are solved depending on the cost structure affecting employers.

$$\begin{aligned}
\bar{R} &= \frac{\bar{\Pi}}{\beta} \\
\bar{Y} &= \bar{Z}\bar{N} \\
\bar{M} &= \rho\bar{N} \\
\bar{v} &= \frac{\bar{M}}{\bar{q}} \\
\bar{s} &= 1 - (1-\rho)\bar{N} \\
\bar{\theta} &= \frac{\bar{v}}{\bar{s}} \\
\bar{p} &= \frac{\bar{M}}{\bar{s}} \\
\bar{u} &= 1 - \bar{N} \\
\eta &= \frac{\bar{M}}{\bar{v}^{\xi}\bar{s}^{1-\xi}} \\
\bar{x} &= \frac{\bar{q}\bar{v}}{\bar{N}} \\
\bar{V}^{s} &= \frac{(1-b)\bar{w}}{1-\beta(1-\rho)(1-\bar{\theta}\bar{q})}
\end{aligned} \tag{23}$$

Letting Γ denote the steady state costs associated with meeting a worker, under the benchmark case of vacancy-posting costs the following equations are also used to compute the steady state

[5] Although we solve the model using an elasticity of substitution ε equal to 11, as is well known its value has not effect on the dynamics of the model, up to first order.

$$
\begin{aligned}
\kappa_v &= \Gamma \frac{\bar{Y}}{\bar{v}} \\
\bar{C} &= \bar{Y} - \kappa_v \bar{v} \\
\bar{w} &= Z - (1 - \beta(1-\rho)) \frac{\kappa_v}{\bar{q}} \\
\bar{V}^J &= \frac{\kappa_v}{\bar{q}} \\
\bar{V}^S &= \frac{(1-b)\bar{w}}{\big(1 - \beta(1-\rho)(1-\bar{\theta}\bar{q})\big)}
\end{aligned}
\tag{24}
$$

whereas under hiring costs we have

$$
\begin{aligned}
\kappa_h &= 2\Gamma \frac{\bar{Y}}{(\bar{x}^2 \bar{N})} \\
\bar{C} &= \bar{Y} - \frac{\kappa_h}{2} \bar{x}^2 \bar{N} \\
\bar{V}^J &= \kappa_h \bar{x} \\
\bar{w} &= \bar{Z} + \beta \frac{\kappa_h}{2} \bar{x}^2 - (1 - \beta(1-\rho)) \bar{V}^J \\
\bar{V}^S &= \frac{(1-b)\bar{w}}{\big(1 - \beta(1-\rho)(1-\bar{\theta}\bar{q})\big)}
\end{aligned}
\tag{25}
$$

In solving the model, we therefore assume that the output shares of the costs involved with matching a worker are the same in order to ensure a proper comparison. The chosen parameter values are shown in Table 2. We set the discount factor as $\beta = 0.99$ and the job separation rate to 0.1, consistent with the evidence in Shimer (2005) for US data. The value for $\bar{q}$ follows den Haan et al. (2000) and the replacement ratio of 0.4 is consistent with Gertler and Trigari (2009). Lastly, we set the steady state fixed cost of hiring as a proportion of GDP (Γ) to 0.66, as estimated in Christiano et al. (2016) and the interest rate to response to inflation is fixed at 1.5. The shocks to technology (ε_{at}), monetary policy (ε_{ut}) and matching efficiency (ε_{mt}) are asssumed to be white noise processes with unit variance.

5 Results

We consider the effects of altering ξ, the elasticity of matches with respect to vacancies across both models with $\xi \in \{0.1, 0.5, 0.9\}$. The impulse responses for each of the

Table 2: Parameterisation

Parameter	Description	Value
β	Discount factor	0.99
σ	Curvature of utility function	2
$\bar{\Pi}$	Steady state (gross) inflation rate	1
ω	Calvo price stickiness parameter	0.75
$\bar{\mu}$	Steady state price mark-up	1
ρ	Job separation rate	0.1
$\bar{N}$	Steady state employment rate	0.9
$\bar{q}$	Steady state degree of labour market tightness	0.7
ε	Elasticity of substitution across goods	11
ϕ_π	Interest rate response to inflation	1.5
ρ_z	Persistence of technology shock	0.9
Γ	Matching costs as a percentage of GDP	0.66

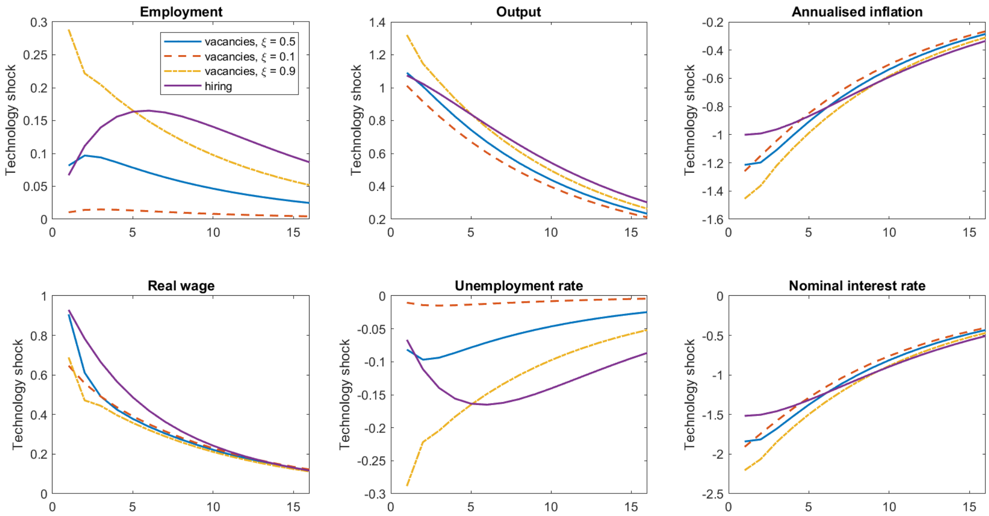
three shocks are shown in Figures 1-3. The first thing to note is that the figures only present one set of responses under hiring costs because under this specification the six variables considered are unaffected by the value of ξ. In contrast, the value of ξ matters for macroeconomic dynamics when the model features search costs: a higher value of ξ makes most of the variables more responsive to each of the shocks, with the exception of the real wage and for the interest rate rule shock, the inflation rate.

Figures (1)-(2) highlight why the hiring cost formulation is often preferred in the literature: it generates a far larger degree of intrinsic persistence and delayed responses to the shocks, which is consistent with much of the empirical evidence.[6] However, the figures also show that this approach to modelling labour market frictions has an important implication: the matching function plays no role in the model. This is most evidently seen in Figure (3), where the response to a matching efficiency shock are shown. In the standard model with search costs this shock is expansionary, raising employment and output at the same time that it results in a decrease in inflation. However, under hiring costs the effects of the shock are zero except for the number of vacancies posted and the

[6]See for example, Christiano et al. (2016).

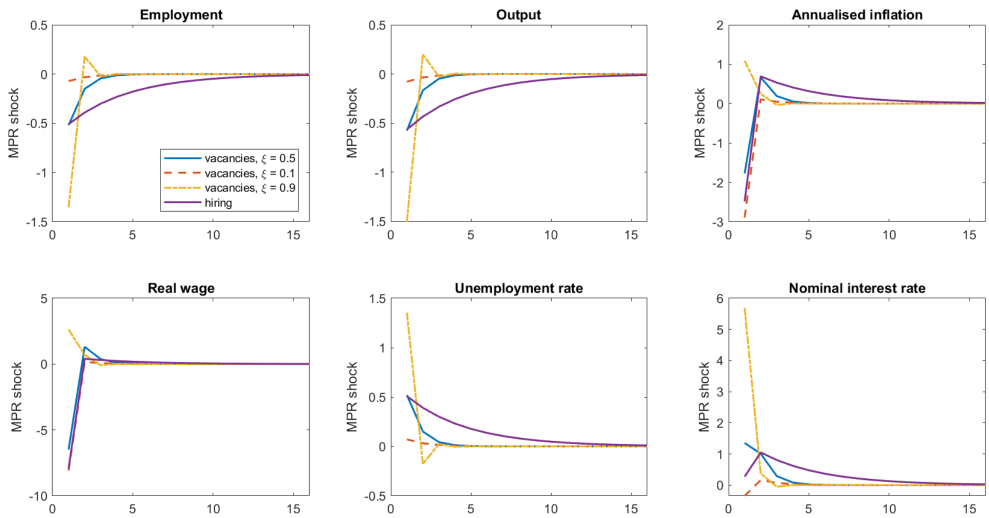
vacancy-filling rate. Although this result was found in Furlanetto and Groshenny (2012), it was not linked to the fact that original cause stems from the irrelevance of the matching function.

Figure 1: Impulse responses to a technology shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ when the model includes either vacancy-posting or hiring costs. Under hiring costs, the response is the same.

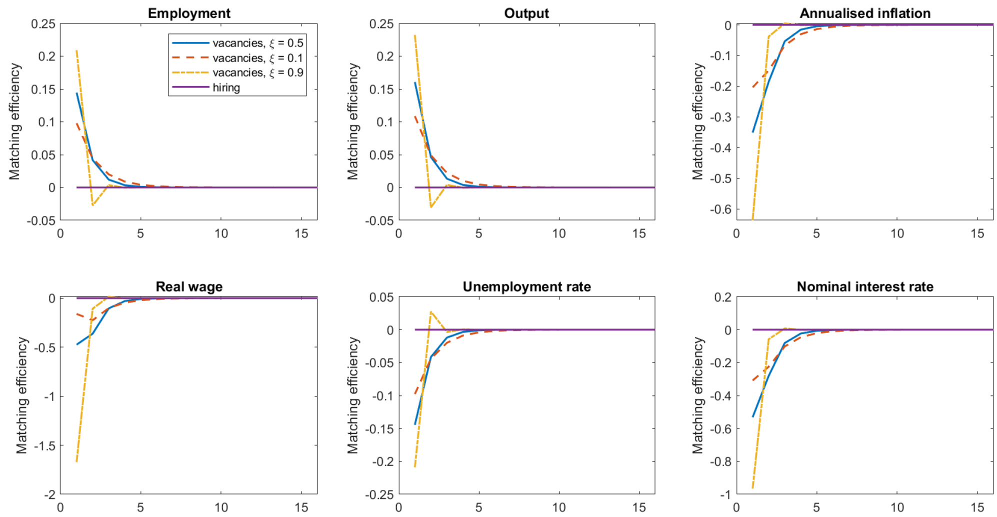
Figure 2: Impulse responses to an interest rate rule shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ when the model includes either vacancy-posting or hiring costs. Under hiring costs, the response is the same..

A potential solution to ensuring that the matching function remains relevant whilst pro-

Figure 3: Impulse responses to a matching efficiency shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ when the model includes either vacancy-posting or hiring costs. Under hiring costs, the response is the same.

ducing dynamics consistent with the empirical literature is to allow both forms of costs simultaneously, as proposed in Furlanetto and Groshenny (2016) and Christiano et al. (2016). Following the latter, we assume that the wholesale firms face a cost of posting vacancies as well as hiring costs. In this case, the free entry condition becomes

$$\kappa_h x_t + \frac{\kappa_v}{q_t} = V_t^J \tag{26}$$

and the job creation condition is given by

$$V_t^J = \frac{Z_t P_t^w}{P_t} - w_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left((1-\rho) V_{t+1}^J + \frac{1}{2} \kappa_h x_{t+1}^2 \right) \tag{27}$$

Lastly, the aggregate resource constraint becomes

$$Y_t = C_t + \frac{1}{2} \kappa_h x_t^2 N_{t-1} + \kappa_v v_t \tag{28}$$

We can then follow Christiano et al. (2016), who find that the bulk of the fixed costs of employment faced by the firm are generated by hiring, rather than search costs as the latter are only three percent of the former. Denoting γ as the steady state size of vacancy-posting costs as a proportion of hiring costs, the relevant steady state equations become

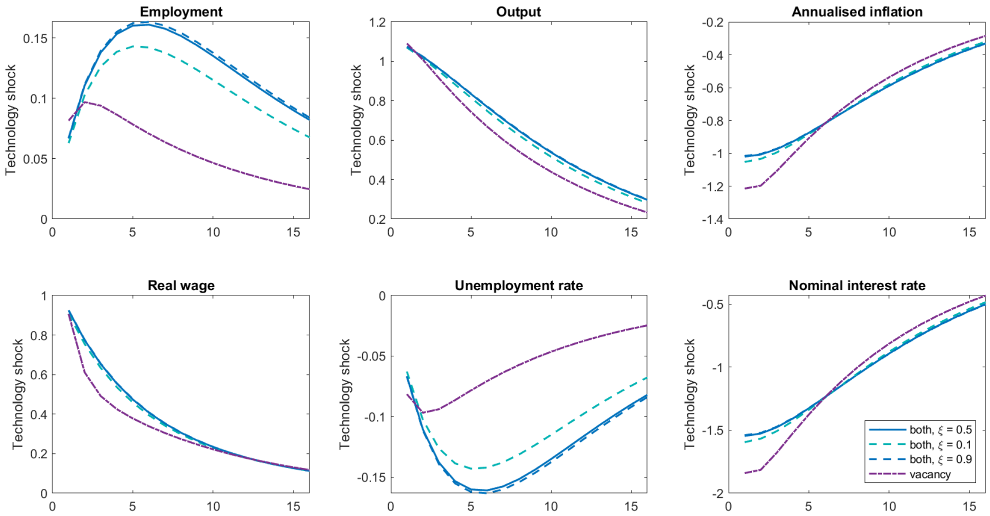
$$\begin{aligned}
\kappa_h &= 2\frac{\Gamma\bar{Y}}{(1+\gamma)\bar{x}^2\bar{N}} \\
\kappa_v &= \gamma\frac{\kappa_h}{2}\frac{\bar{x}^2\bar{N}}{\bar{v}} \\
\bar{C} &= \bar{Y} - \frac{\kappa_h}{2}\bar{x}^2\bar{N} - \kappa_v\bar{v} \\
\bar{V}^J &= \kappa_h\bar{x} + \frac{\kappa_v}{\bar{q}} \\
\bar{w} &= \bar{Z} + \beta\frac{1}{2}\kappa_h\bar{x}^2 - (1-\beta(1-\rho))\,\bar{V}^J \\
\bar{V}^S &= \frac{(1-b)\bar{w}}{\big(1-\beta(1-\rho)(1-\bar{\theta}\bar{q})\big)}
\end{aligned} \tag{29}$$

We solve this version of the model for different values of ξ and compare the results with those of the model with only vacancy-posting costs and $\xi = 0.5$. The results are shown in Figures (4)-(6) and two things stand out: first, changing the value of ξ still has a quantitatively negligible effect on the impulse responses to technology and monetary policy shocks, except for unemployment and employment although even in this case the magnitude is small. Second, matching shocks play no role whatsoever for the six variables shown, with the disturbance only affecting vacancies and the job-filling rate but without a response to the macroeconomic aggregates. This result is not surprising, as the hiring costs comprise the far larger share of the total fixed costs of employing a worker. Assigning a much larger share to the search costs would overcome this conclusion but the result would be achieved by relying on empirically-implausible values.

6 Conclusion

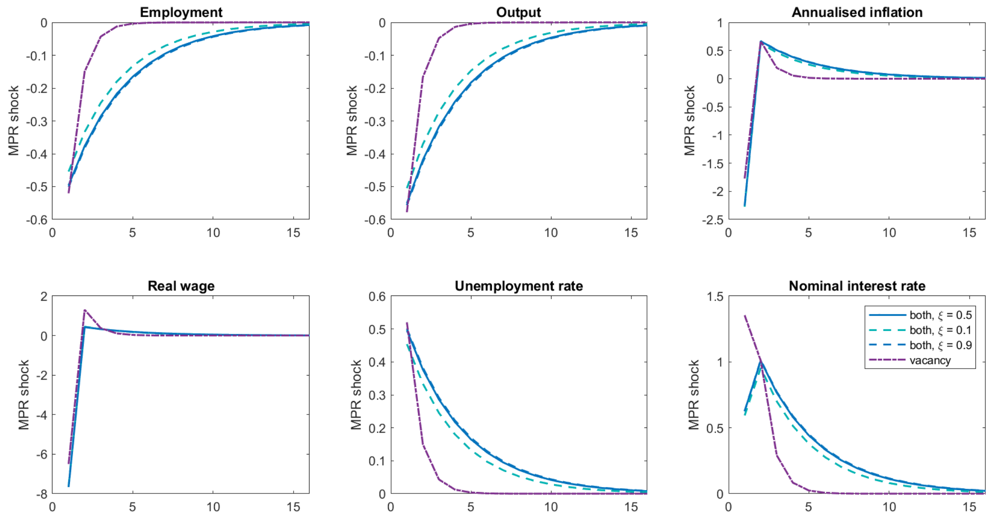
A large proportion of DSGE models of the business cycle include labour market frictions as in Mortensen and Pissarides (1994). Whilst the standard formulation assumes that firms face a cost per vacancy posted, several papers have emphasised replacing this with hiring costs as it improves the empirical fit of the model. However, this modification to

Figure 4: Impulse responses to a technology shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ. Under 'vacancy' the model only contains vacancy-posting costs ($\xi = 0.5$); under 'both' hiring costs are also present.

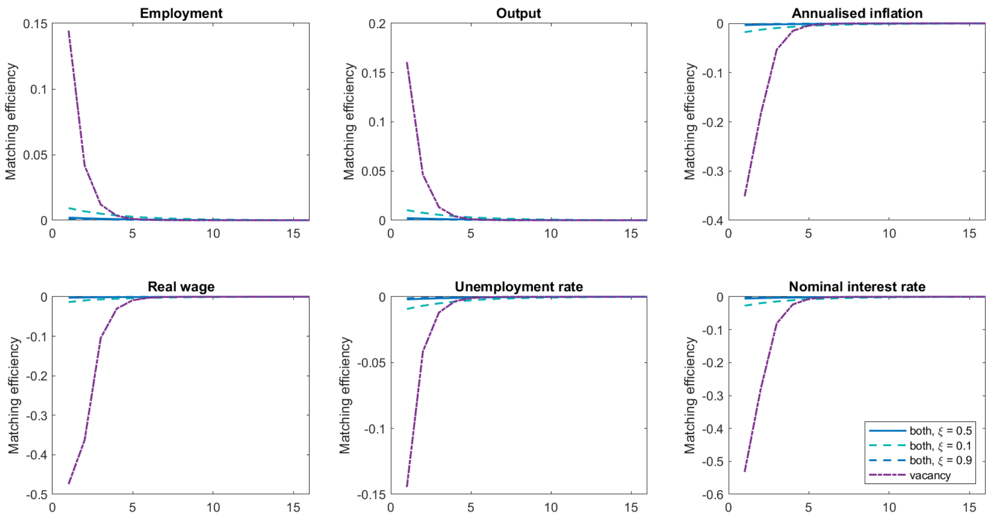
Figure 5: Impulse responses to an interest rate rule shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ. Under 'vacancy' the model only contains vacancy-posting costs ($\xi = 0.5$); under 'both' hiring costs are also present.

the standard search and matching model results in the matching function, a key equation of such models, being irrelevant. Its formulation and parameterisation have no effects on macroeconomic dynamics and it further implies that matching efficiency shocks cannot be drivers of the business cycle.

Figure 6: Impulse responses to a matching efficiency shock



The figure plots the impulse responses to a technology shock for varying degrees of the elasticity of matches to vacancies, ξ. Under 'vacancy' the model only contains vacancy-posting costs ($\xi = 0.5$); under 'both' hiring costs are also present.

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